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An Ω-theorem ...: Addendum

By

Y.-F. S. Pétermann, Genève

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Abstract. It is proved in [1] that the error term associated with the summatory function of the divisor function $\sigma_{-1}(n)$ satisfies (1) $E_{-1}(x) = \Omega_{-}(\log\log x)$ with an implied constant $C_{-} \geqslant (e^{\gamma} - 1)/2$. In fact, the method of [1] yields the better (2) $C_{-} \geqslant e^{\gamma}/2$, and also (3) $E_{-1}(x) = \Omega_{+}(\log\log x)$ with an implied constant (4) $C_{+} \geqslant e^{\gamma}/2$.

The reader is refered to [1] for the notation. In that paper, the proof of (1) is derived from a general estimate of the sum $\Sigma_{n \leqslant x} E_{-1}(An - B)$, where A and B are positive integers satisfying $0 < B < A$ (Lemma 5). A special choice of the parameters A and x, considered as functions of the variable B, is made, and $B \to \infty$.

But a simpler application of Lemma 5 with $B = 1$ also yields (1), with (2). Moreover, I had the impression that the case $B = 0$ was not worth considering; a fallacious impression: indeed this case yields (3), with (4).

Lemma 5*. *If x is a positive integer and $\varepsilon = 0$ or 1, then as $x \to \infty$,*

$$\sum_{n \leqslant x} E_{-1}(An - (1 - (-1)^{\varepsilon})/2) = L_{\varepsilon} x + O((A + (x/A)^{1/2} \log Ax)$$

where

$$L_{\varepsilon} = L_{\varepsilon}(A) = (-1)^{\varepsilon} \frac{\sigma^*(A)\, C(A)}{2A} + C_{\varepsilon}$$

and C_{ε} is a real constant depending only on ε.

The case $\varepsilon = 1$ is Lemma 5 with $B = 1$. For $\varepsilon = 0$ the proof differs only trivially (in the proof of Lemma 3) from that of Lemma 5.

The special choices $A = \prod_{p \leqslant y} p^{\alpha}$, where α is an integral function of y diverging to $+\infty$, and $x = A^2$, yield

$$L_\varepsilon = (-1)^\varepsilon \frac{\sigma(A)}{2A}(1 + o(1)) \quad (y \to \infty),$$

whence, by [1, (46)], if $\log \alpha = o(\log y)$,

$$\frac{1}{x}\sum_{n \leqslant x} E_{-1}(An - (1 - (-1)^\varepsilon)/2)) \sim (-1)^\varepsilon \frac{e^\gamma}{2} \log\log x \quad (y \to \infty) .$$

Reference

[1] An Ω-theorem for an error term related to the sum-of-divisors function. Mh. Math. **103**, 145—157 (1987).

Y.-F. S. PÉTERMANN
Section de Mathématiques
2—4 rue du Lièvre, CP 240
CH-1211 Genève 24, Suisse