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Information, rational expectations and network equilibria – an analytical perspective for route guidance systems

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Abstract. This paper tries to provide with a unified framework for understanding how drivers act in response to exogenously provided route guidance information; and how they form subjective expectations on traffic conditions from repeated learning. The learning problems are placed in the context of iterative adjustment processes which achieve equilibrium if drivers have rational expectations. Route choice models with rational expectations find a new justification since the models appear as the limits of drivers' learning procedures. This paper is also devoted to the question of whether route guidance information can convey substantial information to drivers even if drivers behave with rational expectations of their environment. The author also tries to propose a framework for designing the optimal route guidance systems.

1. Introduction

Most models currently used to represent route choices were developed in the context of assignment modeling where the main concern is with predicting realistic flows on links rather than choices. An important feature of such models is the representation of the interactions between link travel cost and link traffic volume. Most of these models have been concerned with predicting average conditions over a period of time rather than actual conditions on a particular day (Bonsall 1991). Many of them have sought to generate equilibrium flow patterns which might be expected to come about after a period of time. Achievement of a Wardrop equilibrium solution, wherein no driver can unilaterally reduce his travel time by modifying his current routing pattern is a widely used test of the success of such models.

Most route choice models have assumed that drivers are seeking to minimize their travel cost or some weighted combination of time and distance. Recent

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developments in assignment modeling have been more concerned with the incorporation of network dynamics and stochastic choices – which recognizes differences in the perception, or levels of knowledge, of link cost by selecting values for different groups from a certain distribution –; the latter idea is first put forward by Burrell (1968) and subsequently developed in the stochastic user equilibrium (SUE) models. The SUE approach firstly formulated by Daganzo and Sheffi (1977) has been extended to cope with dynamic network modeling by incorporating driver behavior models and network performance models. This framework explicitly treats the distribution of traffic by time-of-day and the drivers' pretrip and en-route adjustment process (e.g., Fisk 1980; Sheffi 1985; Ben-Akiva et al. 1991).

Given what has been learned from the models and empirical works, it is now the time to develop a more comprehensive framework for understanding drivers' route choice behavior with and without route guidance information. The basic form of the model is determined by the need to be able to represent the performance of route guidance information systems in the context of sporadic and dynamically evolving congestion. It follows that the models must represent drivers' perceptions and expectations as they might evolve on a particular day rather than being concerned with average or equilibrium conditions. If the average performance of the system over a period of time is required it will be necessary to consider a number of days and then derive an average performance rather than take an average day. There has been no obvious analytical solution to this and a dynamic simulation of drivers' learning and fluctuations in their route choices over periods therefore seems necessary.

The basic rationale behind this belief is that many drivers possess little or no reliable information concerning travel routes and alternative travel decisions. As drivers' decisions are affected by expected network conditions, the most useful type of information to a driver faced with travel choices would be reliable predictive information. Predictive information must be based on projected traffic conditions which are dependent on the ways in which drivers will respond to the information. The validity of predicted information depends on their consistency with current and future drivers' choices which depend on their use of such information (Arnott et al. 1991). Thus, the relationships among drivers' expectations, information reliability and drivers' behavior need to be modeled in a way which explicitly describes drivers' evolving perceptions and learning mechanisms.

If the driver preference can be scaled in term of utility and subjective probability and conform to the expected utility hypothesis, then relevant probabilities are the conditional ones given the available information. These conditional probabilities express the drivers' expectations regarding travel cost/time. His expected utility conditioned on both his private information and on the information provided by public agent. A concept of equilibrium emerges if we investigate the possibility of consistency among expectations of various drivers. If drivers are using equilibrium expectations to make inferences about the environment, then this equilibrium takes the special form of a rational expectations equilibrium. This paper tries to provide with a unified framework for understanding how drivers act in response to exogenously provided route guidance information; and how they form subjective expectations on traffic conditions from repeated learning. The learning problems are placed in the context of iterative adjustment processes which achieve equilibria if drivers have rational expectations. Route choice models with rational expectations find a new justification since the models appear as the limits of drivers' learning procedures. This paper is also devoted to the question of whether route guidance information can convey substantial information to drivers even if drivers behave with rational expectations of their environment. The author also tries to propose a framework for designing the optimal route guidance systems.

2. Scope of study

2.1. Problem setting

Route guidance information systems have the potential of reducing or eliminating poor route choices and consequently excess travel distance and cost incurred by unaware or uninformed drivers. However, it is very likely to happen the concentration of traffic on the recommended routes and the overreaction of drivers in their response to guidance information. It is also expected that the reliability of the guidance system will be faded away as the fraction of informed drivers increases. Hence, the impacts of the guidance information itself on drivers' perceptions and expectations need to be explicitly taken into consideration if one tries to design navigation systems providing drivers with route guidance information.

Consider a situation in which a small number of drivers start to receive route guidance information. Assume that the information provided is unbiased. The information corresponds to various signals which reduce or eliminate uncertainty. When a driver is able to efficiently use this information he or she is better off. However a driver may be unable to process this information to select the optimal route if he or she may be distracted by the large amount of available information. Thus, information need to be provided drivers with in an understandable (stylized) way. Consider a more complicated situation in which the majority of drivers receive public information on traffic conditions. In this case, it is very likely to happen that drivers overcorrect their beliefs and drivers' overreaction to public information may cause congestion to transfer from one road to another. Overreaction happens if too many drivers respond to public information on current traffic conditions. It may also generate oscillations in road usage (Boyce 1988).

The above descriptions outline a number of important questions which must be addressed in relation to future developments of electronic route guidance systems. Is it possible to provide drivers with reliable predictive information? Do we have the tools to provide predictive information which is consistent with realized traffic conditions? If the above two questions are answered affirmatively, then when, how frequently, and to whom should such information be provided? The fraction of drivers which should be informed is an important policy variable.

More predictive information is most costly, but may decrease the possibility of an overreaction. When drivers with communication devices receive public information and alter their behavior, they affect driving conditions for others, both those with devices and those without. Moreover, if uninformed drivers know that informed drivers are out there they may adjust their behavior too, albeit on a routine rather than daily basis because they lack day-specific information (Arnott et al. 1991). This may cause informed drivers to make further adjustments, and so on. Thus, the reliability of the route guidance information systems are endogenously determined by the whole drivers' behavior within urban networks.

A driver's behavior changes over time, often from day-to-day, due to learning, expectations formation, variable perception of the reliability of the information received, etc. Thus, in broad terms, any framework aimed at analyzing the potential impacts of route guidance information systems should incorporate dynamic models of drivers' behavior and expectations formation.

2.2. Type of information

When making route choices, drivers constantly combine sources of information to form perceptions and expectations of traffic conditions. Conventional sources of information available to drivers include personal experience, word of mouth, and media messages. Drivers who rely solely on such information are likely to have incomplete information about traffic conditions on the network. Information available to drivers may conceptually fall into three categories: (1) historical information – information describing the state of the transportation system during previous time periods; (2) current information – the most up-to-date information about current traffic conditions; (3) predictive information – information concerning during subsequent time periods when travel can occur. Another classification is also useful for our current purpose: a dichotomy of information into that category of common (public) information and private information. Public information, like knowledge about networks, is, in principle, available to the public, and forms a part of the common knowledge for all drivers in Aumann's sense (Aumann 1976). That category of private information may include a broad spectrum of a driver's information totally hidden to others. A driver's preference, characteristic, historical information and prediction may be classified into this category.

If a state of nature is known and the driver's choices of routes are known to all, each driver will know travel time of the available routes and his corresponding utility. In contrast to the single driver decision problem, we consider multi drivers. Thus, a complete description of a state of nature must contain information not only for resolving uncertainties, but also for determining the extent to which each driver knows states of nature. There must be some states of nature that are distinguishable from others if there is incomplete information. The degree to which natural states are indistinguishable will affect drivers' behavior and must be part of the description of a natural state. Recognizing the possible ability of drivers to differentiate among states allows us to analyze asymmetric information beyond that treated by standard decision theories. A complete description of a network should resolve these uncertainties.

In a world not subject to incomplete information, drivers need look no further than their own preferences to be able to make a decision. They need give no thought to the actions of other drivers. However, in a world subject to incomplete information and random fluctuations, this is no longer the case. Drivers are faced with the problem of forecasting travel conditions which are dependent on the actions of other drivers. Rational expectations theories provide for a model of how drivers make these forecasts. Furthermore, in a world of incomplete information, drivers possibly try to acquire information about the future realization of travel conditions. It will, in general, be the case that different drivers have access to different information. The fact that information is dispersed throughout drivers has the potential to cause a misallocation of route choices relative to what would be the case if all drivers know everything. An efficient allocation of route choices will in general require the transfer of information from public agents who have some information about fluctuations of traffic conditions to individual drivers who can take current actions to mitigate avoidable congestions.

2.3. The rational expectations hypothesis

The past decades have witnessed important developments in the study of the expectations formation processes and the problem of decision-making under uncertainty. Of the theories of expectations formation so far advanced, the rational expectations hypothesis has attracted by far the greatest attention. The rational expectations hypothesis (REH) due to Muth (1961) states that subjective expectations held by economic agents will be the same as conditional mathematical expectations based on the true probability model of the economy; or more generally – that the agents' subjective probability distribution coincides with the objective probability distribution of events. Although the REH was advanced by Muth it was work of Lucas (1978), Sargent (1973), Barro (1976) and others that brought it into prominence. This paper makes no attempts to survey the literature on rational expectations. The reader is referred to Shiller (1978), Sargent (1979) for a macroeconomics and Sheffrin (1983), Radner (1980) for a survey of the microeconomics.

In the given context of drivers' route choice behavior, REH assumes that a driver who has a good understanding of a network can efficiently utilize his daily experience to make inferences about the consequence of the route choices taken by other drivers. These inferences are derived, explicitly or implicitly, from an individual's model of the relationship between the information received by himself and the traffic conditions realized in the network. On the other hand, the true relationship is determined by individual drivers' behavior, and hence by their expectations. The drivers have the opportunities to revise their expectations in the light of observations. Hence, there are feedback routes from the true relationship to the individual expectations. An equilibrium of this system, in which the individual expectations are identical to the true distributions, is called a rational expectations equilibrium (REE). In what follows, we characterize a network equilibrium with incomplete information where the respective drivers may form the rational expectations about traffic conditions.

3. Rational expectations equilibria

3.1. Information structure

In this section, a new analytical framework for network equilibria with rational expectations is presented. The basic element of our network equilibrium concept is differential information; different users have different information about the route traffic conditions; they choose their route on the basis of their private (differentiated) information. The purpose of this section is to develop a general equilibrium concept that makes explicit the information or knowledge that a user has as part of his primitive characteristics. The model we describe in this section is a reinterpretation of Harsanyi's model of incomplete information game (Harsanyi 1967–1968). The difference from Harsany's approach is the explicit consideration of the rational expectations formation by drivers (Kobayashi 1990).

Consider N drivers and a set of drivers S. Let us explain how one can formally describe a driver's information about other drivers' characteristics, preferences and route choices. Driver $s \in S$ has his/her own private information, $\omega_s \in \Omega_s$, which is not observable by others including public agents. Let Ω be the set of all possible $\omega = \cup_s \omega_s$. For driver s, let $\Phi_s(\omega)$: $\omega \to \omega_s$ be an onto mapping defined on Ω. Let ω_s be the signal observed by driver s if ω occurs. Driver s can distinguish between ω' and ω'' if $\Phi_s(\omega') \neq \Phi_s(\omega'')$. If $\Phi_s(\omega) \neq \omega$, the private information space of driver s is called incomplete. Let us define the whole space of private information Θ which is defined by a product of drivers' private information spaces:

$$\Theta = \prod_{s=1}^{N} \Phi_s(\Omega) \ . \tag{1}$$

Let us define information structure $\mu \in \Theta$ which is an explicit representation of the incompleteness of all drivers' private information spaces. The realization of driver s's private information and information structure is represented by $\hat{\omega}_s (= \Phi_s(\hat{\omega}))$ and $\hat{\mu} = \prod_{s=1}^{N} \Phi_s(\hat{\omega}) \in \Theta$, respectively. Further, we assume that there are some common measures concerning the distribution of private information.

3.2. Route choice behavior

Travel time of each route varies from time-to-time depending upon fluctuations of local traffic volume and of individual choices. Denote a set of drivers by $S = \{1, \ldots, N\}$ and a set of the admissible routes for driver $s \in S$ by δ_s. Travel time of route a, $\tau_a (a \in \delta_s)$, is a random variable, and each driver is assumed to forecast the probabilistic distribution of τ_a. Driver s's subjective expectations on τ_a can be formalized by a probability density function $\pi_{as}(\tau_a; \phi)$. The symbol ϕ designates the basic case where no route guidance information is provided. Given $\pi_{as}(\tau_a; \phi)$, the expected utility of driver s for route a is defined by

$$V(\omega_{as}; \phi) = \int U(\tau_a, \omega_{as}) \pi_{as}(\tau_a; \phi) \mathrm{d}\tau_a \ , \tag{2}$$

where ω_{as} is driver s's private information about route a and U is a Neumann-Morgenstern type of utility function. Assume that $\partial U/\partial \tau_a \leq 0$ and $\partial^2 U/\partial \tau_a^2 \geq 0$. In this formulation, expectations are assumed to be independent from private information. This assumption implies that private information conveys no information about the realization of traffic conditions. Rather, private information represents local, accidental and non-memorable factors affecting current drivers' route choices. It varies from day-to-day and causes fluctuations of drivers' route choices.

Let us next investigate drivers' behavior when route guidance information is provided to the public. Let $e \in \eta$ be a message announced to all drivers and η be a set of messages. If messages, for example, 'Choose route 1 ($e = 1$)' and 'Choose route 2 ($e = 2$)' are concerned, then the set of messages is denoted by $\eta = (1, 2)$. Let us describe driver s's subjective expectations conditioned on message e by a tuple of mutually independent probability density functions, $\pi_s(e) = \{\pi_{as}(\tau_a; e), a \in \delta_s\}$, and designate the whole spectrum of subjective expectations conditional on message set η by $\Pi_s(\eta) = \{\pi_s(e); e \in \eta\}$. Each density function specifies a driver's subjective belief regarding a conditional distribution of travel time given a message.

Rigorously speaking, the whole structure of subjective expectations should be described by a multi-variate joint probabilistic density function whose marginal density functions represent the respective subjective expectations for each message in η. However, the estimation scheme constructed in this paper only estimates the 'conditional marginal distributions', without regarding to the joint distribution from which they might be derived. That is, we describe the subjective expectations by a tuple of one-dimensional density functions. The further sophistication in describing the whole structure of subjective expectations is reserved for future research.

Consider a situation where a public agent announce drivers message $\hat{e} \in \eta$. Then, given subjective expectations $\pi_{as}(\hat{e})$ in $\Pi_s(\eta)$ and private information $\hat{\omega}_{as}$, the expected utility of driver s for route a, $V(\hat{\omega}_{as}; \pi_{as}(\hat{e}))$, can be represented by

$$V(\hat{\omega}_{as}; \pi_{as}(\hat{e})) = \int U(\tau_a, \hat{\omega}_{as}) \pi_{as}(\tau_a; \hat{e}) \mathrm{d}\tau_a \ . \tag{3}$$

When this driver chooses the route which maximizes his/her expected utility (3), the chosen route is characterized by

$$\gamma^*_{as}(\hat{\omega}_s; \pi_s(\hat{e})) = \arg\max_a \{V(\hat{\omega}_{as}; \pi_{as}(\hat{e}))\} \ , \tag{4}$$

where the symbol arg designates the route which maximizes the R.H.S. of (4). Extend the above discussions for a single driver to all drivers on the network. A Nash equilibrium induced by a situation where all noncooperative drivers compete with each other with incomplete information on a network environment fully characterizes our equilibrium concept with incomplete information. Given the information structure $\hat{\mu}$ and the message $\hat{e}$, the set of the routes chosen by all drivers – a network equilibrium with incomplete information – can be described by $\gamma^*(\hat{\mu}; \pi(\hat{e})) = \{\gamma^*_s(\hat{\omega}_s; \pi_s(\hat{e}))\}_{s \in S}$. Since, as have repeatedly explained, the information structure $\hat{\mu}$ includes the individual drivers' private information $\hat{\omega}_s$, no one can have access ex ante to the whole results of route choices in each period.

3.3. Network equilibria with rational expectations

After each choice is made, each driver is able to record not only his private information and public information, but also the realization of travel time of each run. After m route choices of route a, the s-th driver obtains an m-size empirical sample from the objective distribution of travel time of this route. Based on the empirical samples, driver s forms his/her subjective expectations on travel time conditional on public information. A rational driver, sooner or later, will be motivated to revise his/her expectations, if he/she learns the differences between expectations and experiences. If both all rational drivers' conditional expectations $\pi_{as}(\tau_a; e)$ and conditional objective distributions $v_a(\tau_a; e)$ simultaneously converge upon the rational expectations $\pi_a^*(\tau_a; e)$, let us call that the system reaches a rational expectations equilibrium conditioned on public information.

A formal characterization of network equilibria with rational expectations appears in Kobayashi (1990, 1993). The existence of rational expectations equilibria is guaranteed under fairly weak network conditions (Mertens et al. 1985; Kobayashi 1993). Rational expectations are the conditions of network equilibria rather than being only the condition of individual rationality. In rational equilibria, the information requirements are no greater; drivers need only know the stochastic processes generating travel time. Though the theory of rational expectations equilibria tells public agents about the underlying structural factors that determine the distribution of travel time, in equilibria drivers need not know anything about the structural form of the systems. They need only know the relationships between public information and stochastic factors that may determine network performances.

It must be noted here that the time interval between periods is short relative to the speed of adjustment of expectations. Before expectations can adjust to a temporary network flow the system will already be at the next period, and the environment will have changed. One will then observe a process of repeated incomplete adjustment, together with stochastic changes in the environment, and the system will always be in disequilibrium in the sense that networks will never equilibrate. Nevertheless, even in this case of repeated disequilibrium one would want to distinguish situations in which travel time and drivers' route choices fluctuated in some 'steady' manner around long-run averages, from situations in which travel costs or route choices, or both, fluctuated with greater and greater variance, or increased without bound. To describe the situation it is natural to use the concept of a stationary stochastic process, which is the generalization to the case of uncertainty of the concept of a deterministic equilibrium. However, it is important to emphasize that the stationarity of a stochastic process does not rule out fluctuations of varying period and amplitude. The drivers can learn what is happening around him/her and to form the rational expectations, since their decision environment is subject to a stationary stochastic process.

4. Rational expectations formation

4.1. Expectations formation by learning

In econometric modeling, rational expectations grew out of dissatisfaction with ad hoc models of expectations formation. Nerlove (1958) used an adaptive expectations model. Since econometric studies rarely had enough data to estimate arbitrary distributed lags with any confidence, it became essential to put some a priori restrictions on the form of the lag structure. One restriction, suggested by Muth (1960), was that the distributed lags should be 'rational'. By this he meant that if the sequence of experience is a particular stochastic process, then the anticipated period price should be given by the conditional expectations given all past realization of the stochastic processes.

Adaptive expectations models have also appeared in route choices literature (e.g., Mahmassani 1990; Iida et al. 1992). These studies presume, implicitly or explicitly, that drivers would have to know more than past experience or they would have to have forgotten them. The problem with ad hoc assumptions underlying adaptive expectations models is that it assumes that drivers have too little uncertainty about the structure. Drivers act as if they are certain that the stochastic process generating travel time has some ad hoc form. The rational expectations hypothesis requires drivers to anticipate the current conditions of the routes according to the objective probability distributions of travel time conditioned on all of their current information. From this, it may seem as if rational expectations assume that drivers know a lot more about the process generating travel time than does the adaptive expectations model. This is true but misleading. Driver know something. Whatever it is that they are uncertain about can be modeled from a Bayesian point of view using the rational expectations approach.

Forming the correct conditional distributions of travel time requires some knowledge of the relation between public information and travel time, which in turn depends on all drivers' subjective expectations. Such knowledge is not likely to be directly available to each driver, so it must be gained by experience. Hence the plausibility of the rational expectations hypothesis hinges on the ability of drivers to learn the correct conditional distributions from repeated observations of network data. The problem of learning rational expectations is greatly complicated by the well-known dependence of the 'correct' conditional distributions on the drivers' beliefs. As precisely described in Kobayashi (1993), a rational expectation equilibrium is essentially a fixed point in a space of conditional distributions. Thus the problem of learning rational expectations is partly a problem of consistent estimation and partly a problem of the stability of equilibria (Jordan 1985).

In this paper, we place the learning problem in the context of the dynamic adjustment process constructed in Bayesian estimation procedures to implement rational expectations equilibria. As the environment is repeated over time, each driver revises his/her estimates of the correct probability distributions conditioned on public information. In what follows, let us give the formal definitions of the adjustment process and the learning problems.

4.2. Learning behavior and rational expectations

A useful way to model a driver's learning process is to imagine that at the beginning of period t, driver s has his/her own subjective expectations on travel time of each route and receives message e from a public agent. Suppose, at the period, the driver makes his/her choice y_s^t based on his/her subjective expectations conditioned on message e. At the end of this period, he/she eventually observes travel time $\tau^t_{\gamma_s}$. $\tau^t_{\gamma_s}$ is commonly observed by all drivers having chosen it, but not by other drivers. The drivers may update their subjective expectations, as far as they are motivated to revise it. But, the learning problem is a little bit complicated by the presence of unobserved routes.

We assume that the learning actions described in the above are repeated over periods, and in each sample period $t(t = 0, 1, 2, \ldots)$ the adjustment process takes place, given all drivers' route choices. For adjustment stage t, driver s must use the data $\tau^t_{\gamma_s}$ to form, in sample period $t+1$, new estimates of the conditional distributions of travel time on public message e. A rule for estimating the conditional distribution is called an estimation procedure, and the entire array of estimation procedures for all drivers and all adjustment stages is an estimation scheme.

Let us show that in a given stationary environment, each driver's subjective expectations converge upon rational expectations through learning processes. Define here a set of historical information. Historical information designates the one which a driver has obtained through past experiences. It comprises four types of information: (1) private information ω^t_s, (2) a route choice in period t, γ^t_s, (3) travel time of the chosen route $\tau^t_{\gamma_s}$, and (4) the message provided by the public agent in period t, e^t. Designate the set of historical data which driver s obtains in period t through his/her choice by a tuple of $\sigma^t_s = (\gamma^t_s, \tau^t_{\gamma_s}, \omega^t_s, e^t)$. Let $\Xi^t_s = \prod_{z=1}^{t-1} \{\sigma^z_s\}$ be the whole spectrum of historical information which driver s have compiled up to period t.

At each stage of the learning processes, the drivers' expectations to message e are determined by their estimation procedures and their historical information accumulated through previous sampling periods. Let $\pi^0_s(e) = \{\pi^0_{as}(\tau_a; e)\}_{a\in\delta_s}$ be driver s's initial beliefs for message e. For each $t > 0$ the drivers' subjective expectations for message e are fully regulated by their past experiences Ξ^t_s and initial beliefs $\pi^0_s(e)$:

$$\pi^t_s(\tau; e) = \phi^t_s(\tau, e; \Xi^t_s, \pi^0_s(e)) \ , \tag{5}$$

where ϕ^t_s represents a 'expectations formation mechanism', which explains how driver s forms his/her subjective expectations from his/her past experiences and initial expectations. The recursive nature of the learning process is critical to the expectations formation scheme. At each stage, learning affects the subjective beliefs being learned at the next stage, but there is no feedback from later to earlier stages. With a certain learning rule γ, the expectations formation mechanism $\phi^t_s(\tau, e; \Xi^t_s, \pi^0_s(e))$ can be expanded in a recursive form:

$$\phi^t_s(\tau, e; \Xi^t_s, \pi^0_s) = \gamma\{\sigma^{t-1}_s, \gamma\{\sigma^{t-2}_s, \ldots, \{\gamma\{\sigma^1_s, \pi^0_s(e)\}\} \ldots\}\} \ . \tag{6}$$

An estimation scheme is successful (a.s.) if, for almost every infinite sample, the estimates converge upon the conditional distributions mentioned in our initial description of the adjustment process given above.

The problem of learning rational expectations is greatly complicated by the dependence of the correct conditional distributions on drivers' beliefs. Indeed, if drivers modify their subjective expectations through learning procedures, their route choice behavior will change. Eventually, the objective distribution of travel time will change time after time. Thus, the bilateral relationships exist between the conditional subjective expectations and the conditional objective distribution of travel time.

In order to remark this fact, let us denote the objective distributions of travel time which are realized under subjective expectations $\pi^t(\tau;e)=\{\pi_s^t(\tau;e)\}_{s\in S}$ by a probability density function $v_a(\tau_a;e,\pi^t)$. In a rational expectations equilibrium, both the driver s's conditional subjective beliefs on message $\pi_{as}^t(\tau_a;e)=\phi_{as}^t(\tau_a,e;\Xi_s^t,\pi_s^0(e))$ and the objective distributions of travel time conditioned on message $v_a(\tau_a;e,\pi^t)$ simultaneously converge upon the rational expectations $\pi_a^*(\tau_a;e)$. Apply a learning rule γ recursively. Given a stochastic environment Ω, γ is said to be *successful almost surely* (a.s.), if for each s, route $a\in\delta_s$ and message $e\in\eta$, and for almost every infinite sample $\Xi_s^{(\infty)}=\prod_{z=1}^{\infty}\{\sigma_s^z\}$,

$$\lim_{t\to\infty}\|\phi_{as}^t(\tau_a,e;\Xi_s^t,\pi_s^0)-\pi_a^*(\tau_a;e)\|=0\ , \tag{7}$$

where 'almost every' refers to the distribution ϕ, and $\|\phi_{as}^t(\tau_a;e)-\pi_a^*(\tau_a;e)\|=\sup_\tau\{|\phi_{as}^t(\tau_a;e)-\pi_a^*(\tau_a;e)|\}$. Recall that $\pi_a^*(\tau_a;e)$ denotes the rational expectations for route a.

In order to guarantee the convergence of learning procedures, probabilistic density functions of travel time $v_a(\tau_a;e,\pi^t)$ should satisfy a certain regular condition. Kobayashi and Fujitaka (1993) show that if $v_a(\tau_a;e,\pi^t)$ is Lipschitzian continuous, then there exists, at least, an almost surely successful learning procedure. A probability density function $v_a(\tau_a;e,\pi^t)$ is said to be Lipschitzian continuous if for each $\pi^t(\tau;e)$ there is a neighborhood $\beta[\pi^t(e)]$ and a constant $\varepsilon>0$ with

$$\|v_a(\tau_a;e,\pi^t)-v_a(\tau_a;e,\pi'^t)\|\le\varepsilon\|\pi^t(\tau;e)-\pi'^t(\tau;e)\| \tag{8}$$

for arbitrary $\pi'^t(\tau;e)\in\beta[\pi^t(e)]$ and all $a\in A$, $e\in\eta,\omega\in\Omega$ and $t\ge0$. The Lipschitzian condition (8) implies the continuous differentiability of a probabilistic density function v with respect to $\pi^t(e)$. It is a rather weak condition, and, in a regular environment, the distributions induced by a broad class of performance functions appeared in the previous literature may satisfy this condition. As far as a stationary environment is concerned, there exist learning procedures by which arbitrary expectations converge upon the rational equilibria. In other words, as far as the individual rationality is working, arbitrary subjective beliefs will converge upon the rational expectations in the long run.

There may be a criticism that the theory of rational expectations equilibria that postulates drivers' route choice behavior makes heavy demands on the individual rationality, insofar as it requires drivers not only to make the usual expected utility calculations in the standard model with uncertainty, but also to have a correct model of the joint distribution of travel time, their own initial information, and the eventual observations of travel time. In a theory of adjustment towards a rational expectations equilibrium, what are the appropriate assumptions about drivers' rationality during learning processes? As drivers revise their models, the true model changes in a way that, in principle, depends on the revision rules of all drivers. In our modeling, drivers' rationality is bounded in the sense that the revision rules only require local and personal information. No feedback mechanisms are assumed in the leaning process. To this point, a theory of through-going rationality, more rational than Bayesian learning, would seem to point to a treatment of the learning and adjustment process as a sequential game with incomplete and imperfect information. This treatment is theoretically possible, but it seems to impose too heavy rationality on drivers. A more realistic alternative would envisage some form of bounded rationality during adjustment process, which, if stable, would converge upon a fully rational equilibrium.

4.3. The non-neutrality hypothesis

The rational expectations model is capable of capturing the idea that public information (route guidance information) conveys individual drivers the actions of other drivers. The ability of public information to aggregate private information perfectly is limited by the extent to which individuals must be able to choose the best route as a return for costs they expend on information acquisition. If public information cannot aggregate private information at all, why would any individual expend money on information acquisition? Here may arises the critical question: given public information, drivers repeat route choices and revise their expectations through learning process; then, can public information convey substantial information to drivers even if they behave with rational expectations?

In order to critically formulate the point, let us imagine the most extreme case where public information cannot convey any more additional information than each driver has. If it is the case, public information will lost its reliability at all. Public information is called 'neutral', if for arbitrary e, $e' \in \eta (e \neq e')$ and sufficiently small ε, there hold

$$\| \pi_a^*(\tau_a; e) - \pi_a^*(\tau_a; \phi) \| < \varepsilon$$
$$\| \pi_a^*(\tau_a; e) - \pi_a^*(\tau_a; e') \| < \varepsilon \ . \tag{9}$$

Public information cannot be neutral in order that public information conveys substantially additional information to drivers, i.e., drivers' conditional rational expectations are differentiated by public messages. Conditions (9) postulate rather extreme cases. However, if it is approximately held, the reliability of public information will be faded away in due course. Thus, our main interest can be crystallized into the question of whether public messages have informational powers in differentiating rational expectations.

The debates on the information neutrality hypothesis are never new. In the fields of economics, there has been the accumulation of literature. Especially, Grossman has investigated in his seminal book (1989) how market prices can aggregate the dispersed private information among traders. Grossman pointed out that whenever traders have heterogeneous beliefs there are incentives to open speculative markets, which tends to homogenize their beliefs. The notion that current prices can convey current information is quite complex. This notion was first introduced by Lucas (1972). Green (1977) used the same equilibrium concept but in the context of traders with heterogeneous information. In Green's model there is a class of traders who have some information about the future value of the commodity. Clearly, in a world subject to uncertainty and asymmetric information, agents will attempt to use their information in a speculative market to earn profit. In doing so, information can get transmitted across agents. Though no one knows everything about the economy, each individual's little piece of information gets aggregated and transmitted to others via trading.

If drivers have the information like market prices into which private information could possibly be aggregated, they had no motive for using public information as 'information', so public information do not affect their beliefs in rational expectations equilibria. However, differed from markets, no information which can aggregate the dispersed private information over drivers are available in networks. There are no ways by which private information can be pooled and made available to all drivers. Public information is provided to drivers unilaterally and communications among drivers are forbidden. When information is privately costly then rational expectations cannot be fully revealing. The games where route choices are made close until there are few enough communications by which uninformed drivers cannot completely free-ride on the informed driver's information. If a driver who bears no costs can learn all the information that other drivers acquire, then this will destroy the incentives for the acquisition of public information. But, it is not the case in networks. Thus it is incorrect to expect rational expectations to be fully revealing of information that would be costly for individuals to acquire. However, there is information that would be costly for an arbitrary individual to acquire, but that was costless to the public agent as shown later. In Sect. 6, we will show by simulation experiments how rational expectations are differentiated by public information. Of course, we need more rigorous investigation about this hypothesis, which are reserved for future research.

5. A Bayesian learning model

5.1. Specification of information structure

Consider a discrete network with a finite number of nodes and links. Suppose that driver s chooses route $a \in \delta_s$ with his/her subjective expectations $\pi_{as}(\tau_a; e)$ given private information ω_{as} and public message e. Let use specify the expected utility function of this driver in the additively separable form with respect to private information:

$$V(\omega_{as}; \pi_{as}(e)) = \int U(\tau_a)\pi_{as}(\tau_a; e)\,\mathrm{d}\tau_a + \omega_{as} \,, \tag{10}$$

where ω_{as} is a random variable representing private information about route a.

If route choice probabilities are mutually independent, the distribution of link traffic volume can be approximated by a multi-variate normal distribution function (Sheffi 1985). If a linear link-performance function is applied, travel time is also subject to a normal distribution function. For a class of non-linear link-performance functions, travel time is generally subject to certain skewed distributions. Normal distributions can be regarded as the second-order approximation of arbitrary probabilistic distributions. Assume the independency among private information, i.e., $E[\omega_{as}\omega_{a's'}] = 0(a \in \delta_s, a' \in \delta_{s'})$. This property implies that private information conveys no information about others' behavior. Private information varies from day-to-day and its variation bears fluctuations of drivers' route choices. Let us specify the representative driver's deterministic utility function by

$$U(\tau_a) = 1 - \exp\{\zeta(\tau_a - E_s[\tau_a])\} - E_s[\tau_a] \ , \tag{11}$$

where $E_s[\tau_a]$ is the expected value of τ_a with respect to his subjective expectations $\pi_{as}(\tau_a; e)$ and ζ is a measure of absolute risk aversion. Equation (11) is the first-order approximation of the utility function at $E_s[\tau_a]$. Let us take the Taylor expansion of the expected utility function conditional on message e (see (10)) around $E_s[\tau_a]$. Then, (10) is approximated by the additive sum of means $\pi_{1as}(e)(=E_s[\tau_a])$, variances $\pi_{2as}(e)$ and private information ω_{as}:

$$V(\omega_{as}; \pi_{as}(e)) = -\pi_{1as}(e) - \tfrac{1}{2}\zeta^2 \pi_{2as}(e) + \omega_{as} \ , \tag{12}$$

where $\pi_{1as}(e)$ and $\pi_{2as}(e)$ characterize driver s's subjective expectations conditional on message e. The route chosen by driver s given $\hat{e}$ and $\hat{\omega}_{as}$ with subjective expectations $(\pi_{1as}(\hat{e}), \pi_{2as}(\hat{e}))$ is described by

$$\begin{aligned} \gamma_s^*(\hat{\omega}_s; \pi_s(\hat{e})) &= \arg\max_a \{V(\hat{\omega}_{as} : \pi_{as}(\hat{e}))\} \\ &= \arg\max_a \{-\pi_{1as}(\hat{e}) - \tfrac{1}{2}\zeta^2 \pi_{2as}(\hat{e}) + \hat{\omega}_{as}\} \ . \end{aligned} \tag{13}$$

5.2. Bayesian learning rules

Describe expectations formation mechanisms by a Bayesian estimation method. In a Bayesian framework the probability is defined in terms of a degree of belief. The probability of an event is given by an individual's belief in how likely or unlikely the event is to occur. This belief may depend on quantitative and/or qualitative information, but it does not necessarily depend on the relative frequency of the event in a large number of future experience. Because this definition of the probability is subjective, different individuals may assign different probabilities to the same events.

Assume that both subjective expectations and objective distributions of travel time are simultaneously subject to one-dimensional normal distributions. Let us characterize subjective expectations of driver s on travel time of route a in period t by two parameters (mean $\pi_{1as}(e)$ and variance $\pi_{2as}(e)/2$)

$$\pi^t_{1as}(e) = \phi^t_{1as}(e)(\Xi^t_s, \pi^0_{as}(e))$$
$$\pi^t_{2as}(e) = \phi^t_{2as}(e)(\Xi^t_s, \pi^0_{as}(e)) \ , \tag{14}$$

where $\phi^t_{1as}(e), \phi^t_{2as}(e)$ are the estimation models of the means and variance of driver s's conditional subjective expectations on e for route a in period t, respectively. The mechanisms of expectations formation are, thus, described by a set of functions of a set of historical information Ξ^t_s and initial expectations $\pi^0_{as}(e)$.

For any driver, the true parameters of the objective distributions of travel time are unknown. Assume that driver s obtains $n_a(e)$ observations of travel time of route a under message e up to period t. The driver forms his/her subjective expectations $\pi^t_s(e) = (\pi^t_{1as}(e), \pi^t_{2as}(e))$ based on observations $\tilde{\tau}^t_a(e) = \{\tilde{\tau}_{1a}, \tilde{\tau}_{2a}, \ldots, \tilde{\tau}_{n(e)}\}$. Assume that the driver chooses route a with reference to his/her current $\pi^t_s(e)$. After the route choice is made, a new observation $\tilde{\tau}_{n+1_a}$ is compiled in $\tilde{\tau}^t_a(e)$. This driver updates $\pi^t_a(e)$ based on $\tilde{\tau}^{t+1}_a(e)$ in order to form $\pi^{t+1}_a(e)$. Hereafter, omit subscripts s, a and symbols t, e for the simplicity of expression.

Let us assume that the objective distribution of travel time which is realized under a set of subjective expectations π^t is subject to a normal distribution $N(\theta_1, \theta_2/2)$. Define a vector of unknown parameters by $(\boldsymbol{\theta} = (\theta_1, \theta_2) \in \Theta)$, where $\Theta = \{(\theta_1, \theta_2) \mid \theta_1 > 0, \theta_2 > 0\}$. Observations of travel time are supposed to be samples from a normal distribution $N(\theta_1, \theta_2/2)$. Then, a joint density function $f(\tau \mid \boldsymbol{\theta})$ of $\tilde{\tau}$ is given by

$$f(\tau \mid \boldsymbol{\theta}) = (2\pi)^{-(n/2)} \theta_2^{-(n/2)} \exp\left[-\theta_2^{-1}(n(\theta_1 - \bar{\tau})^2 + s^2)\right] \ , \tag{15}$$

where $\bar{\tau} = 1/n \cdot \sum_{i=1}^{n} \tau_i$ and $s^2 = \sum_{i=1}^{n} (\tau_i - \bar{\tau})^2$. View $f(\tau \mid \boldsymbol{\theta})$ as a function of $\boldsymbol{\theta}$. Assume the true parameter θ_2 is known; then, the conjugate prior distribution of θ_1 in (15) is given by $N(\mu, \theta_2/2\nu)$. By integrating $f(\tau \mid \boldsymbol{\theta})$ on $\boldsymbol{R}$ with respect to θ_1, the conjugate prior distribution of θ_2 is given by an inverted chi-square density function $\chi^{-2}(2\alpha, \beta)$ (DeGroot 1970). Thus, the marginal conjugate prior density functions become:

$$\xi_1(\tilde{\theta}_1 \mid \tilde{\theta}_2 = \theta_2) \sim N(\mu, \theta_2/2\nu)$$
$$\xi_2(\tilde{\theta}_2) \sim \chi^{-2}(2\alpha, \beta) \ , \tag{16}$$

where $\sim$ denotes 'proportional to'. Then, the joint conjugate prior density function for parameters $(\tilde{\theta}_1, \tilde{\theta}_2)$, i.e., $\xi(\tilde{\boldsymbol{\theta}}) = \xi_1(\tilde{\theta}_1 \mid \tilde{\theta}_2) \cdot \xi_2(\tilde{\theta}_2)$, is subject to a normal-inverted chi-square density function $N\text{-}\chi^{-2}(\mu_0, \nu_0, \alpha_0, \beta_0)(\mu_0 > 0, \nu_0 > 0, \alpha_0 > 0, \beta_0 > 0)$ (DeGroot 1970).

The posterior density function for $\boldsymbol{\theta}$ given a set of experience information $\tilde{\tau} = (\tilde{\tau}_1, \ldots, \tilde{\tau}_n)$ is given by a function of $\tilde{\bar{\tau}} = \bar{\tau}\left(= \sum_{i=1}^{n} \tilde{\tau}_i\right)$ and $\tilde{s}^2 = s^2\left(= \sum_{i=1}^{n} \cdot (\tilde{\tau}_i - \tilde{\bar{\tau}})^2\right)$ such that:

$$\xi(\theta \mid \tau) \propto \xi(\theta) f(\tau \mid \theta)$$
$$\propto \theta_2^{-(1/2)} \exp\left[-\frac{\nu_0(\theta_1-\mu_0)^2+n(\theta_1-\bar{\tau})^2}{\theta_2} \right]$$
$$\cdot \theta_2^{-[1+\alpha_0+(n/2)]} \exp\left[-\frac{\beta_0+s^2}{\theta_2} \right] . \tag{17}$$

Considering $\nu_0(\theta_1-\mu_0)^2+n(\theta_1-\bar{\tau})^2 = (\nu_0+n)\left(\theta_1-\frac{\nu_0\mu_0+n\bar{\tau}}{\nu_0+n}\right)^2+\frac{\nu_0 n}{\nu_0+n}(\bar{\tau}-\mu_0)^2$, (17) can be rewritten to

$$\xi(\theta \mid \tau) = \xi(\theta \mid \bar{\tau}, s^2)$$
$$\propto \theta_2^{-1/2} \exp\left[-\frac{\nu_1(\theta_1-\mu_1)^2}{\theta_2} \right] \cdot \theta_2^{-(1+\alpha_1)} \exp\left(-\frac{\beta_1}{\theta_2} \right) . \tag{18}$$

Thus, the posterior distribution of θ is also subject to a normal inverted chi-square density function. These posterior density functions represent drivers' current state of knowledge (prior and sample) about parameters. Between the natural conjugate prior density function and its corresponding posterior function, there holds

$$\xi(\tilde{\theta} \mid (\tilde{\tau}) = \tau) \sim \tilde{\theta} \mid (\tilde{\bar{\tau}} = \bar{\tau}, \tilde{s}^2 = s^2) \sim N-\chi^{-2}(\mu_1, \nu_1, \alpha_1, \beta_1) . \tag{19}$$

Thus, we have the relationship between the parameters of the natural conjugate prior and its corresponding posterior:

$$\mu_1 = \frac{\nu_0\mu_0+n\bar{\tau}}{\nu_0+n} , \quad \nu_1 = \nu_0+n$$
$$\alpha_1 = \alpha_0+\frac{n}{2} , \quad \beta_1 = \beta_0+s^2+\frac{\nu_0 n}{\nu_0+n}(\bar{\tau}-\mu_0)^2 . \tag{20}$$

From (18), we can easily calculate

$$E[\tilde{\theta}_1 \mid \tau] = \mu_1$$
$$E[\tilde{\theta}_2 \mid \tau] = \frac{\beta_1}{\alpha_1} . \tag{21}$$

5.3. *Bayesian recursive formulae*

Drivers can enrich their historical information through their daily route choices. They are motivated to update their subjective expectations, as far as they recognize the differences between their beliefs and experiences. A Bayesian learning procedure is described by a set of updating formulae of $(\pi_{1as}^t(e), \pi_{2as}^t(e))$. This can be achieved in a recursive fashion. Assume that $(\pi_{1as}^t(e), \pi_{2as}^t(e))$ are the posterior parameters estimated based on observations up to period t. In period t, driver s receives message e and chooses route a. Let us again omit subscripts a, s, e at the moment. Then, given the additional observations (a, τ_a), applying (20) and (21), the posterior expectations can be given by

$$\pi_1^{t+1} = \frac{\nu_0 \mu_0 + n^t \bar{\tau}_t}{\nu_0 + n^t} , \tag{22}$$

$$\pi_2^{t+1} = \left\{ \beta_0 + \bar{s}_t^2 + \frac{\nu_0 n^t}{\nu_0 + n^t} (\bar{\tau}_t - \mu_0)^2 \right\} / \alpha_t , \tag{23}$$

where $\alpha_t = \alpha_0 + n^t/2, \bar{\tau}_t = 1/n \cdot \sum_{j=1}^{n^t} \tau_j$, $\bar{s}_t^2 = \sum_{j=1}^{n^t} (\tau_j - \bar{\tau}_t)^2$ and n^t is the number of observations on route a up to t-th period. Once these quantities are obtained it is straightforward to derive the updating formulae. By expanding (22), the learning rule γ of mean π_1^t is described by the following recursive formula:

$$\pi_1^{t+1} = \pi_1^t + \frac{1}{\nu_0 + n^t} \cdot (\tau_t - \pi_1^t) . \tag{24}$$

Thus, forecasting errors $(\tau_t - \pi_1^t)$ are utilized to yield new expectations π_1^{t+1}. Note that $1/(\nu_0 + n^t)$ is no more constant. If n^t becomes large, this weight approaches to zero. In the long-run, forecasting errors play no decisive role in expectations formation. Expectations converge upon the stational one.

Analogous results can be derived for π_2^t. From (23), we have:

$$\pi_2^{t+1} = \pi_2^t + \frac{1}{\alpha_t} \left\{ \frac{\nu_{t-1}}{\nu_t} (\pi_1^t - \tau_t)^2 - \frac{\pi_2^t}{2} \right\} , \tag{25}$$

where $\alpha_t = \alpha_0 + n^t/2, \nu_t = \nu_0 + n^t$. The posterior variance π_2^{t+1} is also revised based on forecasting errors $\nu_{t-1}(\pi_1^t - \tau_t)^2/\nu_t - \pi_2^t/2$. As n^t becomes large, $1/\alpha_t$ approaches to 0. The recursive formulae (24) and (25) embody the assumption that drivers cannot observe travel time of any routes other than the chosen one. Hence, subjective expectations for route $j(\in \delta_s) \neq a$ are not updated until route j will be chosen. That is, for $j(\in \delta_s) \neq a$ we assume that

$$\pi_{1js}^{t+1} = \pi_{1js}^t , \quad \pi_{2js}^{t+1} = \pi_{2js}^t . \tag{26}$$

If t becomes sufficiently large, from (22) and (23), we know that π_1^t, π_2^t can be approximated by

$$\pi_1^t \simeq \bar{\tau}_t , \quad \pi_2^t \simeq \frac{\bar{s}_t^2}{n^t} , \tag{27}$$

where $\bar{\tau}_t$ and $(\bar{s}_t^2/n^t)$ are the sample mean and variance, respectively. As drivers obtain more observations, their subjective expectations asymptotically converge upon the objective one. Thus, given a set of arbitrary initial subjective expectations, the rational expectations appear as the limits of drivers' learning procedures.

6. Route navigation effects of information

6.1. Information systems for route navigation

In order to investigate the impacts of public information on individual drivers' decisions and on expectations formation, simulation experiments are carried out. Given a description of the route choice context, of which are specified in such a way as to bear simultaneously to actual conditions, drivers independently supply decisions of route to destination. These decisions form the time-varying input function to a traffic simulator that yields the corresponding travel time. Information on these consequences is subsequently provided to each driver. By controlling the type and amount of information supplied to drivers, we can study the impacts of alternative information strategies on drivers' behavior as well as their expectations.

This section is devoted to the question of whether route guidance information can convey substantially additional information to drivers even if drivers behave with rational expectations of their environment. At the beginning of period t, a public agent is supposed to observe traffic volumes at monitoring points on a network, and to forecast travel time of each route to be realized in period t. Denote the set of historical data observed at monitoring points up to this period by $\chi^t = \{x_i^k (i = 1, \ldots, n; k = t, t-1, \ldots)\}$. Given message $e \in \eta$, the public agent forecasts the conditional objective distributions of travel time $\Psi(\chi^t; e) = \{\Psi_a(\tau_a; \chi^t, e) : a \in \delta_s)\}$ based on χ^t. The forecasting mechanism is generally described by $\Psi(\chi^t, e) = \Gamma(\chi^t; \Delta(e), \eta)$, where $\Delta(e)$ represents a forecasting model describing how drivers act in response to message e. Thus, the public agent forecasts the conditional objective distributions for each message by use of monitoring information χ^t and a forecasting model $\Delta(e)$. $\Psi(\chi^t, e)$ need not coincide with drivers' subjective or rational expectations, or both. The public agent can have richer information than the drivers, since the drivers have generally no access to monitoring information. There exists information asymmetricity between the public agent and the drivers. This informational advantage put the public agent the way of manipulating, in some ways or another, the drivers' route choices.

Three alternative forecasting mechanisms are applied to simulation experiments: (1) Γ^{**}: the ideal forecasting mechanism, that can describe the full spectrum of drivers' behavior with complete and perfect information; (2) Γ^{*}: the incomplete forecasting mechanism, which incorporates a route choice model with rational expectations (Kobayashi 1993); (3) Γ°: the naive forecasting mechanism, which adopts a standard deterministic model of traffic assignment. The ideal forecasting mechanism Γ^{**} is a fully hypothetical one in the sense that the public agent is assumed to have the full knowledge about drivers' private information.

Though Γ^{**} cannot be constructed in the real world, we can hypothetically constitute it in the experiments. The experiments by Γ^{**} can envisage the ideal situations with which we can compare the informational efficiency of alternative forecasting mechanisms. For Γ^{*}, the public agent is only requested to observe the rational expectations that all drivers share. For Γ^{*}, the public agent need not forecast precisely travel time to be realized in each period, but rather is requested to forecast the conditional objective distributions of travel time for each message. Γ° is the simplest one, and does not incorporate any forecasting models of drivers' behavior into it.

The selection rule Λ designates a mechanism that selects the best message $\hat{e}$ to be announced to all drivers from the set of all messages. Λ is generally described by a system $\hat{e}=\Lambda(\Psi(\chi^{t},e);e\in\eta)$, where η is a set of messages and $\Psi(\chi^{t},e)$ is the outputs of Γ. Let us design three alternative rules: (1) Λ_a: to announce the forecasted travel time of all routes, (2) Λ_b: to recommend the route to be chosen, (3) Λ_c: to announce whether congestion is expected or not in each route.

6.2. *Evaluation of information systems*

Concerning a situation involving a single environment with no uncertainty, the evaluation of the informational efficiency of route guidance information systems $\theta=(\Gamma,\Lambda)$ is rather straightforward. When we add uncertainty, the simplest case is one in which the uncertainty is identical across agents and in which there are no contingent situations. In this case, it has been customary to distinguish between ex ante and ex post efficiency (Hölmstrom and Myerson 1983; Postlewaite et al. 1987). In the case of incomplete information, the situation is much more complex. There may be uncertainty as to a driver's ranking of alternative routes. The question arises whether a comparison of alternatives should be pointwise across his possible private information or in expectation across his information. This latter point can be clarified in terms of the timing of the welfare evaluation with respect to the possible stages of information.

Assume that at the beginning of each period, driver s has observed his/her private information $\hat{\omega}_s$, and it is not changed throughout the period. Assume that the driver has already formed rational expectations and chooses route $\hat{a}$. Then, the ex ante expected utility $V(\hat{\omega}_{\hat{a}s};\pi^{*}_{\hat{a}}(e))$ conditional on message $\hat{e}$ of route $\hat{a}$ is given by

$$V(\hat{\omega}_{\hat{a}s};\pi^{*}_{\hat{a}}(\hat{e}))=-\pi^{*}_{1\hat{a}}(\hat{e})-\frac{1}{2}\zeta^{2}\pi^{*}_{2\hat{a}}(\hat{e})+\hat{\omega}_{\hat{a}s}\ . \tag{28}$$

On the other hand, from (11), the ex post utility function is:

$$U(\tau_{\hat{a}},\hat{\omega}_{\hat{a}s})=1-\exp\left(\zeta(\tau_{\hat{a}}-E^{*}[\tau_{\hat{a}}])\right)-E^{*}[\tau_{\hat{a}}]+\hat{\omega}_{\hat{a}s}\ . \tag{29}$$

In general, a driver's ex ante expected utility does not coincide with his ex post utility after his choice is made.

If all drivers behave with rational expectations, their ex ante expected utility coincides with the long-run average of the ex post utility. If this is the case, the public agent can know both the drivers' ex ante expected utility and the long-run average of the ex post utility by observing the conditional objective distributions of travel time. The long-run average of the ex post utility $EV_s(e;\theta)$ conditioned on message $e \in \theta$, if the environment is stationary, can be defined by

$$\begin{aligned} EV_s(e,\theta) &= E_{\omega_s}[\max_a\{V(\omega_{as};\pi_a^*(e))\};\theta] \\ &= \sum_a \int_{-\infty}^{\infty} V(\omega_{as};\pi_a^*(e))\,\psi(\omega_{as})\mathrm{d}\omega_{as} \prod_{a'\neq a} \int_{-\infty}^{\bar{v}_{a's}(e)} \psi(\omega_{a's})\mathrm{d}\omega_{a's} \;, \end{aligned} \tag{30}$$

where $\bar{v}_{a's}(e) = \bar{c}_a(e) - \bar{c}_{a'}(e) + \omega_{as}$, E_{ω_s} means the expectation with respect to private information ω_s, and $\bar{c}_a(e) = -\pi_{1a}^*(e) - 1/2\cdot\zeta^2\pi_{2a}^*(e)$ is the deterministic part of the ex ante expected utility function. Let $\beta_s(e,\theta)$ denote the relative frequencies that drivers receive message e from information system θ. Then, the social welfare function $SW(\theta)$ is given by

$$SW(\theta) = \sum_s \sum_e \beta_s(e,\theta) EV_s(e,\theta) \;. \tag{31}$$

The route guidance information systems, $\hat{\theta} = \{\hat{\Gamma}, \hat{\Lambda}\}$, should be designed in order to increase the social welfare function (31) as much as possible.

6.3. Simulation experiments

a) Description of the simulation. To simplify the experiments, we consider only one O-D pair connected by two routes with different characteristics as shown in Fig. 1. Drivers are informed public messages at bifurcation point A. The link performance functions are given in the forms of linear functions, whose parameters are also shown in Fig. 1. The drivers' initial expectations for travel time of routes 1 and 2 are assumed to be homogeneous. They are described by normal distribu-

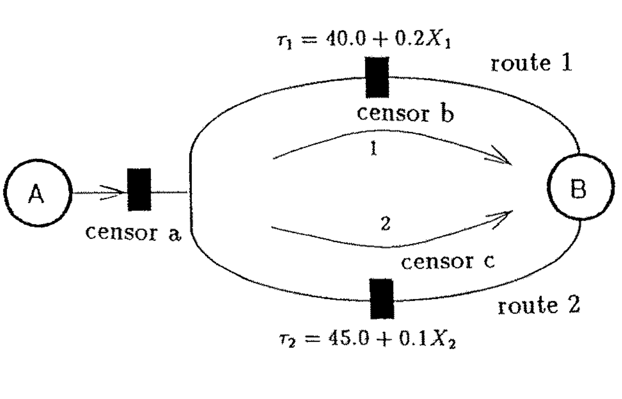

Fig. 1. A hypothetical network for numerical examples

tions $N(50, 0)$. The private information ω_{as} is subject to a Weibull distribution $W(0, 10)$. The overall turbulence of travel time by local traffic is given by a normal distribution $N(25, 10)$. It is assumed that 100 risk-neutral drivers ($\zeta_i = 0$) are motivated to make simultaneous decisions in each iteration. The travel time of both routes is varying over periods due to the fluctuation of local traffic and of all drivers' route choices. At the beginning of each period, the public agent can observe the local traffic volume of the period, but drivers cannot know it.

b) The impacts of the rules for providing information. In order to compare the efficiency of alternative rules for providing public information in the ideal situation, let us first apply Γ^{**} to forecast travel time. In Γ^{**}, the public agent is assumed to know exactly $\pi_s(e)$ and ω_{as}. Γ^{**} enables the public agent to calculate both the ex ante conditional expected utility on e (see (28)) and the ex post conditional utility on e (see (29)). Let us prepare three alternative cases: (1) to apply Λ_a (Case 1); (2) to apply Λ_b (Case 2); (3) to apply Λ_c (Case 3).

Simulation is operated according to the following steps: a) to assume each driver's initial subjective expectations $\pi^0_{as}(e)$; b) to generate normal random numbers representing local traffic volume and Weibull random numbers associated with private information; c) to calculate each driver's conditional expected utility for every message $e \in \eta$ by (12); d) to forecast the individual route choices for each case where the respective message is announced; e) to forecast travel time by aggregating the individual route choices for the respective cases; f) to calculate the conditional ex post utility (29); g) to aggregate the conditional ex post utility over all drivers for the respective cases; h) to select the message which maximizes the aggregated ex post utility; i) to determine the individual drivers' choices in the current iteration; j) to update each driver's subjective expectations by (24).

Simulation continued over 400 periods. The drivers' subjective beliefs on travel time converge upon the rational expectations by around 200-th periods. Figure 2 illustrates the impacts of alternative rules on the objective distributions of travel time. Let us focus on the sample periods when Λ_a selects message e = *(the travel time of route 1 is 60 min, the travel time of route 2 is 55 min).* For each sample period, we calculate the respective average of travel time which has been observed up to the concerned period. Figure 2 shows the changing patterns of thus calculated means of travel time. In this figure, (a) stands for route 1 and (b) does for route 2. For all cases, the differences in travel time of both routes decrease from the case when no public information is provided. Thus, as far as our simulation is concerned, public information is not neutral and conveys substantially additional information to drivers. Λ_a can bear the most informative messages, and Λ_c does the poorest ones. From Fig. 2, we know that Λ_a can provide with the smallest differences between the travel time of both routes. Figure 3 explains the relationship between the information rules and the social welfare indices $SW(\theta)$. It clearly shows that as messages become more informative, $SW(\theta)$ becomes larger.

c) The comparison of forecasting mechanisms. Let us compare the efficiency of alternative forecasting mechanisms: Γ^{**}, Γ^{*}, and Γ°. Simulation experiments by Γ^{*} presume that the public agent cannot observe the drivers' private information.

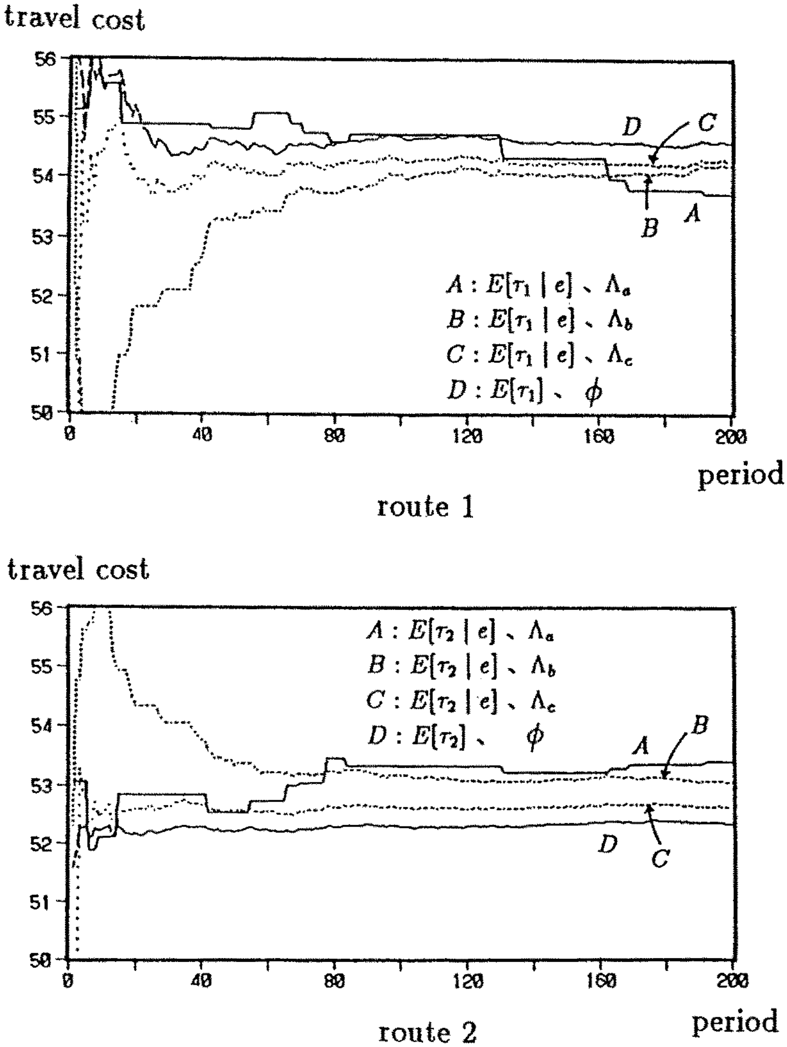

Fig. 2. Public information and learning procedures

The simulation procedures for Γ^* can be made by slightly modifying steps d) and f) of Γ^{**}. If we assume that ω_{as} is distributed according to I.I.D. Weibull distributions with zero mean and variance λ: i.e., $f(\omega_{as}) = \lambda \exp(-\lambda \omega_{as}) \exp(-\exp(-\lambda \omega_{as}))$, the conditional choice probability $p_a(e)$ can be given by the following multinominal logit model:

$$p_a(e) = \frac{\exp\{\lambda[-\pi_{1a}^*(e) - 1/2 \cdot \zeta^2 \pi_{2a}^*(e)]\}}{\sum_{b \in \delta} \exp\{\lambda[-\pi_{1b}^*(e) - 1/2 \cdot \zeta^2 \pi_{2b}^*(e)]\}} . \tag{32}$$

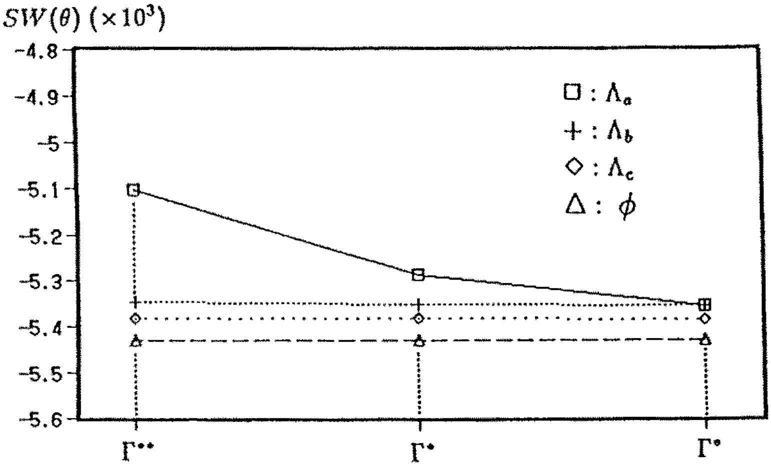

Fig. 3. The comparison of the efficiency of forecasting mechanisms

The public agent forecasts the conditional mean of travel time $\bar{\tau}_a(e)$ by the logit model. $\bar{\tau}_a(e)$ is available only to the public agent. On the other hand, the drivers' private information is totally hidden to the public agent, and the public agent cannot calculate the drivers' conditional ex post utility (29). Instead, it can calculate the ex ante average of the ex post utility $EU_s(e)$ by

$$EU_s(e;\theta)=\sum_{a}\int_{-\infty}^{\infty}U(\bar{\tau}_a;\omega_{as})\psi(\omega_{as})\mathrm{d}\omega_{as}\prod_{a'\neq a}\int_{-\infty}^{\bar{v}_{a's}}\psi(\omega_{a's})\mathrm{d}\omega_{a's}\ , \tag{33}$$

where $U(\bar{\tau}_a;\omega_{as})$ is the ex post utility when $\bar{\tau}_a$ is given. As mentioned earlier, the ex ante conditional expected utility need not coincide with the ex post conditional utility. The public agent chooses the message which maximizes the aggregated sum of (33) over all drivers. If ω_{as} is subject to a Weibull distribution, by integrating (33), we get

$$EU_s(e)=\lambda^{-1}\log\sum_{a}\exp(\lambda\bar{c}_a(e))+\sum_{a}(\bar{c}_a(e)-\bar{u}_a(e))p_a(e)\ , \tag{34}$$

where $\bar{c}_a(e)$ is the deterministic part of the ex ante expected utility (28), $\bar{u}_a(e)=1-\exp(\zeta(\bar{\tau}_a-E^*[\tau_a]))-E^*[\tau_a]$ is the deterministic part of the ex post utility (29) (Kobayashi and Ikawa 1993). The first term of (34) is the log-sum utility and the second term explains the expected deference between the ex ante expected utility and the ex post utility. Though the public agent can calculate $EU_s(e)$, the drivers cannot get it since there is no way for them to know $\bar{\tau}_a(e)$. On the other hand, (31) has ideal properties for welfare measurement, since it can be estimated only by the data available to the drivers.

Figure 3 explains the relationship between the forecasting mechanisms and the social welfare measures $SW(\theta)$. As the forecasting mechanism becomes more noisy, $SW(\theta)$ tends to decrease. It is notable, however, that even noisy forecasting mechanism like Γ° is also able to increase $SW(\theta)$ from the case without public information, though its marginal contribution is not so large. If public information becomes more informative, precise (less noisy) forecasting mechanisms become more powerful in enhancing the social welfare $SW(\theta)$. Thus, we see that the sophistication of forecasting mechanisms is required in order to develop more informative route guidance information systems.

7. Conclusion

This paper has tried to provide with a unified framework for understanding how drivers act in response to exogenously provided route guidance information; and how they form subjective expectations on traffic conditions from repeated learning. Our main result strengthens the intuitive plausibility of expectations equilibrium theory. It asserts that drivers who begin with no knowledge of the probability distributions governing the uncertainty they face, can learn to form correct expectations from repeated observations of travel time. Even though the drivers use public information to form their rational expectations, the drivers' learning procedures do not require any data involving other drivers' private information. Such a result appears to be possible only in recursive adjustment models of expectations formation. We have also attempted qualitative comparisons of alternative route guidance information systems. Our numerical illustration has provided pedagogical insights into the possibility to navigate the drivers' route choices by providing public information.

More work is needed to enlarge the scope of the study and to explore more deeply the drivers' behavior with rational expectations under incomplete and decentralized information. One of the most interesting question is whether government intervention can always guarantee the Pareto improvements of network flows. The author illustrates elsewhere some cases where route guidance information can degrade network performance. Besides this issue, the further items of interests which have not yet been considered include:

- an investigation of the possibility of multi-equilibrium states;
- a rigorous analysis of the global and local stability of the rational expectations equilibrium;
- an empirical investigation of the drivers' process of rational expectations formation;
- an analytical investigation of the pricing of route guidance information;
- an empirical application to social experiments in the real world.

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