## **New formula for indentation toughness in ceramics**

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Lawn *et al.* [1] have provided an analysis of indentation cracking in ceramics based on a half-penny crack model leading to an expression for toughness  $K_0$  of the form  $K_c = \overline{k(E/H)^n P} / c^{3/2} \propto P_r / c^{3/2}$  where k is calibration constant,  $E$  is Young's modulus,  $H$  is hardness,  $P$  is the load applied to the indenter and  $c$  is the length of surface traces of cracks measured from the centre of the impression (see Fig. 1). The residual plastic crack driving force  $P_r \propto (E/H)^n P$  when the Hill expanding cavity model is used to describe the local effects of indentation; the crack length  $c$  is required to greatly exceed  $b$ , the size of the plastic zone, in order for the point loading approximation to be reasonable. This requirement is generally found to be met in ceramics where the crack length  $c$  typically exceeds the plastic zone radius  $b$  by a factor of 3. The exponent  $n$  is estimated by Lawn *et al.* [1], who give  $n = 1/2$ ; however,  $n$  may be determined analytically [2], when  $n = 2/3$ .

Anstis *et al.* [3] validated the Lawn *et al.* form using a wide range of ceramics and glasses for which reliable toughness values were available. Data for a single WC-Co material was also included. Anstis *et aL* [3] found a calibration constant  $k^A = 0.016$  with a coefficient of variation  $v = 0.25$ . The modified form of the analytical determination [2] has also been calibrated using the data of Anstis *et al.*, giving  $k^L = 0.0098$ with a coefficient of variation  $v = 0.26$ . It is seen that these formulae describe the data equally well; an advantage of the approach in [2] lies in its analytic derivation of the exponent  $n$  in the expression for the residual crack driving force,  $P_{\text{r}}$ .

Recent work [4] has shown that unlike the observed behaviour in glasses, indentation cracking in ceramics is not of the assumed half-penny variety. Instead, crack geometry is much more like that known to occur



*Figure 1* Illustration of indentation crack geometry.

in cermets (hard materials containing a ductile phase) and is of the so-called Palmqvist type. Fig. 2 shows a typical crack profile obtained by a serial sectioning technique on a commercial  $Al_2O_3-ZrO_2$  tool material. In view of these findings, it is clear that despite the formal successes of the formulae in [2] and [3], a half-penny model is not a satisfactory basis for analysis, and requires modification. Recent work [4, 5] shows that when cracks are represented by either semi-circles of diameter  $I$  or by rectangles of length  $l$  and depth  $d$ to which cracks approximate more closely, where  $d$  is the crack depth and  $l = c - a$  is the crack length, the stress intensity factors  $K$  controlling surface crack extension differ only by a small numerical term of the order of unity. An analytic expression for the semicircular representation is

$$
K^{SC} = 2\left(\frac{\pi}{2+\pi}\right)^{1/2} \left(\frac{1}{a}\right)^{-1/2} K^{CL}
$$

where  $K^{CLP} = (1/\pi^{3/2})(P_r/c^{3/2})$  is the stress intensity factor for a penny crack centre loaded with point force  $P_r$ ;  $P_r$  is the residual plastic crack driving force referred

TABLE I Indentation toughness data used for calibration taken from Anstis *et al.* [3]

Material	$P/c^{3/2}$ $(MPa \; m^{1/2})$	1/a	E/H	k	$K_c^{\rm A}/K_{\rm lc} = p$	$K_{\rm c}^{\rm L}/K_{\rm lc} = q$	$K_c^P/K_{lc} = r$	$K_{\rm lc}$ (MPa m <sup><math>1/2</math></sup> )
(a) $WC$ - $Co$	210	0.2	43.6	0.00204				12
(b) $Si_3N_4$ (NC132)	60	1.6	16.2	0.0130	0.98	0.95	1.15	4.0
$(c)$ SiC	50	2.8	18.2	0.0191	0.85	0.85	0.78	4.0
(d) $Al_2O_3$ (AD999)	36	2.8	20.2	0.0243	0.67	0.67	0.62	3.9
(e) $Al_2O_3$ (AD90)	31	2.4	29.8	0.0149	0.93	1.03	1.00	2.9
(f) Glass Ceramic	43	1.2	12.8	0.0115	1.00	0.92	1.30	2.5
(g) $Si_3N_4$ (NC350)	33	1.8	17.7	0.0118	1.10	1.10	1.26	2.0
(h) Sapphire	22	2.4	19.5	0.0201	0.76	0.76	0.74	2.1
$(i)$ Glass $(AS)$	19	2.3	13.5	0.0127	1.21	1.21	1.18	0.91
$(i)$ Glass (SL1, II)	14	2.7	12.7, 13.0	0.0158	1.07	1.02	0.95	0.74, 0.75
(k) Glass LA	14	2.5	13.3	0.0136	1.20	1.15	1.10	0.68
$(l)$ Si	13	2.3	15.8	0.0129	1.18	1.16	0.17	0.7

 $K_c^A = k^A (E/H)^{1/2} P/c^{3/2}$ ,  $k^A = 0.016$ ,  $v = 0.25$ ;  $K_c^L = k^L (E/H)^{2/3} P/c^{3/2}$ ,  $k^L = 0.0098$ ,  $v = 0.26$ .  $K_c^P = k^P (1/a)^{-1/2} (E/H)^{2/3} P/c^{3/2}$ ,  $k^P = 0.015, v = 0.26, \bar{p} = 0.99, v = 0.18, \bar{q} = 0.98, v = 0.18, \bar{r} = 1.02, v = 0.22.$ 

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*Figure 2* Typical crack profile resulting from indentation with a Vickers diamond (load  $P = 30$  kg) in an Al<sub>2</sub>O<sub>3</sub>-ZrO<sub>2</sub> ceramic.

to earlier. This may be used in place of the centreloaded penny stress intensity factor in the description of Palmqvist crack extension in ceramics.

Calibration of the new Palmqvist formula may be performed using the data of Anstis *et al.* shown in Table I. The term  $(1/a)^{-1/2}$  shows relatively little variation for the ceramics and glasses listed, having a mean value  $\overline{(1/a)}^{-1/2} = 0.68$ , with a coefficient of variation  $v = 0.14$ , when the WC-Co result is excluded (WC-Co composites do not strictly satisfy the requirement for a point loading approximation, and show somewhat different behaviour accordingly); already, it is clear why an analysis based on half-penny crack geometry was able to reliably describe Palmqvist crack extension. The new formula may be written

$$
K_{\rm c} = k^{\rm p} \left(\frac{1}{a}\right)^{-1/2} \left(\frac{E}{H}\right)^{2/3} \frac{P}{c^{3/2}}
$$

with  $k^p$  = Ave{ $K_{\text{lc}}/[((1/a)^{-1/2} (E/H)^{2/3} P/c^{3/2}]$ } = 0.015 and coefficient of variation  $v = 0.026$ . Toughness values calculated from the Anstis *et al.* formula [3] and its modification [2] and from the above Palmqvist formula may be compared with the experimental values by means of the ratios  $K_c^A/K_{1c} = p$ ,  $K_c^L/K_{1c} = q$ ,  $K_c^p/K_{lc} = r$  listed in Table I. The ability of each of the formulae to describe the experimental data is seen to be quite similar ( $\bar{p} = 0.99, v = 0.18; \bar{q} = 0.98,$  $v = 0.18$ ;  $\bar{r} = 1.02$ ,  $v = 0.22$ ) when WC-Co is excluded.

In conclusion, descriptions of Palmqvist crack extension in ceramics of similar quality are provided by each of the approaches considered  $-$  the Lawn-Anstis half-penny formalism and its modification [2], and finally the analysis [4, 5] based on observed Palmqvist crack profiles. From a theoretical point of view, the latter approach is more satisfactory.

## **References**

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*Received 20 August and accepted 10 September 1986*