Computation of the Seismic Stability of Rock Wedges

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Summary

Newmark's concept of computing the permanent displacement under seismic loads has been combined with the conventional limit equilibrium analysis to compute the displacements of a rock wedge. The rock wedge formed by the intersecting planes may or may not have a tension crack in the upper slope surface. As the static analysis of a rock wedge is available from the literature, only the seismic problem is treated theoretically in more details.

A computer program has been developed to compute the displacements from the digitised input data of the acceleration-time-history. The program can take into account the water pressure on the intersecting planes and on the planes of the tension crack.The effect of rock anchors if present is also taken care of in addition to static surcharge loads. The program calculates the conventional static factor of safety, remaining resistance against sliding, the critical acceleration, exciting force, relative velocity with time and the cumulative displacements.

Two model examples are presented: one with simple sinusoidal acceleration and the other one with actual earthquake data considering the different systems of forces acting on the wedge. The results are critically discussed with respect to the different parameters e. g. anchor forces, water pressure and cohesion influencing the magnitude of displacements under seismic loads. It is shown that the critical acceleration is a better index for the seismic stability than the conventional factor of safety.

The critical acceleration presented in this paper serves as a very handy tool for a site engineer to get the first hand information about the stability of the wedge for a given acceleration-time-history without going into the details of dynamic analysis.

Introduction

The conventional limit equilibrium analysis does not take into account the dynamic load and denotes failure when the value of the factor of safety drops below unity. In the conventional design, lowering the factor of safety below unity is not permitted. On the other hand, an attempt to take into account the dynamic forces due to an earthquake by pseudo static analysis leads to a too conservative design, as such analysis considers the seismic forces to be always present which is far from reality. Moreover in both the above analyses, permanent displacements are not permitted at all.

To overcome these limitations $N \in \mathbb{R}$ w m a r k (1965) while delivering the Fifth Rankine lecture, first proposed to compute permanent displacements of earth and rockfill darns when subjected to strong seismic forces instead of computing the conventional factor of safety. This concept of Newmark is well established today. Credit goes to Sarma (1975), Makdisi and Seed (1977) to bring this concept to the level of common application. Subsequently, work has been done to compute the displacement of blocks and slopes subjected to strong motions by Chang et al. (1984) and Lin et al. (1986). Thun and Harris (1981) computed the displacement of a rock fill dam when subjected to earthquake load utilising this concept.

In the present paper, the displacements of a rock wedge when subjected to earthquake loading has been computed utilising the above concept of N e w m a r k and the limit equilibrium analysis utilising the work presented by $H \circ e k$ and $B \r{r}$ av (1977). The exciting force induced by the seismic acceleration is compared with the remaining resistance against sliding obtained by the static limit equilibrium analysis. If the exciting force is greater than the resistance, the relative acceleration is integrated twice to compute the displacements.

Mechanisms of Sliding

The concept of Newmark (1965) of computing the permanent displacements of a body on an inclined plane due to seismic forces allows the factor of safety to drop below unity as this state exists for only a very short while. During an earthquake, two possible reasons may cause the loss of stability of the body.

Firstly the time dependent earthquake forces together with the static forces may bring down the factor of safety below one for a short while. In other words, earthquake forces may decrease the resisting force and increase the driving force in such a way that ultimately the factor of safety drops below unity and sliding starts. But the stable condition will be regained immediately after the earthquake if the engineering properties of the rock wedge on the failure planes are not altered due to sliding. The described procedure may happen several times or very often during an earthquake.

Secondly, if by the earthquake load effects the pore pressure in the sliding planes of the wedge increases, strength characteristics of the sliding plane will get reduced possibly inducing the wedge to slide even under static conditions. In such a situation, till the excess pore pressure generated is not sufficiently dissipated, the factor of safety even after the earthquake will not rise to its pre-earthquake value. This latter problem being basically a static one it is not considered in this paper.

It is assumed that,

- 1) sliding planes exist or develop during the earthquake,
- 2) the wedge slides as a rigid body,
- 3) the strength characteristics of the sliding plane do not change along with the displacements or velocity of the wedge.

The first two assumptions hold quite well for a sliding rock wedge. The inclined intersecting planes of joints or fissures in the rock forming the boundaries of the wedge $-$ with or without tension crack $-$ are the potential surfaces over which the wedge slides down. The behaviour of the wedge as a rigid body is also not far from reality. Hence this assumption will often be a good engineering approach. However, the validity of the third assumption is mooted on the ground that what actually happens to the strength characteristics of the sliding planes when the wedge starts sliding is not uniquely defined. The problem is very complicated as more often than not gaugy material is present on the sliding planes. A lot of theoretical and experimental work has been done but still more is desirable to understand the complicated behaviour. Coefficients of dynamic friction have been evaluated by Sijing etal. (1981) on the basis of laboratory experiments. However, for easy application of the results in this paper strength characteristics are assumed to be constant. Changes of the values of the friction angle or the cohesion in the sliding planes during the earthquake can be considered by varying these parameters in following calculations.

The limit equilibrium analysis of a rock wedge has been delt with by Ho ek and $Brav$ (1977) in great details. Hence in this paper, the conven-

Fig. 1. Schematic diagram of (a) a block subjected to horizontal acceleration, (b) a block subjected to vertical acceleration

tional way of determining the factor of safety for static conditions is not further outlined. The relevant equations may be found in the above mentioned reference.

From limit equilibrium analysis, the factor of safety *FS* is calculated as the ratio of the resisting force (RF) and the driving force (DF) .

$$
FS = \frac{RF}{DF}.
$$
 (1)

The remaining resisting force against sliding *(RS)* is obtained as the difference between the total resisting force and the driving force

$$
RS = RF - DF.
$$
 (2)

Sliding of the wedge starts, if *RS* becomes negative i. e. *DF* is higher in magnitude than *RF.*

Sliding Due to Earthquake

Fig. 1 a shows a block of weight W resting on a plane inclined at an angle α . The friction between the surface of the block and the plane is characterized by the friction angle ϕ . The block is subjected to a horizontal acceleration a_h which induces a force of magnitude $(W/g) a_h \cos \alpha$ in the direction of motion. This force increases the static driving force $W \sin \alpha$. The component $(W/g) a_h \sin \alpha \tan \phi$ acts opposite to the direction of static resisting force $W \cos \alpha \tan \phi$ hence effectively reducing it. From Fig. 1b one can see that the positive vertical acceleration a_v acts opposite to the gravity g and therefore the component of force $(W/g) a_v \sin \alpha$ due to vertical acceleration is directed opposite to the static driving force $W \sin \alpha$. Furthermore the component (W/g) a_v cos α tan ϕ acts opposite to the direction of the

Fig. 2. Schematic diagram of a wedge

static resisting force $W \cos \alpha \tan \phi$. Hence both static forces effectively are reduced by the vertical seismic forces.

The total remaining resisting force *TRS(7)* due to horizontal and vertical accelerations a_h and a_v in addition to the static loads at any instant of time T is given by

$$
TRS(T) = RS_{st} + RS_{dyn}(T), \qquad (3)
$$

where

$$
RS_{st} = RF_{st} - DF_{st},
$$

\n
$$
RF_{st} = W\cos\alpha\tan\phi,
$$

\n
$$
DF_{st} = W\sin\alpha
$$

\nand
\n
$$
RS_{dyn}(T) = RF_{dyn}(T) - DF_{dyn}(T).
$$

\n(4)

The subscript *st* corresponds to static forces whereas by the subscript *dyn* seismic forces are indicated.

Fig. 2 shows schematically a rock wedge, which is formed by two inclined planes A and B and has a tension crack in the upper slope face. The wedge slides down along the intersecting line *CD* at an angle α with the horizontal. Here after the angle α denotes the dip of the line of intersection *CD.*

For the wedge with two different frictional characteristics ϕ_A and ϕ_B on the intersecting planes A and B the dynamic resisting force is calculated by

$$
RF_{dyn}(T) = -(a_h(T)\sin\alpha + a_v(T)\cos\alpha)(W_A\tan\phi_A + W_B\tan\phi_B)/g. (5)
$$

Similarly, the dynamic driving force is obtained from

$$
DF_{dyn}(T) = (a_h(T)\cos\alpha - a_v(T)\sin\alpha)(W_A + W_B)/g, \qquad (6)
$$

where

 W_A and W_B are the weight of the wedge on the plane A and B respectively,

The dynamic limiting equilibrium now is obtained if

$$
TRS(T) = RF_{st} + RF_{dyn}(T) - (DF_{st} + DF_{dyn}(T)) = 0.
$$
 (7)

Sliding starts at that moment of time T at which *TRS (7)* becomes less than zero.

It is obvious and has been shown from a lot of computations, that the effect of the vertical acceleration is not as pronounced as that of the horizontal one. The reason is that the vertical acceleration reduces or increases both the driving and the resisting forces at the same time which on its own does not effect the stability of the wedge. A positive horizontal acceleration

however reduces the resisting force but increases the driving force. As a result in combination of both seismic components and the gravity the net effect of the horizontal acceleration is much more pronounced for destabilizing the wedge as compared to the vertical acceleration as has also been shown by Kobayashi (1984). Thus if one neglects the vertical acceleration specially if it is considerably smaller than the horizontal one and takes into account only the horizontal acceleration, the analysis of the slope does not suffer from too much of error.

Computation of Displacement

For the computation of displacements the acceleration-time-history must be available in discretised form. Since the time-history is considered to be linear within one time step, the discretisation must be dense enough to describe the true acceleration variation with sufficient accuracy for the highest relevant frequency. This means that the time steps should be in the order of a tenth of the shortest high amplitude vibration period.

The dynamic portions of the resisting and the driving forces are then calculated by Eqs. (5) and (6) at the end of each time step and from Eq. (7) *TRS (T)* is computed. Sliding can start only if RS_{dyn} < 0.0. However, as long as *TRS (T)* remains positive sliding does not occur because the magnitude of the sum of the seismic induced forces $(RF_{dyn}(T) - DF_{dyn}(T))$ is less than the static remaining resisting force $(RS_{st}-DF_{st})$. Once $TRS(T)=0$ the wedge is at the state of dynamic limiting equilibrium. Sliding starts if the seismic induced force RS_{dyn} becomes greater than RS_{st} in magnitude. The precise time of initiation of sliding T_0 --- if not falling at the end of a time $step - is computed by linear interpolation. The relative acceleration of the$ wedge $a_r(T_n)$ at the end of any time step *n* is now calculated from the absolute magnitude of $TRS(T_n)$ divided by the mass $m = W/g$ of the wedge.

Since $RS_{st} = -RS_{dyn}(T_0)$ at the stage of limiting equilibrium, the relative acceleration a_r of the sliding wedge at any instant of time T_n within the time duration of sliding is given by

$$
a_r(T_n) = (a_h(T_n) - a_h(T_0)) (W_A/W(\sin \alpha \tan \phi_A + \cos \alpha) + W_B/W(\sin \alpha \tan \phi_B + \cos \alpha)) + (a_r(T_n) - a_r(T_0)) (W_A/W(\cos \alpha \tan \phi_A - \sin \alpha) + W_B/W(\cos \alpha \tan \phi_B - \sin \alpha).
$$
 (8)

where $a_h(T_0)$ is the critical acceleration a_c because any acceleration beyond this value will cause sliding of the wedge if the vertical component of acceleration is neglected.

Relative velocity v_r increases as long as $a_r(T_n)$ remains positive and decreases as soon as $a_r(T)$ becomes negative and comes to zero at that moment when the integral of $a_r(T)$ becomes zero. The relative displacement increases during the total duration of sliding and obtains a finite magnitude when the relative velocity becomes zero.

As the linear variation of acceleration is assumed within one time step, the relative acceleration between the boundaries of the time step is given by the following expression:

$$
a_r(t) = a_r(T_n) + \frac{a_r(T_{n+1}) - a_r(T_n)}{\delta t} t,
$$
\n(9)

where

 T_0 is the time point of initiation of sliding,

 δt is the uniform time interval between \overline{T}_n and T_{n+1} ,

t is the time elapsed since the previous time step $0 \le t \le \delta t$.

These quantities are schematically shown in Fig. 3.

Fig, 3. Schematic diagram of static and dynamic resistance

First integration of the relative acceleration gives the relative velocity and integrating again the relative velocity the displacement is computed. The integration to obtain the cumulative relative velocity $v_r(T_{n+1})$ from the piecewise linear acceleration is performed by

$$
\nu_r(T_{n+1}) = \nu_r(T_n) + (a_r(T_n) + a_r(T_{n+1})) \delta t/2 \tag{10}
$$

and the cumulative relative displacement $s_r(T_{n+1})$ is then calculated from

$$
s_r(T_{n+1}) = s_r(T_n) + v_r(T_n) \delta t + [2 a_r(T_n) + a_r(T_{n+1})] \frac{\delta t^2}{6}.
$$
 (11)

Hence, the cumulative displacement is computed at the end of every time step. This integration is performed as long as the relative velocity is not equal to zero.

The above mentioned concept is schematically shown below in Fig. 4 with a simple rectangular acceleration function. Though theoretically it is not impossible to have the uphill movement of the wedge, but this problem is of no practical interest. Hence it has not been considered in this paper.

Description of the Program

For computing the factor of safety and the displacements, a computer program has been developed which is capable of analysing the stability of a rock wedge under the following conditions:

- **-- tension crack in the upper slope face,**
- water pressure on the intersecting planes,
- water pressure on the plane of the tension crack,
- dead weight surcharge,
- anchor forces,
- seismic loads.

All the above different situations can be analysed independently or in any combination with each other. For all the above cases the program computes the conventional factor of safety and calculates the displacement of the wedge with time when subjected to earthquake load. The program also computes the anchor forces, their dip and dip directions required to raise the factor of safety to the desired level if the computed factor of safety is less than the desired one. This calculation is always performed independently of the presence of seismic forces. The normal output of the program provides a short summary of the results but optionally a long output can be obtained which comprises of all intermediate results of the computation. For dynamic calculations results at every time step can be printed. The program also computes the induced stress in the anchors due to displacements and checks against their failure due to overstressing. The program is in *Fortran* language and the size is such that it can run on any *PC.*

Model Examples

Since the static stability analysis is a routine task nowadays, in the following two examples effects of seismic forces are presented.

Example 1

In the first example for simplicity sinusoidal acceleration of maximum magnitude of 0.3 g with half cycle of 0.4 sec was fed in the computer for a

Fig. 5. The wedge considered in model examples

wedge with a tension crack in the upper slope face but without water in the crack. The geometrical situation of the wedge and the resisting parameters are shown in Fig. 5. The horizontal acceleration-time-history $a_h(T)$, the computed seismic induced force $RS_{dyn}(T)$, remaining static resisting force against sliding RS_{st} , the relative velocity v_r , and the cumulative relative displacement s_r with time are presented in Fig. 6.

From Fig. 6 one can see that till the exciting force reaches the point I the wedge does not slide, as till that time the exciting force remains less than the resistance against sliding. Beyond point I up to the point II the

Fig. 6. Results of model example 1

exciting force is greater than the resistance against sliding and thereby it induces the wedge to slide. Beyond the point II up to the point III though the exciting force is less than the resistance but the velocity of sliding is greater than zero as a result displacement continues. Beyond the point III velocity is zero and the displacement has a constant value. This state presists up to the point IV as between III and IV exciting force always remains less than the resistance. From point IV onwards the same process is repeated as that of point I.

Example 2

For rigorous computation actual earthquake data obtained from Friuli Earthquake of May 1976, Italy, at Codroipo (CNEN-ENEL, 1976) in the digitised form up to 18 sec was fed in the computer. The accelerogram of the N-S horizontal component is presented in Fig. 7. For better demonstration of the seismic effects, in this example the acceleration has been multiplied by a factor of 10. The same rock wedge considered in the example 1, shown in the Fig. 5 has been used here. Several possible conditions of the wedge have been considered: (1) Wedge without tension crack (TC) in the upper slope face $-$ with or without water in the intersecting planes and with or without stabilising anchors. (2) In the second case a wedge with *TC* in the upper slope and the same conditions with respect to water pressure and anchor forces as before has been analysed. The magnitude of total anchor forces was 18000 kN with dip of 11° and dip direction of 350° . For all the above cases the conventional factor of safety by limit equilibrium analysis as well as the maximum displacements, when subjected to the above mentioned earthquake, have been computed. Results are summarized in Table 1 presenting

- -- factor of safety *FS,*
- $-$ static resisting force RS_{st} ,
- **--** final displacement &,
- $-$ critical acceleration a_{α} .

The wedge is subjected to (1) internal forces, which are induced by gravity and seismic acceleration and therefore depend on the mass of the wedge and to (2) external forces, which are due to cohesion in the inter-

Fig. 7. Acceleration time history used in model example 2

secting planes, water pressure and anchor forces. External forces are independent of the wedge mass.

Table 1 shows the results for the wedge without a tension crack and with a tension crack separately. The different cases of external forces may be taken from the table.

	Wedge without crack				Wedge with crack			
	FS	RS_{st} (kN)	a_{cr}	$S_{\mathcal{T}}$ (m/s^2) $(m \times 10^{-2})$	FS	RS_{st} (kN)	a_{cr} (m/s^2)	S_T $(m \times 10^{-2})$
No cohesion No anchor No water	1.169	15 08 1		0.0836 3039.3	1.169	10 9 26	0.0836	3039.3
No cohesion No anchor 5% water	1.132	11808		0.0655 4633.4	1.131	8 5 3 8	0.0653	4639.8
No cohesion Anchor No water	1.427	35756 (1.169) ⁺ (15.076) ⁺	0.1983	$\frac{1}{2}$ 64.6 $(443.2)^+$	1.535	31 602	0.2419	39.51
Cohesion No anchor No water	1.825	73758	0.4090	1.03	1.709	45 9 36	0.3516	4.87
Cohesion No anchor 35% water	1.569	50 852	0.2820	20.54	1.436	29 215	0.2236	52.7
Cohesion Anchor No water	2.128	94 4 34	0.5237	0.01	2.128	66 610	0.5098	0.02
Cohesion Anchor 35% water	1.855	71 527	0.3966	1.45	1.813	49 891	0.3818	2.23

Table 1. *Results of Model Example 2*

* Anchors failed resulting loss of static stability after 13.98 sec.

 $+$ Due to failure of anchor the new factor of safety, the new resistance against sliding and the total relative displacement.

Attention of the reader is drawn for the very high magnitude of relative displacements of the order of tens of meters in the first two cases of the table. These magnitudes of movement if interpreted in physical terms loose all their meaning as all the assumptions are violated and reader may find the wedge totally out of contact from the intersecting planes. But still, these results have been included to show the effect of various physical parameters influencing the displacements. The magnitude of total displacement acceptable without causing any damage or failure comes from the engineering judgement on the limit of tolerance. Hence, it is

obvious that it can not be prefixed for general use. Therefore neither the model presented, nor the program described in this paper have been provided with inbuilt check to terminate computation beyond a limiting magnitude of displacement.

If no cohesion in the intersecting planes was considered to exist only 5 % of the total water pressure was applied to avoid instability under static loads.

In the cases without any external forces the resisting static forces and the dynamic forces are both proportional to the mass of the wedge thus yielding the same *FS* and ultimate displacement for the wedge with and without tension crack. Although the resisting static forces are smaller for the wedge with crack due to the reduced weight the critical acceleration $a_{cr} = a_h(T_0)$ remains the same because of the proportionally reduced seismic forces. Thus if a_{cr} has the same value the double integration of $a_r(T)$ with respect to time must yield the same displacement.

Since the seismic forces are proportional to the mass of the wedge the ratio of RS_{st}/a_{cr} is always constant for a given wedge independently of the presence of the external forces.

The resisting static forces are influenced by the external forces, e. g. are reduced due to water pressure and are increased by the presence of cohesion in the planes and of anchor forces. In this analysis the anchor forces have been assumed to be the same for both types of wedges, e. g. they increase *RS,,* by an amount of 20 675 kN. Their effect is to increase the critical acceleration at both the wedges but more at the wedge with crack. This leads to a greater effect in reducing the ultimate displacement at this wedge.

The inverse effect may be observed due to water pressure but in this case it has to be considered that at the wedge with crack the water pressure does not only act on the remaining part of the intersecting planes but also on the face of the crack. Therefore the forces due to water pressure are not constant but dependent on the geometry, position of the crack and percentage of filling.

The effect of the cohesion is principally the same as that of anchor forces. However due to the presence of the crack the area of the intersecting planes reduces more than the volume of the wedge. Therefore the forces induced by the cohesion reduce more than proportional to the weight of the wedge. Thus the gain in a_{cr} is smaller at the wedge with tension crack then at the wedge without *TC.*

Any decrease of the critical acceleration does lead to larger displacements not only due to the increased $a_r(T)$ but also due to the fact that the transient instability occurs more often during the total duration of the earthquake.

Neither the *FS* nor RS_{st} nor a_{cr} can be used directly for deducing the actual magnitude of displacement. However, these numbers for a given wedge provide some indication on the expected magnitude of displacement. For different wedges FS and RS_{st} are not comparable as may be seen from Table 1. Almost equal values of FS and RS_{st} for two different

wedges result in quite different magnitude of displacement. However, if the critical acceleration is equal for different wedges they will also have equal amounts of displacement.

The judgement if a given acceleration-time-history has to be considered as dangerous to the stability of the wedge should not be based on a chosen value of the factor of safety but on the engineering decision about a tolerable displacement. Without any doubt displacements of several meters lead to a total disintegration and catastrophic failure of the wedge. On the other hand small displacements of the order of a few centimeters may be acceptable depending on the site conditions. However if the displacements are neither very large nor very small the engineering judgement becomes more critical and more emphasis has to be given to determine precisely the geological conditions and the engineering properties of the rock mass. Special care has to be taken to the limit of tolerance of the displacement concerning the anchors if present in the wedge.

Critical Acceleration

As it is not always possible for a practical engineer to go for rigorous dynamic analysis to evaluate the stability of the wedge under seismic forces, the calculation of the critical acceleration can provide an estimate of the risk of sliding. The approach is based on the assumption that the vertical acceleration can be neglected. From the Eqs. (3) to (6) *TRS(T)* becomes equal to

$$
TRS(T) = -(W/g) ah sin \alpha tan \phi - (W/g) ah cos \alpha + RSst, (12)
$$

where ϕ is the average friction angle.

From this Eq. (12) one gets

$$
a_{cr} = RS_{st}/((W/g)(\cos\alpha + \sin\alpha\tan\phi)).\tag{13}
$$

Fig. 8 shows a plot of $a_{cr}/(RS_{st}/(W/g))$ vs α with the variation of ϕ . From the conventional limit equilibrium analysis the value of RS_{st} and the weight of the wedge W is known. The sliding angle and the friction angle ϕ are as well available. Hence very quickly from Fig. 8 one can estimate the critical acceleration. This identifies the acceleration in the given time-history above which the wedge will start sliding or conversely one can estimate the magnitude of critical acceleration below which no permanent displacement of the wedge takes place. Thus for a practical engineer, for quick evaluation, it becomes a very handy tool.

If the friction angle ϕ is quite different on the plane A and B, the average ϕ may lead to erroneous determination of a_{cr} . Hence for more precise determination of a_{cr} , the Eq. (13) is further modified as

$$
a_{cr} = \frac{RS_{st}}{(W_A/g)(\cos\alpha + \sin\alpha\tan\phi_A) + (W_B/g)(\cos\alpha + \sin\alpha\tan\phi_B)}.
$$
 (14)

Fig. 8. Plot of critical acceleration

Concluding Remarks

The concept of Newmark for computing the permanent displacement under seismic condition and the limit equilibrium analysis have been put together in this paper to evaluate the performance of a rock wedge under different conditions.

The maximum computed displacement of the wedge when compared with the limit of tolerance of deformability of the structure directly resting on the wedge or situated on the adjoining area of the wedge will give the slope design engineer a tool to decide on the safety and stability of the structure. If it is necessary to reduce the permanent displacement of the wedge by improving the resistance against sliding by conventional engineering means, their effect can quickly be evaluated independently or in combination with each other by the computer program described in this paper.

From the field data coupled with the results obtained by limit equilibrium analysis, the critical acceleration can very quickly be determined which will give a first hand information about the seismic stability of the wedge without going into the details of the seismic calculations.

Allowing the factor of safety to go below one for a finite duration of time and taking decision on the basis of displacements is a well calculated risk which a design engineer may like to take depending on the situations.

Notations

Following notations are used in the text presented here.

Acknowledgement

The work presented here has been done while the first author was working as a visiting scientist at the LGA - Grundbauinstitut. Authors acknowledge the valuable suggestions given by Dr. E. Gartung of LGA. The visiting programme has been financed by the DAAD at Bonn, FRG.

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