Wärme- und Stoffübertragung 27, 473–479 (1992)

Wärme- und Stoffübertragung

Convective wall plume in power-law fluid: Second-order correction for the adiabatic wall

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Abstract. This paper considers the steady-state free convection flow arising from an infinitely long horizontal line source of heat embedded in the base of a vertical adiabatic surface when the ambient fluid is a non-Newtonian fluid for moderately large values of the generalized Grashof numbers by the method of matched asymptotic expansions. In particular, the second-order corrections to account for the non-boundary layer effects have been predicted. A family of numerical solutions for the power-law fluid behavior index n ranging from 0.4 to 2.0 and for the Prandtl number $Pr=10$ and 100 are reported.

Auftriebsströmung in einem nicht-Newtonschen Fluid: Korrektur zweiter Ordnung bei adiabater Wand

Zusammenfassung. Diese Arbeit bezieht sich auf die stationäre Auftriebsströmung über einer langen, horizontalen Linienwärmequelle, die in das untere Ende einer senkrechten, adiabaten Fläche eingebettet ist, wobei für das umgebende Fluid nicht-Newtonsches Verhalten unterstellt wird. Unter Voraussetzung mäßig hoher Werte für die verallgemeinerten Grashof-Zahlen kommt die Methode der angepaßten asymptotischen Entwicklung zur Anwendung. Insbesondere wird belegt, daß Korrekturen zweiter Ordnung zur Berücksichtigung von Nichtgrenzschichteffekten erforderlich sind. Die mitgeteilten Ergebnisse umfassen eine Gruppe von numerischen Lösungen im Bereich 0,4 bis 2,0 für den Exponenten n des Potenzansatzes, mit dem das nicht-Newtonsche Fluidverhalten erfaßt wird und jeweils für die Prandtl-Zahlen $Pr=10$ und 100.

Nomenclature

C_f	skin friction coefficient
g	acceleration due to gravity
Gr	generalized Grashof number
Gr_x	generalized local Grashof number
J	second invariant of the strain-rate tensor
K	consistency index
L	reference length
n	index of power-law viscosity model
p	pressure
Pr	generalized Prandtl number
I	non-dimensional heat input by the thermal source
r	radial distance
T	temperature
T_r	reference temperature
u, v	velocity components along (x, y)-axis
x, y	coordinates along and normal to the plate
y	inner variable

Greek symbols

β	thermal expansion coefficient
η	similarity variable
θ	non-dimensional temperature
ϱ	density
τ	shear stress
φ	angular distance
ψ	stream function

Superscripts

$-$	dimensionless variables
	differentiation with respect to η

Subscripts

a	adiabatic
w	wall condition
∞	ambient condition

1 Introduction

The flow resulting from a horizontal line source of heat placed at the base of a vertical surface submerged in a Newtonian fluid has been the subject of many investigations, beginning with Zimin and Lyakhov [1]. A good review of the past work is provided by Ingham and Pop [2]. This configuration, also referred to as the wall plume, is often a convenient and accurate idealization of many electrical and electronics cooling applications. As was pointed out by Jaluria [3] and Joshi [4] such flows are also of importance in studies of boundary layer regimes in transport enclosures.

A growing interest in the dynamics of various polymeric liquids has inspired investigations into the non-Newtonian fluids for several years. In particular, there is a need to understand natural convective flows when the non-Newtonian fluids can be approximated with the power-law viscosity model. Applications for such model can be found in industries processing molten plastics, polymers, food stuff, fiber coating, etc. Thus, considerable attention has been recently directed toward major aspects of this coupled, non-linear power-law viscosity fluid model; see Refs. [5–8].

The scope of this study is to investigate the free convection flow arising from an infinitely long horizontal line source of heat embedded in the base of a vertical adiabatic surface when the ambient fluid is a non-Newtonian power-law fluid for moderately large values of the generalized Grashof numbers by the method of matched asymptotic expansions. In particular, the second-order corrections to account for the non-boundary layer effects have been predicted. Numerical results are given to illustrate the effects of the power law viscosity index and the generalized Prandtl number on the velocity and temperature functions as well as on the local skin friction coefficient. The similarity solution of the classical boundary layer equations for the power-law wall plume problem has been presented recently by Pop, Gorla and Lee [9]. However, very little previous work has been done on free convection plumes in non-Newtonian fluids. To the authors' knowledge, no experimental investigation of this problem has been reported in the literature. Work reported so far by the present authors include theoretical studies on free and mixed convection in wall plumes placed in non-Newtonian power-law fluids.

2 Governing equations

Consider the free convection flow of a power-law fluid adjacent to a vertical surface with a line heat source along the leading edge. The properties of the fluid will be taken as constant excepting the density in the buoyancy term. The coordinate x is the distance measured along the plate from its leading edge in the direction of the flow, y being normal to it. The corresponding velocity components are u and v, p is the pressure, T is the temperature of the fluid inside the plume, T_w and T_∞ are temperatures of the plate and the ambient fluid, respectively.

The governing equations may be written in non-dimensional form as [5, 6, 10]

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{\partial p}{\partial x}+Gr^{-n(n+1)/(4n+1)}\left\{2\frac{\partial}{\partial x}\left(J\frac{\partial u}{\partial x}\right)+\frac{\partial}{\partial y}\left[J\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial y}\right)\right]\right\}+\theta, \tag{2}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{\partial p}{\partial y}+Gr^{-n(n+1)/(4n+1)}\left\{2\frac{\partial}{\partial y}\left(J\frac{\partial u}{\partial y}\right)+\frac{\partial}{\partial x}\left[J\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)\right]\right\}, \tag{3}$$

$$u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}=\frac{1}{Pr}Gr^{-n(n+1)/(4n+1)}\left\{\frac{\partial}{\partial x}\left(J\frac{\partial \theta}{\partial x}\right)+\frac{\partial}{\partial y}\left(J\frac{\partial \theta}{\partial y}\right)\right\} \tag{4}$$

Table 1. Dimensionless variables

x	y	u	v	p	θ	J
$\frac{\bar{x}}{L}$	$\frac{\bar{y}}{L}$	$\frac{\bar{u}}{U}$	$\frac{\bar{v}}{U}$	$\frac{\bar{p}}{\varrho U^2}$	$\frac{T-T_\infty}{T_r}Gr^b$	$\frac{\bar{J}}{U^{n-1}L^{1-n}}$

$b=(6n^2-5n-2)/(4n+1)(n-2)$

where

$$J=\left[2\left(\frac{\partial u}{\partial x}\right)^2+2\left(\frac{\partial v}{\partial y}\right)^2+\left(\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x}\right)^2\right]^{(n-1)/2}. \tag{5}$$

Pr being the generalized Prandtl number and n is the power-law index. These equations will be solved with the boundary conditions,

$$x>0,\ y=0:\quad u=v=0,\quad \theta=(T_w-T_\infty)\,Gr^b/T_r \quad\text{or}\quad \frac{\partial\theta}{\partial y}=0, \tag{6a}$$

$$y\to\infty:\quad u=v\sim 0,\quad \theta\sim 0,\quad p\sim 0, \tag{6b}$$

$$x\le 0,\ y=0:\quad v=\frac{\partial u}{\partial y}=\frac{\partial\theta}{\partial y}=0. \tag{6c}$$

Also, for $x>0$, for an adiabatic surface, the total convected energy downstream $I(x)$ remains constant [11]

$$I(x)=Gr^{-n/(4n+1)}\int_0^\infty u\,\theta\,dy=\text{const.} \tag{7}$$

The non-dimensional quantities in the above equations are defined in Table 1, where L is a characteristic length, ϱ is the density, T_r is a reference temperature, and U is a characteristic velocity

$$U=(\varrho L^n\, Gr^{-n(n+1)/(4n+1)}/K)^{1/(n-2)}$$

and Gr is the generalized Grashof number

$$Gr=g\,\beta\,T_r\,L^{(2+n)/(2-n)}(\varrho/K)^{2/(2-n)}.$$

Further, we shall introduce the strem function ψ defined in the usual way

$$u=\frac{\partial\psi}{\partial y},\quad v=-\frac{\partial\psi}{\partial x} \tag{8}$$

and eliminate pressure p from equations (2), (3). After a little algebra, we get

$$\left(\frac{\partial\psi}{\partial x}\frac{\partial}{\partial y}-\frac{\partial\psi}{\partial y}\frac{\partial}{\partial x}\right)\nabla^2\psi+Gr^{-n(n+1)/(4n+1)}\left\{\frac{\partial^2}{\partial y}\left[J\left(\frac{\partial^2\psi}{\partial y^2}-\frac{\partial^2\psi}{\partial x^2}\right)\right]+\frac{\partial 2}{\partial x^2}\left[J\left(\frac{\partial^2\psi}{\partial x^2}-\frac{\partial^2\psi}{\partial y^2}\right)\right]+4\frac{\partial^2}{\partial x\,\partial y}\left(J\frac{\partial^2\psi}{\partial x\,\partial y}\right)\right\}+\frac{\partial\theta}{\partial y}=0, \tag{9}$$

$$\left(\frac{\partial\psi}{\partial x}\frac{\partial}{\partial y}-\frac{\partial\psi}{\partial y}\frac{\partial}{\partial x}\right)\theta = \frac{1}{Pr}Gr^{-n(n+1)/(4n+1)}\left[\frac{\partial}{\partial x}\left(J\frac{\partial\theta}{\partial x}\right)+\frac{\partial}{\partial y}\left(J\frac{\partial\theta}{\partial y}\right)\right] \quad (10)$$

and Eq. (5) becomes

$$J=\left[4\left(\frac{\partial^2\psi}{\partial x\,\partial y}\right)^2+\left(\frac{\partial^2\psi}{\partial y^2}-\frac{\partial^2\psi}{\partial x^2}\right)^2\right]^{(n-1)/2}. \quad (11)$$

The boundary conditions (6) now read

$$x>0,\ y=0:\quad \psi=\frac{\partial\psi}{\partial y}=0, \quad (12a)$$

$$\theta=(T_w-T_\infty)\,Gr^b/T_r \quad \text{or} \quad \frac{\partial\theta}{\partial y}=0,$$

$$y\to\infty:\quad \frac{\partial\psi}{\partial y}\to 0,\quad \theta\to 0, \quad (12b)$$

$$x\le 0,\ y=0:\quad \psi=\frac{\partial^2\psi}{\partial y^2}=\frac{\partial\theta}{\partial y}=0 \quad (12c)$$

while relation (7) takes the form

$$Gr^{n/(4n+1)}\int_0^\infty \frac{\partial\psi}{\partial y}\,\theta\,dy=I. \quad (13)$$

3 Method of solution

Assuming the Grashof number to be large, we apply the method of matched asymptotic expansions as developed by Van Dyke [12] with $Gr^{-n/(n+1)}$ as a small parameter. The method of solution is as follows. The velocity and temperature fields are divided into two regions: one is the inner region close to the plate and the other is the outer region far from it. Separate, locally valid, expansions of the stream function and temperature are developed for those two regions. These expansions are assumed of the form

$$\psi=Gr^{-n/(4n+1)}\{\psi_0(x,Y)+Gr^{-n/(4n+1)}\psi_1(x,Y) + \text{higher order terms}\}, \quad (14a)$$

$$\theta=\theta_0(x,Y)+Gr^{-n/(4n+1)}\theta_1(x,Y)+\text{higher order terms} \quad (14b)$$

for the inner region where $Y=Gr^{n/(4n+1)}y$ and

$$\psi=Gr^{-n/(4n+1)}\{\tilde{\psi}_1(x,y)+Gr^{-n/(4n+1)}\tilde{\psi}_2(x,y) + \text{higher order terms}\}, \quad (15a)$$

$$\theta\sim 0, \quad (15b)$$

for the outer region, respectively. We note that Eq. (15b) results from the fact that the outer region is isothermal. The equations for (ψ_0,θ_0) and (ψ_1,θ_1) are obtained by substituting Eq. (14) into Eqs. (9), (10) and collecting terms of various powers of Gr. Integrating equations derived from Eq. (9) in this manner once with respect to Y and the resulting arbitrary functions of x set equal to zero, we obtain

i) first-order approximation

$$\frac{\partial\psi_0}{\partial x}\frac{\partial^2\psi_0}{\partial Y^2}-\frac{\partial\psi_0}{\partial Y}\frac{\partial^2\psi_0}{\partial x\,\partial Y}-\frac{\partial}{\partial Y}\left(\left|\frac{\partial^2\psi_0}{\partial Y^2}\right|^{n-1}\frac{\partial^2\psi_0}{\partial Y^2}\right)+\theta_0=0, \quad (16)$$

$$\frac{\partial\psi_0}{\partial Y}\frac{\partial\theta_0}{\partial x}-\frac{\partial\psi_0}{\partial x}\frac{\partial\theta_0}{\partial Y}=\frac{1}{Pr}\frac{\partial}{\partial Y}\left(\left|\frac{\partial^2\psi_0}{\partial Y^2}\right|^{n-1}\frac{\partial\theta_0}{\partial Y}\right), \quad (17)$$

$$Y=0:\quad \psi_0=\frac{\partial\psi_0}{\partial Y}=0,\quad \theta_0=(T_w-T_\infty)\,Gr^b/T_r \ \text{or}\ \frac{\partial\theta_0}{\partial Y}=0, \quad (18a)$$

$$Y\to\infty:\quad \frac{\partial\psi_0}{\partial Y}\to 0,\quad \theta_0\to 0 \quad (18b)$$

and

$$\int_0^\infty \frac{\partial\psi_0}{\partial Y}\,\theta_0\,dy=I. \quad (18c)$$

ii) second-order approximation

$$\frac{\partial\psi_1}{\partial x}\frac{\partial^2\psi_0}{\partial Y^2}-\frac{\partial\psi_0}{\partial Y}\frac{\partial^2\psi_1}{\partial x\,\partial Y}+\frac{\partial\psi_0}{\partial x}\frac{\partial^2\psi_1}{\partial Y^2}-\frac{\partial\psi_1}{\partial Y}\frac{\partial^2\psi_0}{\partial x\,\partial Y} +\frac{\partial}{\partial Y}\left(\left|\frac{\partial^2\psi_0}{\partial Y^2}\right|^{n-1}\frac{\partial^2\psi_1}{\partial Y^2}\right)+\theta_1=0, \quad (20)$$

$$\frac{\partial\psi_0}{\partial Y}\frac{\partial\theta_1}{\partial x}-\frac{\partial\psi_0}{\partial x}\frac{\partial\theta_1}{\partial Y}+\frac{\partial\psi_1}{\partial Y}\frac{\partial\theta_0}{\partial x}-\frac{\partial\psi_1}{\partial x}\frac{\partial\theta_0}{\partial Y} =\frac{1}{Pr}\frac{\partial}{\partial Y}\left(\left|\frac{\partial^2\psi_0}{\partial Y^2}\right|^{n-1}\frac{\partial\theta_1}{\partial Y}\right), \quad (20)$$

$$Y=0:\quad \psi_1=\frac{\partial\psi_1}{\partial Y}=\frac{\partial\theta_1}{\partial Y}=0, \quad (21a)$$

$Y\to\infty$: $\dfrac{\partial\psi_1}{\partial Y}$ and θ_1 match with the outer solution (21b)

and

$$\int_0^\infty\left(\frac{\partial\psi_0}{\partial Y}\theta_1+\frac{\partial\psi_1}{\partial Y}\theta_0\right)dY=0. \quad (21c)$$

3.1 The first-order inner solution

The solution of the first-order inner problem has been shown by Pop, Gorla and Lee [9] to be of the form

$$\psi_0=x^{(2n+1)/(4n+1)}f_0(\eta), \quad (22a)$$

$$\theta_0=x^{-(2n+1)/(4n+1)}h_0(\eta) \quad (22b)$$

and

$$\eta=x^{-(n+1)/(4n+1)}Y. \quad (22c)$$

Functions f_0 and h_0 are determined from

$$(|f_0''|^{n-1} f_0'')' + \frac{2n+1}{4n+1} f_0 f_0'' - \frac{n}{4n+1} f_0'^2 + h_0 = 0\,, \qquad (23)$$

$$\frac{1}{Pr}(|f_0''|^{n-1} h_0')' + \frac{2n+1}{4n+1}(f_0 h_0)' = 0 \qquad (24)$$

with the boundary conditions given by

$$f_0(0) = f_0'(0) = 0, \quad h_0(1) = 0, \quad h_0'(0) = 0\,, \qquad (25\,a)$$

$$f_0'(\infty) = h_0(\infty) = 0\,, \qquad (25\,b)$$

and

$$\int_0^\infty f_0' h_0 \, d\eta = I\,. \qquad (25\,c)$$

Here primes denote differentiation with respect to η. We notice from Eqs. (18 a) and (22 b) that the temperature of the plate is assumed to depend upon x as

$$T_w(x) = T_\infty + Gr^{-b} T_r \, x^{-(2n+1)/(4n+1)}\,.$$

3.2 The second-order outer solution

By substituting Eq. (15) into equation (9), we find that the second-order outer solution is determined by the Laplace equation

$$\nabla^2 \tilde{\psi}_1 = 0 \qquad (26)$$

subject to the matching condition

$$\tilde{\psi}_1(x, 0) = \begin{cases} f_0(\infty)\, x^{(2n+1)/(4n+1)} & x > 0 \\ 0 & x < 0 \end{cases} \qquad (27)$$

and the infinity condition $\partial\tilde{\psi}_1/\partial x = \partial\tilde{\psi}_1/\partial y = 0$. The solution of Eqs. (26), (27) can be shown to be

$$\tilde{\psi}_1 = r^{(2n+1)/(4n+1)} f_0(\infty) \frac{\sin\left[\dfrac{2n+1}{4n+1}(\pi - \varphi)\right]}{\sin\left[\dfrac{2n+1}{4n+1}\pi\right]}\,, \qquad (28)$$

where $r = (x^2 + y^2)^{1/2}$ and $\varphi = \tan^{-1}(y/x)$ are polar coordinates.

3.3 The second-order inner solution

The governing equations and boundary conditions for the second-order inner problem are given by Eqs. (19), (20) with the matching conditions

$$\frac{\partial \psi_1}{\partial Y}(x, \infty) = -x^{-2n/(4n+1)} f_0(\infty) \cot\left(\frac{2n+1}{4n+1}\pi\right), \qquad (29\,a)$$

$$\theta_1(x, \infty) = 0\,. \qquad (29\,b)$$

Note that Eq. (29) are obtained by using Eqs. (14), (15) and (28). It can be shown that this second-order inner problem admits similarity solutions of the form

$$\psi_1 = x^{-(n-1)/(4n+1)} f_1(\eta)\,, \qquad (30\,a)$$

$$\theta_1 = x^{-(5n+1)/(4n+1)} h_1(\eta) \qquad (30\,b)$$

where f_1 and h_1 are determined from

$$(|f_0''|^{n-1} f_1'')' + \frac{2n+1}{4n+1} f_0 f_1'' + \frac{n}{4n+1} f_0' f_1' - \frac{n-1}{4n-1} f_0'' f_1 + h_1 = 0\,, \qquad (31)$$

$$\frac{1}{Pr}(|f_0''|^{n-1} h_1')' + \frac{2n+1}{4n+1} f_0 h_1' + \frac{5n+1}{4n+1} f_0' h_1 + \frac{2n+1}{4n+1} f_1' h_0 - \frac{n+1}{4n+1} f_1 h_0' = 0 \qquad (32)$$

with the boundary and matching conditions,

$$f_1(0) = f_1'(0) = h_1'(0) = 0\,, \qquad (33\,a)$$

$$f_1'(\infty) = -f_0(\infty) \cot\left(\frac{2n+1}{4n+1}\pi\right), \quad h_1(\infty) = 0 \qquad (33\,b)$$

and

$$\int_0^\infty (f_1' h_0 + f_0' h_1)\, d\eta = 0\,. \qquad (33\,c)$$

It is worth mentioning to this end that for $n = 1$, Eqs. (23)–(25) and (31)–(33) reduce to those of Afzal [13] and Ingham and Pop [2] for the problem of convective wall plume in a Newtonian fluid.

An important physical flow quantity of interest in this problem is the local coefficient of skin friction C_f defined as

$$C_f = 2 \frac{\tau_w}{\varrho U^2}$$

where τ_w is the shear stress at the plate given by

$$\tau_w = K\left[J\left(\frac{\partial \bar{u}}{\partial \bar{y}} + \frac{\partial \bar{v}}{\partial \bar{x}}\right)\right]_{\bar{y}=0}.$$

Making use of the variables defined in Table 1 and Eqs. (8), (14), (22) and (30), we obtain

$$C_f\, Gr^{n/(4n+1)} x^{n/(4n+1)} = 2\,|f_0''(0)|^n + 2\, Gr_x^{-n/(4n+1)} |f_0''(0)|^{n-1} f_1''(0) + \text{higher order terms} \qquad (34)$$

where

$$Gr_x = g \beta T_r L^{4(n-1)/(2-n)} \bar{x}^3 (\varrho/K)^{2/(2-n)}$$

is the generalized local Grashof number.

Finally, using Eqs. (14), (22) and (30), we may define the adiabatic wall temperature $\theta_a = (T_w - T_\infty)/T_r$ as

$$\theta_a\, Gr^b x^{(2n+1)/(4n+1)} = h_0(0) + Gr_x^{-n/(4n+1)} h_1(0) + \text{higher order terms}. \qquad (35)$$

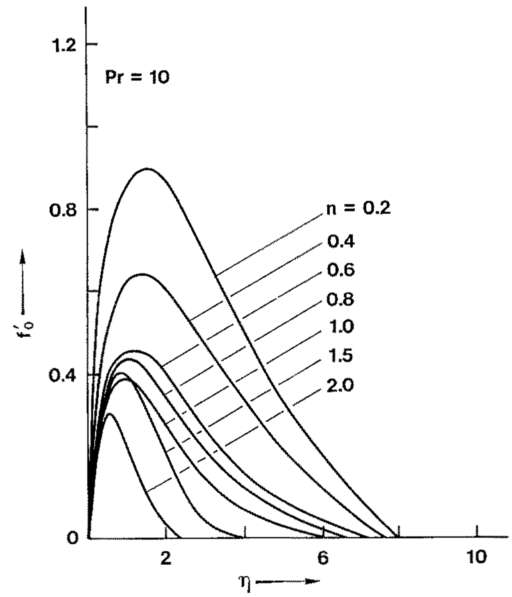

Fig. 1. First order solution for velocity distribution vs. similarity variable, η for various flow behavior index, n ($Pr = 10$)

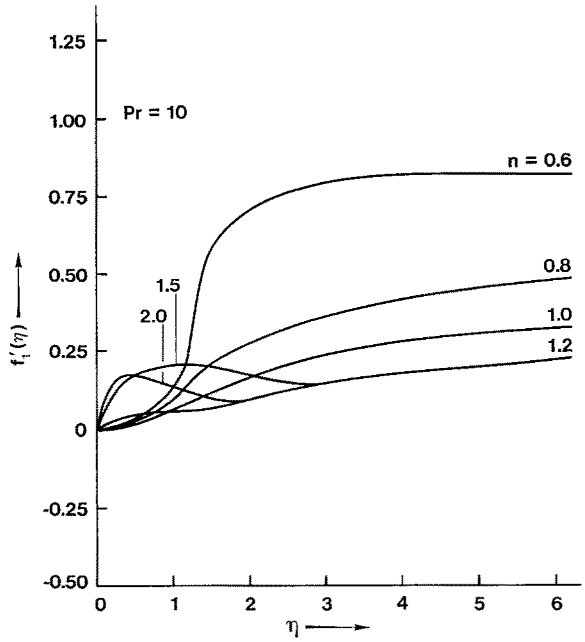

Fig. 3. Second order solution for velocity distribution vs. similarity variable, η for various flow behavior index, n ($Pr = 10$)

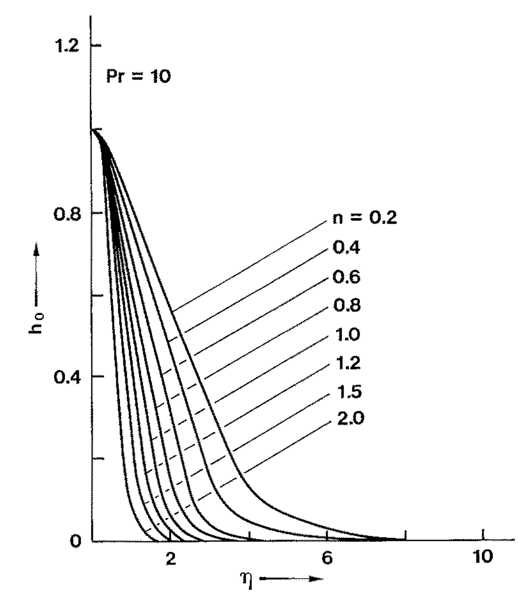

Fig. 2. First order solution for temperature distribution vs. similarity variable, η for various flow behavior index, n ($Pr = 100$)

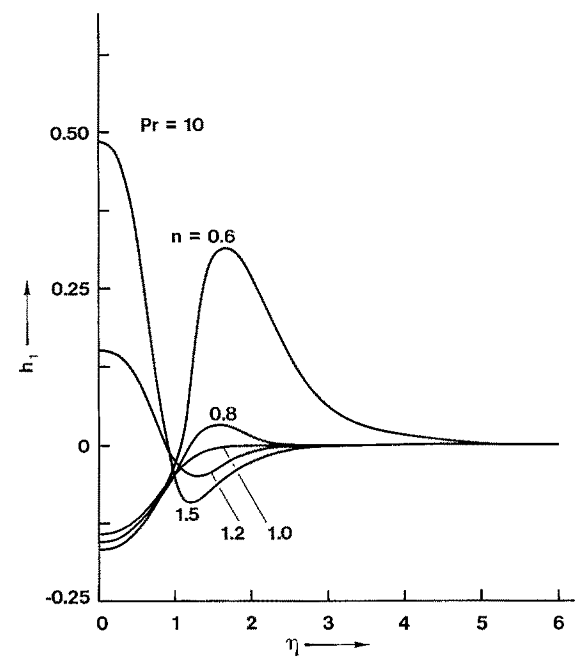

Fig. 4. Second order solution for temperature distribution vs. similarity variable, η for various flow behavior index, n ($Pr = 100$)

4 Results and discussion

Numerical solutions for Eqs. (23)–(25) and Eqs. (31)–(33) were carried out for n ranging from 0.4 to 2.0 and for $Pr = 10$ and 100, respectively. As we have mentioned in Section 2, n is the property of the fluid with $n = 1$ for Newtonian fluids. Non-Newtonian fluids with $n < 1$ are called pseudoplastic (most macromolecular fluids are of that kind with $0.2 < n < 0.6$, see Bird et al. [14]) and those with $n > 1$ dilatant.

The results for $f_0''(0)$, $f_1''(0)$, $f_0(\infty)$, and $h_1(0)$ for selected values of n are tabulated in Tables 2 and 3 for future reference. A complete discussion of the first-order (boundary layer) results has been presented in the previous paper [9]. The results for the first order velocity profile f_0' and temperature profile h_0 are displayed in Figs. 1 and 2, respectively for $Pr = 10$. Figs. 3 and 4 show the numerical results for second order velocity profile f_1' and temperature profile h_1 for $Pr = 10$ and n ranging from 0.4 to 2.0. Fig. 5 shows the vari-

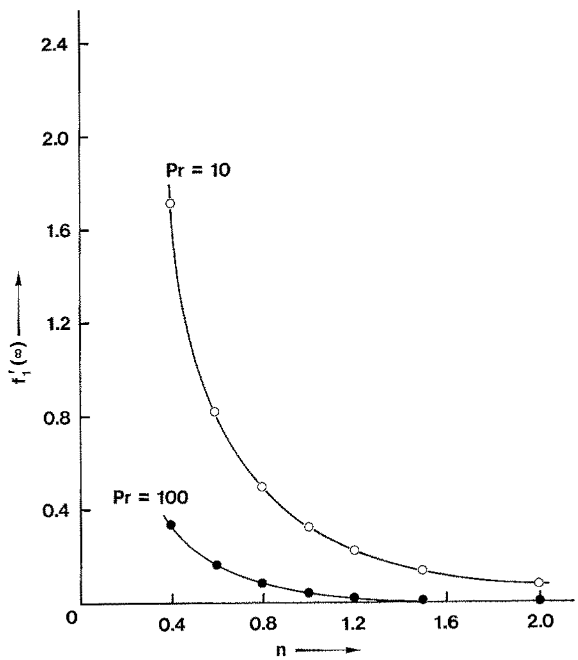

Fig. 5. Second order solution for velocity at the outer edge of the boundary layer, $f_1'(\infty)$ vs. flow behavior index, n for $Pr=10$ and 100

Table 2. $f_0(\infty)$, $f_0''(0)$, $f_1''(0)$, and $h_1(0)$ versus various flow behavior index, n for $Pr=10$.

n	$f_0(\infty)$	$f_0''(0)$	$f_1''(0)$	$h_1(0)$
0.4	2.48399	1.56230	−0.03315	−0.22000
0.6	1.65000	1.08804	0.02196	−0.16500
0.8	1.27000	0.95063	0.01590	−0.15595
1.0	0.99482	0.86123	0.00069	−0.14410
1.2	0.80000	0.90450	0.13457	0.15000
1.5	0.62000	0.82903	0.53149	0.48312
2.0	0.46400	0.76044	0.83138	0.97500

Table 3. $f_0(\infty)$, $f_0''(0)$, $f_1''(0)$, and $h_1(0)$ versus various flow behavior index, n for $Pr=100$.

n	$f_0(\infty)$	$f_0''(0)$	$f_1''(0)$	$h_1(0)$
0.4	0.49000	0.73858	0.00074	−0.00447
0.6	0.32000	0.50828	0.00877	−0.09750
0.8	0.20300	0.41895	−0.00350	−0.23180
1.0	0.12585	0.48062	−0.00500	−0.07948
1.2	0.09210	0.54364	0.00861	0.02513
1.5	0.06500	0.53094	0.05687	0.10000
2.0	0.04200	0.44790	0.20937	0.15000

ation of the second-order velocity at the outer edge of the boundary layer, $f_1'(\infty)$ with n for $Pr=10$ and 100. It is relevant to note that $f_1'(\infty)$ decreases with increasing values of n and Pr. This indicates that the outer potential flow induces a vertical velocity across the boundary layer that decrease when n and Pr increase.

In order to assess the accuracy of numerical results, we have compared our results for $n=1$ (Newtonian fluid) and $Pr=0.72$ with those known from the literature. Thus, the present numerical results for $f_0''(0)$, $f_1''(0)$, and $h_1(0)$ for $n=1$ and $Pr=0.72$ were found to be 1.31005, 0.43805 and 0.49374, while the corresponding values given by Ingham and Pop [2] were 1.31005, 0.43805 and 0.49375. This demonstrates that our numerical results are highly accurate.

From Eqs. (34) and (35) and Tables 2 and 3, we notice that the friction factor and adiabatic wall temperature are under predicted by the first-order boundary layer solution. For values of Grashof number $<0\ (10^5)$, it is observed that the error in using the first order theory amount to about 15%. The second-order correction reinforces the first-order term to augment the friction factor and adiabatic wall temperature. This increase of C_f and θ_a implies a decrease of boundary layers thicknesses.

5 Concluding remarks

In this paper, we have analyzed theoretically the laminar natural convection flow of a power-law fluid generated by a line thermal source embedded in the leading edge of an adiabatic vertical surface with second-order correction in the boundary layer by the method of matched asymptotic expansions. The obtained numerical results allow an accurate evaluation of the velocity and temperature distribution in the boundary layer. It has been shown from the numerical results that the errors in using the first-order theory amount to be 15%. The flow behavior index was varied from 0.4 to 2.0 whereas the Prandtl number was taken as 10 and 100.

Acknowledgements

We appreciate very much the constructive suggestions that were made by Prof. Dr.-Ing. F. Mayinger of the original manuscript.

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Received November 16, 1990