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B_c decays

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Abstract. Theoretical predictions on decay properties of the simplest particle containing more than one heavy quark, the pseudoscalar ($\bar{b}c$) meson B_c^+, are presented. Some channels that should present a distinctive signature are discussed in more detail.

1 Introduction

Given the existence of heavy flavoured quarks, hadron containing two (or more) different heavy quarks should exist. Up to now, we have no experimental confirmation of these particles: as a matter of fact, one expects their production cross section to be small as a consequence of the difficulty of producing (at least) two heavy quarks pairs and of the presumably small probability of recombination into a single hadron.

In the large hadron colliders presently under study (LHC, SSC) one expects however a very copious production of beauty particles ($\sigma(pp \to b\bar{b}X) \geqq 100\,\mu$b at LHC [1]), with an energy high enough to allow a separation of the decay vertex from the primary one. Even if the relative probability for the formation of a particle containing two (or more) different heavy quarks is small, it seems appropriate now to discuss the experimental signatures that could allow the discovery of these particles.

Here we focus our attention on mesons formed by a bottom antiquark and a charmed quark. The lightest state in the family is the pseudoscalar ($c\bar{b}$) meson B_c^+. All the heavier particles should decay, through strong or electromagnetic interactions, to a final state containing B_c^+, which in turn is only allowed to decay weakly.

We will assume in our study the B_c mass as predicted by potential models [2–4]:

$$M_{Bc} \cong 6.27\,\text{GeV}. \tag{1.1}$$

A much larger mass, $M_{B_c} = M_{B_u} + (1.69 \pm 0.18)\,\text{GeV}$, has been obtained using QCD sum rules of the type generally used for mesons made by one heavy and one light quark [5]. The success of potential models in fitting the known spectra of mesons formed by a heavy quark–antiquark pair (charmonium and bottomonium) gives support to our choice.

Relativistic corrections split the masses of the lightest, pseudoscalar and vector, particles. The first excited state, a vector meson, should be heavier by only $70 \to 100$ MeV, so that it would decay emitting a photon: this photon, if detected, could also help in identifying the subsequent B_c weak decay.

In this paper we give theoretical predictions concerning the lifetime and decay branching ratios of B_c^+, considering in particular the decay channels that could provide a distinctive signature for the presence of this particle.

2 Inclusive semileptonic decays

The decay processes of B_c^+ can be divided in three classes: spectator c quark, spectator $\bar{b}$ and annihilation decays (Fig. 1). The inclusive semileptonic rate due to processes of the first class is expected to be approximately equal to the decay rate for a B^+ (or B_d^0) meson. On the contrary, the naive expectations [6, 7] that the spectator model prediction for the D decay rate correctly describes the rate due to spectator $\bar{b}$ decays should fail, for reasons that we will presently discuss. At the same time, the annihilation contribution to the total rate should be relatively important.

In the spectator $\bar{b}$ semileptonic decays, the heavy (and less strongly bound) B_s (or $B_{d,u}$) in the final state has a considerable effect in reducing the phase space: this is shown in Fig. 2, where the kinematical boundaries of the Dalitz plot for these decays are compared with those for c-quark decay, computed using different "standard" values for the quark masses. As one may notice, the endpoint of the lepton spectrum for c-quark decay is roughly equal to the true endpoint

$$E^{\max} = \frac{M_{B_c}^2 - M_{B_s}^2}{2M_{B_c}} \tag{2.1}$$

while the maximum lepton invariant mass q^2 is too big. Also shown in the figure are the kinematical boundaries

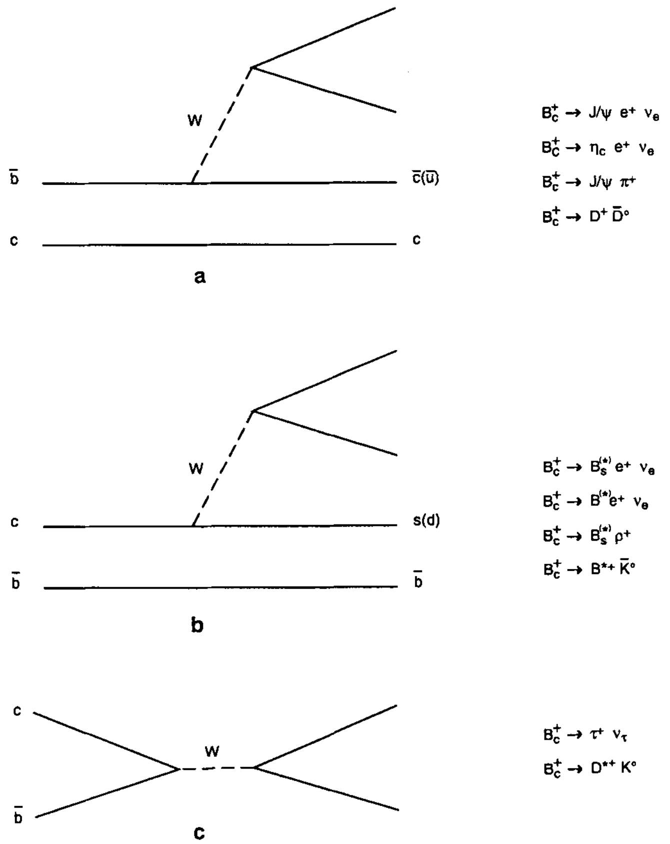

Fig. 1 a–c. Different decay mechanisms for B_c^+, with examples of corresponding exclusive channels. **a** c spectator; **b** $\bar{b}$ spectator; **c** $c\bar{b}$ annihilation

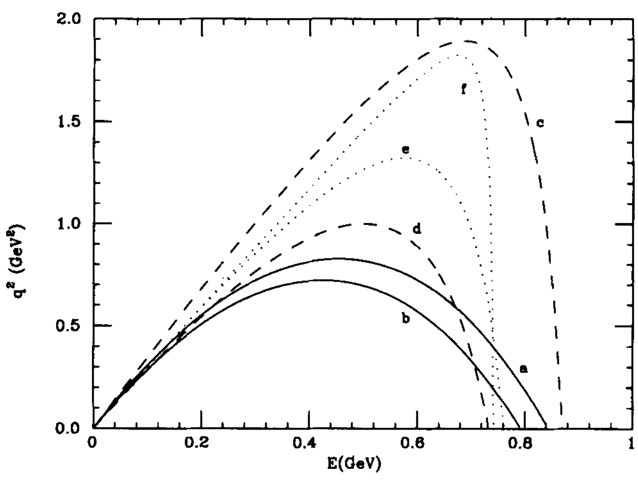

Fig. 2. Dalitz plot for semileptonic decays (E = positron energy, q^2 = invariant square mass of the two leptons). Full lines: a) $B_c^+ \to B_s e^+ \nu$, with $M_{B_s} = 5.39\,\text{GeV}$; b) $B_c^+ \to B_s^* e^+ \nu$, with $M_{B_s^*} = 5.45\,\text{GeV}$. Dashed lines: c) $D^+ \to \bar{K}^0 e^+ \nu$; d) $D^+ \to \bar{K}^{*0} e^+ \nu$. Dotted lines: e) $c \to s e^+ \nu$, with constituent masses ($m_c = 1.7\,\text{GeV}$, $m_s = 0.55\,\text{GeV}$); f) $c \to s e^+ \nu$, with current masses ($m_c = 1.5\,\text{GeV}$, $m_s = 0.15\,\text{GeV}$)

for D semileptonic decays: it is evident that the spectator model may cope better with the experimental situation in this case.

To estimate the rate for B_c decay we make the simplest possible hypothesis, namely we use the spectator model formulae within the limits of the accessible kinematical region, and we obtain

$$\Gamma(B_c^+ \to X_{\bar{b}} e^+ \nu) \cong 0.71\, \Gamma(D^+ \to X_s e^+ \nu). \tag{2.2}$$

Another possible estimate can be made summing the exclusive decay rates in the two channels $B_s e^+ \nu$ and $B_s^* e^+ \nu$. Their sum should be a large fraction of the inclusive rate for $\bar{b}$ spectator semileptonic decays, as suggested by the following simple kinematical argument [8]. If one neglects the spectator quark momentum in the decaying quark rest frame, the mass of the hadrons produced in the inclusive semileptonic decay $Q \to Q' e \nu$ varies in the limited range

$$(m_{Q'} + m_{sp})^2 \leqq \langle M_X^2 \rangle \leqq m_{Q'}^2 + m_{sp}^2 + m_{sp} \frac{m_Q^2 + m_{Q'}^2}{m_Q}. \tag{2.3}$$

Use of this approximate formula (with constituent masses for the quarks) for semileptonic $\bar{B}_u \to X_c$ decays gives a range of M_X values of about 200 MeV, thus providing an argument in favour of the dominance of the final states with lowest mass in the decay rate. In our case, with $m_Q = m_c = 1.7\,\text{GeV}$, $m_{Q'} = m_s = 0.55\,\text{GeV}$ and $m_{sp} = m_b = 5.1\,\text{GeV}$, we obtain a range of M_X values 340 MeV wide, less than the expected [2] mass difference between orbital excitations and ground state in the $(\bar{b}s)$ system.

To evaluate the rates of interest we adopt two different quark models. The ISGW model [9] is basically nonrelativistic and therefore its predictions are most reliable at $q^2 = q^2_{\max} = (M_{B_c} - M_{B_s})^2$. The wave function is a gaussian, whose width can be determined for the B_c (B_s) case according to their prescriptions, minimizing the mean value of the Hamiltonian given in [9] with $\alpha_s = 0.4\,(0.5)$. We obtained $\beta_{B_c} \cong 0.82\,\text{GeV}$ and $\beta_{B_s} \cong 0.51\,\text{GeV}$. These values are larger than $\beta_{B_u} \cong 0.41\,\text{GeV}$, in agreement with the expectation for a system formed by heavier quarks. One has further to extrapolate the form factors to $q^2 = 0$: this has been done both using the q^2 exponential dependence predicted by the ISGW nonrelativistic model and using a pole model extrapolation, with similar results (see Table 2). We obtain for the total rate

$$\Gamma(B_c \to B_s + e + \nu) + \Gamma(B_c \to B_s^* + e + \nu)$$
$$\cong 60 \cdot 10^{-15}\,\text{GeV} \cong \tfrac{1}{2} \Gamma(D^+ \to X_s e^+ \nu). \tag{2.4}$$

The second model we use is the BSW model [10], that predicts form factor values at $q^2 = 0$ in terms of one parameter ω, corresponding to the mean root square transverse momentum of the quarks in the meson. We again expect that for mesons containing two heavy quarks ω be larger than the value (0.4 GeV) appropriate for B and D decays.* Since BSW do not give a

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* In [6] the same ω value has been used for all mesons: this is difficult to justify, since the value of ω for mesons containing a light quark has been already fixed in the BSW model

prescription to evaluate this parameter, we chose its value to reproduce the mean root square p_T estimated in the ISGW model ($\omega_X \cong \beta_X$). The form factor behaviour at $q^2 \neq 0$ is obtained from a pole model extrapolation [10]. We obtain in this case

$$\Gamma(B_c \to B_s + e + \nu) + \Gamma(B_c \to B_s^* + e + \nu) \cong 55 \cdot 10^{-15}\,\text{GeV}. \tag{2.5}$$

The numerical value in (2.5) changes by no more than 30% for reasonable variations of the parameters.

From (2.2), (2.4), (2.5) we conclude

$$\Gamma(B_c^+ \to X_{\bar{b}} e^+ \nu) = (0.6 \pm 0.2)\,\Gamma(D^+ \to X_s e^+ \nu). \tag{2.6}$$

The annihilation contribution to the total rate is not expected to be small in our case. First, the helicity suppression is not very effective if heavy particles (τ lepton or, for nonleptonic decays, c quark) are present in the final state. Morever, the annihilation decays are unsuppressed by colour factors (contrary to B_d decays). Finally, the decay constant f_{B_c}, defined by the relation

$$\langle 0 | \bar{b}\gamma_\mu \gamma_5 c | B_c^+(P) \rangle = i f \hat{B}_c P_\mu, \tag{2.7}$$

is expected to be rather large.

A simple argument, albeit rough, in favour of a large f_{B_c} can be given as follows. Define $f_{J/\psi}$ by

$$\langle 0 | \bar{c}\gamma_\mu c | J/\psi(\varepsilon, P) \rangle = i f_{J/\psi} M_{J/\psi} \varepsilon_\mu. \tag{2.8}$$

From the experimental rate for $J/\psi \to e^+ e^-$ one obtains $f_{J/\psi} = 385$ MeV. Consider now the extreme non relativistic approximation for both B_c and J/ψ, in which the binding potential is approximately coulombic: then one would have

$$\frac{f_{B_c}}{f_{J/\psi}} = \left(\frac{M_{J/\psi}}{M_{B_c}}\right)^{1/2} \frac{|\psi_{B_c}(0)|}{|\psi_{J/\psi}(0)|} = \left(\frac{M_{J/\psi}}{M_{B_c}}\right)^{1/2} \left(\frac{2m_b}{m_b + m_c}\right)^{3/2} \cong 1.3 \tag{2.9}$$

and therefore $f_{B_c} \cong 500$ MeV.

We will use instead the result of [2]

$$f_{B_c} = 570\,\text{MeV} \tag{2.10}$$

(that is also in reasonable agreement with the predictions of [4] and [5]), since the coulombic approximation is probably not good enough for the $c\bar{c}$ potential. Thus we obtain:

$$\Gamma(B_c^+ \to \tau^+ \nu_\tau) = 63 \cdot 10^{-15}\,\text{GeV}. \tag{2.11}$$

3 Nonleptonic decays and lifetime

For nonleptonic decays, as usual in this kind of game, we have to fix the subtraction point μ at a scale of physical relevance for our processes. The coefficients of the operators entering in the effective Hamiltonian vary with μ, and this dependence should be cancelled by a corresponding dependence in the matrix elements of the four fermion operators. Since however we will evaluate the matrix elements via their vacuum-insertion approximation – that knows nothing about μ – we will have to choose the subtraction point, hoping that the choice is good enough (technically, this means that the B-parameters have values close to one at this scale). Our choice will be $\mu \cong m_c$, since the B_c radius should be $O(m_c^{-1})$.

With this choice of μ, we must consider the variation of the coefficients in a range of scales in which the $\bar{b}$ quark can be considered as heavy, and behaves essentially as a static colour source. The anomalous dimensions of four-quark operators containing a b quark are different in the two regions $\mu \gg m_b$ and $\mu \ll m_b$: their explict form in the leading logarithmic approximation (LLA) can be easily obtained from the anomalous dimensions of the corresponding currents [11] and is given in the Appendix, together with the resulting effective Hamiltonian. The further increase of the coefficients for $\mu \ll m_b$, due to the resummation of powers of the large logarithm $\log(m_b^2/\mu^2)$, must be taken into account for decays to exclusive final states. For inclusive decay rates and in LLA, it would only affect subasymptotic corrections (that we are neglecting here), but not the dominant spectator contribution [12]: this is a consequence of the infrared nature of these logarithms [13]. Given the uncertainty in the value of f_{B_c}, we neglect this type of corrections for the annihilation contributions.

In calculating nonleptonic decay rates, we adopt the approximation of neglecting colour suppressed $1/N_c$ terms,* that is phenomenologically successful for B and D decays [14, 15]. The results depend strongly on the values of the quark masses: their values ($m_c = 1.5$ GeV, $m_b = 4.9$ GeV, $m_s = 0.15$ GeV) have been chosen so that the spectator (free quark) predictions for semileptonic decay rates of B and D mesons and for the B total width are in good agreement with the experimental data. Moreover, we approximate the kinematics for $\bar{b}$-spectator nonleptonic decays with that of the corresponding semileptonic decays, therefore assuming that the suppression factor of about 0.6 is also present in this case.

The resulting predictions for inclusive rates are given in Table 1. The rate of annihilation nonleptonic decays is most sensitive to the b and c quark mass values: the number reported in Table 1 is a conservative estimate, that could easily be increased using larger masses or larger f_{B_c}. The three classes of decays in Fig. 1 contribute respectively 37%, 45% and 18% to the total rate; the mean life is estimated to be similar to that of D^0's, namely:

$$\tau_{B_c} \cong 5 \cdot 10^{-13}\,\text{s}. \tag{3.1}$$

4 Exclusive decays

A very clear signature for a B_c (three leptons coming from the same secondary vertex, two of them reconstructing a J/ψ and some missing p_T) could be given by the decay

* Should the reader dislike the neglect of $1/N_c$ terms, he may easily correct the results with the help of formulae in the Appendix, keeping $N_c = 3$ everywhere

Table 1. Inclusive decay rates (in units of 10^{-15} GeV) for free quarks and B_c decays and B_c inclusive branching ratios ($m_c = 1.5$ GeV, $m_b = 4.9$ GeV, $|V_{cb}| = 0.046$). Doubly Cabibbo-suppressed decay channels have been neglected

	free quarks	B_c^+	BR		free quarks	B_c^+	BR
$\bar{b} \to \bar{c} + e^+ + \nu_e$	62	62	4.7%	$c \to s + e^+ + \nu_e$	124	74	5.6%
$\bar{b} \to \bar{c} + \mu^+ + \nu_\mu$	62	62	4.7%	$c \to s + \mu^+ + \nu_\mu$	124	74	5.6%
$\bar{b} \to \bar{c} + \tau^+ + \nu_\tau$	14	14	1.0%	$c \to s + u + \bar{d}$	675	405	30.5%
$\bar{b} \to \bar{c} + \bar{d} + u$	248	248	18.7%	$c \to s + u + \bar{s}$	33	20	1.5%
$\bar{b} \to \bar{c} + \bar{s} + u$	13	13	1.0%	$c \to d + e^+ + \nu$	7	4	0.3%
$\bar{b} \to \bar{c} + \bar{s} + c$	87	87	6.5%	$c \to d + \mu^+ + \nu_\mu$	7	4	0.3%
$\bar{b} \to \bar{c} + \bar{d} + c$	5	5	0.4%	$c \to d + u + \bar{d}$	39	23	1.7%
$B_c^+ \to \tau^+ + \nu_\tau$	—	63	4.7%				
$B_c^+ \to c + \bar{s}$	—	162	12.2%				
$B_c^+ \to c + \bar{d}$	—	8	0.6%	$B_c^+ \to$ all	—	**1328**	**100%**

$B_c^+ \to J/\psi \mu^+ \nu_\mu$, followed by the decay of the J/ψ into a $\mu^+\mu^-$ pair. The estimate for the corresponding *inclusive* decay branching ratio is of 4.7%. We expect [8] that also in this case a considerable fraction of these events will contain a J/ψ in the final state, and proceed to give an estimate of the exclusive decay rate.

In this case, the results obtained in the ISGW and BSW models are different. As it may be seen from Table 2, the two ways to extrapolate form factors in the ISGW model mentioned in the discussion preceding (2.4) give rather different results, in that the pole model has a milder q^2 variation.* The BSW result, using a pole model variation but starting from its prediction at $q^2 = 0$ is the smallest. In fact, it may be noticed that the exponentially extrapolated values of ISGW form factors at $q^2 = 0$ essentially agree with BSW predictions. This, together with the belief that a nonrelativistic treatment should be adequate for both B_c and J/ψ, leads us to assume as our best estimate:

$$\Gamma(B_c^+ \to J/\psi \mu^+ \nu_\mu) = 39 \cdot 10^{-15}\,\text{GeV} \tag{4.1}$$

(with an error of at most 40%). The central value corresponds to a branching ratio of 2.9%. The same result applies to decays with a positron (and its neutrino) in the final state. The combined branching ratio for $B_c \to (l^+ l^-)_{J/\psi} l'^+ \nu (l, l' = e$ or $\mu)$ is therefore $\sim 8 \cdot 10^{-3}$.

Also interesting to confirm the relevance of annihilation decays would be the purely leptonic $B_c^+ \to \tau^+ \nu_\tau$ decay, with a large branching ratio ($\sim 5\%$) and a rather large τ transverse momentum ($\lesssim 2.9$ GeV/c). The unambigous identification of a τ from its subsequent decay will probably be quite difficult, however.

* The uncertainty in the predictions for some other decay channels is very large. Luckily, this happens for strongly Cabibbo-suppressed ($b \to u$) decays, that would anyhow be very difficult to observe

The channels just discussed would not allow the complete B_c^+ reconstruction, due to missing neutrinos. A measure of the B_c mass would be possible observing the decay $B_c^+ \to J/\psi \pi^+$, whose branching ratio is expected to be:

$$\text{BR}(B_c \to J/\psi \pi^+) \cong 2 \cdot 10^{-3}. \tag{4.2}$$

This follows from the estimates for two-body nonleptonic decay rates that we made using the vacuum insertion approximation. Our results, that often disagree with results obtained in previous analyses [6, 7], are presented in Tables 3 and 4. In these tables we have left indicated factors a_1^2 or a_2^2: a_1 and a_2 are combinations

Table 2. Exclusive semileptonic decay rates (in units of 10^{-15}GeV), calculated with $|V_{cb}| = 0.046$ and $|V_{ub}| = 0.1|V_{cb}|$. First column: ISGW model with form factors given in (B1) of [9], with $\kappa = 1$ ($\beta_{B_c} = 0.82$, $\beta_{B_s} = 0.51$, $\beta_{B_u} = 0.41$, $\beta_{J/\psi} = 0.66$ and $\beta_{D_u} = 0.39$ GeV); second column: ISGW model with pole model form factors; third column: BSW model [10] ($\omega_B = \omega_D = 0.4$, $\omega_{B_s} = 0.5$, $\omega_{J/\psi} = 0.6$ and $\omega_{B_c} = 0.8$ GeV)

$B_c^+ \to B_s + e^+ + \nu$	16.4	17.9	11.1
$B_c^+ \to B_s^* + e^+ + \nu$	40.9	46.3	43.7
$B_c^+ \to B_d + e^+ + \nu$	1.0	1.1	0.5
$B_c^+ \to B_d^* + e^+ + \nu$	2.5	3.0	2.9
$B_c^+ \to \eta_c + e^+ + \nu$	10.6	16.1	6.5
$B_c^+ \to J/\psi + e^+ + \nu$	38.5	53.1	21.8
$B_c^+ \to D^0 + e^+ + \nu$	0.033	0.12	0.002
$B_c^+ \to D^{0*} + e + \nu$	0.13	0.32	0.011

Table 3. Two body nonleptonic b-spectator decay rates (in units of 10^{-15} GeV), calculated with $M_{B_c}=6.27$; $M_{B_s}=5.39$; $M_{B_s^*}=5.45$; $m_\pi=0.140$; $m_\rho=0.77$; $m_K=0.495$; $m_{K^*}=0.86$; $M_B=5.27$; $M_{B^*}=5.33$; $f_\pi=0.133$; $f_\rho=0.216$; $f_\omega=0.195$; $f_K=0.162$; $f_{K^*}=0.216$ GeV

	BSW	ISGW
$B_c^+ \to B_s + \pi^+$	$a_1^2 \cdot 31.1$	$a_1^2 \cdot 44.0$
$B_c^+ \to B_s + \rho^+$	$a_1^2 \cdot 12.5$	$a_1^2 \cdot 20.2$
$B_c^+ \to B_s^* + \pi^+$	$a_1^2 \cdot 25.6$	$a_1^2 \cdot 34.7$
$B_c^+ \to B_s^* + \rho^+$	$a_1^2 \cdot 115.6$	$a_1^2 \cdot 152.1$
$B_c^+ \to B^+ + \bar{K}^0$	$a_2^2 \cdot 28.2$	$a_2^2 \cdot 61.4$
$B_c^+ \to B^+ + \bar{K}^{*0}$	$a_2^2 \cdot 10.0$	$a_2^2 \cdot 24.1$
$B_c^+ \to B^{*+} + \bar{K}^0$	$a_2^2 \cdot 31.0$	$a_2^2 \cdot 28.3$
$B_c^+ \to B^{*+} + \bar{K}^{*0}$	$a_2^2 \cdot 147.1$	$a_2^2 \cdot 163.8$
$B_c^+ \to B^0 + \pi^+$	$a_1^2 \cdot 0.97$	$a_1^2 \cdot 1.89$
$B_c^+ \to B^0 + \rho^+$	$a_1^2 \cdot 0.94$	$a_1^2 \cdot 2.14$
$B_c^+ \to B^{*0} + \pi^+$	$a_1^2 \cdot 1.58$	$a_1^2 \cdot 1.28$
$B_c^+ \to B^{*0} + \rho^+$	$a_1^2 \cdot 8.82$	$a_1^2 \cdot 8.86$
$B_c^+ \to B^+ + \pi^0$	$a_2^2 \cdot 0.48$	$a_2^2 \cdot 0.95$
$B_c^+ \to B^+ + \rho^0$	$a_2^2 \cdot 0.47$	$a_2^2 \cdot 1.07$
$B_c^+ \to B^+ + \omega$	$a_2^2 \cdot 0.38$	$a_2^2 \cdot 0.87$
$B_c^+ \to B^{*+} + \pi^0$	$a_2^2 \cdot 0.79$	$a_2^2 \cdot 0.64$
$B_c^+ \to B^{*+} + \rho^0$	$a_2^2 \cdot 4.41$	$a_2^2 \cdot 4.43$
$B_c^+ \to B^{*+} + \omega$	$a_2^2 \cdot 3.60$	$a_2^2 \cdot 3.53$
$B_c^+ \to B_s + K^+$	$a_1^2 \cdot 2.18$	$a_1^2 \cdot 3.28$
$B_c^+ \to B_s^* + K^+$	$a_1^2 \cdot 1.71$	$a_1^2 \cdot 2.52$

Table 4. Decay rates (in units of 10^{-15} GeV) for some two body nonleptonic c-spectator decays, calculated with ISGW form factors

$B_c^+ \to \eta_c + \pi^+$	$a_1^2 \cdot 1.71$
$B_c^+ \to \eta_c + \rho^+$	$a_1^2 \cdot 4.04$
$B_c^+ \to J/\psi + \pi^+$	$a_1^2 \cdot 1.79$
$B_c^+ \to J/\psi + \rho^+$	$a_1^2 \cdot 5.07$
$B_c^+ \to \eta_c + K^+$	$a_1^2 \cdot 0.127$
$B_c^+ \to \eta_c + K^{*+}$	$a_1^2 \cdot 0.203$
$B_c^+ \to J/\psi + K^+$	$a_1^2 \cdot 0.130$
$B_c^+ \to J/\psi + K^{*+}$	$a_1^2 \cdot 0.263$

of the coefficients in the effective Hamiltonian (see the Appendix for their precise definition). For $\mu = m_c$ and in the large N_c limit that we used in Table 1 their numerical values are $a_1 = 1.24\,(1.19)$ and $a_2 = -0.33\,(-0.39)$ for $\bar{b}(c)$ weak decays, respectively.

A few comments on the content of Tables 3 and 4 are in order. For decays to a final state containing a beauty hadron (Table 3) both the annihilation and the "penguin" operators contributions are suppressed by a very small factor, $O(\sin^{10}\theta_C)$, and can be safely neglected. Summing up the exclusive rates, one finds a total close to the independent prediction for inclusive decays given in Table 1. Moreover, as it can be seen from Table 2, the predictions of ISGW and BSW models are in reasonably good agreement here. It is unfortunate that these decay channels are very difficult experimentally, since they require the reconstruction of a B meson from its subsequent decay products. For this reason we found useless to report predictions for doubly Cabibbo-suppressed decays (for example, $B_c \to B_d^0 K^+$).

Among the decays to a charm containing final state we present in Table 4 those for which the ISGW and BSW form factors essentially agree* and the annihilation contribution should be small or vanishing. To avoid too large uncertainties, even in the framework of the model that we adopt, we do not present predictions for those exclusive channels for which the form factors differ widely in the two models used (like decays to $D\bar{D}$, see Table 2), nor for those that receive a potentially large annihilation contribution that it is difficult to estimate reliably: this happens for example for the decay $B_c \to J/\psi\, D_s$ (that also receives contribution from "penguin" operators). If one neglects annihilation and "penguin" contributions, the decay rate in this channel would be similar to that in $J/\psi\pi^+$.

As a general characteristic of B_c decays, the presence in the final state of a J/ψ is rather frequent. The inclusive $B_c \to J/\psi X_{u\bar{d}(\bar{s})}$ rate can be evaluated approximately in the large N_c limit, simply multiplying the known semileptonic rate by a factor of $3a_1^2 = 4.6$. The resulting branching ratio for $B_c \to (J/\psi +$ light quarks or leptons) is 19%. This corresponds to the most interesting channels, in that the J/ψ could have a momentum up to $p_{\max} \sim 2.4$ GeV in the B_c rest frame, to be compared with a $p_{\max} \sim 1.7$ GeV for B_u decays.** A rough estimate of the decay rate to other, phase space suppressed, final states containing a J/ψ leads to the total branching ratio:

$$\mathrm{BR}(B_c \to J/\psi + X) \cong 0.24. \qquad (4.3)$$

This differs from the prediction in [7], where only a few exclusive final states in hadronic channels have been included and (on the other hand) a very large semileptonic decay rate has been assumed.

* This happens since the form factors are evaluated here at q^2 near to 0, see the remarks preceding (4.1)

** Since however the B_c produced in the process of fragmentation of a $\bar{b}$ quark could be softer than B_u [7, 16] it will be difficult to tell a B_c only from the observation of a J/ψ in the decay products

5 Conclusions

In this paper, we have given theoretical estimates of the decay properties of B_c^+, the lightest $\bar{b}c$ meson, based on a QCD-corrected effective Hamiltonian and operator matrix elements evaluated in the vacuum insertion approximation and in the large N_c limit.

The results can be summarized as follows. From a study of inclusive decays, we predict the contribution to the total rate of different decay mechanisms to be 37% from c-spectator decays, 45% from $\bar{b}$-spectator decays and 18% from $c\bar{b}$ annihilation. The B_c lifetime is expected to be $\tau_{B_c} = 5\cdot 10^{-13}$ s. The inclusive semileptonic branching ratios should be 10.6% for decays to e (or μ) and 5.7% for decays to τ, dominantly due to the direct annihilation decay $B_c \to \tau\nu_\tau$.

We have also presented in Tables 2–4 predictions for exclusive semileptonic and two-body nonleptonic decays (in vacuum insertion approximation), based on the use of two different models for the form factors. The most clear signature for a B_c decay should be given by the decay chain

$$B_c^+ \to J/\psi l'^+ \nu \to (l^+ l^-) l'^+ \nu \quad (l', l = e \text{ or } \mu)$$

with a combined branching ratio (for e and μ) of $(8\pm 3)\cdot 10^{-3}$. Probably the best decay allowing complete reconstruction is $B_c \to J/\psi \pi^+$, again followed by $J/\psi \to l^+ l^-$, with a combined branching ratio of about $3\cdot 10^{-4}$, although other decays could be used as well.

The smallness of these numbers calls for a large production rate of B_c to be able to observe it. For e^+e^- colliders, a calculation in a simple and naive nonrelativistic model [17] gives at the Z peak a cross section corresponding (with a luminosity $\mathscr{L} = 10^{31}\,\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$) to a meagre 400 B_c^+ produced in 10^7 s.

The abundant production of b and c quarks at the LHC and SSC makes the hadron colliders more promising for B_c search, although the present estimate [16, 7] of the production cross section, $\sigma(pp \to B_c^+ X) \cong 5\cdot 10^{-3}\sigma(pp \to b\bar{b}X)$, is only an educated guess, based on assumptions on unknown phenomenological parameters. This point will be discussed elsewhere [18].

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Appendix

The effective hamiltonians for c nonleptonic decays have the form ($q_i = d$ or s)

$$\mathscr{H}^c_{\text{eff}} = \frac{G}{2\sqrt{2}} V_{uq_2} V^*_{cq_1} [C^c_+(\mu) O^c_+ + C^c_-(\mu) O^c_-] + \text{h.c.} \qquad (A.1)$$

$$O^c_\pm = (\bar{q}_{1\alpha}\gamma_\mu(1-\gamma_5)c_\beta) \cdot (\bar{u}_\gamma \gamma^\mu (1-\gamma_5) q_{2\delta})(\delta_{\alpha\beta}\delta_{\gamma\delta} \pm \delta_{\alpha\delta}\delta_{\gamma\beta}). \qquad (A.2)$$

The anomalous dimensions of the operators O^c_+ and O^c_- for $\mu > m_c$ are

$$\gamma_\pm = -\frac{\alpha_s}{2\pi}\frac{3}{N_c}(1 \mp N_c). \qquad (A.3)$$

As a consequence, in leading logarithmic approximation and for $\mu \geqq m_c$ one finds [19]

$$C^c_+(\mu) = \left(\frac{\alpha_s(M_W^2)}{\alpha_s(m_b^2)}\right)^{6/23} \left(\frac{\alpha_s(m_b^2)}{\alpha_s(\mu^2)}\right)^{6/25}, \qquad (A.4)$$

$$C^c_-(\mu) = [C^c_+(\mu)]^{-2} \qquad (A.4)$$

and (for $\alpha_s(m_c^2) = 0.27$, $\alpha_s(m_b^2) = 0.19$, $\alpha_s(M_W^2) = 0.11$) one has $C^c_+(m_c) = 0.80$ and $C^c_-(m_c) = 1.57$.

Nonleptonic decays of a b quark in three quarks with different flavours are described by the effective hamiltonians ($q_1, q_2 = c$ or u; $q_3 = d$ or s)

$$\mathscr{H}^b_{\text{eff}} = \frac{G}{2\sqrt{2}} V_{q_1 b} V^*_{q_2 q_3} [C^b_+(\mu) O^b_+ + C^b_-(\mu) O^b_-] + \text{h.c.} \qquad (A.5)$$

$$O^b_\pm = (\bar{q}_{1\alpha}\gamma_\mu(1-\gamma_5)b_\beta) \cdot (\bar{q}_{3\gamma}\gamma^\mu(1-\gamma_5)q_{2\delta})(\delta_{\alpha\beta}\delta_{\gamma\delta} \pm \delta_{\alpha\delta}\delta_{\gamma\beta}) \qquad (A.6)$$

and for $\mu > m_b$ the anomalous dimensions are the same as in the previous case.

However to determine $C^b_+(\mu)$ and $C^b_-(\mu)$ for $\mu = m_c$ it is necessary to calculate anomalous dimensions also for $m_c \leqq \mu \leqq m_b$, that are different from (A.3), since at these scales the b quark behaves as a static colour source. Using the results of [11] on currents containing one heavy quark field to evaluate the corresponding anomalous dimension of four-fermion operators with one heavy quark, we have obtained:

$$\gamma_\pm = -\frac{\alpha_s}{2\pi}\left[3\frac{N_c^2-1}{4N_c} + \frac{3}{2N_c}(1 \mp N_c)\right], \qquad (A.7)$$

$$C^b_+(\mu) = \left(\frac{\alpha_s(M_W^2)}{\alpha_s(m_b^2)}\right)^{6/23} \left(\frac{\alpha_s(m_b^2)}{\alpha_s(\mu^2)}\right)^{-3/25}, \qquad (A.8)$$

$$C^b_-(\mu) = \left(\frac{\alpha_s(M_W^2)}{\alpha_s(m_b^2)}\right)^{-12/23} \left(\frac{\alpha_s(m_b^2)}{\alpha_s(\mu^2)}\right)^{-12/25}, \qquad (A.9)$$

and one has in this case $C^b_+(m_c) = 0.90$ and $C^b_-(m_c) = 1.57$.

The more complicated case in which a quark–antiquark pair (and therefore "penguin" operators) is present is not needed for the decays that we have considered in this paper: we will discuss it elsewhere.

The factors a_1 and a_2 for nonleptonic two body decays (Tables 3, 4) are defined as

$$a_1 = C_+ \frac{N_c+1}{2N_c} + C_- \frac{N_c-1}{2N_c};$$

$$a_2 = C_+ \frac{N_c+1}{2N_c} - C_- \frac{N_c-1}{2N_c}, \qquad (A.10)$$

and in the large N_c limit are replaced by

$$a_1 \cong \tfrac{1}{2}(C_+ + C_-); \quad a_2 \cong \tfrac{1}{2}(C_+ - C_-). \qquad (A.11)$$

The enhancement factor for nonleptonic spectator inclusive rates is

$$3\cdot\left[C_+^2\frac{N_c+1}{2N_c}+C_-^2\frac{N_c-1}{2N_c}\right], \tag{A.12}$$

which corresponds in the large N_c limit used in Table 1 to

$$3\cdot\left[\frac{C_+^2+C_-^2}{2}\right]. \tag{A.13}$$

For annihilation decays the enhancement factor is instead $3\cdot a_1^2$.

For c-spectator and annihilation inclusive decays we used for C_+^b and C_-^b the values $C_-^b(m_b)=1.33$ and $C_+^b(m_b)=0.87$.

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