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Naturalness in Supersymmetric GUTS

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Abstract. Requiring the naturalness as defined by Veltman (radiative corrections should not disturb the gauge hierarchy in the Born term), supersymmetric grand unified theory based on the standard SU(5) is constructed. Supersymmetry must be unbroken above TeV energies. Renormalization-group analysis gives $\sin^2\theta_W(M_W)$ and α_c in agreement with experimental values. The unification mass M_{GUT} is larger than the standard model and hence proton lives longer.

1. Introduction

Grand unification of strong and electroweak interactions [1] (GUT) requires an enormous ratio of the GUT and electroweak mass scales [2]. It was argued that unnatural fine tuning of parameters is needed to achieve this gauge hierarchy if one uses elementary Higgs bosons [3]. Recently a simple criterion of naturalness was introduced by Veltman: If a theory with an ultraviolet cut-off Λ gives radiative corrections to masses and couplings larger than the actually observed values, it is called unnatural [4]. If the theory is to be applicable beyond this naturalness breaking mass scale [5] Λ, fine tuning between the Born term and radiative corrections becomes necessary.* The naturalness of the gauge hierarchy (radiative corrections do not upset the hierarchy in the Born term) requires that no quadratic divergence be present in masses and couplings. Since quadratic divergence occurs in two-point functions of spinless particles, one should either replace elementary Higgs bosons by composite ones (Technicolor scenario [3, 6, 7]), or cancel boson and fermion loop contributions (supersymmetry [8]). The underlying supersymmetry idea was particularly emphasized by Veltman [4]. The purpose of this paper is to stress in this supersymmetry alternative that the naturalness requires supersymmetry being unbroken above TeV energies and to construct a realistic supersymmetric SU(5) GUT which gives good results for $\sin^2\theta_W(M_W)$, α_c, and the unification mass scale M_{GUT} from the renormalization-group analysis [2] in spite of the presence of many new particles (supersymmetric partners of gauge bosons, Higgs bosons, quarks and leptons).

In this paper, we require the naturalness in the limited sense: The radiative corrections in the cut-off theory should not disturb the hierarchy in the Born term (preservation of naturalness in higher orders). However, 't Hooft has defined the naturalness to mean a more ambitious demand: if a theory contains small parameters, it has to have larger symmetry for vanishing value of the parameters [5]. This definition requires the absence of fine tuning not only in higher orders but also in the Born term. Therefore all dimensionful parameters much smaller than the grand unification mass scale (or perhaps the Plank mass) have to arise dynamically. In writing up this paper, we have received a preprint [9] attempting this scenario. This ambitious approach is very interesting and worth pursueing, but it does not seem easy to achieve the unification of strong and electroweak interactions. In this paper we will not explore the possibility of the dynamical breakdown of supersymmetry. In writing up the paper, we were told that Dimopoulos and Georgi have recently obtained similar results as ours [10].

In Sect. 2 we describe general consequences of requiring the naturalness and construct a supersymmetric SU(5) GUT with the naturalness. In Sect. 3 we show that it is possible to achieve the grand unification consistent with the renormalization group analysis of $\sin^2\theta_W$, α_C, and M_{GUT}.

2. Supersymmetric GUTS

The naturalness requires that radiative corrections in cut-off theories should not disturb the hierarchy

* As was emphasized by several people, especially by Weinberg [6] this fine tuning does not mean a logical inconsistency. In this respect naturalness as formulated here is an aesthetic requirement

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in the Born term. Therefore no quadratic divergence is allowed in masses and couplings. It has been shown that renormalizable supersymmetric theories involving chiral scalar and vector supermultiplets have no quadratic divergences [8]. In order to exploit this feature for achieving the naturalness, Higgs bosons, gauge bosons, quarks and leptons have to accompany supersymmetric partners. Since no exact supersymmetry is observed in nature, the symmetry has to be broken either spontaneously or explicitly. Renormalization of spontaneously broken supersymmetric theories was discussed and no quadratic divergence has been found [11]. Power counting argument supplemented by mass-independent renormalization scheme can be used to show that the cancelation of quadratic divergence in scalar two-point functions is not spoilt by explicit breaking of supersymmetry provided the breaking is soft (operator of dimension less than four) [12]. Therefore unobserved supersymmetry partners such as the scalar partner of electron can be heavy without destroying the cancelation of quadratic divergences. However, if we give an arbitrarily large mass splitting among members of supersymmetric multiplets, we lose the cancelation of quadratic divergences. Since the naturalness breaking mass scale of the gauge model before the supersymmetrization is effectively replaced by the mass scale of the supersymmetry breaking, we conclude that the supersymmetry should be unbroken above TeV energies.

Let us construct an example of supersymmetric model of grand unification explicitly. As a first approximation, we take symmetry breaking mass scales of the electroweak gauge group and of the supersymmetry to be negligible compared to the grand unification mass scale M_{GUT}. We take the standard SU(5) model [1, 13] as the simplest possibility and introduce the following supersymmetric multiplets for gauge- and Higgs-boson system: Vector multiplet $V^a (a=1,\dots,24)$ containing SU(5) gauge bosons, left-handed chiral multiplet* $\Phi^a_- \ (a=1,\dots,24)$ containing Higgs bosons in the adjoint representation, left-handed chiral multiplet H^i_- and right-handed one $H'^i_+ \ (i=1,\dots,5)$ containing complex Higgs bosons in 5 representation, and a left-handed chiral multiplet M_- as a singlet of SU(5). The singlet M_- is a convenient technical device to form an appropriate Higgs potential because the renormalizability allows only up to third power in chiral supermultiplets instead of the fourth power in the usual scalar field. Quarks and leptons are contained in $\underline{10}$ and $\underline{5}^*$ left-handed chiral superfields. As is well-known, a vector supermultiplet contains a vector and a Majorana spinor fields both massless, and a chiral supermultiplet contains a Majorana spinor, a scalar and a pseudoscalar fields [14, 15].

The supersymmetric Lagrangian for gauge- and Higgs supermultiplets consists of several pieces

$$L = L_{\mathrm{gauge}} + L_\Phi + L_H + L_M + L_{\mathrm{int}}. \tag{2.1}$$

The kinetic term for the gauge supermultiplet in the Wess–Zumino gauge [16] reads*

$$\begin{aligned} &L_{\mathrm{gauge}} = 2\mathrm{Tr}\left[-\tfrac{1}{4}(V_{\mu\nu})^2 + \tfrac{1}{2}\bar{\lambda} i \nabla\!\!\!/ \lambda + \tfrac{1}{2}D^2\right] \\ &V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu - ig[V_\mu, V_\nu] \\ &\nabla_\mu \lambda = \partial_\mu \lambda - ig[V_\mu, \lambda] \\ &V_\mu = V^a_\mu T^a, \ \mathrm{Tr}(T^a T^b) = \tfrac{1}{2}\delta^{ab} \end{aligned} \tag{2.2}$$

where g is the gauge coupling and V_μ, λ and D are gauge field, Majorana spinor, and auxiliary field of the vector supermultiplet as 5×5 matrices of SU(5). The kinetic term for the singlet supermultiplet M_- is given by

$$L_M = \partial_\mu A^*_{-M} \partial^\mu A_{-M} + \bar{\psi}_{-M} i \partial\!\!\!/ \psi_{-M} + F^*_{-M} F_{-M} \tag{2.3}$$

where A_{-M}, ψ_{-M}, and F_{-M} are spinless, spinor, and auxiliary fields of the left-handed supermultiplet M_-. The kinetic term of the Higgs supermultiplet Φ_- contains interactions with the gauge multiplet and reads in the Wess–Zumino gauge

$$\begin{aligned} L_\Phi = 2\mathrm{Tr}[&\nabla_\mu A^+_- \nabla^\mu A_- + \bar{\psi}_- i \nabla\!\!\!/ \psi_- + F^+_- F_- \\ &+ i\sqrt{2}(A^+_- g\bar{\lambda}\psi_- - \bar{\psi}_- g\lambda A_-) + A^+_- gDA_-] \end{aligned} \tag{2.4}$$

where we omit the suffix ϕ for component fields A_-, ψ_-, and F_- of the supermultiplet Φ. We have a similar kinetic term L_H for the complex Higgs supermultiplet H_- and H'_+ in the 5 representation. The most general renormalizable Lagrangian for chiral supermultiplets Φ_-, H_-, H'_+ and M_- is conveniently written down in terms of the superfield notation [15].

$$\begin{aligned} L_{\mathrm{int}} = -\tfrac{1}{2}\bar{D}D\{&h_1 \mathrm{Tr}(\Phi^3_-) + h_2 \mathrm{Tr}(\Phi^2_-)M_- + m_1 \mathrm{Tr}(\Phi^2_-) \\ &+ h_3 H'^+_+ \Phi_- H_- + h_4 H'^+_+ H_- M_- + m_2 H'^+_+ H_- \\ &+ sM_- + m_3 M^2_- + h_5 M^3_- + \mathrm{h.c.}\} \end{aligned} \tag{2.5}$$

where $-\bar{D}D/2$ is the projection operator of the F-component. The couplings h_i are dimensionless and of order unity, m_i are of order M_{GUT}, and s is of order M^2_{GUT}, and all couplings and masses are assumed to be real for simplicity.

The equations of motion for auxiliary fields are given by

$$\begin{aligned} -F^+_- = {}&3h_1\left(A^2_- - \frac{\mathrm{Tr}A^2_-}{5}\right) + 2h_2 A_- A_{-M} \\ &+ 2m_1 A_- + h_3\left(A_{-H}A^+_{+H'} - \frac{A^+_{+H'}A_{-H}}{5}\right) \end{aligned} \tag{2.6}$$

$$-F_{+H'} = (h_3 A_- + h_4 A_{-M} + m_2)A_{-H} \tag{2.7}$$

$$-F^+_{-H} = A^+_{+H'}(h_3 A_- + h_4 A_{-M} + m_2) \tag{2.8}$$

* The suffix — refers to the left-handedness

* In general we adopt the notation of Salam and Strathdee [15] except $\gamma_5^{\mathrm{ours}} = i\gamma_5^{\mathrm{salam}}$. Therefore our metric and γ-matrices are the same as Bjorken and Drell

$$- F^*_{-M} = h_2 \operatorname{Tr} A_-^2 + h_4 A^+_{+H'} A_{-H} + s$$
$$+ 2m_3 A_{-M} + 3h_5 A^2_{-M} \tag{2.9}$$

It is well-known that the supersymmetry is not broken (spontaneously) if and only if auxiliary fields have vanishing vacuum expectation values [17, 14]. We would like to maintain the supersymmetry down to low mass scales, hence we need to find solutions* for $\langle F_- \rangle = \langle F_{+H'} \rangle = \langle F_{-H} \rangle = \langle F_{-M} \rangle = 0$. To obtain the desirable pattern of the symmetry breaking $SU(5) \to SU(3) \times SU(2) \times U(1)$, we would like to find a solution with the vanishing vacuum expectation value for the $\underline{5}$ plet Higgs field**

$$\langle A_{-H} \rangle = \langle A_{+H'} \rangle = 0 \tag{2.10}$$

which satisfies $\langle F_{-H} \rangle = \langle F_{+H'} \rangle = 0$ automatically. By SU(5) rotations we can choose the representation in which the adjoint Higgs $\langle A_- \rangle$ is diagonal

$$\langle A_- \rangle_{ij} = x_i \delta_{ij} \tag{2.11}$$

Denoting $\langle A_{-M} \rangle \equiv M$ we obtain from $\langle F_- \rangle = \langle F_{-M} \rangle = 0$

$$3h_1 (x_i^2 - \frac{1}{5} \sum_{j=1}^{5} x_j^2) + 2(h_2 M + m_1) x_i = 0 \tag{2.12}$$

$$h_2 \sum_{j=1}^{5} x_j^2 + s + 2m_3 M + 3h_5 M^2 = 0 \tag{2.13}$$

Equation (2.13) cannot be satisfied by vanishing x_i's, if we demand

$$m_3^2 - 3h_5 s < 0. \tag{2.14}$$

Therefore we necessarily obtain the spontaneous breakdown of SU(5). Equation (2.12) implies that x_i can take only two values x_+ and x_-. If we choose $x_i = x_+$ for $i = 1, 2, 3$ and $x_i = x_-$ for $i = 4, 5$, we obtain the desired breaking pattern*** $SU(5) \to SU(3) \times SU(2) \times U(1)$ with

$$x_+ = V, \quad x_- = -\tfrac{3}{2} V \tag{2.15}$$

$$V = \frac{4(h_2 M + m_1)}{3h_1} \tag{2.16}$$

The vacuum expectation value M of the singlet field is finally determined by inserting (2.15) and (2.16) into (2.13). Since the above solution is supersymmetric, it is an absolute minimum of the effective potential [14, 17]. In this supersymmetric solution we can work out the mass matrix of Higgs multiplets labeled as [13]

$$A_- - \langle A_- \rangle$$
$$= \frac{1}{\sqrt{2}} \begin{bmatrix} \frac{\lambda^a}{\sqrt{2}} H_8^a + \sqrt{\frac{2}{15}} H_0, H_X \\ H_X^+, \frac{\tau^i}{\sqrt{2}} H_3^i - \sqrt{\frac{3}{10}} H_0 \end{bmatrix} \begin{matrix} a = 1, \ldots, 8 \\ i = 1, 2, 3 \end{matrix} \tag{2.17}$$

$$A_{-H} = \begin{bmatrix} \underset{\sim}{H} \\ \varphi \end{bmatrix}. \tag{2.18}$$

We find that H_X and H_X^+ are massless Goldstone bosons and eaten by gauge bosons whereas H_8^a and H_3^i have mass $= 15h_1 V/2$. The singlet Higgs boson $A'_M \equiv A_{-M} - \langle A_{-M} \rangle$ mixes with H_0 and both masses are of order V. In order to assure the gauge hierarchy and prepare for the symmetry breaking $SU(2) \times U(1) \to U(1)_{em}$ at a much smaller energy scale, we require that φ should not have mass of order V:

$$m_\varphi = \sqrt{2}(-\tfrac{3}{2} h_3 V + h_4 M + m_2) = 0 \tag{2.19}$$

This condition at the tree level is the only fine tuning which we need for the gauge hierarchy*. The Higgs boson mediating the proton decay becomes superheavy (mass $= h_3 V/\sqrt{2}$).

The massless complex doublet φ is not a Goldstone boson and hence it would become superheavy by radiative corrections if the theory were not supersymmetric (pseudo-Goldstone boson [18]). However Zumino has shown that vacuum energy (value of effective potential at stationary points) in supersymmetric theories vanish to all orders if the supersymmetry is not broken [19]. The Zumino's theorem implies that non-Goldstone massless bosons remain exactly massless to all orders if the supersymmetry is unbroken. Since spontaneous or explicit soft breaking of the supersymmetry occurs at smaller mass scales (TeV or electroweak mass scale) in our model, the non-Goldstone boson φ-cannot receive radiative corrections of order M_{GUT}, but becomes the Higgs doublet which is responsible for the spontaneous breakdown of $SU(2) \times U(1) \to U(1)_{em}$.

Yukawa couplings of matter supermultiplets (quarks and leptons) and Higgs supermultiplets H_-, H'_+ are precisely analogous to the ordinary Yukawa couplings which give rise to quark and lepton masses. Mechanism of baryon or lepton number conservation in this model is precisely the same as the nonsupersymmetric standard model (violation is suppressed by powers of $1/M_{\mathrm{GUT}}$). However separate conservation of quark and lepton flavours can be violated at a certain level because supersymmetric

* Vacuum expectation values of auxiliary fields of gauge- and matter supermultiplet also vanish in the solution which we find

** If $\langle A_{-H} \rangle \neq 0$, one obtains undesirable symmetry breaking pattern such as $SU(5) \to SU(3) \times U(1)$ at M_{GUT}. One can show that there is a continuous range of parameter space where $SU(5) \to SU(3) \times SU(2) \times U(1)$ is allowed, but $\langle A_{-H} \rangle \neq 0$ is not

*** For certain values of parameters one can also get another solution with a breaking pattern $SU(5) \to SU(4) \times U(1)$. Both solutions are supersymmetric and have same (zero) vacuum energy. It is an interesting question whether one or the other breaking pattern might be preferred as the temperature drops in the early universe

* All parameters of explicit soft breaking of supersymmetry should be much smaller than M_{GUT} and hence they can also be regarded as finely tuned

partners may have different degree of the GIM cancelation [20] and different $K-M$ mixing angles [21]*. It is interesting to constrain the model further by experimental data on rare processes.

As for the triangle anomaly of the gauge current, new fermions in the Higgs supermultiplets do not give rise to anomalies because gauge interactions of the Higgs supermultiplets are assigned left-right symmetrically [22]. Anomalies associated with matter supermultiplets cancel between quarks and leptons in each generation as usual.

3. Renormalization-Group Analysis

Let us summarize the particle content of our model after the spontaneous breakdown $SU(5) \to SU(3) \times SU(2) \times U(1)$ at M_{GUT}. Superheavy particles (mass is of order M_{GUT}) are the following: twelve vector supermultiplets corresponding to the broken generators of SU(5), color octet and SU(2) triplet chiral supermultiplets containing H_8^a and H_3^i, chiral supermultiplets containing H_0 and A'_M, and color triplet chiral supermultiplet $\underset{\sim}{H}$. All of them decouple below M_{GUT}. Surviving supermultiplets below M_{GUT} are the following: vector supermultiplets containing SU(3), SU(2) and U(1) gauge bosons, chiral supermultiplets containing complex Higgs doublets φ, and quark and lepton chiral supermultiplets. To make the renormalization-group analysis of running coupling constants [2], we take M_W as the typical mass scale of both breakings of electroweak gauge group and the supersymmetry. Therefore as a crude approximation we assume that all the surviving supermultiplets are effective between M_{GUT} and M_W, whereas only massless $SU(3) \times U(1)_{em}$ vector supermultiplets** and ordinary quarks and leptons (without superpartners) are effective below M_W.

The one-loop β-functions β_3, β_2 and β_1 for SU(3), SU(2) and U(1) gauge couplings between M_{GUT} and M_W are given by

$$\beta_i = \frac{g^3}{16\pi^2} b_i \tag{3.1}$$

$$b_3 = -3 \times 3 + 2n_g \tag{3.2}$$

$$b_2 = -3 \times 2 + 2n_g + 1 \tag{3.3}$$

$$b_1 = 2n_g + \tfrac{3}{5} \tag{3.4}$$

where n_g is the number of generations of quarks and leptons.*** In terms of the fine structure constants $\alpha(M_W)$ and $\alpha_c(M_W)$ of $U(1)_{em}$ and QCD at M_W, the weak mixing angle $\sin^2\theta_W(M_W)$ at M_W and the grand unification mass M_{GUT} are given by [2].

$$\sin^2\theta_W(M_W) = \frac{5(b_1-b_2)\alpha(M_W)/\alpha_c(M_W) + 3(b_2-b_3)}{-8b_3+3b_2+5b_1} \tag{3.5}$$

$$\ln\frac{M_{GUT}}{M_W} = \frac{6\pi}{-8b_3+3b_2+5b_1}\left(\frac{1}{\alpha(M_W)} - \frac{8}{3\alpha_c(M_W)}\right) \tag{3.6}$$

The QCD fine structure constant at μ below M_W is given by

$$\alpha_c(\mu) = \left(\frac{1}{\alpha_c(M_W)} + \frac{b'_3}{2\pi}\ln\frac{M_W}{\mu}\right)^{-1}$$

$$b'_3 = -9 + \tfrac{4}{3}n_g \tag{3.7}$$

We take the electromagnetic fine structure constant at M_W as [23]

$$(\alpha(M_W))^{-1} = 128 \tag{3.8}$$

We tabulate for several values of $(\alpha_c(M_W))^{-1}$ the result of the weak angle $\sin^2\theta_W$, unification mass scale M_{GUT}, and QCD fine structure constant α_c at $\mu = 10$ GeV. The number of generation is relevant only for α_c(10 GeV), and $n_g = 3$ and 4 cases are shown. Neutral current data give [24]

$$\sin^2\theta_W|_{\exp} = 0.224 \pm 0.015 \tag{3.9}$$

We see that the grand unification can in fact be achieved before the Planck mass beyond which unification would be presumably meaningless without incorporating gravity. We see also that $\sin^2\theta_W(M_W)$ is in good agreement with the experimental one and α_c is quite consistent with conventional values from hard scattering analysis.* As a general feature of supersymmetric grand unified models we observe that M_{GUT} tend to be larger and hence proton lives longer than nonsupersymmetric models. This is

Table 1. Results of the renormalization-group analysis of the weak angle $\sin^2\theta_W(M_W)$ at M_W, the unification mass scale M_{GUT}, and the QCD fine structure constant α_c at 10 GeV for several values of $(\alpha_c(M_W))^{-1}$ and for three and four generations of quarks and leptons

$(\alpha_c(M_W))^{-1}$	4	6	8	10
$\sin^2\theta_W(M_W)$	0.215	0.222	0.229	0.236
M_{GUT} GeV	7.8×10^{17}	1.5×10^{17}	2.8×10^{16}	5.2×10^{15}
α_c(10 GeV)				
for $n_g = 3$	0.421	0.229	0.157	0.119
for $n_g = 4$	0.356	0.208	0.147	0.114

* The author is indebted to T. Yanagida for a discussion on this point

** Superpartners of gluons and photons (gluinos and photino) can have Majorana masses when the supersymmetry is broken. However we assume as a simplest plausible possibility that they have relatively small masses compared to M_W

*** In general β-function for supersymmetric $SU(N)$ gauge theory with n and m chiral supermultiplets in adjoint and fundamental representations is given by $b = -(3-n)N + m/2$

* There is a considerable uncertainty in the QCD threshold effect around and below M_W in our model

because β-functions become less asymptotic free due to richer spectra of spinors and scalars. In fact if one introduces more Higgs scalars or allows more surviving supermultiplets below M_{GUT},* one often find M_{GUT} larger than the Planck mass.

It is an interesting problem to construct supersymmetric GUT based on the spontaneous breakdown of supersymmetry as has been done for $SU(3) \times SU(2) \times U'(1) \times U''(1)$ model [25]. Spontaneous breakdown can reduce the number of (finely tuned) parameters of supersymmetry breaking substantially. A number of experimental signatures below and around M_W have also been worked out [26]. However it seems unlikely that the present approach based on global supersymmetry can shed much light on the question of quark and lepton masses, which might need supergravity or something else.

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Note Added in Proof

Immediately after submitting the paper, I have obtained a preprint by S. Dimopoulos and H. Georgi (HUTP-81/AO22) which describes a very similar supersymmetric SU(5) model. Since they did not introduce the singlet M_-, their model contains unbroken SU(5) vacuum which is degenerate with the desirable vacuum for $SU(5) \rightarrow SU(3) \times SU(2) \times U(1)$ symmetry breaking. I also obtained a preprint "Dynamical breaking of supersymmetry" by E. Witten which contains much material relevant to our discussion.

* For instance, imposing the R-invariance [14] would make supermultiplets H_8^a and H_3^i massless at M_{GUT} and hence M_{GUT} tends to be larger than the Planck mass