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Communications, transmission, and transportation networks, by H. Frank and I. Frisch, Addision-Wesley, Reading, Massachusetts, 1971, 479 pages, 191 illustrations.

As a discipline, network flow theory, as distinguished from the more descriptive areas of graph theory, is quite young. Originating with studies of the transportation and assignment problems in the 1940's, network theory has rapidly become an essential tool in almost every area of the physical, management and social sciences as well as in combinatorial theory. As a result, many network techniques and models have developed in separate fields and are still scattered among several literatures. In their recent text, Frank and Frisch have attempted to unify some of this literature by giving a thorough discussion, mostly theoretical, of those aspects of graph theory and especially network flow theory that relate directly to physical systems. In doing so, they have drawn primarily from the electrical engineering and applied mathematics literatures, although most of the results presented are applicable to each of the areas mentioned above.

The text treats both deterministic and probabilistic networks. The deterministic sections are prescriptive (almost every result is accompanied by an algorithm), whereas the probabilistic sections are more descriptive. Material is presented, though informally, in a lemma, theorem, algorithm format supplemented by extensive examples and illustrations. It is organized with respect to classes of physical problems. Consequently, some solution techniques reappear at various places throughout the text; also, some material within the same chapter is analyzed by substantially different methods.

The topics discussed in the book may be categorized as either (i) analysis of deterministic systems, (ii) analysis of probabilistic systems, or (iii) synthesis of networks with certain characteristics (mostly for deterministic systems.)

The first category includes the max-flow min-cut theorem (three proofs are given) and the corresponding maximum flow problem followed by problems of multicommodity flow (especially the two commodity case), maximum flow between every pair of nodes in a network, shortest and N^{th} shortest routes, minimal spanning trees, and minimum cost flow with linear and then convex costs. It also includes analysis of distances on graphs, location of (weighted) centers and (weighted) medians of graphs, networks with flow gain on arcs, networks with memory, and several problems concerning arc and node connectivity of networks.

Probabilistic graphs are primarily analyzed under two separate assumptions: (a) that random flow exists in the branches of a given graph and (b) that the branch structure of a graph is not known, but rather is given by some known probability distribution. Assuming (a), the text considers using both statistical and probabilistic procedures to determine what additional flow the system can attain and also how best to redesign branch capacities to meet certain required flows. Assuming (b), it discusses finding the expected number of components and number of descendents of a given node. Under both assumptions, asymptotic results are given and simulation is discussed. In the concluding chapter, some basic queueing problems in networks are also introduced.

For the synthesis problems discussed, data such as the maximum flow between every pair of nodes, the distance structure between nodes or information regarding either arc or node connectivity is given. The objective is to design a network with these characteristics. In some cases, the general problem is unsolved and results are given for a restricted class of networks. For example, when maximum flow data is given, a solution is presented for pseudosymmetric graphs (i.e., the in-degree of each node equals its out-degree).

As is true of many texts, some of the strengths of Frank and Frisch's book are also its weaknesses. The book provides a wealth of material on networks and the exposition is quite detailed, especially in the proofs. Thus, hopefully, it would be suitable as both (i) a reference source to those working in the field, and (ii) a good place for the novice to be introduced to networks. The text does much to meet these dual objectives, but is not entirely successful. The expert will probably find the detailed exposition somewhat of a hindrance; the novice may feel overwhelmed by the amount of material (though this can surely be overcome in a lecture-course environment).

For the most part, the authors have succeeded in giving a complete picture of networks as related to physical systems. In addition to the contents as outlined above, a closing section at the end of each chapter gives a summary of the chapter and comments on related results. Several new topics are also introduced in the exercises. The most glaring omission is the currently active research area of matching theory, which has obvious applications to communication and transmission. Furthermore, it is regrettable that no algorithm is given for the shortest route problem with non-negative distances (though one is cited). This algorithm would have direct application for the reliability problem discussed in the text as well as for the material presented on distances in graphs.

The book contains a number of typographical errors, most of which are easy to pick up. Some may lead to confusion. For example, to be consistent with the material following it, the term $(-k/l)$ in Theorem 8.4.1 should be in the exponent. In addition, there are errors of a more serious nature. Theorem 7.5.2 and the discussion following it are not valid. The statement of that theorem should only be that k is a lower bound on θ, the minimum number of branches whose removal disconnects the graph. In Theorem 6.6.1, the hypothesis that $d(v_i, v_j)>0$ for $v_i \neq v_j$ seems to be required, and on page 27 the text refers to a positive semi-definite quadratic form when concavity requires a negative semi-definite form.

Nevertheless, this book makes a valuable addition to the network flow literature. Due to its scope alone, it belongs in the reference library of anyone with a serious interest in networks. Many of the results, especially those on probabilistic networks and network synthesis, appear in book form for the first time. It should also serve as a valuable text for a graduate network course, especially within an electrical engineering department; or, function as a supplementary text for a broader graph theory course.

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