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# The extended Nambu-Jona-Lasinio model with separable interaction: low energy pion physics and pion-nucleon form factor

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**Abstract.** A Lagrangian formulation of the Nambu-Jona-Lasinio model with separable interaction is given. The electromagnetic interaction is introduced in a non-minimal way to the nonlocal quark current. Various choices of the vertex form factors characterizing the composite structure of mesons and baryon are investigated. We find that the physical observables depend very weakly on form factor shapes. We calculate the $\pi NN$ form factor considering nucleon as a three-quark system.

## I Introduction

The Nambu-Jona-Lasinio (NJL) model [1], and other models motivated by it [2–10] are powerful instruments for the study of the composite structure of hadrons. Actually, the first success of this model has been related to the explanation of the spontaneous breaking of chiral symmetry and the small pion mass [2].

To give a sense to various loop integrals arising in this approach, a momentum cutoff is usually introduced and the detailed momentum dependence of the hadronic vertices which characterize the composite structure of hadrons is neglected. In this rough approximation it was shown that the NJL-model reproduces the standard formulation of the $\sigma$-model [2].

More realistic generalizations of the NJL-model use the nonlocal four-quark interactions, usually in a separable form. In this way hadron wave functions and global hadronic characteristics can be connected [3–5].

There exist more fundamental approaches [6] which realize the NJL mechanism starting from the QCD bosonization, but this requires to introduce bilocal hadronic fields producing equations that are difficult to solve. Any simplifications of this approach yield a kind of NJL-model with nonlocal interactions and/or modified quark propagators.

A special formulation of quark confinement has been introduced in [9, 10]. It was assumed that the hadron-quark vertices are local but the quark propagators inside the quark loop are described by entire analytical functions providing both a quark confinement and ultraviolet convergence of all diagrams.

The main goal of this paper is to give a Lagrangian formulation of the NJL-model with separable interaction both for mesons and for the first time for baryons. We check the Goldstone theorem in this approach which means that a zero-mass pion appears in the chiral limit.

In fact, we do not pay much attention to the Schwinger-Dyson (SD) equation for constituent quark masses and the Bethe-Salpeter (BS) equation for hadron masses because they have too many free parameters to be predictive. Actually, these equations may be considered only as the self-consistent constraints which connect the quark and hadron masses with the NJL coupling strength.

All important information about the composite structure of hadrons is concentrated in the matrix elements of the physical processes, in particular in the electromagnetic form factors characterizing the response of a bound state to the interaction with a photon. Here, we introduce the electromagnetic interactions by means the time-ordering $P$-exponent in the nonlocal quark currents. This reproduces automatically the Ward-Takahashi identities and electromagnetic gauge invariance in each step of calculation.

One of the principal goals of this paper is to investigate the dependence of the physical properties on the choice of the various form factors of the separable interaction. There are two adjustable parameters, a range parameter $\Lambda$ appearing in the separable interaction and a constituent quark mass $m_q$. As in the papers [4, 5], the weak decay constant $f_\pi$, the two-photon decay width $\Gamma_{\pi^0\to\gamma\gamma}$, as well as the charge form factor $F_\pi(q^2)$ and the $\gamma^*\pi^0\to\gamma$ transition form factor $F_{\gamma\pi}(q^2)$ are calculated. Here we consider both monopole and dipole, Gaussian, and screened Coulomb form factors.

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For the first time we calculate the $\pi NN$-constant within the NJL-model with separable interaction considering a nucleon as a three-quark system. It gives us a hope to construct an unified scheme for hadronic interactions which allows to take into account the quark structure of hadrons at low energies.

The paper is organized in the following way.

In Sect. II we give the Lagrangian formulation of the NJL-model with separable interaction based on the so-called *compositeness condition* in quantum field theory. Such approach may be used not only for the simplest two-quark states (mesons) what is shown to be equivalent to the QCD-bosonization but for the arbitrary quark states, for example, baryons. As a result we get the nonlocal interaction Lagrangian with form factor characterizing the distribution of quarks inside a hadron. We introduce the interaction with electromagnetic field into this Lagrangian using the time-ordering P-exponent that allows to control the gauge invariance of the matrix elements on each step of calculation.

In Sect. III we calculate the basic pion decay constants ($f_\pi$ and $g_{\pi\gamma\gamma}$) and define the model parameters (a parameter $\Lambda$ characterizing the interaction range and a constituent quark mass) by fitting the experimental data for the four kinds of form factors: monopole, dipole, Gaussian, and screened Coulomb.

In Sect. IV we calculate the charge pion form factor and the $\gamma^*\pi^0 \to \gamma$ transition form factor. Our results are in good agreement with available experimental data and depend very weakly on form factor shapes.

In Sect. V we calculate the fundamental strong $\pi NN$ constant using a Gaussian form factor both for $\pi qq$ vertex and $Nqqq$ one. We find that the value $g^2_{\pi NN}/4\pi = 14.4$ which is usually used in the literature is reproduced if a range parameter for nucleon is equal to $\Lambda_N \approx 2\Lambda_\pi = 2$ GeV.

In conclusion, we discuss the obtained results and give their perspective.

## II The NJL-model with separable interaction

For the convenience of the reader we give the Lagrangian of the NJL-model with separable interaction (SI) [3]

$$L^{\rm SI}_{\rm NJL} = \bar q i\not\partial q + \frac{G}{2}\{J_S^2 + J_P^2\} \qquad (1)$$

with $J$ given by

$$J_S = \int dy \bar q(x+y/2) f(y^2) q(x-y/2)$$
$$J_P^i = \int dy \bar q(x+y/2) f(y^2) i\gamma^5 \tau^i q(x-y/2). \qquad (2)$$

Here the form factor $f(y)$ characterizes a region of a quark-antiquark interaction. In the original NJL-model the form factor was chosen to be a $\delta$-function (or unity in momentum space). The Lagrangian (1) is invariant under the global axial ($q \to e^{i\gamma^5\tau\theta}q$) and vector ($q \to e^{i\tau\theta}q$) transformations.

The standard way of the bosonization of the NJL-model may be found in many papers (see for instance [2, 3] ) so that we just give a short sketch of some points which will be needed further. Let us consider the vacuum generating functional

$$Z = \int \delta q \int \delta \bar q \exp\{i\int dx L^{\rm SI}_{\rm NJL}\} \qquad (3)$$

(an infinite renormalization constant is omitted).

Using the Gaussian transformation for the quadratic interaction of quark currents and then integrating over quark fields one obtains

$$Z = \int \delta\sigma \int \delta\boldsymbol{\pi} \exp\{iW_{\rm eff}[\sigma, \boldsymbol{\pi}]\} \qquad (4)$$

with the effective action $W_{\rm eff}$ given by

$$W_{\rm eff}[\sigma, \boldsymbol{\pi}] = -\frac{m_0^2}{2}\int dx[\sigma^2(x) + \pi^2(x)] - iN_c \operatorname{tr}\ln[i\not\partial - \tilde\sigma - i\gamma^5\tilde\pi]. \qquad (5)$$

where $N_c$ is a number of colors, $m_0^2 = 1/G$ is a bare meson mass, and the fields $\tilde\sigma$ and $\tilde\pi$ are given by

$$\tilde\sigma(x_1, x_2) = \sigma\left(\frac{x_1+x_2}{2}\right) f((x_1-x_2)^2)$$

$$\tilde\pi = \tau\boldsymbol{\pi}\left(\frac{x_1+x_2}{2}\right) f((x_1-x_2)^2). \qquad (6)$$

Assuming that the field $\sigma$ has a nonvanishing vacuum expectation $\sigma_0$

$$\sigma(x) = s(x) + \sigma_0 \qquad (7)$$

and varying the action (5) $\delta W_{\rm eff}[\sigma_0, 0]/\delta\sigma_0 = 0$, one obtains a gap equation

$$1 = 4GN_c N_f i\int \frac{d^4k}{(2\pi)^4}\frac{f^2(k^2)}{k^2 - \Sigma^2(k^2)} \qquad (8)$$

with the quark mass operator defined by

$$\Sigma(k^2) = \sigma_0 f(k^2) \qquad (9)$$

where $f(k^2)$ is the Fourier-transform of the vertex form factor.

In the calculation of physical values [4, 5], the momentum-dependent mass operator is approximated by an effective mass $\langle\Sigma(k^2)\rangle = m_q$ (we neglect here the bare quark masses). The integral (8) is calculated by transition to the Euclidean region $k^0 \to ik_4$, so that $k^2 \to -k_E^2$. This procedure is well-defined for a wide class of form factors $f(k^2)$ decreasing rapidly in the Euclidean region (see, for details [9]).

Further we would like to show how to extract the kinetic terms from (5). To do this, consider the leading order in the series of (5)

$$W^{(2)}_{\rm eff} = -\frac{m_0^2}{2}\int dx(s^2 + \boldsymbol{\pi}^2) + \frac{i}{2}N_c \operatorname{tr}[S(\tilde s + i\gamma^5\tilde\pi)]^2, \qquad (10)$$

where we have introduced the notation for the quark propagator

$$S(x) = [i\not\partial - \Sigma(-\partial^2)]^{-1}\delta(x). \qquad (11)$$

After simple transformations, one obtains

$$W^{(2)}_{\rm eff} = \frac{1}{2}\sum_{\varphi = s,\pi}\int dx_1 \int dx_2 \phi(x_1)\{-m_0^2\delta(x_1-x_2) + \Pi_\varphi(x_1-x_2)\}\phi(x_2) \qquad (12)$$

with $\Pi_\varphi(x)$ given by

$$\Pi_\varphi(x) = iN_c N_f \int dy_1 \int dy_2 f(y_1^2) f(y_2^2) \,\mathrm{tr}\left[ S\left( x - \frac{y_1+y_2}{2} \right.\right.$$
$$\left.\left. \times \Gamma_\varphi S\left( -x - \frac{y_1+y_2}{2}\right)\Gamma_\varphi \right], \qquad (13)$$

where

$\Gamma_\varphi = I(\phi = s)$, or $i\gamma^5(\phi = \pi)$.

Further we represent the Fourier-transform of the two-point function of Eq. (13) in the form

$$\Pi_\varphi(p^2) = \int dx e^{ipx}\Pi_\varphi(x) = \Pi_\varphi(m_\varphi^2) + \Pi'_\varphi(m_\varphi^2)(p^2 - m_\varphi^2) + \Pi_\varphi^{\mathrm{ren}}(p^2)$$

where $m_\varphi^2$ is the physical meson mass. Using this expansion one obtains

$$W_{\mathrm{eff}}^{(2)} = \frac{1}{2}\sum_{\varphi=s,\pi}\{\int dx\phi(x)[(-m_0^2 + \Pi_\varphi(m_\varphi^2)) + (\Box - m_\varphi^2)\Pi'_\varphi(m_\varphi^2)]\,\phi(x) + \int dx_1 \int dx_2 \phi(x_1)\Pi_\varphi^{\mathrm{ren}}(x_1 - x_2)\phi(x_2)\}. \qquad (14)$$

It is readily seen that if we require fulfilment of the condition

$$1 = G\Pi_\varphi(m_\varphi^2) = iGN_c N_f \int \frac{d^4k}{(2\pi)^4} f^2(k^2) \times \mathrm{tr}\left\{\Gamma_\varphi\left[\frac{1}{\not k + \not p/2 - \Sigma((k+p/2)^2)}\right] \times \Gamma_\varphi\left[\frac{1}{\not k - \not p/2 - \Sigma((k-p/2)^2)}\right]\right\}_{p^2 = m_\varphi^2} \qquad (15)$$

the physical pole appears in the meson Green function.

Putting the pion mass in (15) to zero one has the gap equation (8), thereby reproducing the Goldstone theorem.

Scaling the fields $\phi$ ($s$ or $\pi$) in (5) by the factor $1/\sqrt{\Pi'_\varphi(m_\varphi^2)}$ one obtains $W_{\mathrm{eff}}$

$$W_{\mathrm{eff}}[s,\pi] = \frac{1}{2}\sum_{\varphi=s,\pi}\{\int dx\phi(x)(\Box - m_\varphi^2)\phi(x) + \sum_{n=2}^{\infty}\frac{1}{n}\mathrm{tr}\left[S\frac{\tilde{\phi}}{\sqrt{\Pi'_\varphi(m_\varphi^2)}}\right]^n\}. \qquad (16)$$

One can see that the only connection of this expression with the original NJL-Lagrangian is via the quark mass operator $\Sigma(k^2)$ in the gap equation (8).

We would like to remark that the effective action (16) can be obtained from the quantum field theory defined by the following Lagrangian

$$L = L_0 + L_{\mathrm{int}} \qquad (17)$$

where

$$L_0 = \bar{q}(i\not\partial - m_q)q + \frac{1}{2}s(\Box - m_s^2)s + \frac{1}{2}\pi(\Box - m_\pi^2)\pi \qquad (18)$$

$$L_{\mathrm{int}} = \frac{g_s}{\sqrt{2}}s(x)J_S(x) + \frac{g_\pi}{\sqrt{2}}\pi(x)J_P(x) \qquad (19)$$

if the renormalization constants of the meson fields are set equal to zero

$$Z_\varphi = 1 - \frac{g^2}{2}\Pi'_\varphi(m_\varphi^2) = 0. \qquad (20)$$

This condition reflects the composite nature of the hadrons (dressed states in quantum field theory). It is the so-called *compositeness condition* discussed in many papers (see, for instance [9–12]).

Our formulation of the NJL-model with separable interaction may be extended to describe the interactions of any physical states. For instance, we give here the Lagrangians describing octets of vector (axial), pseudoscalar (scalar) mesons, and baryons.

1. Mesons $M = \frac{1}{\sqrt{2}}\sum_0^8 \lambda^i\phi^i$.

$$L_M^0(x) = \pm\frac{1}{2}\mathrm{tr}M(x)(\Box - m_M^2)M(x)$$
$$(+\text{ for S, P } - \text{ for V, A}) \qquad (21)$$

$$L_M^{\mathrm{int}}(x) = g_M \int dy_1 \int dy_2 f((y_1 - y_2)^2) \times \delta\left(x - \frac{y_1+y_2}{2}\right)\bar{q}(y_1)\Gamma_M M(x) q(y_2) = g_M \int dy f(y^2)\bar{q}(x + y/2)\Gamma_M M(x) q(x - y/2). \qquad (22)$$

2. Baryons $B = \frac{1}{\sqrt{2}}\sum_1^8 \lambda^i\psi^i$.

$$L_B^0(x) = \mathrm{tr}\bar{B}(x)(i\not\partial - m_B)B(x), \qquad (23)$$

$$L_B^{\mathrm{int}}(x) = \bar{B}^{km}(x)\int dy_1 \int dy_2 \int dy_3 \times \delta\left(x - \frac{y_1+y_2+y_3}{3}\right) F\left(\frac{1}{18}\sum_{i<j}(y_i - y_j)^2\right) \times \{ig_V J_V^{mk}(y_1,y_2,y_3) + g_T J_T^{mk}(y_1,y_2,y_3)\} + \mathrm{h.c.}$$
$$= \bar{B}^{km}(x)\int d\xi_1 \int d\xi_2 F(\xi_1^2 + \xi_2^2) \times \{ig_V J_V^{mk}(x - 2\xi_1, x + \xi_1 - \sqrt{3}\xi_2, x + \xi_1 + \sqrt{3}\xi_2) + g_T J_T^{mk}(x - 2\xi_1, x + \xi_1 - \sqrt{3}\xi_2, x + \xi_1 + \sqrt{3}\xi_2)\} + \mathrm{h.c.} \qquad (24)$$

where

$$J_V^{mk}(y_1, y_2.y_3) = \lambda_i^{mm_1}\gamma^\mu\gamma^5 q_{a_1}^{m_1}(y_1) \times (q_{a_2}^{m_2}(y_2)\varepsilon^{km_2n}\lambda_i^{nm_3}C\gamma^\mu q_{a_3}^{m_3}(y_3))\varepsilon^{a_1a_2a_3}, \qquad (25)$$

$$J_T^{mk}(y_1, y_2.y_3) = \lambda_i^{mm_1}\sigma^{\mu\nu}\gamma^5 q_{a_1}^{m_1}(y_1) \times (q_{a_2}^{m_2}(y_2)\varepsilon^{km_2n}\lambda_i^{nm_3}C\sigma^{\mu\nu}q_{a_3}^{m_3}(y_3))\varepsilon^{a_1a_2a_3}. \qquad (26)$$

The notation implied is as follows: $k$, $m$, $n$ and $a$ are flavor and color indices, $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu, \gamma^\nu]$, and $C$ is the charge conjugation matrix, respectively. The choice of variables in the form factor of the separable interaction implies the use of the center of mass frame

$$y_1 = x - 2\xi_1 \quad y_2 = x + \xi_1 - \sqrt{3}\xi_2 \quad y_3 = x + \xi_1 + \sqrt{3}\xi_2$$

so that

$$\xi_1 = \frac{y_2 + y_3 - 2y_1}{6} \quad \xi_2 = \frac{y_3 - y_2}{2\sqrt{3}}.$$

The Fourier-transform of the vertex form factor is defined as

$$F(\xi^2) = \int \frac{d^8k}{(2\pi)^8} e^{-ik\xi} F(k^2)$$

where $\xi^2 = \xi_1^2 + \xi_2^2$ and $k^2 = k_1^2 + k_2^2$. This means that there is an analogy between the meson form factor $f(k^2)$ in the four-dimensional space and the baryon one $F(k^2)$ in the eight-dimensional space.

Now we introduce the electromagnetic interaction into this scheme. Note that this was done in [4] by using the minimal substitution $\partial^\mu \to \partial^\mu - ie_q A^\mu$ both in the free quark Lagrangian and in the interaction part which has a form factor. Restoring gauge invariance in this case requires a complicated procedure which is fairly arbitrary.

Here we would like to suggest to introduce the electromagnetic fields to the interaction Lagrangian using the time-ordering $P$-exponent. In this case the gauge invariant meson-quark vertex has the form

$$\begin{aligned} L_M^{\rm int}(x) &= g_M \int dy_1 \int dy_2 \delta\left(x - \frac{y_1 + y_2}{2}\right) f((y_1 - y_2)^2) \\ &\times \bar{q}(y_1) P \exp\left\{ieQ \int_{y_1}^{x} dz^\mu A^\mu(z)\right\} \Gamma_M M(x) \\ &\times P \exp\left\{ieQ \int_{x}^{y_2} dz^\mu A^\mu(z)\right\} q(y_2) \end{aligned} \tag{27}$$

where $Q = \frac{1}{2}(\lambda^3 + \frac{1}{\sqrt{3}}\lambda^8) = \mathrm{diag}(2/3, -1/3, -1/3)$.

For neutral mesons one obtains

$$\begin{aligned} L_{M^0}^{\rm int}(x) &= g_M \int dy_1 \int dy_2 \delta\left(x - \frac{y_1 + y_2}{2}\right) f((y_1 - y_2)^2) \bar{q}(y_1) \\ &\times P \exp\left\{ieQ \int_{y_1}^{y_2} dz^\mu A^\mu(z)\right\} \Gamma_M M^{(0)}(x) q(y_2). \end{aligned} \tag{28}$$

For baryons this interaction is introduced in a similar way.

We shall use the S-matrix defined by

$$S = T \exp\{i \int dx L^{\rm int}(x)\} \tag{29}$$

to derive one-loop quark diagrams describing the physical processes. The $T$-product is defined in a standard manner

$$\langle 0| T(q(x)\bar{q}(y))|0\rangle = \int \frac{d^4k}{(2\pi)^4 i} e^{-ik(x-y)} \frac{1}{m_q - \not{k}}. \tag{30}$$

The hadron-quark coupling constants $g_M$ in Eq. (22) and $g_B$ in Eq. (24) are defined from the compositeness condition (20).

## III Model parameters and pion decay constants

First we would like to discuss the model parameters. Of course, the form factor $f(k^2)$ characterizing the composite structure of hadron is an unknown function. In principal its shape could be related to a quark-antiquark potential but here we are going to test the sensitivity of physical observables on the choice of this form factor. With this in mind we consider four kinds of widely used form factors:

- monopole $f(k^2) = \frac{\Lambda^2}{\Lambda^2 - k^2}$
- dipole $f(k^2) = \frac{\Lambda^4}{(\Lambda^2 - k^2)^2}$
- Gaussian $f(k^2) = \exp\{\frac{k^2}{\Lambda^2}\}$
- screened Coulomb $f(k^2) = -\frac{\Lambda^2}{k^2}(1 - \exp\{\frac{k^2}{\Lambda^2}\})$

All Feynman diagrams are calculated in the Euclidean region ($k^2 = -k_E^2$) where the form factors decrease rapidly so that no ultraviolet divergences arise. For convenience the form factors are chosen to be dimensionless.

The three-dimensional Fourier-transforms of the form factors can be considered as nonrelativistic potentials (in Born approximation). Putting $k^0 = 0$ one can get

$$V(r) = \int \frac{d^3\mathbf{k}}{(2\pi)^3} e^{i\mathbf{k}\mathbf{r}} f(-\mathbf{k}^2) = \Lambda^3 v(r\Lambda). \tag{31}$$

We obtain

- monopole $v(r) = \frac{1}{4\pi r} e^{-r}$
- dipole $v(r) = \frac{1}{8\pi} e^{-r}$
- Gaussian $v(r) = \frac{1}{8\pi\sqrt{\pi}} e^{-r^2/4}$
- screened Coulomb $v(r) = \frac{1}{4\pi r}\left(1 - \frac{2}{\sqrt{\pi}} \int_0^{r/2} dt e^{-t^2}\right)$

Thus there are two adjustable parameters, $\Lambda$ characterizing the region of quark-antiquark interaction, and the constituent quark mass $m_q$. We shall define these parameters by fitting the experimental pion decay constant $f_\pi$ ($f_\pi^{\rm expt} = 132$ MeV) and $g_{\pi\gamma\gamma}$ ($g_{\pi\gamma\gamma}^{\rm expt} = 0.276\ \mathrm{GeV}^{-1}$).

We shall imply that all masses and momenta inside the Feynman integrals are expressed in the unit $\Lambda$. Also we shall neglect the pion mass when calculating the physical pion decay constants.

### 1 Pion-quark coupling constants

As mentioned above the pion-quark coupling constants are defined from the compositeness condition (20) with the pion mass operator given by

$$\begin{aligned} \Pi_\pi(p^2) &= 6 \int \frac{d^4k}{(2\pi)^4 i} f^2(k^2) \\ &\times \mathrm{tr}\left\{\gamma^5 \left[\frac{1}{m_q - \not{k} + \not{p}/2}\right] \gamma^5 \left[\frac{1}{m_q - \not{k} - \not{p}/2}\right]\right\}. \end{aligned} \tag{32}$$

Neglecting the pion mass one has

$$\left(\frac{3g_\pi^2}{4\pi^2}\right)^{-1} = \frac{1}{4} \int_0^\infty du\, u f^2(-u) \frac{(3m_q^2 + 2u)}{(m_q^2 + u)^3}. \tag{33}$$

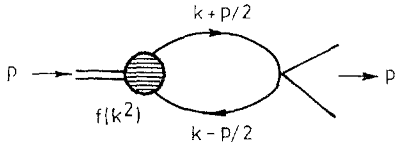


Fig. 1. The diagrams describing the weak pion decay

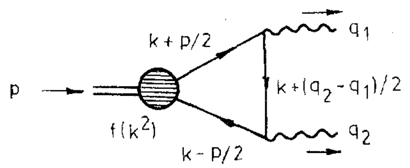


Fig. 2. The diagrams describing the pion two-photon decay

## 2 Pion weak decay

The weak decay of the pion is defined by the diagram of Fig. 1. After simple transformations of the Feynman integral, we have

$$f_\pi = \frac{3g_\pi}{4\pi^2} m_q \int \frac{d^4k}{\pi^2 i} \frac{f(k^2)}{[m_q^2 - (k+p/2)^2][m_q^2 - (k-p/2)^2]}$$
$$\simeq \frac{3g_\pi}{4\pi^2} m_q \int_0^\infty duuf(-u) \frac{1}{(m_q^2+u)^2}. \tag{34}$$

## 3 Pion two-photon decay

The two-photon decay of the pion is defined by the diagram of Fig. 2. After similar transformations we have

$$G_{\pi\gamma\gamma}(p^2, q_1^2, q_2^2) = \frac{g_\pi}{2\sqrt{2}\pi^2} \frac{m_q}{\Lambda^2}$$
$$\times \int \frac{d^4k}{\pi^2 i} \frac{f(k^2)}{[m_q^2 - (k+p/2)^2][m_q^2 - (k-p/2)^2]}$$
$$\times \frac{1}{[m_q^2 - (k + (q_1 - q_2)/2)^2]}. \tag{35}$$

The two-photon decay coupling constant is obtained from Eq. (35) where both photons are on the mass shell:

$$g_{\pi\gamma\gamma} = G_{\pi\gamma\gamma}(m_\pi^2, 0, 0) \simeq \frac{g_\pi}{2\sqrt{2}\pi^2} \frac{m_q}{\Lambda^2} \int_0^\infty duuf(-u) \frac{1}{(m_q^2+u)^3}. \tag{36}$$

The numerical results for the physical observables for the best fit are shown in Table 1 for different choices of form factors. Inserting the best values for $\Lambda$ and $m_q$ into the gap equation (8), gives $G = 3.039\pi^2\Lambda^2$ for the monopole form factor. One can check also that the low-energy relation $f_\pi g_{\pi\gamma\gamma} = 1/(2\sqrt{2}\pi^2)$ is reproduced with good accuracy ($\lesssim 7\%$).

**Table 1.** The best fit of the physical observables

| Form | | | $f_\pi$(MeV) | | $g_{\pi^0\gamma\gamma}$(GeV$^{-1}$) | |
|---|---|---|---|---|---|---|
| Factors | $\Lambda$(MeV) | $m_q$(MeV) | NJL SI | EXP | NJL SI | EXP [14] |
| monopole | 400 | 267 | 132 | | 0.251 | |
| dipole | 1000 | 245 | 132 | 132 | 0.263 | 0.276 |
| Gaussian | 1000 | 237 | 132 | | 0.261 | |
| Coulomb | 450 | 250 | 132 | | 0.262 | |

# IV Pion electromagnetic form factors

## 1 The $\gamma^*\pi^0 \to \gamma$ form factor

The form factor for the $\gamma^*\pi^0 \to \gamma$ transition was measured for space-like momentum transfer $Q^2 > 0$ of the virtual photon [13] by making use of the two-photon process $\gamma\gamma \to \pi^0$, where the two photons are radiated virtually by colliding $e^+e^-$ beams.

In the extended NJL model this form factor is expressed as

$$F_{\gamma\pi}(Q^2) = e^2 G_{\pi\gamma\gamma}(m_\pi^2, -Q^2, 0)$$
$$\simeq e^2 \frac{g_\pi}{2\sqrt{2}\pi^2} \frac{m_q}{\Lambda^2} R_{\pi\gamma}(Q^2/\Lambda^2) \tag{37}$$

with the structure function $R_{\pi\gamma}$ given in the Appendix.

Our results for various form factors are shown in Fig. 3. The experimental data are described by the monopole fit with

$$F(Q^2) = \frac{e^2 g_{\pi\gamma\gamma}}{1 + Q^2/\Lambda_\pi^2} \quad \Lambda_\pi = 0.77 \text{ GeV}. \tag{38}$$

The radius for $\gamma^*\pi^0 \to \gamma$ transition is defined by

$$\langle r_{\pi\gamma}^2 \rangle = -6 \frac{F'_{\pi\gamma}(0)}{F_{\pi\gamma}(0)} \tag{39}$$

where

$$F_{\pi\gamma}(0) = \int_0^\infty duu \frac{f(-u)}{(m_q^2+u)^3}$$

$$F'_{\pi\gamma}(0) = -\frac{1}{2} \frac{m_q^2}{\Lambda^4} \int_0^\infty duu \frac{f(-u)}{(m_q^2+u)^5} \tag{40}$$

The numerical results for the radius $r_{\pi\gamma}$ are given in Table 2. Excellent agreement with the available experimental data is reached. Our results practically do not depend on the choice of vertex form factors $f(k^2)$.

## 2 The pion charge form factor

The pion charge form factor is defined by the diagrams of Fig. 4. These diagrams are not gauge invariant separately.

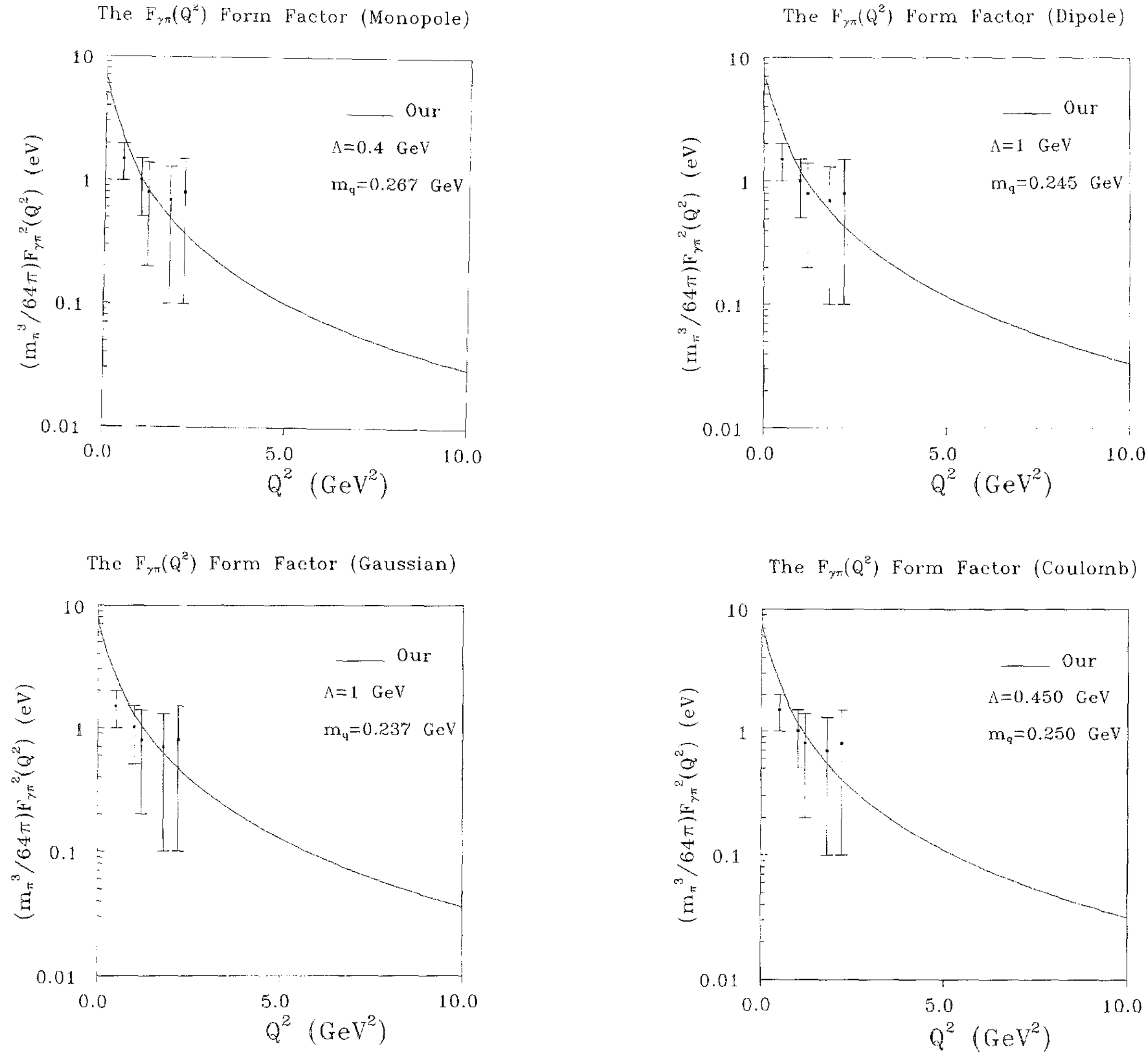


**Fig. 3.** The form factor of the $\gamma^*\pi^0 \to \gamma$ transition for spacelike photons $0 \leq Q^2 \leq 10$ GeV$^2$. The dashed line is the result of the monopole fit with $\Lambda = 770$ MeV. The solid lines are our predictions for different choices of the form factors. The experimental data are from [13]

The sum of the diagrams can be written as

$$\begin{aligned}\Lambda^\mu(p,p') &= \frac{q^\mu}{q^2}[\Pi_\pi(p^2)-\Pi_\pi(p'^2)]\\ &\quad + \frac{3g_\pi^2}{4\pi^2}\int\frac{\mathrm{d}^4k}{4\pi^2 i} f\left(\left[k+\frac{p}{2}\right]^2\right) f\left(\left[k+\frac{p'}{2}\right]^2\right)\\ &\quad \times \mathrm{tr}\left[\gamma^5 S(k+p')\left(\gamma^\mu - \frac{q^\mu \not{q}}{q^2}\right)S(k+p)\gamma^5 S(k)\right]\\ &\quad + \frac{\eta^\mu}{\eta^2}\frac{3g_\pi^2}{4\pi^2}\int\frac{\mathrm{d}^4k}{4\pi^2 i}\int_0^1 \mathrm{d}t f(k^2) f'((k+qt/2)^2)\\ &\quad \times k\eta\cdot\left(\mathrm{tr}\left[\gamma^5 S\left(k+\frac{p'}{2}\right)\gamma^5 S\left(k-\frac{p'}{2}\right)\right]\right.\\ &\quad \left. - \mathrm{tr}\left[\gamma^5 S\left(k+\frac{p}{2}\right)\gamma^5 S\left(k-\frac{p}{2}\right)\right]\right)\end{aligned} \tag{41}$$

where we use the following notation

$$\eta^\mu = P^\mu - q^\mu \frac{P\cdot q}{q^2} \quad P = p + p'.$$

The Ward-Takahashi identity directly follows from (41)

$$q_\mu \Lambda^\mu(p,p') \equiv \Pi_\pi(p^2) - \Pi_\pi(p'^2). \tag{42}$$

Taking $q = 0$, one obtains on the one hand

$$\Lambda^\mu(p,p) = \frac{\partial\Sigma_\pi(p^2)}{\partial p^\mu} = 2p^\mu \frac{\partial\Sigma_\pi(p^2)}{\partial p^2} \tag{43}$$

where $\Sigma_\pi(p^2) \equiv (3g_\pi^2/4\pi^2)\Pi_\pi(p^2)$ , and, on the other hand

$$\Lambda^\mu(p,p) = 2p^\mu F_\pi(0) \tag{44}$$

**Table 2.** The radius of the $\gamma^{\star}\pi^0 \to \gamma$ form factor

| Vertex | $r_{\pi\gamma}(fm)$ | |
|---|---|---|
| Function | NJL SI | EXP [14] |
| monopole | 0.655 | |
| dipole | 0.658 | |
| Gaussian | 0.654 | $0.65 \pm 0.03$ |
| Coulomb | 0.659 | |

**Table 3.** The electromagnetic radius of pion

| Form | NJL SI | | | EXP [14] |
|---|---|---|---|---|
| Factor | $\langle r_\pi^2\rangle^{\Delta}$ | $\langle r_\pi^2\rangle^{\circ}$ | total | fm$^2$ |
| monopole | 0.545 | $-0.012$ | 0.533 | |
| dipole | 0.461 | $-0.005$ | 0.456 | 0.430 |
| Gaussian | 0.409 | $-0.002$ | 0.407 | |
| Coulomb | 0.488 | $-0.006$ | 0.482 | |

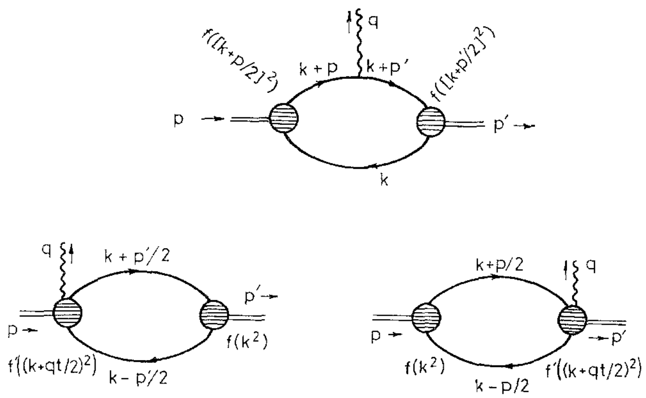


**Fig. 4.** The diagrams describing the pion charge form factor

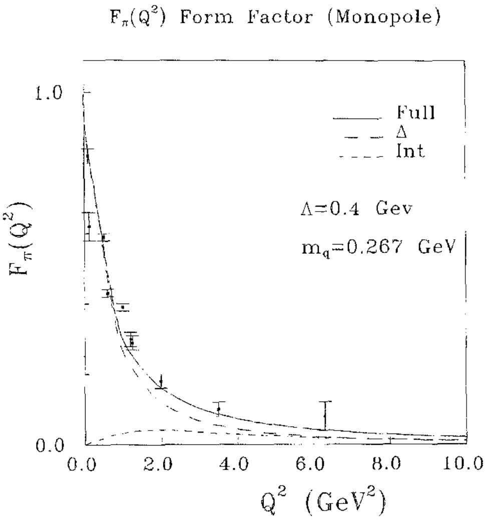


**Fig. 5.** The pion charge form factor $F_\pi(Q^2)$ for spacelike photons $0 \le Q^2 \le 10\,\mathrm{GeV}^2$. Separate contributions from the triangle and bubble diagrams are shown as dashed and dotted lines, respectively. The solid line is the total result. The experimental data are from [14]

where $F_\pi(0)$ is the pion charge form factor at the origin. It follows from the comparison of (43) and (44) on the pion mass shell that the compositeness condition $\Sigma'(m_\pi^2) = 1$ is equivalent to the normalization of the pion form factor at the origin $F_\pi(0) = 1$.

Note that the implementation of gauge invariance in the context of the minimal substitution [4] leads to

$$\int_0^1 \mathrm{d}t f'((k+qt/2)^2 + q^2 t(1-t)/4) = \frac{f((k+q/2)^2) - f(k^2)}{kq + q^2/4}$$

in (41) while the gauge invariant vertex (27) leads to

$$\int_0^1 \mathrm{d}t f'((k+qt/2)^2).$$

For practical purposes this difference is not important in our case.

The numerical computation of the pion charge form factor is performed in the Breit frame

$$q = (0, \mathbf{q}),\ p = (E, \mathbf{q}/2),\ p' = (E, -\mathbf{q}/2),\ E = \sqrt{m_\pi^2 + \mathbf{q}^2}. \tag{45}$$

The analytical expressions for the form factor are given in the Appendix.

The contributions to the pion charge radius coming from the triangle ($\Delta$) and bubble ($\bigcirc$) diagrams are written down

$$\langle r_\pi^2\rangle^{\Delta} = -6\frac{1}{\Lambda^2}\frac{\Phi_1(0)}{\Phi_0(0)},\quad \langle r_\pi^2\rangle^{\circ} = -6\frac{1}{\Lambda^2}\frac{\Phi_2(0)}{\Phi_0(0)} \tag{46}$$

where

$$\Phi_1 = -\int_0^\infty \mathrm{d}u u \left\{\frac{f^2(-u)}{(m_q^2+u)^5}\left[\frac{1}{12}u^2 + \frac{3}{8}m_q^2 u + \frac{3}{8}m_q^4\right] + u\frac{[f'(-u)]^2}{(m_q^2+u)^3}\left[\frac{7}{96}u + \frac{3}{32}m_q^2\right]\right\}$$

$$\Phi_2 = \int_0^\infty \mathrm{d}u u \left\{\frac{1}{16}\frac{f^2(-u)}{(m_q^2+u)^5}m_q^2[m_q^2 - u] - \frac{1}{48}\frac{[f'(-u)]^2}{(m_q^2+u)^3}u^2\right\}$$

$$\Phi_0 = \int_0^\infty \mathrm{d}u u \frac{f^2(-u)}{(m_q^2+u)^3}\left[\frac{3}{4}m_q^2 + \frac{u}{2}\right]$$

The numerical results for the radius are given in Table 3. One can see that our results are in good agreement with the available experimental data and depend very weakly on the choice of vertex form factors $f(k^2)$.

The behavior of charge form factor for monopole vertex is shown in Fig. 5.

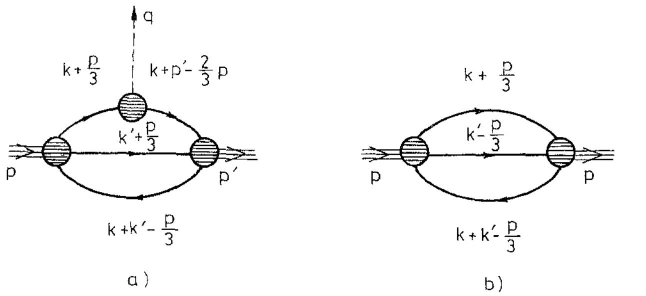


**Fig. 6.** The diagrams discribing **a** the $G_{\pi NN}$ - form factor and **b** the nucleon mass operator

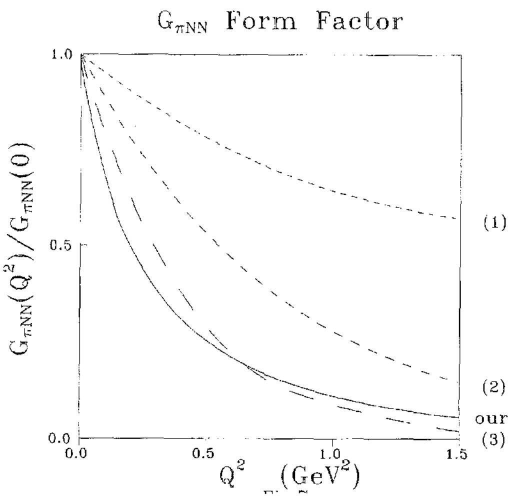


**Fig. 7.** The strong $\pi NN$ form factor. The solid line is our prediction ($\Lambda_N = 1.95$ GeV). The dashed lines are the result of [15, 20–21] for various shapes of the $\pi NN$-form factor. (1) – Hard Monopole ($\Lambda_{\pi NN} = 1.30$ GeV), (2) – Soliton model, (our) – our result, (3) – soft Gaussian ($\Lambda_{\pi NN} = 0.64$ GeV)

## V Strong $\pi NN$ form factor

The $\pi NN$ form factor plays a crucial role in the analyses of low-energy $\pi N$, $NN$ scattering and in the reaction of $\pi$-meson photo-production. Usually this form factor is introduced phenomenologically with a shape being chosen from the best description of the experimental data. Many attempts have been done to get the $\pi NN$ form factor from more fundamental representations. For example, realisitic calculations of strong pion-nucleon vertex function have been done within the chiral soliton models [15]–[18] and within the quark confinement model (QCM) [19].

Here, we will use the Lagrangian (24) which describes a nucleon as relativistic three-quark system to calculate the $\pi NN$ form factor. For simplicity, we will choose the Gaussian form factor

$$F(k^2) = \exp\left(\frac{k^2}{\Lambda_N^2}\right)$$

with $\Lambda_N$ being an adjustable parameter characterizing the distribution of quark inside a nucleon. Also, we will use the tensor three-quark current (26) since it provides more reliable description of the static baryon characteristics (see, [9]).

The $\pi NN$ form factor is defined by the vertex diagram of Fig.6a with the tensor coupling constant $g_T$ being determined from the compositeness condition (20) where the nucleon mass operator is defined by the diagram of Fig.6b. The matrix element is written as

$$M^i_{\pi NN} = \bar{u}(p')\tau^i \Lambda_{\pi NN}(p, p')u(p),$$

$$\Lambda_{\pi NN}(p, p') = \gamma^5 G_{\pi NN}(q^2) \tag{47}$$

where $u(p)$ and $u(p')$ are the spinors corresponding to initial and final nucleon states with momentum $p$ and $p'$, respectively, $q = p - p'$.

The vertex function $\Lambda_{\pi NN}(p, p')$ is written as

$$\begin{aligned}
\Lambda_{\pi NN}(p, p') &= g_\pi g_N^2 \int \frac{d^4k}{\pi^2 i} \int \frac{d^4k'}{\pi^2 i} \\
&\times \exp\left(\frac{12k^2 + 12kk' + 12k'^2 - 6kq + 2q^2}{9\Lambda_N^2}\right) \\
&\times \exp\left(\frac{(k + p'/2 - p/6)^2}{\Lambda_\pi^2}\right)\left[36S_q\left(k + p' - \frac{2p}{3}\right)\gamma^5 S_q\left(k + \frac{p}{3}\right)\right. \\
&\times \mathrm{Tr}\left[\gamma^5 S_q\left(k + k' - \frac{p}{3}\right)\gamma^5 S_q\left(k' + \frac{p}{3}\right)\right] \\
&+ 36\gamma^5 S_q\left(k + p' - \frac{2p}{3}\right)\gamma^5 S_q\left(k + \frac{p}{3}\right) \\
&\times \gamma^5 \mathrm{Tr}\left[S_q\left(k + k' - \frac{p}{3}\right)S_q\left(k' + \frac{p}{3}\right)\right] \\
&- \sigma^{\mu\nu}\gamma^5 S_q\left(k + p' - \frac{2p}{3}\right)\gamma_5 S_q\left(k + \frac{p}{3}\right) \\
&\left.\times \sigma^{\alpha\beta}\gamma^5 \mathrm{Tr}\left[\sigma_{\mu\nu} S_q\left(k + k' - \frac{p}{3}\right)\sigma_{\alpha\beta} S_q\left(k' + \frac{p}{3}\right)\right]\right]
\end{aligned}$$

After cumbersome calculations, we have

$$G_{\pi NN}(q^2) = 16\sqrt{6}\pi \frac{\Lambda_\pi}{\Lambda_N} \mu e^{\mu^2} \frac{R_{\pi NN}(\mu, r; Q^2/\Lambda_N^2)}{R_{NN}(\mu, r)\sqrt{R_{\pi\pi}(\mu)}} \tag{48}$$

where we introduce the notation

$$\mu = \frac{m_q}{\Lambda_\pi}, \quad r = \frac{\Lambda_\pi}{\Lambda_N}.$$

The integrals $R_{NN}$ and $R_{\pi NN}$ are given in Appendix. The integral $R_{\pi\pi}$ is defined by (33). We make calculations on the threshold, i.e. we assume that $m_N = 3m_q$.

The only adjustable parameter $\Lambda_N$ is defined by fitting $G_{\pi NN}(0)$ to the widely used value 13.45. We find that the best value is $\Lambda_N = 1.95$ GeV $\approx 2\Lambda_\pi$.

The behavior of the form factor $G_{\pi NN}(Q^2)$ in the Euclidean region up to 1.5 GeV$^2$ is shown in Fig.7. For comparison we plot the $\pi NN$ form factors obtained in other approches [15], [20], [21]. One can see that the pion-nucleon form factor obtained in our approach falls off somewhat faster than in the OBE-model [20] and is closed to the result of soliton model [15].

## VI Summary

We have formulated the Nambu-Jona-Lasinio model with separable interaction using the Lagrangian with the compositeness condition and non-minimal inclusion of the electromagnetic interaction. This allows to calculate any low-energy physical processes on one-loop level maintaining the relativistic and electromagnetic gauge invariance in each step of calculation. On one hand the form factors in the hadron-quark vertices take into account the composite structure of hadrons thereby being related to a quark-antiquark potential, on the other hand, they make the Feynman integrals convergent.

We have calculated the pion weak decay constant, the two-photon decay width, as well as the form factor of the $\gamma^*\pi^0 \to \gamma$-transition, and the pion charge form factor. The two adjustable parameters, the range parameter $\Lambda$ appearing in the separable interaction and the constituent quark mass $m_q$, have been fixed by fitting the experimental data for the pion decay constants.

The main goal of this paper has been to investigate the sensitivity of the physical observables to the choice of the vertex form factors defining the composite structure of hadrons. We have considered the following form factors: monopole, dipole, Gaussian and screened Coulomb, and found that the numerical results depend very little on these shapes.

We have calculated the $\pi NN$ form factor and found that it has a "soft" behavior.

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## Appendix

(1) The form factor of the $\gamma^*\pi^0 \to \gamma$ transition is written down

$$R_{\pi\gamma}(Q^2) = \int\limits_0^\infty du u \frac{f(-u)}{m_q^2+u} \int\limits_0^1 d\alpha \left\{ \frac{1}{\sqrt{R(u,\alpha,Q^2)}\left[\sqrt{R(u,\alpha,Q^2)} + M_+(u,\alpha)\right]} + \frac{1}{\sqrt{R(u,\alpha,Q^2)+4uQ^2\alpha(1-\alpha)}\left[\sqrt{R(u,\alpha,Q^2)+4uQ^2\alpha(1-\alpha)} + M_+(u,\alpha)\right]} \right\}$$

where $M_\pm(u,\alpha) = m_q^2 + u \pm Q^2\alpha/2$, $R(u,\alpha,Q^2) = M_-^2(u,\alpha) + 2Q^2 m_q^2 \alpha$.

(2) The contributions coming from the triangle ($\Delta$) and bubble ($\bigcirc$) diagrams to the pion charge form factor are written as

$$F_\pi^\Delta(Q^2) = \frac{\Phi_1(Q^2)}{\Phi_1(0)}, \qquad F_\pi^\circ(Q^2) = \frac{\Phi_2(Q^2)}{\Phi_1(0)}$$

$$\Phi_1(Q^2) = \frac{3}{\pi}\int\limits_0^\infty dk k^3 \int\limits_0^1 dx \frac{x^2}{\sqrt{1-x^2}} \int\limits_{-1}^1 dy f(-k^2) \times \frac{f(-k^2 - \frac{Q^2}{4} - kxy\sqrt{Q^2})}{P_1(k,x,y,Q^2)} \times \left\{ 2\frac{P_2(k,x,y,Q^2)P_3(k,x,y,Q^2)}{Q_-(k,x,y,Q^2)} - 1 \right\}$$

$$\Phi_2(Q^2) = \frac{4}{\pi}\sqrt{Q^2}\int\limits_0^\infty dk k^6 \int\limits_0^1 dx x^3\sqrt{1-x^2} \int\limits_{-1}^1 dy y \int\limits_0^1 dt f(-k^2) \times f'\left(-k^2 - \frac{Q^2t^2}{4} - kxyt\sqrt{Q^2}\right) \times \frac{m_q^2+k^2}{Q_+(k,x,y,Q^2)Q_-(k,x,y,Q^2)}$$

where

$$P_1(k,x,y,Q^2) = \left[m_q^2 + k^2 + \tfrac{3}{2}kxy\sqrt{Q^2} + \tfrac{Q^2}{2}\right]^2 + \tfrac{k^2Q^2}{4}(1-x^2)$$

$$P_2(k,x,y,Q^2) = m_q^2 + \tfrac{1+2x^2}{3}k^2 - \tfrac{1}{2}kxy\sqrt{Q^2}$$

$$P_3(k,x,y,Q^2) = m_q^2 + k^2 + \tfrac{1}{2}kxy\sqrt{Q^2} + \tfrac{Q^2}{4}$$

$$Q_\pm(k,x,y,Q^2) = \left[m_q^2 + k^2 \pm \tfrac{1}{2}kxy\sqrt{Q^2}\right]^2 + \tfrac{k^2Q^2}{4}(1-x^2)$$

(3) The structure integrals defining the nucleon mass operator and the $\pi NN$ vertex are written down

$$R_{NN}(\mu,r) = \int\limits_0^\infty\int\limits_0^\infty \frac{d\beta_1 d\beta_2}{D^3(\beta_1,\beta_2,0|3,2,1)} \times \exp\left[-12\mu^2r^2\frac{F(\beta_1,\beta_2,0)}{D(\beta_1,\beta_2,0|3,2,1)}\right] \times \left[D(\beta_1,\beta_2,0|3,0,-1) + 48\mu^2r^2\frac{\beta_2(1+\beta_1)^2}{D(\beta_1,\beta_2,0|3,2,1)}\right]$$
$$+ 2\int\limits_0^\infty\int\limits_0^\infty\int\limits_0^\infty \frac{d\beta_1 d\beta_2 d\beta_3}{D^3(\beta_1,\beta_2,\beta_3|3,2,1)} \times \exp\left[-48\mu^2r^2\frac{F(\beta_1,\beta_2,\beta_3)}{D(\beta_1,\beta_2,\beta_3|3,2,1)}\right]$$
$$\times\left[1+\beta_1 + 48\mu^2r^2\left(2D(\beta_1,\beta_2,0|15,13,11) - \frac{D(\beta_1,\beta_2,0|1,1,1)D(\beta_1,\beta_2,0|10,7,4)}{D(\beta_1,\beta_2,\beta_3|3,2,1)}\right)\right.$$
$$+ 2592\mu^4r^4\beta_3 D(\beta_1,\beta_2,0|1,1,1) \times\left(\frac{2}{3} - \frac{D(\beta_1,\beta_2,0|1,1,1)}{D(\beta_1,\beta_2,\beta_3|3,2,1)} + \frac{9D(\beta_1,\beta_2,0|1,1,1)}{D(\beta_1,\beta_2,\beta_3|3,2,1)}\right.$$
$$\left.\left.\times\left(1 - \frac{2D(\beta_1,\beta_2,0|1,1,1)}{D(\beta_1,\beta_2,\beta_3|3,2,1)}\right)\right)\right]$$

$$
\begin{aligned}
&R_{\pi NN}(\mu, r, Q^2) \\
&= -64 \int_0^\infty \int_0^\infty \int_0^\infty \frac{d\beta_1 d\beta_2 d\beta_3}{D^2(\beta_1 + r, \beta_2, \beta_3 | 3, 2, 1)} \exp\left[ -12\mu^2 r^2 \right. \\
&\left. \frac{F(\beta_1 + r, \beta_2, \beta_3)}{D(\beta_1 + r, \beta_2, \beta_3 | 3, 2, 1)} - 3Q^2 \frac{N(\beta_1 + \frac{r}{2}, \beta_2, \beta_3 | 1 + r)}{D(\beta_1 + r, \beta_2, \beta_3 | 3, 2, 1)} \right] \\
&+ 160 \int_0^\infty \int_0^\infty \int_0^\infty \frac{d\beta_1 d\beta_2 d\beta_3}{G^2(\beta_1 + r, \beta_2, \beta_3, 0 | 3, 2, 1)} \exp\left[ -12\mu^2 r^2 \right. \\
&\left. \frac{E(\beta_1 + r, \beta_2, \beta_3, 0)}{G(\beta_1 + r, \beta_2, \beta_3, 0 | 3, 2, 1)} - 3Q^2 \frac{M(\beta_1 + \frac{r}{2}, \beta_2 + \frac{r}{2}, \beta_3, 0)}{G(\beta_1 + r, \beta_2, \beta_3, 0 | 3, 2, 1)} \right] \\
&+ 32 \int_0^\infty \int_0^\infty \int_0^\infty \int_0^\infty \frac{d\beta_1 d\beta_2 d\beta_3 \beta_4}{G^2(\beta_1 + r, \beta_2, \beta_3, \beta_4 | 3, 2, 1)} \\
&\times \exp\left[ -12\mu^2 r^2 \frac{E(\beta_1 + r, \beta_2, \beta_3, \beta_4)}{G(\beta_1 + r, \beta_2, \beta_3, \beta_4 | 3, 2, 1)} \right. \\
&\left. - 3Q^2 \frac{M(\beta_1 + \frac{r}{2}, \beta_2 + \frac{r}{2}, \beta_3, \beta_4)}{G(\beta_1 + r, \beta_2, \beta_3, \beta_4 | 3, 2, 1)} \right] \\
&\times \left[ 120 \left( 2 - 3 \frac{D(\beta_3, \beta_4, 0 | 1, 1, 1)}{G(\beta_1 + r, \beta_2, \beta_3, \beta_4 | 3, 2, 1)} \right) \right. \\
&- \frac{1 + \beta_1 + \beta_2 + r}{G(\beta_1 + r, \beta_2, \beta_3, \beta_4 | 3, 2, 1)} \\
&+ Q^2 \left( 1 - 4 \frac{D(\beta_3, \beta_4, 0 | 1, 1, 1)}{G(\beta_1 + r, \beta_2, \beta_3, \beta_4 | 3, 2, 1)} \right. \\
&+ 11 \frac{M(\beta_1 + \frac{r}{2}, 0, \beta_3, \beta_4)}{G(\beta_1 + r, \beta_2, \beta_3, \beta_4 | 3, 2, 1)} \\
&- 6 \frac{(1 + \beta_3)^2 (\beta_1 - \beta_2)^2}{G(\beta_1 + r, \beta_2, \beta_3, \beta_4 | 3, 2, 1)} \\
&\left. + 3 \frac{(D(\beta_3, \beta_4, 0 | 1, 1, 1) + M(\beta_1 + \frac{r}{2}, 0, \beta_3, \beta_4))^2}{G(\beta_1 + r, \beta_2, \beta_3, \beta_4 | 3, 2, 1)} \right) \\
&+ \frac{Q^2}{G(\beta_1 + r, \beta_2, \beta_3, \beta_4 | 3, 2, 1)} \\
&\left. \times (6D(\beta_3, \beta_4, 0 | 1, 1, 1) - 9M(\beta_1 + \tfrac{r}{2}, 0, \beta_3, \beta_4)) \right]
\end{aligned}
$$

where

$$
\begin{aligned}
D(\beta_1, \beta_2, \beta_3 | x_1, x_2, x_3) &= x_1 + x_2(\beta_1 + \beta_2 + \beta_3) \\
&\quad + x_3(\beta_1\beta_2 + \beta_2\beta_3 + \beta_1\beta_3) \\
F(\beta_1, \beta_2, \beta_3) &= (1 + \beta_1)(\beta_2 - \beta_3)^2 + (1 + \beta_2)(\beta_3 - \beta_1)^2 \\
&\quad + (1 + \beta_3)(\beta_1 - \beta_2)^2 \\
G(\beta_1, \beta_2, \beta_3, \beta_4 | x_1, x_2, x_3) &= x_1 + x_2(\beta_1 + \beta_2 + \beta_3 + \beta_4) \\
&\quad + x_3(\beta_1 + \beta_2)(\beta_3 + \beta_4) + \beta_3\beta_4 \\
E(\beta_1, \beta_2, \beta_3, \beta_4) &= (1 + \beta_1 + \beta_2)(\beta_3 - \beta_4)^2 \\
&\quad + (1 + \beta_3)(\beta_1 + \beta_2 - \beta_4)^2 + (1 + \beta_4)(\beta_1 + \beta_2 - \beta_3)^2 \\
M(\beta_1, \beta_2, \beta_3, \beta_4) &= (1 + 2\beta_1)(1 + 2\beta_2)(2 + \beta_3 + \beta_4) \\
N(\beta_1, \beta_2, \beta_3 | x) &= x(1 + 2\beta_1)(2 + \beta_2 + \beta_3).
\end{aligned}
$$

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