where c is the cardinality of the continuum, and for each unbounded sequence $\{S_n\}$ with the set of the different partial limits of cardinality γ, there exists a regular matrix that sums the sequence $\{S_n\}$, but does not sum any divergent sequence with the set of different partial limits of cardinality different from γ.

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BEHAVIOR OF SOLUTIONS OF FUNCTIONAL AND DIFFERENTIAL-FUNCTIONAL EQUATIONS WITH SEVERAL TRANSFORMATIONS OF THE INDEPENDENT VARIABLE

G. A. Derfel'

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In this paper which deals with the same subject as [1-5], we study the behavior (at the origin) of the solutions of the functional equation

$$\sum_{j=0}^{l}\sum_{k=0}^{m} a_{jk}p^{\beta_k}f(\alpha_j p) = 0, \tag{1}$$

where the a_{jk} are complex constants, and α_j and β_j are real numbers such that $1 = \alpha_0 < \alpha_1 < \ldots < \alpha_l$; $0 = \beta_0 < \beta_1 < \ldots < \beta_m$. For Eq. (1) to contain a deviating argument, it will be assumed in the following that there exist quantities $0 \le k_1$ and $k_2 \le m$ such that $a_{0k_1} \neq 0$, $a_{lk_2} \neq 0$. Whereever it is not stipulated otherwise, we shall consider solutions of Eq. (1) defined in a punctured neighborhood of the origin $(0 < |p| < r)$, such a solution being a complex-valued function f(p) of a real variable p that is defined and continuous for $0 < |p| < \alpha_l r$, and that causes Eq. (1) to be an identity for any $0 < |p| < r$.

If the β_k are rational numbers and the α_j are multiplicatively commensurable, i.e., $\alpha_j = q^{r_j}$, where $q > 1$ and the r_j are rational, then Eq. (1) reduces to a so-called q-difference equation that has been studied in [6, 7].

With the aid of Theorems 1 and 2 below, it is possible to extend certain results of the theory of q-difference equations to an equation of the form (1).

These results are applied (Theorem 3) to the proof of the existence of fast decreasing (at infinity) solutions of the differential-functional equation

$$y^{(m)}(x) = \sum_{j=0}^{l}\sum_{k=0}^{m} b_{jk}y^{(k)}(\lambda_j x), \tag{2}$$

where the b_{jk} are complex constants, and the λ_j are real, with

$$1 < \lambda_0 < \cdots < \lambda_l \tag{3}$$

(it is assumed that there exists a $0 \le j_1 \le l$ such that $b_{j_1 0} \neq 0$).

Let us introduce the necessary notations and definitions. By k_j we shall denote the smallest index k such that $a_{jk_j} \neq 0$. In the plane we plot the points with coordinates $(\ln\alpha_j, \beta_{k_j})$. The Newton diagram of these points is defined by the characteristic polygonal line L of Eq. (1). By μ we shall denote the tangent of the angle of inclination of the extreme right segment of the characteristic polygonal line L (i.e., μ is the angular coefficient of the straight line on which this segment lies).

Theorems 1 and 2 that follow are characterizing the behavior of the solutions of Eq. (1) at the origin.

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THEOREM 1. If $\mu = 0$, then Eq. (1) has in a neighborhood $0 < |p| < r$ of the origin a one-parameter family of solutions of the form

$$f(p) = Cp^{\sigma} \sum_{\nu_1,\ldots,\nu_m=0} c_{\nu_1,\ldots,\nu_m} (p^{\beta_1})^{\nu_1} \ldots (p^{\beta_m})^{\nu_m}, \tag{4}$$

where the series is convergent for $|p| < r$; σ is a root of the quasipolynomial $g(s) = \sum_{j=0}^{l} a_{j0} e^{s \ln \alpha_j}$, and $\sigma + \nu_1\beta_1 + \ldots + \nu_m\beta_m$ is not a root of g(s) for any natural $\nu_1, \ldots, \nu_m$; the coefficient $c_{0,\ldots,0}$ can be taken arbitrarily.

Theorem 2 yields a necessary and sufficient condition of existence of a solution that is bounded at the origin for $\mu \neq 0$.

THEOREM 2. 1) If $\mu > 0$, then Eq. (1) has at least one nontrivial solution bounded at the origin. Moreover, there exists a solution that has in a neighborhood of the origin a bound

$$|f(p)| \leqslant C \exp\{-\mu_1 \ln^2 |p|\}, \qquad 0 \leqslant |p| < r \tag{5}$$

with a positive C and any $\mu_1 < \mu/2$.

2) If $\mu < 0$, then Eq. (1) has a unique solution $f(p) \equiv 0$ bounded at the origin. If the solution f(p) has in a neighborhood of the origin a bound

$$|f(p)| \leqslant C \exp\{\mu_2 \ln^2 |p|\}, \qquad 0 < |p| < r \tag{6}$$

with a positive C and $\mu_2 < -0.5\mu \ln^2(\alpha_l/\alpha_{l-1})/\ln^2 \alpha_l$, then f(p) will vanish identically.

Proof of Theorem 1. By reducing (if necessary) all the terms of Eq. (1) by the common multiplier p^{β}, where $\beta = \min_{0 \leqslant j \leqslant l} \beta_{k_j}$, it is always possible to achieve that at least one point of the characteristic line L should lie on the x axis. Hence if $\mu = 0$, then it can be assumed without loss of generality that the extreme right segment of the line L lies on the x axis, i.e.,

$$a_{l_0} \neq 0 \tag{7}$$

and there exists a $j_0 = 0, \ldots, l-1$ such that

$$a_{j_0 0} \neq 0; \ a_{j0} = 0 \quad \text{for any} \quad j = 0, \ldots, j_0 - 1. \tag{8}$$

Let us write $\alpha_j = e^{\gamma_j}$ and substitute (4) into (1). For the quantities $c_{\nu_1,\ldots,\nu_m}$ we hence obtain the difference equation

$$\left[\sum_{j=0}^{l} a_{j0} e^{(\sigma+\nu_1\beta_1+\ldots+\nu_m\beta_m)\gamma_j}\right] c_{\nu_1,\ldots,\nu_m} + \sum_{k=1}^{m} \left\{\sum_{j=0}^{l} a_{jk} e^{[\sigma+\nu_1\beta_1+\ldots+(\nu_k-1)\beta_k+\ldots+\nu_m\beta_m]\gamma_j}\right\} c_{\nu_1,\ldots,\nu_k-1,\ldots,\nu_m} = 0 \tag{9}$$

with initial conditions

$$c_{\nu_1,\ldots,\nu_{k-1},\ldots,\nu_m} = 0, \quad \text{for} \quad \nu_1 \geqslant 0, \ldots, \nu_k = 0, \ldots, \nu_m \geqslant 0. \tag{10}$$

For the σ occurring in (4), let us now take a number such that

$$\sum_{j=0}^{l} a_{j0} e^{\sigma\gamma_j} = 0; \tag{11}$$

$$\sum_{j=0}^{l} a_{j0} e^{(\sigma+\nu_1\beta_1+\ldots+\nu_m\beta_m)\gamma_j} \neq 0, \tag{12}$$

if at least one of the numbers $\nu_1, \ldots, \nu_m$ is nonzero. For proving the existence of such a σ, let us note that according to (7) and (8) the quasipolynomial $g(s) = \sum_{j=0}^{l} a_{j0} e^{s\gamma_j}$ has infinitely many roots, all of them being located in a strip $d_2 \leq \operatorname{Re} s \leq d_2$ that is parallel to the imaginary axis ([8, Sec. 12.5]). By assuming that d_1 is an exact infimum, and d_2 an exact supremum of the real parts of these roots, we find that there exists at least one root $s = \sigma$ such that $d_2 - \operatorname{Re}\sigma < \beta_1$. This root evidently satisfies the conditions (11) and (12).

It follows from (9), (10), and (11) that the coefficient $c_{0,\ldots,0}$ can be taken as desired. If $c_{0,\ldots,0}$ has been selected, then Eq. (9) makes it possible to determine the $c_{\nu_1,\ldots,\nu_m}$ for which

$$\nu_1+\ldots+\nu_m=1; \quad \nu_1\geqslant 0,\ldots,\nu_m\geqslant 0 \tag{13}$$

[by virtue of (12) they are uniquely determined].

After finding all the $c_{\nu_1,\ldots,\nu_m}$ whose multiindex $(\nu_1,\ldots,\nu_m)$ belongs to the (m − 1)-dimensional simplex $S_k^{m-1}=\{\nu_1\geq 0,\ldots,\nu_m\geq 0 \mid \nu_1+\ldots+\nu_m=k\}$, it is possible to determine [uniquely, by virtue of (12)] with the aid of Eq. (9) all the $c_{\nu_1,\ldots,\nu_m}$ for which $(\nu_1,\ldots,\nu_m)\in S_{k+1}^{m-1}=\{\nu_1\geq 0,\ldots,\nu_m\geq 0 \mid \nu_1+\ldots+\nu_m=k+1\}$. In this case we have the bound

$$|c_{\nu_1,\ldots,\nu_m}|\leqslant M(mD)^{\nu_1+\ldots+\nu_m}, \qquad \nu_1\geqslant 0,\ldots,\nu_m\geqslant 0, \tag{14}$$

where $M=|c_{0,\ldots,0}|$, D being a positive number.

For proving (14), we shall prove the validity of the inequality

$$|c_{\nu_1,\ldots,\nu_m}|\leqslant D\sum_{k=1}^{m}|c_{\nu_1,\ldots\nu_k-1,\ldots,\nu_m}|, \; \nu_1\geqslant 0,\ldots,\nu_m\geqslant 0. \tag{15}$$

Indeed,

$$L_0(\nu_1,\ldots,\nu_m)=\Big|\sum_{j=0}^{l}a_{j0}e^{(\sigma+\nu_1\beta_1+\ldots+\nu_m\beta_m)\gamma_j}\Big|$$

$$=|a_{l_0}|\,|e^{\sigma\gamma_l}|\,|e^{(\nu_1\beta_1+\ldots+\nu_m\beta_m)\gamma_l}|\,\Big|1+\sum_{i=0}^{l}\frac{a_{l-i,0}}{a_{l_0}}e^{(\sigma+\nu_1\beta_1+\ldots+\nu_m\beta_m)(\gamma_l-\gamma_{l-i})}\Big|. \tag{16}$$

If the number $\nu_1\beta_1+\ldots+\nu_m\beta_m>R$ is sufficiently large, then the last factor in (16) will be larger than 1/2 and, therefore,

$$L_0(\nu_1,\ldots,\nu_m)>D_R e^{(\nu_1\beta_1+\ldots+\nu_m\beta_m)\gamma_l}, \text{where } D_R=\frac{1}{2}|a_{l_0}|\,|e^{\sigma\gamma_l}|. \tag{17}$$

But if $\nu_1\beta_1+\ldots+\nu_m\beta_m\leq R$, then

$$L_0(\nu_1,\ldots,\nu_m)=D_{\nu_1,\ldots,\nu_m}e^{(\nu_1\beta_1+\ldots+\nu_m\beta_m)\gamma_l}, \tag{18}$$

and it follows from (12) that $D_{\nu_1,\ldots,\nu_m}\neq 0$. Let us denote

$$D_1=\min_{\{\nu_1\geqslant 0,\ldots,\nu_m\geqslant 0 \mid \nu_1\beta_1+\ldots+\nu_m\beta_m\leqslant R\}}(D_{\nu_1,\ldots,\nu_m}, D_R)>0.$$

It then follows from (17) and (18) that

$$L_0(\nu_1,\ldots,\nu_m)\geqslant D_1 e^{(\nu_1\beta_1+\ldots+\nu_m\beta_m)\gamma_l}, \quad \nu_1\geqslant 0,\ldots,\nu_m\geqslant 0. \tag{19}$$

Moreover,

$$L_k(\nu_1,\ldots,\nu_m)=\Big|\sum_{j=0}^{l}a_{jk}e^{[\sigma+\nu_1\beta_1+\ldots+(\nu_k-1)\beta_k+\ldots+\nu_m\beta_m]\gamma_j}\Big|\leqslant\sum_{j=0}^{l}|a_{jk}|\,|e^{\sigma\gamma_j}|\,|e^{(\nu_1\beta_1+\ldots+\nu_k\beta_k+\ldots+\nu_m\beta_m)\gamma_l}|\leqslant D_2 e^{(\nu_1\beta_1+\ldots+\nu_m\beta_m)\gamma_l}, \tag{20}$$

where $D_2=\sum_{j=0}^{l}|a_{jk}|\,|e^{\sigma\gamma_j}|$. Hence, formula (15) will follow from (19) and (20).

Let us assume that the bound (14) holds for all $(\nu_1,\ldots,\nu_m)$ that belong to the simplex S_k^{m-1}. By virtue of the induction hypothesis, the initial conditions (10), and the inequality (15), we then conclude that for all $(\nu_1,\ldots,\nu_m)\in S_{k+1}^{m-1}$ we have $|c_{\nu_1,\ldots,\nu_m}|\leq Dm(mD)^k M=(mD)^{k+1}M$. Thus we have proved (14). By virtue of the fact that (14) shows that the series in (4) is convergent in a neighborhood of the origin $|p|<r$, we have o completely proved Theorem 1.

Proof of Theorem 2. Let f(p) be a solution of Eq. (1). Let us write

$$f(p)=e^{-\frac{\mu}{2}\ln^2 p}g(p). \tag{21}$$

Then the function g(p) will satisfy the equation

$$\sum_{j=0}^{l}\sum_{k=0}^{m} a_{jk}e^{-\frac{\mu}{2}\gamma_j^2}p^{\beta_k-\mu\gamma_j}g(\alpha_j p), \tag{22}$$

where, as before, $\gamma_j = \ln \alpha_j$. This equation is of the same type as (1). Its characteristic polygonal line L' can be obtained from the characteristic polygonal line L of Eq. (1) by a transformation F: $(\gamma, \beta) \to (\gamma, \beta - \mu\gamma)$ of the (γ, β) plane into itself. If a segment of L lies on a straight line $\beta = \eta\gamma + b$ with an angular coefficient η, then the corresponding segment of L' will lie on a straight line $\beta = (\eta - u)\gamma + b$ whose angular coefficient is smaller by a quantity μ. In particular, the extreme right segment of the polygonal line L' will be horizontal. Hence Eq. (22) will satisfy the conditions of Theorem 1, and this equation will have a solution of the form (4). Thus the assertion 1 of Theorem 2 follows directly from (21).

Below we shall prove that if a function f(p) satisfies Eq. (1) and it has a bound (6) at least for $0 < p < r$, then this yields $f(p) \equiv 0$.

If $\mu < 0$, then it can be assumed without loss of generality that

$$a_{l_0} \neq 0,\ a_{l_0} = 0,\ \frac{\beta_{k_j}}{\ln\alpha_l - \ln\alpha_j} \geqslant -\mu \ \text{ for } \ j = 0, \dots, l-1 \tag{23}$$

(as above, k_j is the smallest subscript k such that $a_{jk_j} \neq 0$). Hence

$$\beta_{k_j} \geqslant \delta = -\mu(\ln\alpha_l - \ln\alpha_{l-1}), \tag{24}$$

and in a sufficiently small neighborhood $0 \leq p < r$ of the origin we have the inequalities

$$\left|\sum_{k=0}^{m} a_{lk}p^{\beta_k}\right| > \frac{1}{2}|a_{l_0}|; \tag{25}$$

$$\left|\sum_{k=0}^{m} a_{jk}p^{\beta_k}\right| \leqslant |a_{jk_j}|\,|p^{\beta_{k_j}}|\left|1 + \sum_{k=k_j+1}^{m}\frac{a_{jk}}{a_{jk_j}}p^{\beta_k-\beta_{k_j}}\right| \leqslant D_1 p^{\delta},$$

$$j = 0, \dots, l-1, \tag{26}$$

where $D_1 = 2\max\limits_{\substack{j=0,\dots,l\\k=0,\dots,m}}|a_{jk}|$. Let us denote

$$a_j(p) = \left(\sum_{k=0}^{m} a_{jk}p^{\beta_k}\right)\Big/\sum_{k=0}^{m} a_{l_k}p^{\beta_k}\Big), \quad b_{j+1}(p) = a_j\left(\frac{p}{\alpha_l}\right),$$

$$\xi_{j+1} = \alpha_j/\alpha_l, \quad 0 < \xi_1 < \dots < \xi_l < 1, \qquad j = 0, \dots, l-1, \tag{27}$$

then we obtain from (1) the bound

$$|f(p)| \leqslant \sum_{j=1}^{l}|b_j(p)f(\xi_j p)|, \tag{28}$$

and by virtue of (25) and (26) we have

$$|b_j(p)| \leqslant D_2 p^{\delta}. \tag{29}$$

Let $0 < p_0 < r$, let us denote $I_n = [\xi_1^{n+1}p_0, \xi_1^n p_0]$, $M_n = \max\limits_{p \in I_n}|f(p)|$, $n = 0, 1, \dots,$ and let us prove the inequality

$$M_n \geqslant D^n \exp\left\{-\frac{\mu}{2}\ln^2\frac{\alpha_{l-1}}{\alpha_l}n^2\right\}M_0, \tag{30}$$

where D is a positive number. For this purpose let us note that for any $0 < p < p_0$ and $n = 0, 1, \dots$ it is possible [by iteration of (28)] to estimate the value of the function f at the point p in terms of the value of this function in the interval I_n with the aid of the inequality

$$|f(p)| \leqslant \sum_{\substack{j_1,\dots,j_s=1 \\ \xi_{j_1}\dots\xi_{j_s}p\in I_n}}^{l} b_{j_1}(p)\, b_{j_2}(\xi_{j_1}p)\dots b_{j_s}(\xi_{j_1}\dots\xi_{j_{s-1}}p)\, f(\xi_{j_1}\dots\xi_{j_s}p)|, \tag{31}$$

where the sum is taken over all possible collections $j_i = 1, \dots, l$; $i = 1, \dots, s$ such that $\xi_{j_1}\dots\xi_{j_s}p \in I_n$. The subscript s in the right-hand side of (31) reaches its minimum value s_{min} for $\xi_{j_1} = \dots = \xi_{j_s} = \xi_1$, and its maximum value s_{max} for $\xi_{j_1} = \dots = \xi_{j_s} = \xi_l$. For $p \in I_n$ we hence obtain

$$s_{\min} = n - 1; \quad s_{\max} \leqslant (n+1)\frac{\ln \xi_1}{\ln \xi_l}. \tag{32}$$

The sum in (31) consists of not more than $l^{s_{max}}$ terms each of which has in accordance with (29) a bound

$$|b_{j_1}(p)\, b_{j_2}(\xi_{j_1}p)\dots b_{j_s}(\xi_{j_1}\dots\xi_{j_{s-1}}p)\, f(\xi_{j_1}\dots\xi_{j_s}p)| \leqslant D_2^s p^\delta (\xi_{j_1}p)^\delta \dots (\xi_{j_1}\dots\xi_{j_s}p)^\delta M_n \leqslant D_3^{s_{\max}} \xi_l^{\frac{\delta}{2}s_{\min}^2} M_n \tag{33}$$

with a positive D_3. From (31), (32), and (33) it follows that

$$M_0 \leqslant D_4^n \xi_l^{\frac{\delta}{2}n^2} M_n. \tag{34}$$

By replacing in (34) the quantities δ and ξ_l by the values from (24) and (27), we obtain the sought inequality (30).

Let us note that for $p \in I_n$ we have

$$(\ln p - \ln p_0)/\ln \xi_1 - 1 \leqslant n \leqslant (\ln p - \ln p_0)/\ln \xi_1. \tag{35}$$

Moreover, for any nontrivial solution f(p) of Eq. (1) we have $M_0 \neq 0$. Therefore, assertion 2 of Theorem 2 follows from (30), (35), and the relation $\xi_1 = 1/\alpha_l$.

By virtue of Part 1 of Theorem 2 we can prove the following theorem.

THEOREM 3. If condition (3) holds, then for any positive c and $\mu_3 < m/(2 \ln \lambda_l)$ Eq. (2) will have a nonfinite solution defined on the entire axis and having a bound

$$|y(x)| \leqslant C \exp\{-\mu_3 \ln^2(1+|x|)\}, \qquad -\infty < x < \infty. \tag{36}$$

In [5] we have proved Theorem 3 under the additional assumption of multiplicative commensurability of λ_j ([5, Theorem 1.4]). The method of proof used in the present paper makes it possible to get rid of this restriction.

Proof of Theorem 3. After having proved Theorem 2, we can complete the proof of Theorem 3 in the same way as the proof of [5, Theorem 1.4]. Therefore, we shall outline here only the scheme of the proof.

The sought nonfinite solution of Eq. (2) that satisfies (36) and the additional initial conditions $y^{(k)}(0) = 0$, $k = 0, \dots, m-1$ can be obtained with the aid of the Laplace transform. For this purpose the function

$$f(p) = \int_0^\infty y(x) e^{-px} dx \tag{37}$$

must satisfy the equation

$$p^m f(p) + \sum_{j=0}^{l} \sum_{k=0}^{m-1} c_{jk} p^k f\left(\frac{p}{\lambda_j}\right) = 0 \tag{38}$$

with constants c_{jk}. Equation (38) is an equation of the form (2), and in this case we have $\mu \geq m/\ln \lambda_l$. By virtue of Part 1 of Theorem 2, Eq. (38) has a nontrivial solution in a neighborhood of the origin that satisfies (5). It is easy to see that this solution can be analytically continued in the entire complex plane with a cut along the negative real axis in such a way that the bound (5) is retained in the cut neighborhood of the origin ($0 < |p| < r$; $-\pi < \arg p < \pi$). Moreover, the inequality

$$|f(p)| \leqslant C_4 \exp\{-\mu_4 \ln^2|p|\}, \quad |p| > r, \quad -\pi < \arg p < \pi, \tag{39}$$

is also satisfied; here C_4 and μ_4 are positive constants. It follows from (37), (5), and (39) that y(x) satisfies (36).

By an analysis similar to that used in the proof of Theorem 3 (based on Part 1 of Theorem 2), it is easy to see that the condition of multiplicative commensurability of the transformations of the argument can be dropped also in [5, Theorem 4.2].

The solution y(x) constructed in the proof of Theorem 3 satisfies not only (36), but also the estimate $|y(x)| \leqslant C \exp\{-\mu_s \ln^2|x|\}$, $-\infty < x < \infty$, with some $0 < \mu_5 \leq \mu_3$ (see [4, Theorem 3]).

Under the conditions of Theorem 1 there exist solutions of Eq. (1) that cannot be written in the form (4) (see [6]).

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SYLOW 2-SUBGROUPS OF THE COUNTABLE ALTERNATING GROUP

Yu. V. Dmitruk

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In this paper we describe the structure of the Sylow 2-subgroups of the countable alternating group, i.e., the group of all even permutations of a countable set which move only finitely many points. The representation of Sylow p-subgroups of finite symmetric groups as polynomial tableaux introduced in [3] plays an important role in our description; it generalizes in a natural way to the case of countable permutation groups.

Let V be a k-dimensional vector space over the field F_p with p elements; let S(V) and A(V) be the symmetric and alternating groups of degree p^k acting on the set V; let $e_1, e_2, \ldots, e_k$ be some fixed basis for V. Let V_i be the subspace of V spanned by the vectors $e_1, e_2, \ldots, e_i$, $i = 1, 2, \ldots, k$; $V_0 = \{0\} \subset V_1 \subset \ldots \subset V_k = V$ is the flag consisting of the subspaces V_i. The group of all permutations in S(V) preserving this flag (term-by-term) and inducing the identity permutation on each quotient V_{i+1}/V_i is a Sylow p-subgroup of the symmetric group $S(V) = S_{p^k}$. It was shown in [3] that this group (which we denote by P_{p^k} below) has a convenient representation by sets of truncated polynomials of the type

$$a = [a_1, a_2(x_1), \ldots, a_k(x_1, x_2, \ldots, x_{k-1})], \quad (1)$$

where the $a_i(x_1, x_2, \ldots, x_{i-1}) \in F_p[x_1, x_2, \ldots, x_{l-1}]$ are truncated polynomials, i.e., representatives of minimal degree of the coset class modulo the ideal $I = (x_1^p - x_1, x_2^p - x_2, \ldots, x_{i-1}^p - x_{i-1})$. The tableau (1) corresponds to the permutation in S(V) given by $t \to ta = (t_1 + a_1, t_2 + a_2(t_1), \ldots, t_k + a_k(t_1, \ldots, t_{k-1}))$, where $t = (t_1, t_2, \ldots, t_k)$ is a point in the space V. The product of two tableaux with coordinates $a_i(x_1, x_2, \ldots, x_{i-1})$ and $b_i(x_1, x_2, \ldots, x_{i-1})$, respectively, is the tableau with coordinates

$$a_i(x_1, x_2, \ldots, x_{i-1}) + b_i(x_1 + a_1, x_2 + a_2(x_1), \ldots, x_{i-1} + a_{i-1}(x_1, \ldots, x_{i-2})), \quad i = 1, 2, \ldots, k. \quad (2)$$

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