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Laminar-Film Condensation on a Flat Plate in the Absence of a Body Force

By ROBERT D. CESS, Pittsburgh 35, Pa., U.S.A.[1]

1. Introduction

Analyses of laminar-film condensation generally deal with condensation of a saturated vapor on a surface located within a gravitational field and with negligible vapor velocity. This problem was originally studied by NUSSELT [1][2] for the case of a vertical flat plate, and both fluid acceleration and thermal convection within the liquid film were neglected. SPARROW and GREGG [2] have recently obtained a solution to this problem without neglecting acceleration and convection effects. This was accomplished through application of boundary-layer techniques and subsequent utilization of a similarity transformation.

A further consideration of laminar-film condensation on vertical surfaces involves a finite vapor velocity. Here the motion of the liquid film is affected by both the gravitational body force and the sweeping effect of the vapor. In this situation, a similarity transformation does not exist, and a complete solution to the problem would be quite complicated. Only approximate solutions have been obtained based upon the assumption that the shear stress at the liquid-vapor interface is a constant known quantity[3].

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[1]) Heat Transfer and Flow Section, Mechanics Department, Westinghouse Research Laboratories.

[2]) Numbers in brackets refer to References, page 433.

[3]) For example, see Reference [3].

The purpose of the present paper is to investigate laminar-film condensation on a flat plate for the case in which no body force is present. The motion of the liquid film therefore results solely from the sweeping effect of the adjacent vapor. This problem is illustrated in Figure 1. A saturated vapor at temperature T_∞ flows

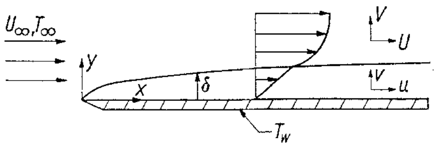

Figure 1
Physical model and coordinate system.

with a free-stream velocity U_∞ across a cooled flat plate having a constant surface temperature T_w. The vapor condenses on the plate surface producing a liquid film of thickness δ.

Physically, such a situation can occur in one of two ways. First, if the plate is oriented normal to any acting body force; for example, if the plate is placed horizontally in a gravitational field. The second situation involves the complete absence of a body force as would occur in a nonrotating space vehicle.

2. Basic Equations

It will be assumed that a continuous laminar film of condensate exists. Let u and v be the velocity components of the liquid, and U and V be those of the vapor. Temperature is denoted by T. For $y < \delta$ the equations expressing the conservation of mass, momentum, and energy may be written as

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 , \tag{1}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2} , \tag{2}$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \alpha \frac{\partial^2 T}{\partial y^2} , \tag{3}$$

while for the vapor ($y > \delta$) the continuity and momentum equations become

$$\frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} = 0 , \tag{4}$$

$$U \frac{\partial U}{\partial x} + V \frac{\partial U}{\partial y} = \nu_v \frac{\partial^2 U}{\partial y^2} . \tag{5}$$

The quantities α and ν are the thermal diffusivity and kinematic viscosity, respectively. Further, the nomenclature will be adopted that a physical property with no subscript refers to the liquid, whereas the subscript v denotes the vapor.

The boundary conditions at the plate surface are $u = v = 0$ and $T = T_w$, while $T = T_\infty$ for $y = \delta$ and $U = U_\infty$ for $y = \infty$. Three additional boundary conditions are necessary. Across the liquid-vapor interface both velocity and shear stress must be continuous, so that within the framework of boundary-layer theory $u = U$ and $\mu\, \partial u/\partial y = \mu_v\, \partial U/\partial y$ for $y = \delta$, where μ is the dynamic viscosity. In addition, continuity of mass across $y = \delta$ requires that

$$\varrho \left(u \frac{d\delta}{dx} - v \right)_{y=\delta} = \varrho_v \left(U \frac{d\delta}{dx} - V \right)_{y=\delta}$$

with ϱ denoting density.

Equations (1) and (4) are satisfied by introducing stream functions ψ and Ψ defined by the relations

$$u = \frac{\partial \psi}{\partial y}, \quad v = -\frac{\partial \psi}{\partial x}, \quad U = \frac{\partial \Psi}{\partial y}, \quad V = -\frac{\partial \Psi}{\partial x}.$$

Defining further

$$f(\eta) = \frac{\psi}{\sqrt{\nu_v U_\infty x}}, \quad F(\eta) = \frac{\Psi}{\sqrt{\nu_v U_\infty x}}, \quad \Theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}, \quad \eta = y \sqrt{\frac{U_\infty}{\nu_v x}},$$

equations (2), (5), and (3) transform, respectively, to the ordinary differential equations

$$f''' + \frac{1}{2} \left(\frac{\nu_v}{\nu} \right) f f'' = 0, \tag{6}$$

$$F''' + \frac{1}{2} F F'' = 0, \tag{7}$$

$$\Theta'' + \frac{Pr}{2} \left(\frac{\nu_v}{\nu} \right) f \Theta' = 0. \tag{8}$$

The boundary conditions become

$$f(0) = f'(0) = 0, \quad \Theta(0) = 1, \tag{9a}$$

$$f(\eta_\delta) = \left(\frac{\varrho_v}{\varrho} \right) F(\eta_\delta), \quad f'(\eta_\delta) = F'(\eta_\delta), \quad f''(\eta_\delta) = \left(\frac{\varrho_v \nu_v}{\varrho \nu} \right) F''(\eta_\delta), \quad \Theta(\eta_\delta) = 0, \tag{9b}$$

$$F'(\infty) = 1, \tag{9c}$$

where $\eta_\delta = \delta \sqrt{U_\infty / \nu_v x}$.

One may note that equations (7) and (9b) are analogous to vectored blowing or suction, where in this case the vectored blowing or suction is specified at $\eta = \eta_\delta$ rather than $\eta = 0$. In view of this result, consider a function $G(\eta)$ defined by

$$G''' + \frac{1}{2} G G'' = 0 \tag{10}$$

with the boundary conditions

$$G(0) = G_w, \; G'(0) = 0, \; G'(\infty) = 1.$$

This corresponds to the problem of normal blowing or suction as treated by EMMONS and LEIGH [4], and it has been shown that vectored blowing or suction can be determined from the solution of equation (10) with its boundary conditions through use of a fictitious origin for η. Consequently, the solution of equation (7) may be written as

$$F(\eta) = G(\eta - \gamma), \tag{11}$$

where γ and the appropriate value for G_w are constants to be determined from the boundary conditions at $\eta = \eta_\delta$.

The quantity η_δ remains to be evaluated, and this may be accomplished in the same manner as considered by SPARROW and GREGG [2]. An over-all energy balance on the liquid film yields

$$k \int_0^x \left(\frac{\partial T}{\partial y}\right)_{y=0} dx = \varrho\, h_{fg} \int_0^\delta u\, dy + \varrho\, c_p \int_0^\delta u\, (T_\infty - T)\, dy ,$$

where c_p is the specific heat of the liquid at constant pressure, k is the thermal conductivity, and h_{fg} denotes the latent heat of condensation. Writing this in terms of the boundary-layer variables, and letting $\Delta T = T_\infty - T_w$, there is obtained

$$\left(\frac{\nu}{\nu_v}\right)\left(\frac{c_p \Delta T}{Pr\, h_{fg}}\right) = -\frac{1}{2}\, \frac{f(\eta_\delta)}{\Theta'(\eta_\delta)} \tag{12}$$

with Pr representing the Prandtl number ($Pr = \nu/\alpha$). Thus, a fixed value of η_δ defines the parameter on the left-side of this equation.

3. Approximate Solution

The solution of the preceding equations can be obtained by numerically integrating equation (6) and matching this solution with equation (11) through use of the boundary conditions at $\eta = \eta_\delta$. Equation (8) may then be solved through a separate numerical integration. Such a procedure would, however, be somewhat tedious in view of the number of parameters involved (ϱ_v/ϱ, ν_v/ν, η_δ, Pr), and an approximate solution will instead be considered.

It may readily be verified that series solutions of equations (6), (7), and (8) are of the form

$$f(\eta) = \left(\frac{\nu}{\nu_v}\right)\left[\frac{\beta}{2}\, \eta^2 - \frac{1}{2}\, \frac{\beta^2}{5!}\, \eta^5 + \cdots\right], \tag{13}$$

$$F(\eta) = G_w + \frac{G''_w}{2}\, (\eta - \gamma)^2 - \frac{1}{2}\, \frac{G_w G''_w}{3!}\, (\eta - \gamma)^3 + \cdots , \tag{14}$$

$$\Theta(\eta) = 1 - \sigma\, \eta + \frac{\beta\, Pr}{2}\, \frac{\sigma}{4!}\, \eta^4 + \cdots , \tag{15}$$

where equation (14) follows from equations (10) and (11), $G''_w = G''(0)$, and β and σ are constants. For all practical purposes, it will be necessary to evaluate equation (14) only in the neighborhood of η_δ so as to satisfy the boundary conditions at $\eta = \eta_\delta$. The assumption will now be made that η_δ and $(\eta_\delta - \gamma)$ are sufficiently small such that the preceding equations may be rewritten as

$$f(\eta) = \left(\frac{\nu}{\nu_v}\right) \frac{\beta}{2}\, \eta^2 , \tag{16}$$

$$F(\eta) = G_w + \frac{G''_w}{2}\, (\eta - \gamma)^2 , \tag{17}$$

$$\Theta(\eta) = 1 - \sigma\, \eta . \tag{18}$$

Physically, the neglect of higher-order terms in equation (13) is the same as neglecting acceleration effects within the film, since equation (16) gives $\partial^2 u/\partial y^2 = 0$. In turn, equation (18) neglects thermal convection within the film, and the resulting temperature profile is linear.

Equations (16), (17) and (18) contain five unknowns (β, γ, σ, G_w, G_w''), and four of these may be evaluated from equations (9b) giving $\sigma = 1/\eta_\delta$ and

$$\beta = \left(\frac{\varrho_v}{\varrho}\right)\left(\frac{\nu_v}{\nu}\right)^2 G_w'' , \tag{19a}$$

$$\gamma = \eta_\delta \left(1 - \frac{\varrho_v \nu_v}{\varrho \nu}\right), \tag{19b}$$

$$G_w = \frac{G_w''}{2} \left(\frac{\nu_v}{\nu}\right)\left[1 - \left(\frac{\nu_v}{\nu}\right)\left(\frac{\varrho_v}{\varrho}\right)^2\right] \eta_\delta^2 . \tag{19c}$$

The remaining unknown, G_w'', has been tabulated as a function of G_w by EMMONS and LEIGH [4], and an abbreviated listing of their results is given in Table 1.

Table 1. Values of G_w'' [4]

G_w	0	0·1	0·2	0·4	0·6	0·8	1·0	1·5	2	5	10
G_w''	0·332	0·369	0·406	0·483	0·563	0·645	0·729	0·945	1·169	2·590	5·049

Now, from equations (16) and (18), equation (12) gives

$$\eta_\delta^3 = \frac{4}{\beta}\left(\frac{c_p \Delta T}{Pr\, h_{fg}}\right). \tag{20}$$

Further, for most physical applications ($\varrho_v/\varrho \ll 1$) equation (19c) may be rewritten as

$$\frac{G_w}{G_w''} = \frac{1}{2}\left(\frac{\nu_v}{\nu}\right)\eta_\delta^2$$

and combining this with equations (19a) and (20)

$$\frac{G_w}{(G_w'')^{1/3}} = \frac{(4)^{2/3}}{2}\left\{\left(\frac{\nu}{\nu_v}\right)^{1/2}\left[\left(\frac{\varrho}{\varrho_v}\right)\left(\frac{c_p \Delta T}{Pr\, h_{fg}}\right)\right]\right\}^{2/3} . \tag{21}$$

Employing Table 1, the quantity G_w'' is completely determined for a given value of the parameter

$$\left(\frac{\nu}{\nu_v}\right)^{1/2}\left[\left(\frac{\varrho}{\varrho_v}\right)\left(\frac{c_p \Delta T}{Pr\, h_{fg}}\right)\right].$$

Asymptotic expressions may also be obtained for G_w''. If the right-side of equation (21) is very small, then $G_w \simeq 0$ and

$$G_w'' = 0{\cdot}332 . \tag{22}$$

On the other hand, if the right-side of equation (21) is large, then $G_w \simeq 2\,G_w''$ [4] and equation (21) reduces to

$$G_w'' = \frac{1}{2}\left(\frac{\nu}{\nu_v}\right)^{1/2}\left[\left(\frac{\varrho}{\varrho_v}\right)\left(\frac{c_p \Delta T}{Pr\, h_{fg}}\right)\right]. \tag{23}$$

4. Results

The results of usual practical importance are the shear stress and heat transfer at the plate surface. Consider first the local shear stress τ, which is given by

$$\tau = \mu \left(\frac{\partial u}{\partial y}\right)_{y=0} = \frac{\varrho\, U_\infty^2\, \nu}{\sqrt{\nu_v\, U_\infty\, x}}\, f''(0) .$$

From equations (16) and (19a), it is found that

$$\left(\frac{\tau}{\varrho_v\, U_\infty^2}\right) \sqrt{Re} = G_w'' , \tag{24}$$

where Re is the vapor Reynolds number defined as $Re = U_\infty\, x/\nu_v$.

The shear stress results are illustrated in Figure 2. For small values of the abscissa, a constant value is asymptotically approached given by equation (22). Equation (23) represents the asymptotic result for large values of the abscissa. One may note that since acceleration effects have been neglected the shear stress is constant throughout the film, and the results for τ also apply at the liquid-vapor interface.

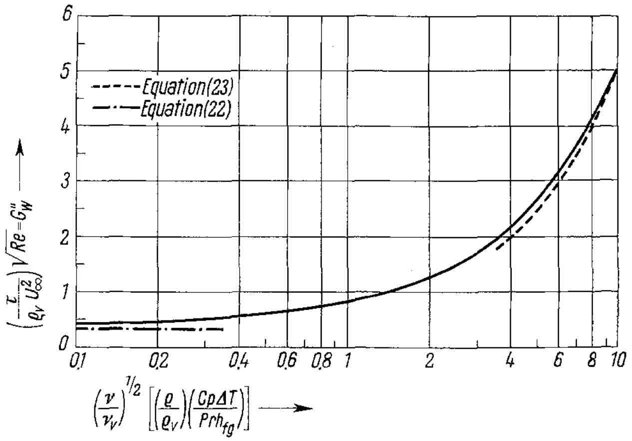

Figure 2

Variation of local shear stress.

With respect to heat transfer at the plate surface, the local heat flux per unit area q is given by

$$q = -k\,(T_\infty - T_w)\left(\frac{\partial\Theta}{\partial y}\right)_{y=0} = -k\,(T_\infty - T_w)\sqrt{\frac{U_\infty}{\nu_v\, x}}\;\Theta'(0)\,.$$

From equation (18), and writing the result in terms of a local Nusselt number

$$Nu = \frac{q\,x}{(T_\infty - T_w)\,k} = \frac{1}{\eta_\delta}\sqrt{Re}\,.$$

Combining this with equations (19a) and (20), then

$$\frac{Nu}{\sqrt{Re}}\left(\frac{\nu}{\nu_v}\right)^{1/2} = \left(\frac{G_w''}{4}\right)^{1/3}\left\{\left(\frac{\nu_v}{\nu}\right)^{1/2}\left[\left(\frac{\varrho_v}{\varrho}\right)\left(\frac{Pr\,h_{fg}}{c_p\,\Delta T}\right)\right]\right\}^{1/3}. \tag{25}$$

A plot of the Nusselt number is shown in Figure 3. For small values of the abscissa, the asymptotic relation is found from equations (22) and (25) to be

$$\frac{Nu}{\sqrt{Re}}\left(\frac{\nu}{\nu_v}\right)^{1/2} = 0{\cdot}436\left\{\left(\frac{\nu_v}{\nu}\right)^{1/2}\left[\left(\frac{\varrho_v}{\varrho}\right)\left(\frac{Pr\,h_{fg}}{c_p\,\Delta T}\right)\right]\right\}^{1/3}. \tag{26}$$

Upon combining equations (23) and (25)

$$\frac{Nu}{\sqrt{Re}} \left(\frac{\nu}{\nu_v}\right)^{1/2} = 0{\cdot}5 \,, \tag{27}$$

which is the asymptotic expression for large values of the abscissa.

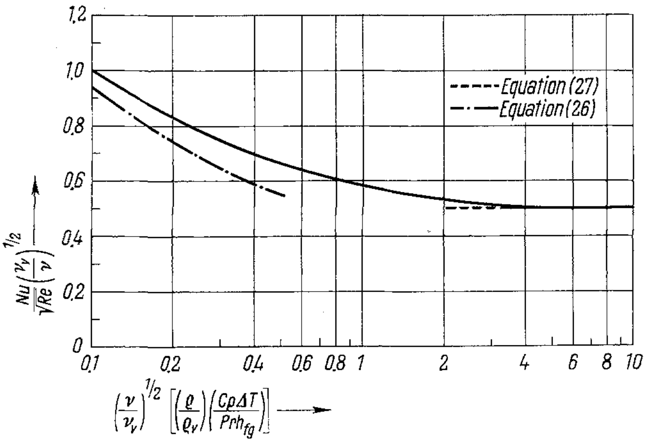

Figure 3

Variation of local Nusselt number.

5. Discussion of Results

It will be recalled that higher-order terms have been neglected in equation (13), and that this corresponds to negligible acceleration effects within the liquid film. The neglect of acceleration terms is therefore permissible if the condition

$$\frac{\beta^2}{5!}\,\eta_\delta^5 \ll \beta\,\eta_\delta^2$$

is satisfied. From equation (20), this may be rewritten as

$$\frac{c_p \Delta T}{Pr\,h_{fg}} \ll 30\,.$$

In a similar manner, thermal convection within the film will be negligible only if higher-order terms are small in equation (15), and this gives

$$\frac{c_p \Delta T}{h_{fg}} \ll 12\,.$$

A final requirement follows from equation (14) to be

$$\frac{c_p \Delta T}{Pr\,h_{fg}} \ll 3\,.$$

Therefore, the present analysis is applicable if the last two conditions are satisfied.

Admittedly, the present results neglect any ripple effects within the liquid film, and this could be an important factor in the actual physical process. Nevertheless, the analysis gives an insight into the physical phenomena involved, and illustrates the effect of the various parameters.

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Zusammenfassung

Die laminare Strömung von gesättigtem Dampf über eine gekühlte ebene Fläche wird untersucht für den Fall der Kondensation eines Films an der Oberfläche. Es wird angenommen, dass keine Gravitationskraft vorliegt und dass die Bewegung des Kondensats lediglich durch die Reibung am strömenden Dampf hervorgerufen wird. Ergebnisse für die Scherkraft und die Wärmeübertragung an der Oberfläche werden mitgeteilt. Die wesentlichen Rechenergebnisse konnten in zwei Diagrammen zusammengefasst werden: Figur 2 gibt die zu erwartenden Schubspannungen wieder, Figur 3 den Wärmeübergang.

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Signalfluss und Analogiedarstellung der Neutronenökonomie beim Abbrand von Reaktorbrennstoffen

Von WALTER HÄLG, Zürich[1]).

1. Allgemeine Bemerkungen

Bei sämtlichen Untersuchungen, welche die Gestehungskosten der Atomenergie zum Gegenstand haben, bildet die Frage nach der ökonomischen Verwendung der im Reaktor vorhandenen Neutronen eine ausschlaggebende Rolle. Jedes Neutron, welches aus dem Reaktorkern entweicht oder welches nicht zur Einleitung einer Kernspaltung oder zur Umwandlung von nichtspaltbarem Stoff in spaltbare Kerne Verwendung findet, stellt ein Ausgabenposten in der Bilanz dar. LEWIS[2]) hat verschiedentlich gezeigt, dass es zweckmässig ist, die Neutronenbilanz der Kernbrennstoffe von derjenigen der übrigen Reaktorbaustoffe einschliesslich Leckanteil separat zu betrachten. Man erreicht dadurch eine weitgehend universelle Behandlungsweise der verschiedenen Reaktorkonzeptionen, weil sich der komplexe zeitliche Ablauf von Neutronenproduktion und Neutronenkonsumation im Spaltstoff gleichartig darstellt.

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[1]) ETH und Eidg. Institut für Reaktorforschung, Würenlingen.

[2]) Vergleiche zum Beispiel W. B. LEWIS, *Low cost fuelling without recycling*, AECL, Nr. 382, oder AECL Nr. 651.