Articles 4

CONGRESSIONAL COMMITTEE ASSIGNMENTS:

AN OPTIMIZATION MODEL WITH INSTITUTIONAL CONSTRAINTS

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> "Scheduling falls more severely than most operations research studies into the two groups of studies: simple optimally soluble examples, and large intractable realistic cases."
>
> T. A. J. Nicholson

It has become rather common in the scholarly literature on the United States House of Representatives to focus on the activities of the Chamber's subunits – its standing committees. Woodrow Wilson's intuition of nearly a century ago that "Congress in session is Congress on public exhibition, while Congress in its committee rooms is Congress at work" has been sustained in numerous studies. There has been a recent upsurge of interest in the intricate process by which members come to be appointed to committees. Empirical studies of the committee

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assignment process, following Master's (1961) classic piece, have exhausted the data of the public record in detailing matters of committee personnel. Descriptive studies by Achen and Stolarek (1974), Bullock (1971, 1972, 1973), and Clapp (1964, Chap. 5), as well as theoretical pieces by Bullock and Sprague (1969), Cohen (1974), Rohde and Shepsle (1973), Shepsle (1973, 1974), Uslaner (1971), and Westefield (1974), have acknowledged the import of the committee structure for an understanding of Congressional politics and have provided some insight about the committee personnel of that structure.

While I shall not review the fruits of this research here, I should, before beginning, detail several ways in which this paper differs from the research cited above. Although this study is very much an examination of the committee assignment process, it is, first and foremost, a formal theoretical piece. I have made little effort to reproduce the richness of detail found in other studies; description will be a decidedly secondary concern here. Rather I shall be concerned with analytical categories and their theoretical consequences.

Second, the underlying gestalt of this paper is economic rather than sociological. The committee assignment process is viewed as an allocation phenomenon in which scarce but valued committee slots are allocated among a well-defined clientele group according to carefully specified rules. The sociological nexus, expecially between party leaders, members of the committees charged with making assignments, other congressmen, and outside interest groups is given only indirect attention.

Third, though the model presented here is economic and relatively abstract, I am fully aware that it is a *real* institutional process I am studying. I have, consequently, attempted to defend a middle ground between the highly abstract choice-theoretic literature and the often atheoretical descriptive literature on Congress. The resulting product undoubtedly does a disservice to both sets of studies, but may nonetheless provide an approach that possesses both theoretical power and empirical utility. This, in any event, is my intention.

I. Temporal Sequence of the Committee Assignment Process

Committee assignments are a party responsibility. Each party has created a Committee on Committees (CC) to parcel out committee slots to party members in the chamber.[1] At the beginning of each Congress the party CCs are faced with the task of filling vacancies in the twenty-one standing committees.

[1] The Democratic CC is composed of the 15 (when the Democrats are the majority party) Democratic members of the Ways and Means committee. In recent Congresses the Speaker, Majority Leader and Chairman of the Party Caucus have been added. They are responsible for filling vacancies on all other committees while the entire party caucus fills vacancies on Ways and Means. There have been several major changes in this procedure in 1975. The Republican CC is composed of one member from each state with some Republican representation in Congress. Each member has as many votes as there are Republicans in his delegation. As a result of this majority rule – weighted voting arrangement, Republican appointments are dominated by a subcommittee of representatives from states with large Republican delegations. The Republican leader chairs his party's CC.

Directly following the November election, newly elected and returning congressmen submit to their respective CCs their requests for committee assignments. For newly elected congressmen, the request list is in the form of a *preference ordering*. The typical preference ordering contains three requests (i.e., three committees ordered according to preference), though there is considerable variation. Some reveal a preference for only a single committee, while others rank-order as many as nine committees (see Rohde and Shepsle, 1973).

For returning members, on the other hand, an *informal property right* is operative: nonfreshmen, whenever feasible, may retain committee assignments held in the previous Congress if they wish. If a change is desired, however, a returning member may request a transfer to another (presumably more preferable) committee, in which case he voluntarily yields his property claim on his previously held committee slot; or he may request a dual assignment, in which case he retains his previously held slot and is given an additional assignment as well. Only under extreme conditions (so long as the property right norm operates) can a returning member be forced to resign a previously held committee slot, although an occasional voluntary resignation occurs.

After requests are made lobbying for assignments begins. For some the effort is rather casual and uninvolved. For others the effort is much more active. Congressmen write letters to members of their CC, setting forth arguments in behalf of their requests; pay personal visits to members of the CC, party leaders, committee chairmen and their state delegation dean; and solicit letters of recommendation on their behalf from their state delegations, from party leaders outside the House, and from relevant interest groups.

At the opening of a new Congress, leaders of the majority and minority parties negotiate a committee structure. At this point they determine the *size* of each of the twenty standing committees[2] and the *distribution* of slots on each committee between the majority and minority parties. On each of these decisions party leaders are given legislative guidance by the Legislative Reorganization Act (LRA) of 1946 and subsequent amendments. That act specifies committee sizes (which have consistently been revised upward) and recommends a division of slots between majority and minority closely in accord with the party ratio in the chamber. The sizes and party ratios on committees negotiated often reflect an attempt by leaders to accommodate new demands for committee slots and to avoid "bumping" returning members from committees as a consequence of dramatic changes in the chamber party ratio (this is the extreme condition, alluded to above, in which the property right norm is inoperative).

The final stage of the assignment process is the actual allocation of slots to new and returning congressmen by party CCs. Their final recommendations must be

[2]The Committee on Standards of Official Conduct, created in the wake of the scandal surrounding Representative Adam Clayton Powell, is omitted from this analysis. Although it is the twenty-first standing committee, it appears to have a rather special status that distinguishes it from other standing committees.

ratified by their respective party caucuses, but this is typically *pro forma*.[3]

For our purposes, then, the committee assignment process may be characterized by the following temporal sequence:

1. the committee configuration in the (t-1)st Congress;
2. an "exogenous" shock – an election;
3. the submission of requests by freshmen and returning Congressmen;
4. the negotiation of a committee structure for the tth Congress, by the respective party leaders;
5. the creation of a committee configuration for the tth Congress by the party CCs.

This temporal sequence suggests that the committee assignment process is, in fact, composed of three distinct but interrelated processes, each involving different sets of actors (or the same actors in different roles). In order to provide a theoretical account of *request behavior, negotiated structure,* and *committee assignments*, it is necessary to examine actor goals and motives.

II. Actor Motives

Elsewhere (Shepsle, 1973) I have provided a rather detailed account of actor objectives, focusing on maximizing behavior in an environment of scarce commodities, formal procedures, and competing maximizers. Here I give only a brief summary:

Actor	Behavior	Objective
members	submission of requests for initial committee assignment or transfer	"good" committee assignments
party leaders	negotiation of a committee structure: committee sizes and party ratios established	strategy of "accommodation"
party CC	committee assignments	*quid pro quos* and "pipelines" into committees

Since committees lie at the heart of Congressional life, any brief description of the actors, their behavior, and the objectives toward which that behavior is aimed is bound to ignore relevant institutional details. Nevertheless let me simply observe that:

1) Member requests are motivated by a desire for a "good" assignment, where "good" is determined by introspective value judgments, an assessment of the likelihood of obtaining a particular assignment, and the opinions, advice, and preferences of interested others, e.g., state

[3]For the remainder of this paper Democratic assignments are the focus. Much of the model will obviously carry over to Republican assignments, though procedural differences will require the rationale for some of the assumptions to be altered.

delegations, party leaders, outside advisors. The *revealed preferences* of members are the culmination of a complex set of interactions.

2) party leaders are chiefly interested in accommodating member requests, though they do take more particular interest in the money committees (Appropriations, Ways and Means) and the agenda committee (Rules). The "strategy of accommodation" is succinctly stated by Westefield (1974): ". . . committee positions are given the status of a currency, a basis of exchange between leaders and followers. The leaders . . . perceive they can use the currency to accommodate the members and thereby induce the members to behave in ways the leaders desire. Indeed, the leaders can 'manufacture' this currency and add to the resource base at their disposal."

3) The members of the party CC – the allocation instrument – appear to want to induce member cooperation and assistance for their own particular projects and aspirations. They, too, attempt to accommodate member requests with an eye to eliciting *quid pro quo* behavior.

In the next section I present a linear programming model of committee assignments. Requests and negotiated structure are taken as exogenous inputs to the actual assignment process. Let us turn, then, to a specification of the CC objective function and the institutional rules which constrain CC behavior.

III. A Programming Model of Committee Assignments

In the last section I have attempted to specify a context in which actors seek to realize goals or objectives. Applicants seek "good" committee assignments; party leaders seek "responsive" followers with, in some instances, "correct" policy preferences; members of the CC pursue "profitable" trades, bargains, and *quid pro quos*; and other actors, e.g., state delegation leaders, committee chairmen, policy coalition leaders, and interest group representatives, through their interactions with the principal actors, attempt to influence the latter's interpretations, respectively, of "good," "responsive," "correct," and "profitable." Thus goal-seeking is the principal mode of behavior and a specific temporal sequence provides the behavioral context.

If this were all we had – well-defined goals and a specific context – we would be limited to a considerable extent in what we could say with confidence. One of the fortunate things about institutional analysis, however, is that the feasible range of behavior in pursuit of goals is greatly delimited by *institutional constraints*. Some of these constraints are "natural" in the sense that they follow directly from a definition of scarcity. Others are simply agreed-upon rules of behavior, e.g., the property-right norm. Together these constraints define *feasible outcomes*. The domain of goal-seeking, then, is restricted to a feasible set defined by institutional constraints. Our task in this section is to identify these institutional constraints and to give them a formal characterization. Before that, some notational conventions are stated.

Let $M = \{1, 2, \ldots, m\}$ be a set of m applicants for committee assignments.[4] Let $C = \{c_1, c_2, \ldots, c_n\}$ be the set of n committees partioned into the following subsets:

$E = \{c_1, c_2, \ldots, c_e\}$ is the subset of *exclusive* committees;

$S = \{c_{e+1}, c_{e+2}, \ldots, c_s\}$ is the subset of *semiexclusive* committees; and

$N = \{c_{s+1}, c_{s+2}, \ldots, c_n\}$ is the subset of *nonexclusive* committees.[5]

Finally let $v = (v_1, v_2, \ldots, v_n)$ be the *committee vacancy vector*. The v_i are a function of the election returns and the decisions made at the stage of negotiated structure. Obviously, $v_i \geqslant 0$ for all i.

Define an m x n *assignment matrix* A with typical element a_{ij}. The element a_{ij} gives the disposition of the i^{th} congressman vis-a-vis the j^{th} committee. If $a_{ij} = 1$ then i is assigned to committee j; if $a_{ij} = 0$ he is not (we havenot, as yet, interpreted values of a_{ij} other than zero or unity). The assignment matrix A is a formal characterization of a *decision* by the CC.

The CC is not unrestricted in the assignments it can make. Some of these restrictions, as I observed above, are "natural"; others are formal rules imposed by the Legislative Reorganization Act and its amendments or by the party caucus. Restrictions fall neatly into two categories: *apportionment constraints* and *service restrictions.*

Appointment Constraints:

[1] The number of assignments to the j^{th} committee may not exceed v_j: $\sum_{i=1}^{m} a_{ij} \leqslant v_j$, for all j. There are n constraints of this variety. If they are satisfied as equalities then all vacancies are filled; otherwise some vacancies remain unfilled. Empirically, vacancies are occasionally left unfilled (see Shepsle (1973, 1974)).

[II] Every congressman must serve on at least one committee: $-\sum_{j=1}^{n} a_{ij} \leqslant -1$, for all i.

[III] No congressman is permitted to serve on more than two committees: $\sum_{j=1}^{n} a_{ij} \leqslant 2$, for all i.

[4] As I noted earlier, I focus in this paper on *Democratic* committee assignments. Moreover, it will be less complicated to deal initially with freshmen congressmen. Thus, I do not examine committee transfer phenomena.

[5] During the 86th through 90th Congresses, the period for which empirical examination of this model is conducted (Rohde and Shepsle (1973) and work in progress), the committees in each status category were: *exclusive:* Appropriations, Rules, Ways and Means; *semiexclusive*: Agriculture, Armed Services, Banking and Currency, Education and Labor, Foreign Affairs, Interstate and Foreign Commerce, Judiciary, Public Works, Science and Astronautics; *nonexclusive*: District of Columbia, Government Operations, House Administration, Interior and Insular Affairs, Merchant Marine and Fisheries, Un-American Activities, and Veterans Affairs. Post Office and Civil Service was changed from semiexclusive to nonexclusive status at the beginning of the 88th Congress. Since the time period of this study, Science and Astronautics has been changed to Nonexclusive Status and the name of Un-American Activities was changed to Internal Security. In the 94th Congress, Internal Security has been eliminated.

Service Restrictions:

[IV] A congressman may serve on at most one exclusive committee; if he does he may serve on no other committee. A congressman serving on a semiexclusive committee may serve on at most one nonexclusive committee: $3 \sum_{j=1}^{e} a_{ij} + 2 \sum_{j=e+1}^{s} a_{ij} + \sum_{j=s+1}^{n} a_{ij} \leqslant 3$, for all i.

[V] A congressman may serve on no more than one semiexclusive committee: $\sum_{j=e+1}^{s} a_{ij} \leqslant 1$, for all i.

[VI] A congressman may not receive a multiple assignment to the same nonexclusive committee: $a_{ij} \leqslant 1$, for all i, and for j = s+1, . . . , n.

Constraint classes [I]-[VI] define the set of feasible assignments as specified by the formal rules of the game. For technical as well as obvious substantive reasons, one additional class of contraints is included:

[VIII] nonnegativity: $-a_{ij} \leqslant 0$, for all i and j.

Having characterized the domain of feasible A- matrix values formally, I conclude this section with a mathematical statement of the CC objective: the management goal.

Objective Function: Recall that the management goal has the CC attempting to maximize the satisfaction of its applicant clientele by matching, to the extent feasible, assignments with requests. Let us, then define a preference matrix P = $[p_{ij}]$, of the same order as the assignment matrix A, where

p_{ij} = 1 if applicant i lists committee j in his preference ordering
= 0 otherwise

CC Objective Function: $\max_{A} \sum_{i=1}^{m} \sum_{j=1}^{n} p_{ij} a_{ij}$

That is, the CC's objective is to select a configuration of assignments – an A matrix – that maximizes the "correlation" between expressed preferences and actual assignments. This is accomplished by the A matrix that maximizes the product of its elements and the corresponding elements of P. Notice that this operational definition ignores the *order* in which requests are listed, relying instead on a crude dichotomy (a committee is either listed or not). This appears to do no great disservice to actual data.

The task, then, of the CC is to select a matrix consisting of mn variables so as to maximize an objective function linear in those variables, subject to 2mn - ms + 4m + n constraints linear in those variables. What we have is a linear programming problem of a very special sort: it is a variant of the *general assignment problem.*

IV. Assignment Problems[6]

Assignment problems constitute a general class of problems concerned with the *efficient allocation of indivisible resources*. They are treated briefly in most linear programming texts, e.g., Gale (1960). A general, detailed survey is provided by Motzkin (1956); algorithmic discussions (especially as it relates to the classic *transportation problem*) are found in Balinski (1968), Kuhn (1955, 1956), and Tornqvist (1953); relevant theoretical treatments appear in Gale (1956), Heller and Tompkins (1956), Hoffman and Kruskal (1956), and von Neumann (1953); finally, applications of assignment problems include optimal college admissions policies and pairing of marriage partners (Gale and Shapley, 1962), optimal location of production facilities (Koopmans and Beckmann, 1957), and exchange economies (Shapley and Shubik, 1972).

In the remainder of this section I present the *simple assignment problem* in order to motivate the results pertaining to constraints [I]-[VII] and the objective function given in the previous section. The reader is cautioned to observe that the simple assignment differs in significant ways from the *committee assignment problem* of the last section. These differences are spelled out below. Nevertheless, the results reported in this section apply to the committee assignment problem as well.

Suppose there is a legislature consisting of n members and n committee slots. Assignments to committees are governed by the following simple rules, where a_{ij} is the *extent* to which the i^{th} individual is assigned to the j^{th} slot:

(C.1) Each legislator is "completely assigned":
$\sum_{j=1}^{n} a_{ij} = 1$, for all i.

(C.2) Every vacancy is filled: $\sum_{i=1}^{N} a_{ij} = 1$, for all j.

(C.3) Nonnegativity: $a_{ij} \geqslant 0$, for all i and j.

(C.1) through (C.3) define $n^2 + 2n$ constraints, although only $n^2 + 2n - 1$ of them are independent. Since $\sum_i (\sum_j a_{ij}) = \sum_j (\sum_i a_{ij})$, one of the constraints from (C.1) or (C.2) can be derived from the 2n - 1 others.

Definition: An assignment is said to be *feasible* if it satisfies (C.1) through (C.3).[7]

The *set of feasible points* in n^2 - space (since there are n^2 variables a_{ij}) is characterized by the intersection of 2n - 1 hyperplanes (C.1 and C.2) with n^2 half-spaces (C.3). Since (C.1) through (C.3) require $0 \leqslant a_{ij} \leqslant 1$,[8] for all i and j, i.e.,

[6]This section follows the very excellent paper by Koopmans and Beckmann (1957).

[7]When I discuss the *committee* assignment problem the definition of feasibility is the same, except (C.1) through (C.3) is replaced by [I] through [VII].

[8](C.3) requires nonnegativity. To show that the a_{ij}'s are bounded from above, assume the contrary: $a_{i^*j^*} > 1$. Then from the i^{*th} equation of (C.1) or the j^{*th} equation of (C.2), it must be the case that some other a_{ij} is negative, contrary to (C.3). This completes the proof.

the a_{ij}'s are bounded, and since the constraints are linear in the a_{ij}'s, the set of feasible points is a *convex polyhedron.*

Definition: A *vertex (extreme point)* of a convex set is an element of the set which cannot be expressed as a convex combination of any two other points in the set.

Extreme points play an important role in characterizing convex polyhedra. In fact, it may be demonstrated that any point in a closed, bounded, convex set is some convex combination of the set's extreme points. The extreme points play an especially prominent role in the problem at hand, as the theorem below suggests:

THEOREM 1 (Koopmans and Beckmann, 1957): A linear objective function defined on a convex polyhedron reaches its maximum at a vertex. If it does not reach a maximum at a vertex, then it reaches a maximum at no other point in the polyhedron. If it reaches a maximum at more than one vertex, then it achieves a maximum at every point on the face of the polyhedron defined by those vertices.

An extremely rigorous proof of this theorem is given in Koopmans (1951, p. 88, note 17). Some geometric intuition will give the reader a feel for this result. Consider the simple linear system, consisting of five constraints in two space, depicted in Figure 1:

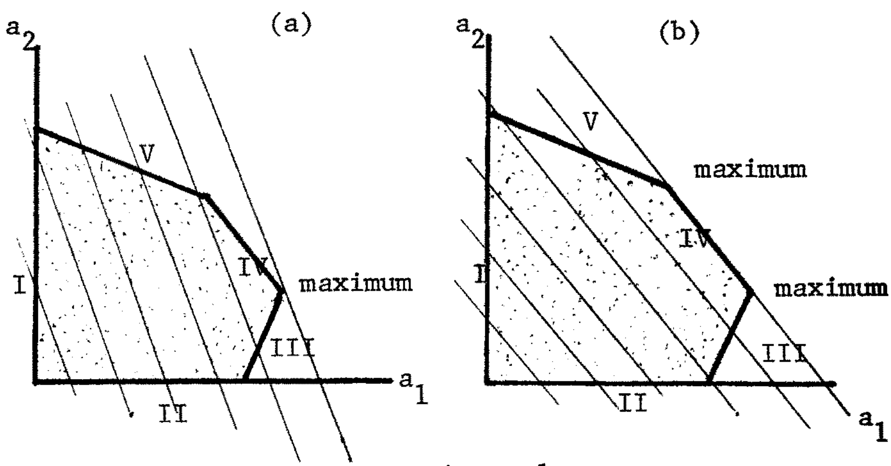

Figure 1

I and II are nonnegativity constraints and III, IV, and V are linear constraints. The dotted region – a convex polyhedron – is the feasible set of values of a_1 and a_2. Projections of the objective function, which is linear in a_1 and a_2, are also given in Figure 1. For any two points on the same projection, the values of the objective function are identical. Any point on a higher projection (one to the "northeast") implies a larger value for the objective function than a point on a lower projection (one to the "southwest"). If the contours of the objective function are not parallel to any of the constraint hyperplanes, as in (a), then a unique vertex of the convex polyhedron maximizes the objective function. If, on the other hand, the contours are parallel to a constraint hyperplane (other than the nonnegativity constraints), as in (b), then several vertices and the face defined by them maximize the objective function.[9]

Theorem 1 tells us that an optimal assignment, for a given linear objective function, occurs at (at least) one of the vertices of the feasible set. The next task, then, is to identify the vertices of (C.1) through (C.3) and to examine their characteristics.

Definition: A feasible assignment, A, is called a *permutation* if and only if it is a doubly stochastic matrix with $a_{ij} = 0$ or 1 for all i and j.
The permutation matrices, for any n, are quite literally the permutations of individuals among slots (or slots among individuals). For n = 2 they are

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

For any n there are n! permutations.

THEOREM 2 (Birkhoff, 1946): Let $P = \left\{ A^1, A^2 \ldots, A^{n!} \right\}$ be the set of permutation matrices. A feasible assignment, A*, can be written as a weighted average, with nonnegative weights, of the n! permutations:

$$A^* = \sum_{k=1}^{n!} \lambda_k A^k \quad ,$$

$$\text{where } \lambda_k \geq 0, \quad \sum_{k=1}^{n!} \lambda_k = 1, \text{ and } A^k \varepsilon P.$$

A proof of this theorem is provided by Koopmans and Beckmann (1957, Appendix A).

Theorem 2 establishes a one-to-one correspondence between the vertices of the convex polyhedron defined by (C.1) to (C.3) and the set, P, of permutations. That is, the vertices of the feasible set are permutation matrices. Together with Theorem 1, this has a nonobvious consequence: the maxima (if they exist at all) of *any* objective function, linear in assignments, result in an *integral* assignment. Thus, even if fractional assignments made substantive sense, there would never be any need to resort to them.

The economic consequences of these theorems, especially as they relate to decentralized markets and price systems, are traced by Koopmans and Beckmann (1957) and will not concern us here. The method, however, will prove most useful.

[9]Throughout I do not worry about the *existence* of optima. The structure of the problems with which I am concerned (closed, bounded feasible sets and well-behaved objective functions) insures the existence of optimum points. Rarely, however, are these optima *unique.* Thus Figure 1b, where multiple optimum points are illustrated, is more typically the case in the committee assignment problem.

Before returning to the "real world" – I take constraints [I] through [VII] and the objective function of the previous section to be a close approximation of the "real world" – some major differences between the simple assignment problem of (C.1) through (C.3) and the *committee assignment problem* of the last section should be made explicit. First, while it is apparent that the simple assignment problem has a feasible set, the same may not be said about the committee assignment problem. The simple assignment problem, at the outset, provides n slots for n members. The constrainst require the n slots to be allocated so that each member receives, in total, exactly a "full" assignment. The committee assignment problem, on the other hand, gives no assurances of feasibility.

Constraints (C.1) to (C.3) of the simple assignment problem suggest an extremely simple structure. Because of the simple structure, e.g., the coefficients in all constraints equations are zeros and ones, Birkhoff's Theorem readily identifies the vertices as the simple permutations. The structure of the committee assignment problem is much less simple and elegant – one of the high costs of dealing with *real* institutional processes (see this paper's headnote). It will take considerably more analytical effort to achieve interesting results.

A third important difference between (C.1) and (C.3) and [I] through [VI] (excluding the common nonnegativity conditions) is seen at a glance. The former are *equality* constraints while the latter are *inequalities.* As a result, the *committee* assignment problem opens up an additional possibility: incomplete assignments. And empirically (see Shepsle (1973, 1974)) there are numerous instances of committee slots left vacant by the party CCs.

Finally, there is the problem of *multiple assignments*. Constraints [IV] - [VI] provide for the possibility of multiple assignments in the committee assignment problem. This feature, perhaps more than any other, distinguishes the committee and simple assignment problems.

V. Committee Assignment Problem: Results

Feasibility Theorem: The condition both necessary and sufficient for feasibility is $\sum_{j=1}^{n} v_j \geq m$.

Proof: (1) *Necessity.* From [I]

$$\text{feasibility} \rightarrow \sum_{i=1}^{m} a_{ij} \leq v_j \qquad j = 1, \ldots, n$$

$$\rightarrow \sum_{j=1}^{n} \sum_{i=1}^{m} a_{ij} \leq \sum_{j=1}^{n} v_j$$

$$\text{From [II], feasibility} \rightarrow \sum_{j=1}^{n} a_{ij} \geq 1 \qquad i = 1, \ldots, m$$

$$\rightarrow \sum_{i=1}^{m} \sum_{j=1}^{n} a_{ij} \geq m$$

By transitivity the condition follows and necessity is established.

(2) *Sufficiency:* Suppose $\sum_{j=1}^{n} v_j \geqslant m$. Then arbitrarily select m of the vacancies, assign them (again arbitrarily) one to each i, and leave the remaining vacancies (if any) unfilled. [I] and [II] are clearly satisfied, and [III] through [VI], which apply to multiple assignments, are trivially satisfied. [VII], too, is satisfied since $a_{ij} = 1$ or 0. Sufficiency is established. Q.E.D.

I assume in the remainder of this paper that the condition in the Feasibility Theorem is satisfied. That is, at the stage of negotiated structure, allowance is made for the number of applicants in the creation of vacancies. Empirically the condition is always satisfied (see Shepsle, 1974, Table II). For a similar theorem as applied to transportation problems, see Gale (1960, p. 5).

In order to underscore the complexity of our problem, it is useful, now that feasibility problems have been disposed of, to focus on some of the other features that distinguish it from the simple assignment problem. The first is *completeness.*

Definition: An assignment is said to be *complete* if and only if $\sum_{i=1}^{m} a_{ij} = v_j$ for every j.

That is, a complete assignment is one in which each class [I] constraint is satisfied as an *equality*.

THEOREM 3: For some configurations of vacancies, every feasible assignment satisfies some constraint in [I] as an inequality. That is, for some vacancy vectors, no feasible assignment is complete.

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Proof A simple example establishes the result. Whenever $\sum_{j=1}^{e} v_j > m$, no complete assignment is feasible (since [IV] is violated otherwise). Other conditions rendering complete assignments infeasible are:

(1) $\sum_{j=e+1}^{s} v_j > m$ (2) $\sum_{j=e+1}^{n} v_j > 2m$

and so on. Q.E.D.

Even if we were to suppose that party leaders, at the stage of negotiated structure, attempt to provide the CCs with a vector of vacancies that can be completely allocated, the CC objective function may not be compatible with a complete assignment.

THEOREM 4: Even when complete assignments are feasible, they may not be optimal.

Proof: An example illustrates this result. Let us suppose that only three committees have vacancies: $v = (v_e, v_s, v_n) = (2, 3, 3)$, where c_e, c_s, and c_n are, respectively, exclusive, semi-exclusive, and non-exclusive. Let $M = \{1,2,3,4,5\}$ be the members seeking assignments. Their requests are recorded in the following preference matrrix:

$$P = \begin{array}{c} \\ \underline{1} \\ \underline{2} \\ \underline{3} \\ \underline{4} \\ \underline{5} \end{array} \begin{array}{c} \begin{array}{ccc} \underline{e} & \underline{s} & \underline{n} \end{array} \\ \begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \end{array}$$

Notice that c_s is in excess demand whereas c_e is in excess supply. An optimal assignment (not necessarily unique) is

$$A^o = \begin{array}{c} \\ \underline{1} \\ \underline{2} \\ \underline{3} \\ \underline{4} \\ \underline{5} \end{array} \begin{array}{c} \begin{array}{ccc} \underline{e} & \underline{s} & \underline{n} \end{array} \\ \begin{bmatrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix} \end{array}$$

For this assignment $v(A^o) = \sum_i \sum_j p_{ij} a_{ij}^o = 7$. A quick glance at P should convince the reader that A^o does indeed maximize $v(\cdot)$. Only eight preferences are expressed in P and one of them (in the *s* column) is impossible to satisfy because of a supply constraint ($v_s = 3$); therefore, at most seven requests may be satisfied and A^o, above, does precisely that. *Notice that A^o is incomplete* – $\sum_i a_{ie} = 1 < v_e$. All complete assignments require assigning a ***nonrequestor*** to c_e and removing him from at least one committee he requested and received in A^o. Consequently, complete assignments reduce the objective function. Q.E.D.

A summary to this point is in order. Having given the necessary and sufficient condition for feasibility (Feasibility Theorem), I have demonstrated that for some vacancy configurations no complete assignment is feasible (Theorem 3). For others, complete assignments may be feasible, but they are not optimal (Theorem 4). These results have underscored three important characteristics of the assignment process: *feasibility, completeness,* and *optimality*. One last characteristic – that of an *integral assignment* – is considered in Theorem 5. But first two important lemmas. Their proofs are straighforward so they are omitted.

LEMMA 1: Any convex combination of complete assignments is itself a complete assignment.

LEMMA 2: A complete assignment is either a convex combination of complete assignments or it is an extreme point.

We have not, to this point, restricted the a_{ij}'s to zero or one (though substantively this is all that makes sense). In fact, in the *simple* assignment problem, Theorem 2 demonstrates that integral assignments emerge as a consequence of goal-seeking behavior. It is, however, somewhat more problematic in the committee assignment problem.

Definition: An assignment is said to be integral if a_{ij} = 0 or 1 for all i and j.

Clearly, if nonintegral A's are extreme points (vertices) of the polyhedron C(A), defined by [I] through [VII], then they will be optimal for *some* objective functions (see Figure 1). Unfortunately,

THEOREM 5: For some vacancy configurations, nonintegral points are extreme.

Proof: Suppose there are two committees with three vacancies, an exclusive committee with one slot (v_e=1) and a nonexclusive committee with two slots (v_n= 2). Let m = 2. By the Feasibility Theorem, feasible assignments exist. With these parameters, however, no *integral* assignment is complete: either one applicant receives the exclusive slot and the other one of the nonexclusive slots ([VI] prohibits him from receiving both) or both receive nonexclusive slots ([IV] prohibits an applicant from serving on any other committee if he receives an exclusive committee slot). *Yet there do exist nonintegral complete assignments:*

$$\text{applicant} \quad \begin{matrix} \\ \\ 1 \\ 2 \end{matrix} \overset{\text{committee}}{\begin{matrix} c_e & c_n \\ \begin{bmatrix} X & 1 \\ 1-X & 1 \end{bmatrix} \end{matrix}}$$

where $1/3 \leqslant X \leqslant 2/3$. Note that [IV] is satisfied by both applicants. From Lemma 2 a complete assignment is either a convex combination of complete assignments or is an extreme point. In either case, some nonintegral points are extreme. In fact,

$$\begin{bmatrix} X & 1 \\ 1-X & 1 \end{bmatrix} = \lambda_1 \begin{bmatrix} 1/3 & 1 \\ 2/3 & 1 \end{bmatrix} + \lambda_2 \begin{bmatrix} 2/3 & 1 \\ 1/3 & 1 \end{bmatrix}$$

where λ_1 = 2-3X and λ_2 = 3X-1, and the latter two A-matrices are extreme points. Q.E.D.

We must conclude, then, that nothing in the structure of the problem precludes fractional assignments. The example in Theorem 5 is our first sure illustration that nonintegral assignments may qualify as extreme points.

At this point there are several possible courses of action. One is to move away from the general linear programming model, opting instead for a more restrictive approach: *integer programming*. A second, and I think more appealing, course of action is to draw on a descriptive feature alluded to earlier. Most descriptive studies of committee assignments indicate that freshmen are rarely assigned to exclusive committees. Moreover, when freshmen are assigned to these committees, it is typically at the behest of the party leadership. Since more senior members are often co-opted to serve on exclusive committees, since the leadership plays an active role in recruiting for these committees, and since freshmen are rarely involved in exclusive committee recruitment, it makes sense in a study of freshmen assignments to separate out the process by which exclusive committee vacancies are filled and focus, instead, on the remaining committees. Hence:

ASSUMPTION 1: $\sum_{j=1}^{e} v_j = 0$

This assumption has a very salutary effect as seen in the following consequence. With Assumption 1, constraint [IV] becomes

$$2 \sum_{j=e+1}^{s} a_{ij} + \sum_{j=s+1}^{n} a_{ij} \leq 3 \qquad i = 1, \ldots, m$$

which, in turn, may be written as

$$\sum_{j=e+1}^{s} a_{ij} + \sum_{j=e+1}^{n} a_{ij} \leq 3 \qquad i = 1, \ldots, m$$

Moreover, since $\sum_{j=1}^{e} a_{ij} = 0$ by Assumption 1, the second term on the left-hand side is recast:

$$\text{[IV*]} \qquad \sum_{j=e+1}^{s} a_{ij} + \sum_{j=1}^{n} a_{ij} \leq 3 \qquad i = 1, \ldots, m.$$

But this is simply the sum, for each i = 1, . . . , m, of constraints [III] and [V]. This result and its consequence are given in Lemma 3 and Theorem 6.

LEMMA 3: Let X be the set of points satisfying the system of constraints

$$\text{(1)} \qquad \sum_{j=1}^{n} \alpha_{ij} x_j \leq \gamma_i \qquad i = 1, \ldots, m.$$

Then X satisfies

$$(2) \qquad \sum_{j=1}^{n} \beta_j x_j \leq b$$

where $\beta_j = \sum_{i=1}^{m} \lambda_i \alpha_{ij}$, $b = \sum_{i=1}^{m} \lambda_i \gamma_i$, and $\lambda_i \geq 0$.

Lemma 3 simply asserts that any point satisfying a system of constraints (1) also satisfies any nonnegative linear combination of those constraints.

Proof: Suppose $x = (x_1, \ldots, x_n)$ is a point in X satisfying (1). Then

$$\sum_j \alpha_{ij} x_j \leq \gamma_i \qquad i = 1, \ldots, m$$

Multiplying by λ_i: $\quad \sum_j \lambda_i \alpha_{ij} x_j \leq \lambda_i \gamma_i \qquad i = 1, \ldots, m$

$$\sum_i \sum_j \lambda_i \alpha_{ij} x_j \leq \sum_i \lambda_i \gamma_i$$

$$\sum_j x_j (\sum_i \lambda_i \alpha_{ij}) \leq \sum_i \lambda_i \gamma_i$$

$$\sum_j \beta_j x_j \leq b$$

and (2) is satisfied. Q.E.D.

THEOREM 6: The convex polyhedra C(a) and C′(a), defined by [I] through [VII] and [I] through [III] – [V] through [VII], respectively, are identical.

Proof: By Assumption 1, [IV] becomes [IV*]. In particular, for the k^{th} applicant (i = k), [IV] becomes:

$$\sum_{j=e+1}^{s} a_{kj} + \sum_{j=1}^{n} a_{kj} \leq 3.$$

But, for the k^{th} applicant constraints [III] and [V] are, respectively:

$$\sum_{j=1}^{n} a_{kj} \leq 2 \qquad \text{and}$$

$$\sum_{j=e+1}^{s} a_{kj} \leq 1$$

Letting [III] and [V] represent the constraint system (1), [IV*] represent constraint (2), and the relevant λ's be unity, Lemma 3 establishes that [IV*] is satisfied by all points in C′(a); hence it is identical to C(a). Q.E.D.

The salutary effect of Assumption 1 is the following: the class [IV] constraints are redundant (Theorem 6). Not only is the magnitude of the problem reduced with the elimination of these m constraints; as is seen below, precisely the "right" constraints have been eliminated, i.e., class [IV] turns out to be the culprit that permits fractional extreme points.

LEMMA 4 (Hoffman - Kruskal, 1956): If, in a system of linear inequalities with integral coefficients and constant terms, every non-singular square submatrix of the coefficient matrix has determinant ± 1, then every extreme solution is integral.

Lemma 4 provides a useful sufficient condition for the determination of *integral* extreme points. Notice that [I]-[III], [V]-[VII] is a system of linear inequalities with integral coefficient and constant terms. Also note that a necessary condition for a system to satisfy the premises of the lemma is that every coefficient be 0, +1, or -1 (since each coefficient is a *minimal*, i.e., 1 x 1, submatrix of the coefficient matrix). This is true of the committee assignment problem by virtue of Assumption 1 and Theorem 6; otherwise the coefficients of [IV] would have violated the premises of Lemma 4.

The property that every submatrix of the constraint coefficient matrix have a determinant of 0, +1, or -1 is known as the *unimodular property* (Hoffman – Kruskal, 1956).

We have, in effect, replaced one problem with another. Lemma 4 tells us when a constraint system has integral extreme points. We need, however, some means of determining if the premises of Lemma 4 are satisfied. The next two lemmas and a definition provide such a strategy.

LEMMA 5 (Heller-Tompkins, 1956): Let A be an m x n matrix whose rows can be partitioned into two disjoint sets B and C, with the following properties:

(1) every column of A contains at most two non-zero entries;

(2) every entry in A is 0, +1, or -1;

(3) if two non-zero entries in a column of A have the same sign, then the row of one is in B, and the other in C;

(4) if two non-zero entries in a column of A have opposite signs, then the rows of both are in B, or both in C.

Then every minor determinant of A is 0, +1, or -1.

Definition: A matrix B is said to be *Dantzig sufficient* for A if A is unimodular whenever B is.

That is, B is Dantzig sufficient for A if, after determining that B is unimodular, it is necessarily the case that A is unimodular too.

LEMMA 6: Dantzig sufficiency in transitive.

This Lemma follows directly from the previous definition. It asserts that if B is Dantzig sufficient for A, and C Dantzig sufficient for B, then C is Dantzig sufficient for A.

My stragegy is as follows: beginning with the constraint coefficient matrix defined by [I]-[III], [V]-[VII], a series of Dantzig sufficiencies are employed to produce a matrix to which Lemma 5 is applied directly. With its unimodularity established, it follows from Lemma 6 that the original constraint matrix is unimodular and, from Lemma 4, that every extreme solution is integral.

These results and proofs are found in the Appendix. They allow us to state two summary theorems:

THEOREM 7: The extreme points of C are integral.

Proof: This follows from Assumption 1, Theorem A, and Lemma 4.

THEOREM 8: Any linear function of assignments is maximized at an integral assignment.

Proof: This is a direct implication of Theorems 1 and 7.

Theorems 7 and 8 are important. Along with the Feasibility Theorem, they characterize three of the four important features of the committee assignment problem: *feasibility, integral assignments, optimality*. These theorems notwithstanding, however, Theorem 4 should not be forgotten. Some of the integral extreme points represent *incomplete* assignments.

In fact, a simple corollary of Theorems 1, 4, 7, and 8 is:

COROLLARY: The extreme points of the convex polyhedron C, defined by [I] - [III], [V] - [VII], are a proper subset of all "permutations" of complete and incomplete assignments. In particular, every integral assignment, in which at least m vacancies are filled and distributed in accord with the remaining contraints, is optimal for some objective function.

Thus, for even moderate m, n, and v, the number of extreme points is large indeed.[10]

VIII. Discussion

This essay has begun the task of mapping the *operating characteristics* of an important institutional process. The theoretical posture articulated in the first few sections emphasized goal-seeking or maximizing behavior in a context in which feasible behavior is constrained by institutional rules of the game. In the latter part of this essay I have abstracted away much empirical detail (though I have tried to persuade the reader, on the basis of evidence, that the formulation here is

[10]In Shepsle (1974), a computing formula is given for the number of extreme points as a function of the parameters m and n and the particular distribution of v_j's. To give an example, there are nineteen 87th Congress Democratic freshmen (m = 19) applying for 30 vacancies ($\sum_j v_j$ = 30) on 11 committees (n = 11). With these parameters there are approximately 1.28×10^{19} extreme points. These points are located in a space of dimensionality mn = 209.

reasonable) in order to capture the basic interaction between goals, constraints, and characteristics of outcomes. I shall not review the results here; the reader may wish to reread the principal theorems.

The next task – one that proves the ultimate worth of the preceding methodology – is to tease out some *behavioral* consequences. The first thing to note in this regard is that, formal constraints notwithstanding, there is an unimaginably large number of options available to the party CCs (see Shepsle, 1974). The *discretionary authority* of the party CCs, that is, is impressive – hence their "clout" in Congressional life. Having said this, it is all the more surprising to report that the simple optimizing model proposed here works remarkably well. Cohen's recent analysis (1974), as well as the author's own empirical work in which the objective function value of actual assignments reaches as high as 80% of optimal, suggest that it is the technological incapacities of the actors, not serious model misspecifications, that account for the deviations between actual and optimal. Second, the work of Rohde and Shepsle (1973) that inspired this model (also see Achen and Stolarek (1974)) provides substantial empirical support for the optimization paradigm proposed here. Finally, elsewhere I have exploited some results from duality theory in mathematical programming (see Shepsle, 1973, Theorems 10, 11, 12, and 13) to deduce consequences related to the *shadow prices* associated with committee slots. It will be interesting to see whether these "prices" play a signalling role to the party leadership akin to the roles ordinary prices and shadow prices play in market and planned economies, respectively.

The strategy of analysis employed here – conceptualizing goals and constraints in a formal structure and deducing consequences – is limited and incomplete. Its ultimate worth depends on further theoretical and detailed empirical examinations. It does, however, have the virtue of turning attention away from descriptive detail at particular points in time and toward general institutional operating characteristics. In this fashion, I believe, we will secure more realiable knowledge about institutions and, for the normatively inclined, learn how to change them or design better ones.

Appendix
Establishment of The Unimodularity of The Convex Polyhedron C(A)

Before beginning this task, it is convenient to transform the system to matrix notation.

Instead of writing assignments as a matrix $A = [a_{ij}]$, it is convenient to write it as a column vector (which I call *a*) by attaching the rows in A end-to-end in sequence:

$$\underline{a}' = [a_{11} \ldots a_{1n} a_{21} \ldots a_{2n} \ldots a_{m1} \ldots a_{mn}]$$

Similarly, the weight matrix $P = [p_{ij}]$ is written as a row vector: $p = [p_{11}, p_{12}, \ldots, p_{1n}, p_{21}, p_{22}, \ldots, p_{2n}, \ldots, p_{m1}, p_{m2}, \ldots, p_{mn}]$. Write the coefficients from the constraint inequalities as rows of a matrix C and, finally, write the constant terms of the constraint system as a column vector:

$$\underline{d}' = [\underbrace{v_1, \ldots, v_m}_{[I]}, \underbrace{-1, \ldots, -1}_{[II]}, \underbrace{2, \ldots, 2}_{[III]}, \underbrace{1, \ldots, 1}_{[V]}, \underbrace{1, \ldots, 1}_{[VI]}, \underbrace{0, \ldots, 0}_{[VII]}]$$

Notice, that class [IV] has been eliminated. For convenience semiexclusive committees are now indexed $c_1, \ldots, c_s$, and nonexclusive committees $c_{s+1}, \ldots, c_n$ (i.e., there are still n committees indexed).

The committee assignment problem may now be written as a *primal linear programming problem* in matrix form:

$$\max_{a} \underline{p} \cdot \underline{a} \quad \text{subject to}$$
$$C \cdot \underline{a} \leq \underline{d}\ .$$

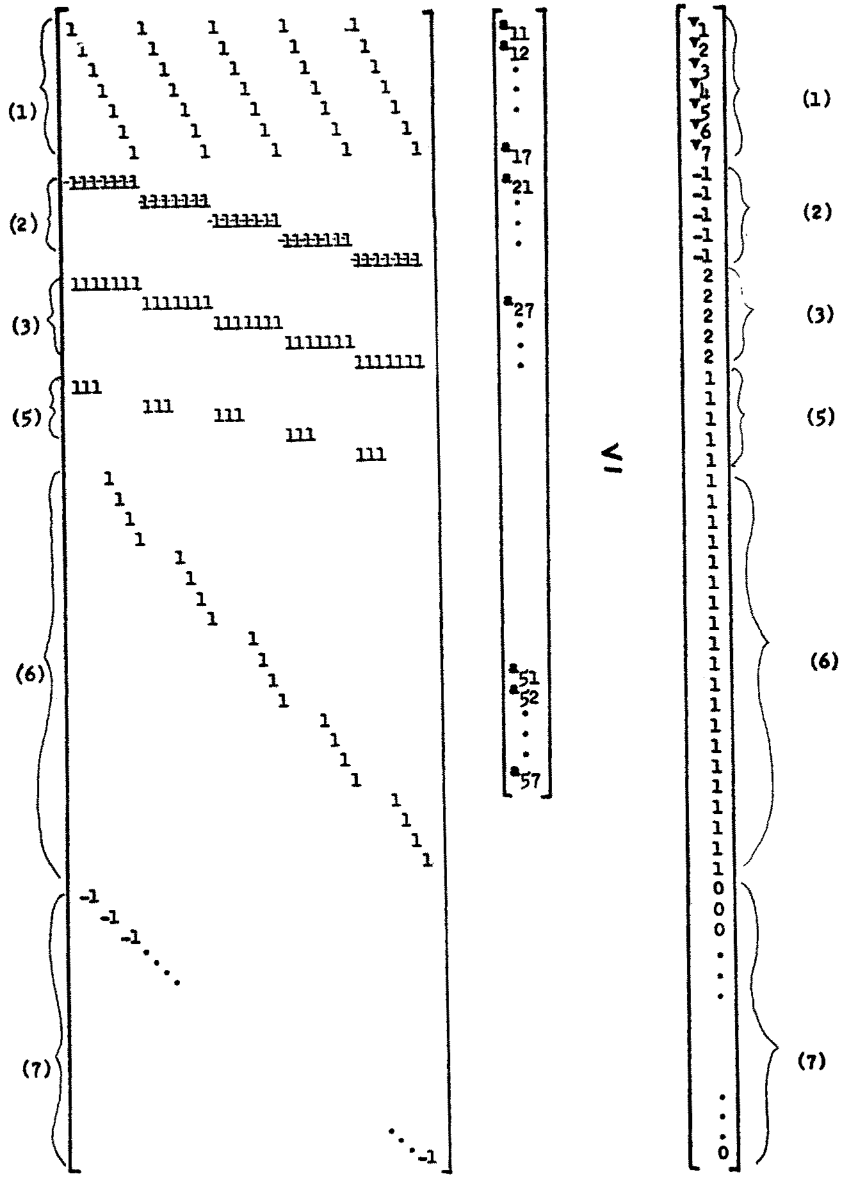
The constraint system for five applicants and seven committees (three semiexclusive and four nonexclusive) is illustrated in Figure 2.

LEMMA A: The matrix composed of coefficients from [I] and [V] is Dantzig sufficient for the entire constraint coefficient matrix, C.

Proof: (a) Let C_1 be the submatrix of C with the mn rows defined by [VII] deleted. *C_1 is Dantzig sufficient for C*: the coefficients of [VII] appear as mn rows, each of which has (mn-1) entries of zero and one entry of -1. Consider an arbitrary (k x k) sumatrix, K, of C, one (or more) of whose rows is from [VII]:

case 1: The column of this row with the -1 entry is in K. Then compute Det (K) by expanding along this row. Det (K) = -Det (K_1) where K_1 (in C_1) is the (k-1) x (k-1) submatrix of K with the type [VII] row and the column in which the -1 appears deleted. Since type [VII] constraints affect the determinant by a factor of -1, if K_1 is unimodular, so is K.

Figure 2: Polyhedron Defined By Constraints (Minus (4)) For Five Applicants, Three Semiexclusive Committees and Four Nonexclusive Committees*



*Cell entries in matrix are zero unless otherwise indicated.

case 2: Now the column of the type [VII] constraint row with the -1 entry is not in K. The row, then, is composed entirely of zeros. Cleraly if K_1 (as defined above) is unimodular, so is K.

Together these two cases establish the proposition and the mn type [VII] rows may be deleted. (b) By Theorem 6, the submatrix C_2 of C_1 with the m type [IV] constraints deleted is Dantzig sufficient for C_1.

(c) Let C_3 be the submatrix of C_2 with the m rows of [II] deleted. C_3 is Dantzig sufficient for C_2: since any row of [II], say the r^{th} ($1 \leqslant r \leqslant m$), is the negative of the r^{th} row of [III], any submatrix of C_2 containing both has a zero determinant. Moreover, any submatrix of C_2 containing the r^{th} row of [II] and the s^{th} row ($s \neq r$) of [III] has a determinant differing only in sign from the submatrix containing the r^{th} and s^{th} rows of [III]. Therefore, if C_3 is unimodular so is C_2, and we may delete the type [II] coefficients.

(d) Let C_4 be the submatrix of C_3 with the m rows of [III] deleted. It is easy to see (consult Figure 2) that any row of [III] is simply the sum of the appropriate row of [V] and the (n-s) appropriate rows of [VI]. By virtue of this linear dependency (and the detailed reasoning set forth in Hoffman and Kuhn, 1956, pp. 205-206), C_4 is Dantzig sufficient for C_3.

(e) Let C_5 be the submatrix of C_4 with the m (n-s) rows of [VI] deleted. By reasoning identical to (a), C_5 is Dantzig sufficient for C_4. C_5 is an (m + n) x mn matrix whose elements are the coefficients of classes [I] and [V] exclusively. Since Dantzig sufficiency is transitive by Lemma 6, C_5 is Dantzig sufficient for C. Q.E.D.

It is now possible to prove

THEOREM A: The constraint matrix C is unimodular.

Proof: By Lemma A attention is focused exclusively on the coefficients in [I] and [V]. It is shown that this matrix (C_5 of Lemma A) satisfies the Heller-Tompkins condition of Lemma 5. Condition (4) is trivially satisfied since no coefficients in [I] and [V] are negative. Condition (2) is satisfied since all coefficients are 0 or +1. Observe in [I] – the reader may wish to consult Figure 2 – that each column has *excactly* one non-zero element. In [V] each column has *at most* one non-zero element. Therefore, in C_5 no column has more than two non-zero elements. Hence, (1) is satisfied. Finally, to satisfy (3) simply partition C_5 into C_5^* and C_5^{**}. The former is the n x mn matrix defined by [I] alone; the latter is the m x mn matrix defined by [V] alone. With (1)-(4) of Lemma 5 satisfied, C_5 is unimodular. By Lemma A, C is unimodular. Q.E.D.

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