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# The Datum Flow Chain: A Systematic Approach to Assembly Design and Modeling

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**Abstract.** *Current CAD systems are 'part-centric' and do not capture the underlying logic of an assembly at an abstract level. We need to make CAD systems 'assembly-centric'. To be able to lay out, analyze, outsource, assemble and debug complex assemblies, we need ways to capture their fundamental structure in a top-down design process, including the designer's strategy for constraining the parts kinematically and locating them accurately with respect to each other. We describe a concept called the 'Datum Flow Chain' to capture this logic. Most assembly problems occur due to ineffective datum logic or the choice of assembly procedures that are not consistent with the datum logic, if any, that was used to design the parts. The DFC relates the datum logic explicitly to the product's key characteristics, assembly sequences, and choice of mating features, and provides the information needed for tolerance analyses. Two types of assemblies are addressed: Type-1, where the assembly process puts parts together at their pre-fabricated mating features, and Type-2, where the assembly process can incorporate in-process adjustments to redistribute variation. Two types of assembly joints are defined: mates that pass dimensional constraint from part to part, and contacts that merely provide support. The scope of DFC in assembly planning is presented using several examples. Analysis tools to evaluate different DFCs and select the ones of interest are also presented.*

**Keywords:** Assembly; Datum flow chain; Fixtureless assembly; Kinematic constraint; Tolerances

## 1. Introduction

Assembly is the point in a product's life cycle where parts from disparate sources come together and the product first comes to life. Fit-up problems are often discovered during final assembly when trying to put these parts and sub-assemblies together. Finding the source of these fit-up problems is a very difficult and time-consuming task, and most of the time the exact causes cannot be identified. The time and cost involved to make engineering changes, in-process adjustments, etc., to fix these problems increase rapidly as the product development process evolves. Early anticipation and avoidance of these problems can have a huge impact in reducing the product development time, cost, and production fit-up problems, and can improve final product quality.

Many fit-up problems occur due to a part-centric approach to product design that ignores assembly and system issues. The assembly process should be viewed as a proxy for a wide range of decisions, events, and relationships between different stages of the product development process. It is common to view assembly as a process that merely fastens parts together. Assembly is really the chaining together of dimensional relationships and constraints. The success of these chains determines the success of the product's quality from an assembly point of view. The goal of assembly modeling is to permit these chains to be defined first, and followed by design of individual parts. We propose a concept called the Datum Flow Chain (DFC) to implement this approach to assembly modeling and design of assemblies.

Current CAD systems provide rudimentary assembly modeling capabilities once part geometry exists, but these capabilities basically simulate an assembly drawing. Most often, the dimensional relations that are explicitly defined to build an assembly model in CAD are those most convenient to construct the CAD model and are not necessarily the ones that need to be controlled for proper functioning of the assembly. What is missing is a way to represent and display the designer's strategy for

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locating the parts with respect to each other, which amounts to the underlying structure of dimensional references. The DFC is intended to capture this logic.

Ideally, the design of a complex assembly starts with a general description of the top level design requirements (key characteristics[1], KCs [1]) for the whole assembly. These requirements are then systematically formalized and flowed down to subassemblies and finally down to individual parts. During these early stages of design, the following major elements of the design process have to be considered:

1. Systematically relate the identified KCs to important datums on assemblies, parts, and fixtures at the various assembly levels.
2. Design consistent dimensional and tolerance relationships (locating schemes) among elements of an assembly so as to deliver these KC relationships.
3. Identify assembly procedures that best deliver the KCs repeatedly without driving the costs too high.

Current CAD systems do not support this approach to designing assemblies and instead encourage premature definition of part geometry, allowing designers to skip the consideration of the above three elements. The proper consideration of these elements at an early stage of the design can avoid potential problems during final assembly.

This paper is an extension of the work introduced by the authors [2], and presents techniques that formalize this approach to designing assemblies. The goal is to provide a common mathematical basis for establishing three basic kinds of information about an assembly:

- 'Location responsibility': which parts or fixtures locate which other parts
- Constraint: which degrees of freedom of a part are constrained by which features on which other parts, including checking for inappropriate over- or under-constraint
- Variation: how much uncertainty there is in the location of each of the parts relative to some base part or fixture which represents the reference dimension

as well as to enable this information to be established, explored, and verified early in design, perhaps even before detailed geometric models of the parts are available.

Section 2 provides background on some of the terms and concepts used in the paper. The concept of the DFC and its properties and relationship to KCs are described in Section 3. Most assembly modeling techniques in the past have tried to apply a single technique to model all assemblies and have not tried to make the distinction between some fundamentally different types of assemblies. As a result, there are some assembly problems that cannot be explained by existing modeling techniques. We have identified two types of assemblies that require distinct modeling methods. These types of assemblies are described in Section 4. The DFC provides for a structured method of planning out the assembly procedures which is described in Section 5. Section 6 presents analysis techniques to choose between alternate DFCs. Section 7 summarizes the modeling approach and presents directions for future research. Details concerning the topics in this paper may be found in Mantripragada [23].

## 2. Background and Prior Work

Assemblies have been modeled systematically by Lee and Gossard [3], Sodhi et al. [4], Srikanth and Turner [5] and and Roy et al. [6], among others. Such methods are intended to capture relative part location and function, and enable linkage of design to functional analysis methods like kinematics, dynamics, and in some cases tolerances. Almost all of them need detailed descriptions of parts to start with, in order to apply their techniques. Gui and Mantyla [7] have attempted to apply a function-oriented structure modeling to visualize assemblies and represent them in varying levels of detail. Other researchers have studied methods to generate assembly sequences [8–10] for the assembly starting from the descriptions of the parts and their mating features constituting the assembly. While some have tried to generate all possible sequences, others have tried to identify a single optimal sequence subject to certain constraints. There has not been much effort to identify assembly sequences at a concept stage where the geometry of the parts is not certain yet and use them to influence the design of the assembly.

Top-down design emphasizes the shift in focus from managing individual part design to managing the design in terms of mechanical 'interfaces'

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[1] A key characteristic is property of a product which is required for function, safety, or other important need, and which is at risk for not being achieved due to variation. In this paper, KCs are expressed as geometric dimensions, but in general they could be any measurable item such as a voltage, a length of time, etc.

between parts. Smith [11] proposes eliminating or at least minimizing critical interfaces in the structural assembly rather than part-count reduction as a means of reducing costs. The process of interface control is also known as dimensional management. He emphasizes that at every location in the assembly structure, there should only be one controlling element that defines location, and everything else should be designed to 'drape to fit'. This is similar to the idea of mates and contacts introduced by Mantripragada et al. [2]. Muske [12] describes the application of dimensional management techniques on 747 fuselage sections. He describes a top-down design methodology to systematically translate key characteristics to critical features on parts and choose consistent assembly and fabrication methods. No computer or conceptual tools to support these processes are described.

Shah et al. [13] proposed an attributed graph model to interactively allocate tolerances, perform tolerance analysis and validate dimensioning and tolerancing scheme at the part level. This model defines chains of dimensional relationships between different features on a part and can be used to detect over and under dimensioning of parts. Wang and Ozsoy [24] provided a method for automatically generating tolerance chains based on assembly features. Shalon et al. [25] showed how to analyze complex assemblies, including detecting inconsistent tolerancing datums, by adding coordinate frames to assembly features and propagating the tolerances by means of $4 \times 4$ matrices. Zhang and Porchet [26] present the Oriented Functional Relationship Graph which is similar to the DFC, including the idea of a root node, propagation of location, checking of constraints, and propagation of tolerances. The DFC is an extension of these ideas, emphasizing the idea of designing assemblies by designing the DFC first, defining the interfaces between parts at an abstract level, and then providing detailed geometry. A number of new concepts are introduced here.

## 3. Datum Flow Chain

### 3.1. The Concept

An assembly is characterized by a set of Key Characteristics (KC) that it has to deliver upon final assembly. These are assembly level dimensions relating a datum or feature on one part to that on another part in the assembly. An example KC is the size and straightness of the gap between a car hood and fender. Typically, such KCs are achieved (or delivered) when several different parts are made and assembled correctly. Correct assembly means that each part is located (or constrained) kinematically in at most six degrees of freedom (dof) at the correct spatial location measured in each of these dof. A typical part in the assembly has multiple joints with other parts in the assembly. Not all of these joints transfer locational and dimensional constraint, and it is essential to distinguish the ones that do from the ones that are redundant location-wise and merely provide support or strength. We define the joints that establish constraint and dimensional relationships between parts as *mates*, while joints that merely support and fasten the part once it is located are called *contacts*. Hence, mates are directly associated with the KCs for the assembly because they define the resulting assembly spatial relationships and dimensions. The process of assembly is not just of fastening parts together but should be thought of as a process that first defines the location of parts using the mates and then fastens the parts together once their location has been defined. The mates are fastened first and only then can the contacts be fastened.

Explicit identification and definition of the mates in the assembly is an integral part of assembly design and is a pre-requisite to assembly process planning and variation stackup analysis. The choice of which joints will be *mates* and which ones will be *contacts* is made by the designer at the conceptual design stage. Joints directly involved with the delivery of KCs are declared as mates. If these distinctions can be expressed carefully and mathematically, then we can construct directed graph representations for dimensional transfer from mate to mate in a declarative way, providing a basis for synthesizing constraint, location, and tolerance achievement. We call this directed graph of mates the Datum Flow Chain (DFC) [2]. It assigns a hierarchy to the joints between parts by defining which part(s) or fixture locates which other part(s) in the assembly. As explained in Section 4, in some assemblies, *mates* are accomplished wholly or partly by supporting fixtures that have to be included in the DFC.

Assembly design involves designing the datum flow chain explicitly to determine the location strategy before performing any kind of analysis. DFCs express the designer's logical intent concerning how parts are to be related to each other geometrically to deliver the KCs repeatedly. When defining the DFC, the designer must define explicitly the surfaces or reference axes on mating features which are intended to carry dimensional constraint to the mating part. This approach makes it unnecessary, even counter-productive, to construct algorithms that 'identify' tolerance chains or loops, since the DFC equips the designer to define them purposefully as a main objective of assembly design.

Standard methods to define such relationships inside a part exist today (ANSI Y14.5M standards), but no such standards exist for creating DFCs at an assembly level as described here. In a manner similar to how the dimensioning scheme within a part determines the procedures that can be employed to fabricate the part, the DFC severely constrains the permissible assembly procedures that can be followed to build the assembly.

### 3.2. Assumptions

The following assumptions are made to model the assembly process using a DFC:

1. All parts in the assembly are assumed rigid. Hence, each part is completely located once its position and orientation in the three dimensional space are determined.
2. Each assembly operation completely locates the part being assembled with respect to existing parts in the assembly or an assembly fixture. Only after the part is completely located is it fastened to the remaining parts in the assembly.

Assumption 1 states that each part is considered to be fully constrained once three translations and three rotations are established. We thus rule out assemblies that contain locked-in stress that is achieved by over-constraining the parts. In fact, allied research [22] describes a method based on screw theory for examining an assembly containing specific features to see whether each part is fully, over- or under-constrained. Assumption 2 is included in order to rationalize the assembly process and to make incomplete DFCs make sense. An incomplete DFC represents a partially completed assembly. If the parts in a partially completed assembly are not completely located, by each other or by fixtures, it is not reasonable to expect that they will be in position for receipt of subsequent parts, in-process measurements, transport, or other actions that may require an incomplete assembly to be dimensionally coherent and robust. This assumption enables us to critique alternate assembly sequences, as explained in Section 5.4.

### 3.3. Properties of a DFC

A datum flow chain is a directed acyclic (a graph with no cycles) graphical representation of an assembly with nodes representing the parts and arcs representing *mates* between them [2]. Every node represents a part or a fixture and every arc transfers dimensional constraint along one or more dof from the node at the tail to that at the head. Loops or cycles in a DFC would mean that a part locates itself once the entire cycle is traversed, and hence are not permitted. Every arc constrains certain degrees of freedom depending upon the type of mating conditions it represents. The sum of the degrees of freedom constrained by all the incoming arcs to a node (called the in-degree) in a DFC should be equal to six unless there are some kinematic properties in the assembly or designed mating conditions such as slip joints which can accommodate some amount of pre-determined motion. Each arc has an associated $4 \times 4$ transformation matrix that represents mathematically how the part at the head of the arc is located with respect to the part at the tail of the arc. A typical DFC has only one root node that has no arcs directed towards it, which represents the part from which the assembly process begins. This could either be the base part or a fixture.

Consider the aircraft horizontal stabilizer skin assembly shown in Fig. 1. It consists of four main parts: Plus-chord, Aft-skin, Fwd-skin and 11 stringers. Stringer 3 is called the splice stringer because

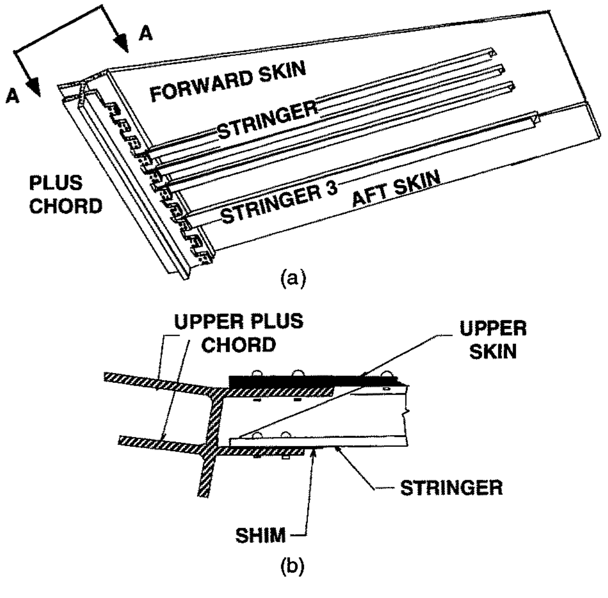

**Fig. 1.** Example assembly (Airplane horizontal stabilizer upper skin assembly). (a) The assembly. (b) Section A-A.

it splices the forward and aft skins to each other. A traditional representation of the assembly using a liaison diagram is shown in Fig. 2(a). A Liaison diagram is an undirected graph where the nodes represent parts and the arcs represent liaisons (contacts or mates) between them [8]. A candidate DFC for the assembly is shown in Fig. 2(b).

The DFC in Fig. 2(b) states that the location of Stringers 1–2 and splice stringer-3 is determined completely by mating features on the Aft skin. The splice stringer locates the Forward skin relative to the Aft skin. The Aft and Forward skin together locate the Plus-chord. Mating features on the Forward skin locate stringers 4–11. Liaisons 2, 4, 6, 7, 8, and 9 are thus mates while liaisons 1, 3, and 5 are contacts. The features used to assemble the stringers to the plus chord (liaisons 1, 3, and 5) should allow for absorption of part variations and avoid forming an over constrained assembly. In case the chosen features at the plus chord-stringer interface are holes, they should be over sized. A suitable set of mating features for this assembly is described in Section 5 and a complete discussion of the process is in Cunningham et al. [14].

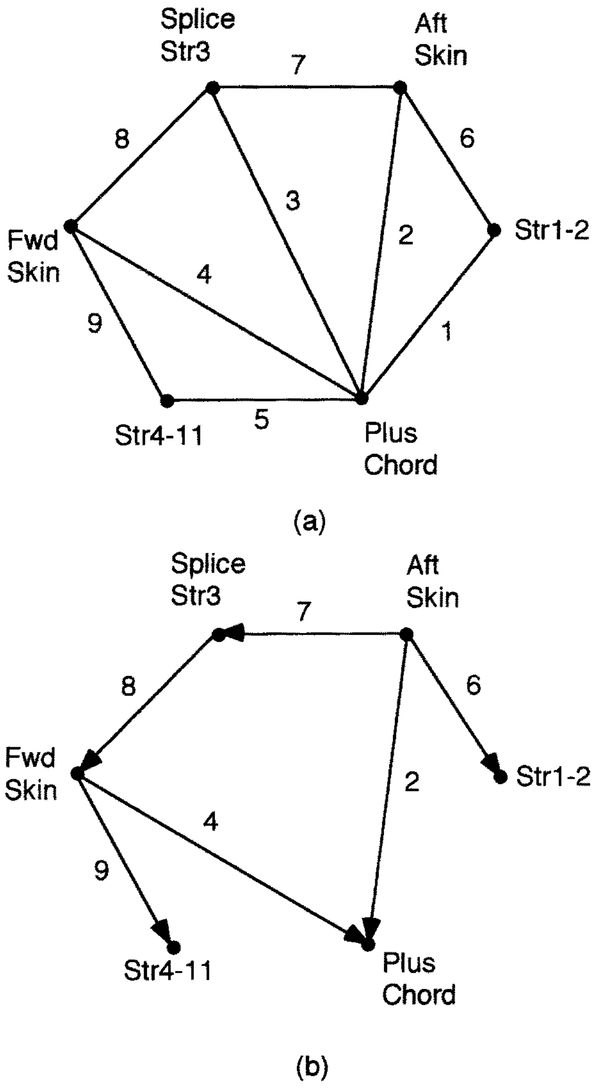

**Fig. 2.** (a) Liaison diagram, and (b) Datum flow chain for the assembly in Fig. 1.

A suggestive analogy between DFCs and electric circuits is given in Appendix 1.

## 4. Types of Assemblies

Most models of assemblies represent the assembly as complete, i.e. with all its parts in place and all mates and contacts fastened. Therefore, these models are not capable of addressing issues that occur during the act of assembling. Assembly planning involves considering a series of successively more complete assemblies. Incomplete assemblies may have unconstrained degrees of freedom that will be constrained when the assembly is complete. They may be subject to shape or size variations that the final assembly will not be subject to. Yet these uncontrolled degrees of freedom or variations may cause the next assembly step to fail or may result in a mishapen final assembly, and thus have to be considered during design. In order to manage these issues systematically, we distinguish two types of assemblies.

### 4.1. Type-1 Assemblies

Type-1 comprises typical machined or molded parts that have their mating features fully defined by their respective fabrication processes prior to final assembly. We also call these *part-defined* assemblies, because the variation in the final assembly is determined completely by the variation contributed by each part in the assembly, assuming that all the 'rules' of assembly (correct bolt torque, cleanliness, etc.) are followed. The assembly process merely puts the parts together by joining their predefined mating features. The assembly process is thus passive and cannot influence the distribution of variation in the assembly. The mating features are almost always defined by the desired function of the assembly, and the designer of the assembly process has little or no freedom in selecting mating features.

Defined in terms of the DFC, a type-1 assembly is one where every part has at least one mate with at least one other part in the assembly. Fixtures, if present, merely immobilize the base subassembly and present it to the part being assembled in the desired position and orientation.

### 4.2. Type-2 Assemblies

The second type of assembly includes aircraft and automotive body parts that are usually given some or all of their assembly features or relative locations during the assembly process. Assembling these parts requires placing them in proximity and then drilling holes or bending regions of parts, as well as riveting or welding. The locating scheme for these parts must include careful consideration of the assembly process itself since function by no means is a sufficient guide. Final assembly quality depends crucially on achieving desired final relative locations of the parts, something that is by no means assured because at least some of the parts lack definite mating features that tie them together unambiguously. A different datum flow logic, assembly sequence, etc. will result in quite different assembly configurations, errors and quality. It is possible to build a perfect assembly out of imperfect parts and vice versa by choosing an appropriate or inappropriate datum flow chain logic.

In type-2 assemblies, some mating features can be chosen specifically to meet assembly requirements. Features can be chosen to selectively propagate variation along certain directions and absorb in other directions. This can be illustrated in Fig. 3 by simple 1D slip plane type features.[2]

The slip plane in Fig. 3(a) will absorb variation along the $x$ direction and transmit it along $y$ direction while those in Figs 3(b) and (c) will absorb variation in the $y$ direction and transmit it along the $x$ direction. Hence, in type-2 assemblies, assembly design involves design of joints that transfer constraint and those that do not.

Defined in terms of the DFC, a type-2 assembly is one where it is possible to have only *contacts* between all parts in the assembly. In such cases, the parts will have *mates* with fixtures used to locate them. In-process measurements can be made to adjust the location of these parts during assembly to tune out the effects of variation caused by the manufacturing processes used to fabricate the parts, as described by Frey et al. [27]. Alternatively, rigid fixtures can be relied on to establish the dimensional relations between the parts, passively accomplishing the measurement and adjustment steps. Hence we call these *assembly-defined* assemblies, as the assembly process can redistribute the variation. Typically, a type-2 assembly will have a mixture of mates and contacts, making in-process adjustments or absorption possible only at certain locations and not at others. In the extreme that there are mates between every part in the assembly, type-2 reduces to type-1.

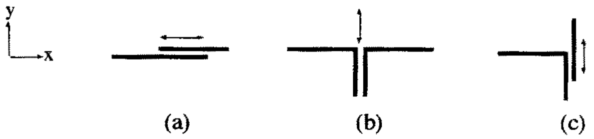

**Fig. 3.** Some simple assembly features illustrating selective absorption of variation along the arrows.

## 5. Assembly Design and Planning using DFC

Most assembly planning systems developed in the past have treated assembly planning as an activity separate from product design. Assembly plans are developed after all the individual parts are designed. Often a problem with this approach is that the choice of assembly method is not consistent with the design of the product because assembly considerations were not made during product design. This leads to fit-up problems during assembly that are hard to diagnose. The DFC allows for a top-down approach to designing assemblies. This approach starts with carefully identifying the assembly requirements from the top level customer requirements down to the fabrication of individual parts using a method called Key Characteristics (KC). These resulting specifications are then used to define candidate Datum Flow Chains (DFC) for the assembly. Next, the mating and assembly features that carry the datums and establish the relationships imposed by the DFC are designed. Different procedures are employed for type-1 and type-2 assemblies. For the former, the selection of mating features is usually determined by considering function and may not be determined by the assembly process. The assembly process can however affect the tolerancing and dimensioning of these features and the fixturing options during assembly. For the latter, feature type and location selection are a crucial part of the assembly process design. The DFC is then used to classify assembly sequences based on the order of establishment of these dimensional

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[2] Slip planes are well-known and widely used in the automobile industry.

references and identify assembly sequences that will deliver KCs consistently. The following sections describe this approach in detail.

### 5.1. Key Characteristics and DFC Design

Key Characteristics are a product's geometric features and material properties that are highly constrained or for which minute deviations from nominal specifications (regardless of manufacturing capability) have a significant impact on the product's performance, function, and form at each product assembly level [1]. KCs come directly from the customer requirements and are flowed down in a systematic manner from assembly to subassembly to part level. Identification of all the KCs and their flowdown for the assembly is a pre-requisite to DFC design. The decision of which liaisons are to be mates and which ones are to be contacts is a conscious decision made by the designer based on the KC requirements for the assembly. Joints directly associated with the delivery of KCs should be designated as mates and tightly toleranced during the design and monitored during assembly process. The choice of assembly procedures and fixturing methods also contribute to KC delivery.

### 5.2. Mating Feature Design: Designing Locating Schemes

In type-1 assemblies, the datum flow and mating features between parts are determined by considering almost exclusively the desired function of the assembly. Hence, the designer may not have enough freedom to design the mating features specifically to suit assembly needs. However, in the case of type-2 assemblies, the designer has a lot more freedom to design these assembly mating features to locate parts with respect to each other or a fixture. In these assemblies, the choice of assembly features and DFC design are tightly coupled. Designing a locating scheme for these assemblies involves first determining at a very high level what part(s) locates what other part(s) in the assembly and then iteratively designing the assembly features that will accomplish the location. Since each arc in a DFC is a mate, an appropriate feature has to be chosen to accomplish the mate. This feature may need to constrain some dof to perform its function as a mate and absorb uncertainties in other directions normal to the constrained ones, such as thermal expansion, etc. This is illustrated by the following example.

Figure 4(b) shows one possible mating feature set implementation of the DFC in Fig. 4(a). The numbers on the arcs in the DFC indicate the number of dof determined by the mate. Mates are solid lines in the DFC while contacts are dashed lines. These features are formed during fabrication and control the location of functional but non-mating features such as edges. The location requirements for these edges in turn are driven by various KCs as described in the next subsection. The holes provide full planar location. The slots provide planar location perpendicular to their long axes and accommodate variation caused by thermal expansion and shot peen growth along these axes. The mating features are joined with temporary rivets until permanent ones are installed. All slots are drilled out to become full size holes for permanent fastening.

The assembly strategy in Fig. 4 is emerging in the aircraft industry and is variously called precision assembly, determinate assembly, and hole-to-hole assembly [28]. It is an effort to convert typical fixture-based or Type-2 methods of aircraft fuselage assembly into Type-1. Figures 7(b), 10 and 11 below use DFC vocabulary to describe the currently-used Type-2 process for these parts.

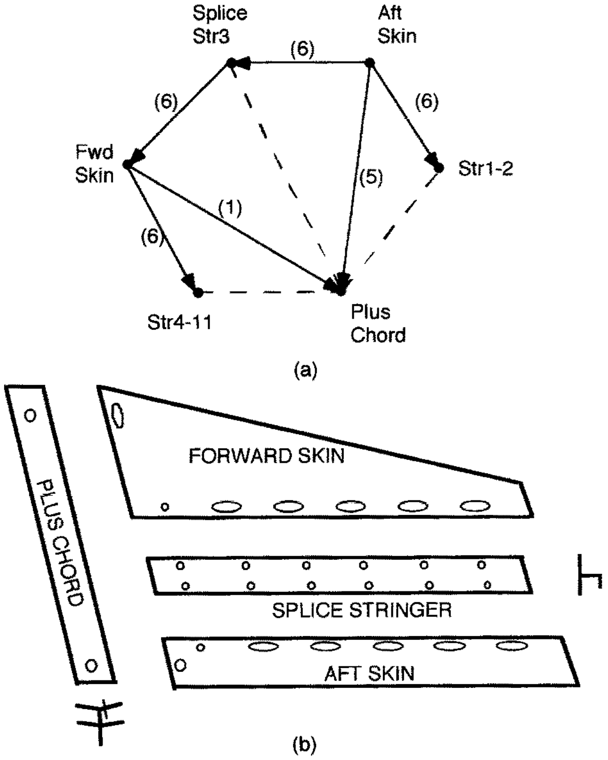

**Fig. 4.** (a) A DFC for the assembly in Fig. 1, (b) possible feature set implementation to carry the datum logic defined by the DFC in (a). Dashed lines are contacts.

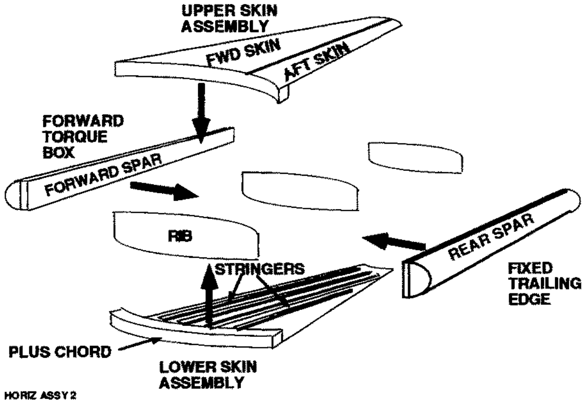

**Fig. 5.** Horizontal stabilizer assembly.

The DFC provides guidance on how to dimension and tolerance individual parts based on their presence in the DFC. The portion of the DFC that passes through different mating features within a part determines the critical chain within the part that needs to be controlled tightly during design and fabrication. The tolerancing of this chain internal to the part will determine the error contributed by the part to the final assembly KC error distribution.

### 5.3. Decomposition and Subassembly Design

The choice of which liaisons are to be mates and which ones are to be contacts determines the decomposition of the assembly and the permissible set of subassemblies. This choice is usually driven by the KCs for the assembly but can sometimes also be driven by assembly limitations.

This situation is illustrated here using the horizontal stabilizer shown in Figs 5 and 6. The horizontal stabilizer is affected by load certification and aerodynamic requirements and these two issues produce three KCs of importance:

- Inboard joint strength requires the Plus Chord, Forward Torque Box, and Fixed Trailing Edge be accurately aligned (nominal ± 0.005in), which flows down to:
  **KC #1**: plus chord alignment to spar end fittings.
- The skin gaps must be accurate and consistent (nominal ± 0.030in), which flows down to:
  **KC #2**: gaps between the skins on the upper skin assembly and those on the Forward Torque Box and Fixed Trailing Edge, and
- **KC #3**: gap between the Forward and Aft skins of the upper skin assembly.

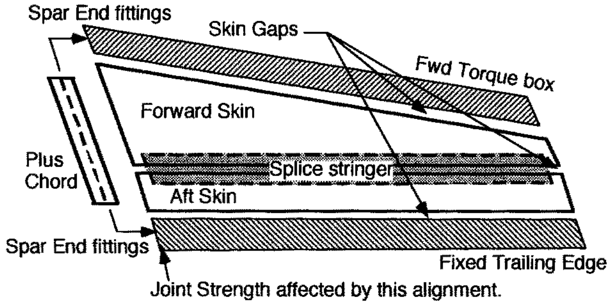

**Fig. 6.** KCs for the assembly shown in Fig. 5.

A desired DFC to repeatedly deliver these KCs independently is shown in Fig. 7(a). Independent delivery is achieved because each KC is associated with its own six dof arcs in the DFC. The plus-chord-skin and plus-chord-stringer joints are labelled as contacts as they are not directly associated with the delivery of any KCs. The assembly sequence resulting from this DFC was physically not realizable. Once the plus chord was assembled to the fixed trailing edge and forward torque box, there was no access left to assemble the skin-stringer assembly to the plus-chord. Hence, a modified DFC shown in Fig. 7(b) had to be designed to successfully assemble the parts. To achieve the KCs, the joints between the plus-chord and the skins are now defined as mates and controlled carefully during design and assembly. This resulted in a different decomposition of the assembly. Plus-chord is now part of the upper-skin assembly that is assembled as one unit to the fixed trailing edge and forward torque box. The two KCs are no longer independent in this decomposition but are tightly coupled so that delivery of both cannot be guaranteed. The new DFC assigns a higher priority to the strength KC, as indicated by the fact that it is associated with its own 6 dof arcs in the DFC. The quality of the skin-gap KC is determined by the mates between the plus chord and end fittings and the mates between the plus chord and the skins, as indicated by the fact that it is associated with contacts.

If we tried to associate both KCs with arcs in the DFC, we would have to convert some contacts to mates. This would result in some parts being over-constrained. Such a design would merely contain locked-in stress but it would not likely succeed in delivering both KCs to equal tolerances.

### 5.4. Assembly Sequence Planning

Typically, for any assembly there can be a large number of feasible assembly sequences. De Fazio and Whitney [8], Homem De Mello [15] and Bour-

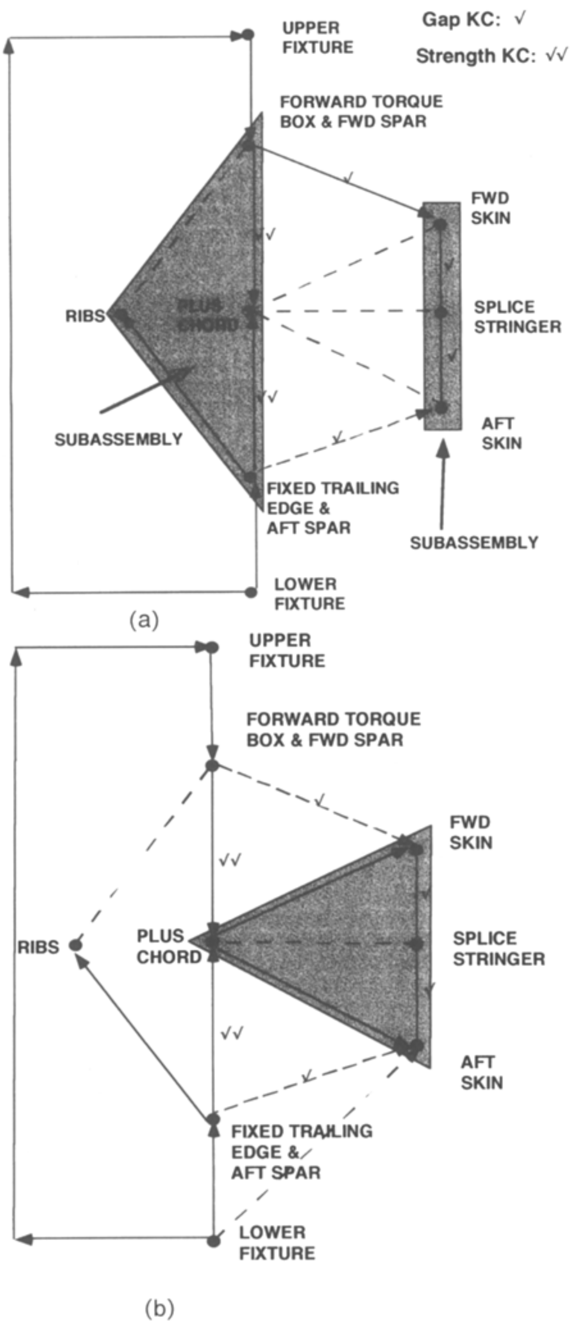

**Fig. 7.** (a) Desired DFC for delivering the KCs for the assembly, (b) DFC that was actually implemented due to assembly constraints. Grey areas are subassemblies, dashed lines are contacts.

jault [16] have developed algorithms to generate assembly precedence relations based on the geometric reasoning and interference analysis. These precedence constraints were then used to generate a complete set of feasible assembly sequences. The complete set of assembly sequences is represented by an assembly sequence graph [8]. It is not clear from this assembly sequence graph which sequences will deliver all the KCs repeatedly, and evaluation of the complete set of assembly sequences is a time consuming task. We describe an approach to assist pruning the assembly sequence graph into a smaller manageable set of assembly sequences using the DFC. We call this set a *family* of assembly sequences.

### *5.4.1 Assembly Precedence Constraints*

Traditionally, methods employed to generate all possible assembly sequences for a given assembly have treated all liaisons to be of the same type and have not made the distinction between *mates* and *contacts* as pointed out in Section 3. Generating all possible assembly sequences is done by representing all the geometric and mechanical assembly constraints as assembly precedence constraints. For example, the cut-set method [17] generates all possible part combinations and tests the connectivity of the subgraph formed by a combination of parts or nodes in the liaison diagram. All connected subgraphs are possible subassemblies in this approach. We argue that not all possible subassemblies are desirable and emphasize the need to only consider the assembly sequences that generate the desirable ones. We define desirability of subassemblies in terms of the ability of assembly process to deliver a dimensional tolerance on KCs. The following describes the procedure to identify desirable subassembly states using the DFC.

The design of a DFC involves the conscious decision of designing mates and contacts. As mentioned earlier, contacts do not define any dimensional relationships between parts and have to be established only after the mates that define the dimensional relationships are made. Using this argument, the following rule is imposed by the DFC:

**Contact rule**: Only connected subgraphs in a DFC can form permissible subassemblies

Subassemblies with only ‘contacts’ between any two parts are not permitted because contacts do not contribute to a KC. This rule will thus generate additional assembly precedence constraints that eliminate subassemblies whose parts do not establish part of a DFC.

If the location of a part is defined by more than one part in the assembly, all the defining parts should be present in the subassembly before the part can be assembled. This argument is captured in the following rule:

**Constraint rule**: Subassemblies with incompletely located (under-constrained) parts are not permitted.

The constraint rule imposes the condition that the in-degree of all but one of the nodes in a subassembly must add up to six. The one exceptional node could represent either a base part or a fixture, and has an in-degree equal to zero. This rule ensures that every subassembly has fully located parts.

Both these rules are translated by a computer program into assembly precedence constraints connecting liaisons with ordering operators. The precedence constraints take the following form, similar to the approach followed by De Fazio and Whitney [8]:

$$(i \,\&\, j) >= (k \,\&\, l)$$

The operator '>=' means 'must precede or concur with'. The above constraint is read as: liaison $i$ and $j$ must be completed before or concurrently with, completion of (both) liaisons $k$ and $l$ (but not necessarily before or concurrently with either liaison $k$ or $l$).

The following algorithm generates these extra constraints. The liaison diagram and the DFC for the assembly are represented using their incidence matrices. In these matrices, rows represent the nodes in the graphs and columns indicate the liaisons (mates in the case of the DFC). Figure 8 shows these matrices for the assembly in Fig. 2.

A computer program reads these matrices as inputs and applies the Contact and Constraint rules as follows:

- Contact rule: to eliminate the possibility of subassemblies with only contacts between parts, the incidence matrices for the liaison diagram and the DFC are compared to determine which liaisons are contacts. Then, for each contact, a precedence relation is generated stating that all mates in the DFC pointing to the parts the contact connects must be completed before the contact can be completed. For example, in Fig. 2, liaison 3 joining Plus-chord and splice stringer-3 is a contact. Incoming mates to Plus-chord and splice stringer-3 include liaisons 2 and 4. Thus, liaisons 2 and 4 must be completed prior to or simultaneously with liaison 3 (2 & 4 >= 3). This type of precedence relation will ensure that subassemblies with only contacts between parts will not be allowed. Subassemblies involving only plus-chord and stringers are not permitted by this rule, as there are no designed mating features between these parts.
- Constraint rule: to ensure that subassemblies with incompletely constrained parts are not allowed, each row in the DFC matrix is examined one at a time. If a part (row) has more than one incoming mate (element with value '−1'), then all incoming mates must be simultaneously completed to ensure that the part be fully constrained when assembled. For example, looking at the first row of the DFC matrix in Fig. 8, the Plus-chord has two incoming mates, liaisons 2 and 4. Thus, liaisons 2 and 4 must be completed simultaneously (2 >=4 and 4 >= 2). The Constraint rule prevents subassemblies such as (Fwd-skin, Plus-chord, Aft-skin) subassembly since it has incompletely constrained parts.

| | Liaison Diagram | | | | | | | | | Datum Flow Chain | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Nodes/ liaisons | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 2 | 4 | 6 | 7 | 8 | 9 |
| Plus-chord | 1 | 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 | -1 | -1 | 0 | 0 | 0 | 0 |
| Str 1-2 | 1 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | -1 | 0 | 0 | 0 |
| Aft-Skin | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 1 | 0 | 1 | 1 | 0 | 0 |
| Splice-Str_3 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 0 | 0 | 0 | -1 | 1 | 0 |
| Fwd-Skin | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 1 | 0 | 1 | 0 | 0 | -1 | 1 |
| Str 4-11 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | -1 |

**Fig. 8.** Incidence matrices for the liaison diagram and DFC shown in Fig. 2. In the liaison diagram matrix, a 1 in a row means that a liaison connects to the part in that row. Similarly, in the DFC matrix, a 1 indicates that an arc leaves that part, while a -1 indicates that an arc comes into that part.

There are thus two sets of assembly precedence constraints: Geometric precedence constraints, and precedence constraints generated by the DFC. For a given assembly, each candidate DFC design will generate a different set of precedence constraints. But the geometric precedence constraints remain the same for a given assembly unless there are major changes in mating features between parts. For the DFC shown in Fig. 2, the precedence constraints imposed by the *Contact* and *Constraint* rules are shown in Fig. 9. Note that the first three precedence constraints come from the *Contact* rule and the last two from the *Constraint* rule. The number of operations computed by this algorithm depends on the number of liaisons in the assembly and so the complexity of the algorithm scales accordingly.

The reduction in the number of assembly sequences can be very great, as indicated in Fig. 9, for Type-2 assemblies because they are likely to have numerous contacts. Less reduction may be expected from Type-1 assembies. However, one can easily impose constraints on Type-1's mandating that the assembly build the DFC from the root out for quality assurance purposes, for example, effectively eliminating most alternative sequences.

### *5.4.2 Family of Assembly Sequences*

The precedence constraints imposed by the DFC (by way of the contact and constraint rules) are applied in addition to the ones generated based on the geometric reasoning and interference analysis to

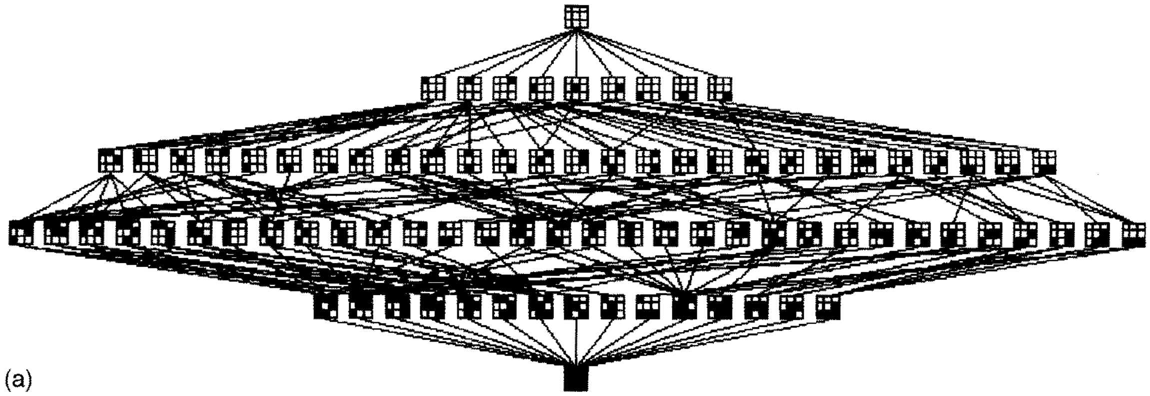

**Fig. 9.** (a) Complete set of 312 assembly sequences, (b) resulting family of 28 sequences after applying constraints imposed by the DFC.

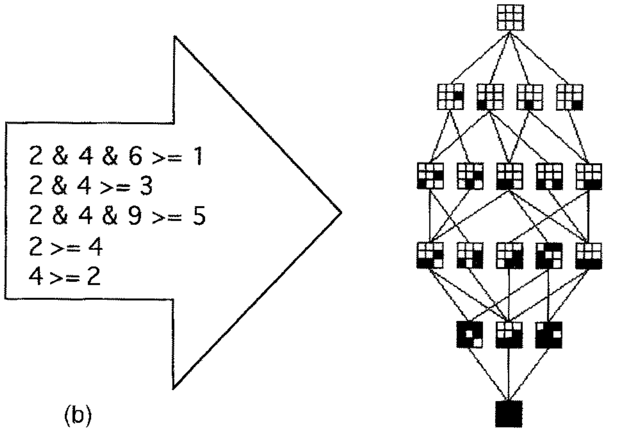

**Fig. 9.** Continued.

generate a reduced set of assembly sequences. We call this set a DFC-family of assembly sequences for the given DFC. Assembly sequences in a family share some common properties since they satisfy the same locating scheme defined by the DFC. These properties, described in detail in Section 6, are slightly different for type-1 and type-2 assemblies. A typical assembly has a large number of assembly sequences and it is not practical to evaluate every possible assembly sequence. The DFC helps reduce the search space by creating families of assembly sequences that share common properties. As will be shown in the following sections, in type-1 assemblies a family forms an equivalence class of assembly sequences that will all have the same probability of delivering the KCs. In such cases only one sequence from the entire family need be evaluated. In type-2 assemblies, however, sequences within a family also need to be evaluated.

The assembly shown in Fig. 1 has 312 feasible assembly sequences, as shown in the assembly sequence graph in Fig. 9(a). Every box in the graph represents a feasible assembly state and every path from the top to bottom of the graph represents a feasible assembly sequence [18]. At every level, one part is added or one process is performed on the assembly. After applying the constraints imposed by the DFC, the family consists of only 28 assembly sequences shown in Fig. 9(b).

### 5.5. Modeling Fixtures in Assembly Operations

Fixtures are an integral part of any assembly process. In an automated type-1 assembly process, they immobilize the base subassembly and present it to the part being assembled in the desired location. On the other hand, in type-2 assemblies fixtures define the location of one part with respect to another during assembly. Most assembly planning approaches in the past have modeled the assembly process strictly as adding parts and have not included fixtures in the modeling process. Our approach to including fixtures is to represent the fixture as a part. It forms the node with zero in-degree (base node) that roots the DFC and starts the assembly process.

Figure 10 shows the method currently used to

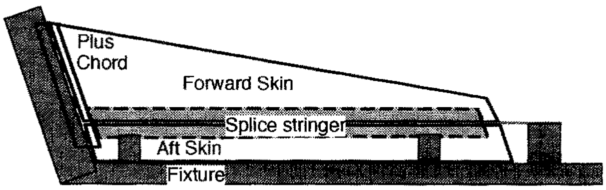

**Fig. 10.** Method of assembling skin subassembly using a fixture. It supports and aligns all of the parts by means of various surfaces and appendages, some of which are shown relatively realistically while others are shown schematically.

assemble the skin subassembly of the horizontal stabilizer. It depends on a large fixture, which is shown schematically. All the parts are located with respect to the fixture. The parts are placed in the fixture and joined to each other in the following order: splice stringer, 12 other stringers (not shown), aft skin, forward skin, join skins and stringers, plus chord, fasten plus chord to skins, fasten plus chord to stringers. See Cunningham et al. [14] for details. Figure 11 shows the DFC for this process. As expected, all of the arcs emanate from the node labelled 'fixture'.

Final assembly of the horizontal stabilizer takes place in another fixture. The DFC for this process is shown in Fig. 7(b). Again, the root of the DFC is in the lower fixture. The sequence of operations is: place fixed trailing edge and forward torque box in fixture; place ribs between these subassemblies and fasten, using the fixed trailing edge as the main locator; place the skin subassemblies onto this subassembly, trying to align the plus chord to the ends of the forward torque box and fixed trailing edge at the same time as trying to align skin edges to the skins of the forward torque box and fixed trailing edge.

In general, assemblies of this type can contain a mix of mates and contacts between the parts as well as mates between parts and the fixture. The DFC can represent these cases without difficulty.

To accomplish final assembly requires, as described above, building skin subassemblies in one fixture and the final assembly in another. We model assembly process for type-2 assemblies and type-1 assemblies involving multiple fixtures as series of clusters of assembly operations. Each cluster has one fixture and one or more associated DFCs that control all the assembly operations performed at the fixture, as shown in Fig. 12. By modeling this way, we avoid the problem of fixtures coming in and going away whenever there is re-fixturing. Precedence constraints (both geometric and imposed by the DFC) and resulting assembly sequences are generated for each cluster. During a re-fixturing, we must be aware of datum shifts which occur when the subassembly is located differently in one cluster compared to another. The entire assembly process is thus modeled using a piece-wise continuous chain formed by tracing the DFCs through one cluster, along the datum shift line, and into the next cluster. This chain determines how the KCs are delivered by the assembly process in multiple assembly stations and is an input to tolerance analysis.

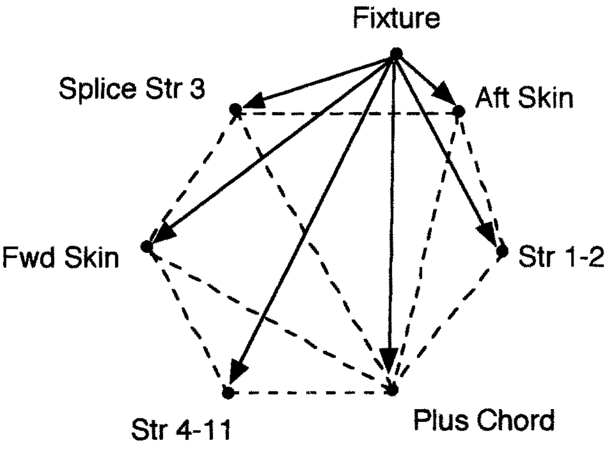

**Fig. 11.** Datum Flow Chain for the Fixture-based Assembly Method for Wing Subassembly Shown in Fig. 9. The root node is the fixture and, as indicated in Fig. 9, the datum flow chain emanates from that node and provides location to all the parts. The actual joints between the parts are all contacts and are indicated in the DFC by dashed lines.

In the case of the horizontal stabilizer, the process begins in the fixture shown in Fig. 10 and is completed as shown in Fig. 7(b). In Fig. 10, the base datum is the aft edge of the aft skin. In Fig. 7 the coupling of the gap and strength KCs prevents the assemblers from using the lower edge of the aft skin as the datum during final assembly, and they are forced to adjust it into position to satisfy the coupled KCs as best they can. The fixtureless process indicated in Fig. 4 was created in part to eliminate problems of this kind.

## 6. Evaluating Alternate DFCs

A DFC highly constrains both assembly design and process. The design of mating features at part interfaces, tolerances on individual features and subassembly configurations are limited by which joints are mates and which ones are contacts in the DFC. The locating scheme, tolerance chains for the KCs, family of permissible assembly sequences and quality of resulting KCs are also determined by the

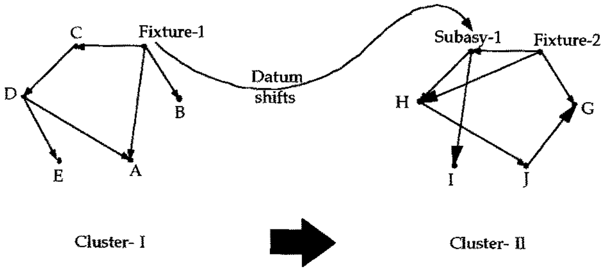

**Fig. 12.** Modeling multiple assembly station assembly processes using DFC.

design of the DFC. Hence, it is essential to develop analysis tools to evaluate and compare alternate DFCs. Some of the analysis can be qualitative, as there is no such thing as an optimal DFC or optimal assembly sequence. Different DFCs can be preferred under different operating conditions. Some of these analysis tools will be described below.

### 6.1. Decomposition and Subassembly Analysis

The decomposition of the assembly into subassemblies and the design of the DFC are related, as described in Section 5.3. The subassemblies deliver segments of KC chains and can be used as indicators to monitor the status of KCs during the assembly process. The percentage error contributed by individual subassemblies can be used for error budgeting and tolerance allocation purposes. Some subassemblies are more desirable than others in making these observations. For example, subassemblies where the status of the KCs is not predictable until the last few assembly operations can be undesirable for this reason. Since different DFCs will result in different permissible sets of subassemblies, the relative desirability of subassemblies can be used as a metric to evaluate alternate DFCs.

### 6.2. Constraint Analysis

Part of designing assemblies is deciding what features to use for mates and contacts. Feature geometry can be quite varied, and different shapes constrain different degrees of freedom between 1 and 6. Combinations of several features can result in over-constraint. It is surprising how many mistakes are made in which assemblies are unintentionally over-constrained [29]. In Adams [22] a method for detecting over-constrained assemblies is described, based on screw theory. Such a test appears to be a necessary part of assembly design because it critiques an important step in converting a notional DFC into a complete featurized design.

### 6.3. Variation Propagation Analysis

An important metric used to choose between alternate DFC designs and assembly sequences is the variation associated with the final assembly KCs accumulated from individual part and fixture variations, assembly errors, etc. The goodness of a locating scheme imposed by a DFC is evaluated by performing variation propagation analysis on families of assembly sequences, instead of individual assembly sequences. In type-1 assemblies, accumulated variation causes assemblability problems due to interference, at an intermediate assembly operation [19]. In type-2 assemblies, it is possible to make in-process adjustments during assembly and hence re-define the distribution of variation [20]. Variation propagation algorithms that will determine the resulting variation distribution of assembly dimensions, in the presence of these variation absorption sites, must be applied to determine if all KCs can be delivered by the assembly process. Choice of the locating scheme and assembly sequence have a profound effect in these types of analyses. A different DFC will impose a different locating scheme and resulting family of assembly sequences. These analyses can be used as metrics to choose between alternate DFCs.

Since the designer defines explicitly the relationships between different parts while constructing a DFC, it has all the information needed to perform any kind of variation propagation analysis. The tolerance chains for any KC can be readily derived by traversing the DFC between the nodes (parts) of interest. The DFC is an acyclic graph and all the paths between any two nodes collectively define the tolerance chain between the two nodes. Hence, there is a unique tolerance chain in the DFC for every KC. A graph similar to the DFC can be constructed at a part level to determine how to define all the datums within each node (part) that this tolerance chain will pass through so that the KC will be achieved. The extent to which these nodes are broken down into lower level graphs depends on the stage of the design process and level of detail desired. Further along in the design process, tolerance chains can be constructed to higher levels of detail.

#### *6.3.1 Assemblability Analysis*

In type-1 assemblies, all parts come to the assembly stations in their final form and the assembly process merely puts them together. Each new part contributes more variation to the assembly. A frequent problem is not being able to assemble a part due to interference caused by accumulated variation, especially in the case of automated assembly. For example, the interference could be caused due to limits imposed by part mating conditions: wedging and jamming [21]. Hence, an assemblability check needs to be performed at each assembly operation to determine the success or failure of an assembly operation.

At each assembly operation, a closed loop tolerance chain is completed that passes through the fixture locating the subassembly, the subassembly itself, the part being assembled, the tool gripping the part being assembled and finally back to the fixture. This tolerance chain, called the 'in-process assemblability tolerance chain', can be derived directly by tracing the mates in the DFC. In addition, there can be other parallel chains being completed by an assembly operation. In such cases, these other parallel chains establish contacts and hence can be seen in the liaison diagram and not the DFC. The contacts must have sufficient clearances to permit assembly after the mates have been established. Tolerance analysis using any standard tolerance analysis system can be performed on this assembly tolerance chain to determine if the assembly operation will fail or succeed. This problem is described in detail in Whitney et al. [19].

By working with a family of sequences, we restrict our analysis to subassembly configurations that are defined by mates present in the controlling DFC. Each family of assembly sequences forms an equivalence class of sequences that share a common locating scheme. Hence, each assembly sequence in the family of assembly sequences builds different elements of the same in-process assemblability tolerance chain. If any one assembly sequence fails to perform an assembly operation, all the assembly sequences in the family will also fail to perform that particular assembly operation. These sequences will however fail at different assembly stations depending upon when the particular operation is performed. Hence, it is not necessary to examine every assembly sequence in a family of assembly sequences. It is sufficient to analyze any one assembly sequence from the family and if this sequence fails to perform any particular assembly operation, the entire family of sequences can be rejected. On the other hand, if this sequence successfully completes all assembly operations, every sequence in the family of assembly sequences will also complete all assembly operations successfully. For a type-1 assembly, the state of the in-process assemblability tolerance chain at a particular assembly station is independent of the path taken to arrive at that assembly state. If all assembly operations are successful, all the KCs for the assembly will be delivered regardless of the assembly sequence.

### *6.3.2 KC Deliverability Analysis*

In type-2 assemblies, there is freedom to consciously select at least some of the features that define mates and contacts. The contact features can be designed to selectively absorb variation along certain directions and propagate certain others (for example, slip planes, peg-slot joints, designed gaps, etc.) [20]. The ability to make in-process adjustments in type-2 assemblies is due to the presence of mates that are completed by fixtures and contacts that allow for variation absorption in the assembly. The amount of variation tuned out and the directions along which the variation can be tuned out is determined by the type of the contact feature. Rigid body motion between parts based on in-process measurements is possible along selected directions. Variations accumulated in mates between two parts cannot be tuned out directly. However, it may be possible to tune out their effects on final assembly dimensions when some contacts are established in downstream assembly operations.

Although we have stated above that mates carry dimensional transfer information between parts while contacts carry none, it is true in general that joints can carry dimensional information along some directions while carrying none along others. Thus joints, in general, can have some of the properties of both mates and contacts. We assume that every part in the assembly is definitively located (within the limits of variation) by the DFC, via mates to other parts or mates to fixtures. This can occur in two ways: (1) a single arc carrying six DOF points to the part; (2) two or more arcs whose DOF add to 6 point to the part. In case (2), these DFC arcs are members of an acyclic loop that coincides with a loop in the liaison diagram. Since all the members of this loop are mates, there are no contacts in this loop. In case (1), the single arc cannot be a contact because the part at the end will not, by definition, be definitively located. However, case (1) can include situations where the underlying liaison diagram contains a loop, of which one or more arcs will by definition not be elements of the DFC. These extra arcs are contacts (otherwise the parts they connect would be over-constrained, since the existing mates' DOF already add to 6). Therefore, contacts will be found exclusively in liaison diagram loops that do not correspond to DFC loops. Therefore, to identify where and when to absorb variation, one needs to identify and monitor, during an assembly sequence, the closure of specific, easily identified loops in the liaison diagram. Figure 11 shows a DFC for the fixture based assembly process that is currently used to build the assembly shown in Fig. 1. As can be seen in the figure, quite a few parts have mates with the fixture and contacts between each other. The contacts are shown as a dashed line in the DFC for illustration purposes.

Variation propagation algorithms that can determine resulting variation distribution of KCs in the presence of in-process adjustments have been developed to evaluate different assembly sequences [20]. To simplify matters and to permit us to study variation absorption, these algorithms consider any absorption zone in a joint as a contact. This simplification permits us to describe a way to definitively identify all the contacts in a liaison diagram by inspection, by comparing it to the DFC that lies on top of it. Different assembly sequences will establish contacts at different stages of the assembly process and hence will have different resulting variation distributions. Thus, for type-2 assemblies, the state of the tolerance chains at any assembly station is a function of the path taken to arrive at that station. We call this property of type-2 assemblies as *path dependency.*

### 6.4. Assembly Sequence Analysis

Evaluation of assembly sequences within a family resulting by applying DFC constraints can also be done using traditional evaluation methods. The most basic is inspecting different assembly states and transitions and interactively deleting the undesirable ones [18]. The editing could be based on conditions such as: deletion of moves where a particular set of liaisons is made, specification that a particular move must immediately precede another, subassemblies hard to assemble due to accessibility problems, etc. These editing techniques quickly reduce the number of sequences to a handful that can be subject to more detailed analysis.

## 7. Conclusions and Future Work

This paper presents a top-down design approach to link logical design of assembly layouts with KC flowdown, assembly sequence, and tolerance analyses, and create assembly sketchers and analyzers capable of analyzing assembly processes before detailed geometry has been designed. The DFC permits layout designers to think through possible hierarchies of dimensional datums and then to design chains of these datums to control how parts are located with respect to each other. It is useful for selecting dimensional datum strategies and assembly processes that are best able to meet final assembly requirements. The DFC emphasizes the need to distinguish the joints that define dimensional constraint from the ones that are redundant location wise and merely provide support once the parts are located.

The paper describes the role of the DFC in subassembly design, assembly modeling and planning. Algorithms to translate the hierarchy imposed by a DFC into assembly precedence constraints were developed and presented. These algorithms are used to generate families of assembly sequences that share the same locating scheme. This reduces the design space to a small set of workable assembly sequences that are consistent with part design. In type-1 assemblies, since there are no in-process adjustments permitted during assembly, all sequences in a family have the same probability of delivering the key characteristics. However, in type-2 assemblies these sequences have different probabilities of success due to the ability to make in process adjustments during assembly. The DFC provides for a common environment to address a broad range of assembly planning issues for both types of assemblies. It is our intention to demonstrate the DFC idea in a commercial CAD system. For this purpose, a DFC editor is being created to interactively define DFCs, visualize them on the computer screen, and connect them to required analyses such as tolerance buildup and assembly sequencing.

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# Appendix 1

This appendix presents a suggestive analogy between DFCs and electric circuits. The analogy is not perfect but it suggests that it is possible to apply some of the methods of circuit analysis to DFCs.

**Table A1.** Suggestive analogies between electric circuits and Datum Flow Chains (*Note: loops are allowed in a DFC but cycles (directed loops) are not)

| | Circuits | DFCs |
|---|---|---|
| node | joint between elements | an element |
| arc | passage for current between elements | 'passage' for constraint between elements |
| root | ground, voltage reference | base datum, location reference |
| node 'value' | voltage (scalar) | spatial location (6-vector) |
| arc 'value' | current carried | denotes dof carried |
| 'node law' | incoming currents add to zero | incoming dof add to 6 |
| 'loop law' | voltage changes around a loop add to zero | location changes around a loop add to zero* |