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Contributions of Parity-Violating Baryon Matrix Elements to Nonleptonic Charmed-Baryon Decays[1]

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Abstract. Baryon-baryon matrix elements of the parity-violating charm-changing weak Hamiltonian are calculated using the MIT bag model. The changes in the s- and p-wave amplitudes of Cabibbo-favored nonleptonic decays of charmed baryons due to these parity-violating baryon transition elements are in general not substantially significant. However, they can overwhelm or be comparable to the contributions arising from the parity-conserving matrix elements for some reactions such as the s-wave amplitude of $\Lambda_c^+ \to \Xi^0 K^+$ and $\Delta^{++} K^-$ and the p-wave amplitude of $\Lambda_c^+ \to \Sigma^0 \pi^+$.

I. Introduction

Although the study of nonleptonic hyperon decays is an old subject, it recently received a lot attention in the literature. It has been known for quite some time, that though the standard current-algebra approach (i.e., commutator term for s-waves and ground-state baryon pole for p-waves) successfully reproduces the relative sign of the s- and p-wave amplitudes, it fails to predict their relative magnitudes. A fit to s-waves yields too small p-wave amplitudes (except for $\Lambda \to N\pi$ decays) when compared with experiments*. Likewise, a fit to p-waves predicts too large s-waves [3]. The absolute magnitudes of both s- and p-wave amplitudes have been calculated in the nonrelativistic quark model [4] and the MIT bag model [5–7]. To have a better agreement between theory and experiment the importance of including meson-pole contributions [8] (or equivalently, the factorizable contributions [6, 7, 9]), which vanish in the soft-pion limit, was also recognized and emphasized. It was then observed by Le Yaouanc et al. [10] that the $\frac{1}{2}^-$ low-lying baryon resonance can contribute considerably to s-wave amplitudes and the evidence for such corrections was shown by Pham [11], Milosević et al. [12] also pointed out that the $\frac{1}{2}^+$ excited baryon poles can help to explain p-wave amplitudes. Taking into account these $\frac{1}{2}^-$ and $\frac{1}{2}^+$ poles and appropriate meson-pole contributions, Bonvin [13] concluded that individual s- and p-wave amplitudes can be reproduced to an accuracy of about 10%.

The discussion of two-body charmed baryon nonleptonic weak decays is complicated by some other features. If the final meson is not light, in principle one cannot relate the three-hadron amplitude $\langle BM|H_w|B_c\rangle$ to the baryon-baryon transition matrix $\langle B|H_w|B_c\rangle$ through the use of current algebra and PCAC. To study all two-body decays exclusively, it would be ideal to have a reliable and direct theoretical evaluation of the three-body matrix elements*. However, a direct calculation of $\langle BM|H_w|B_c\rangle$ within the quark model involves some uncertainties and is not as reliable as the standard current-algebra technique and PCAC [15]. The $B_c \to B + M$ decay (where M is a pion or a kaon) has been studied by several authors [14, 16, 17] in the framework of the conventional soft-meson technique and it was found that in general the theoretical prediction of the branching ratios is larger than that of experimental results, in particular for $\Lambda_c^+ \to \Lambda\pi^+$ decays. Two potentially important corrections may

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* See, for instance, [1] and [2]. As shown in [2] the p-wave amplitude is not only too small by roughly a factor of 2, it varies differently for different reactions

* Indeed, this is the starting framework employed by [14]

arise: As the hyperon decays the low-lying $\frac{1}{2}^-$ baryon poles for s-wave and excited $\frac{1}{2}^+$ poles for p-wave may change considerably the conventional current-algebra results. Secondly, baryon matrix elements of the parity-violating Hamiltonian, which can be safely disregarded in hyperon nonleptonic decays, are not necessarily negligible in charmed-baryon decays since $SU(4)$ symmetry is badly broken. Parity-violating baryon transition amplitudes for hyperon decays away from the limit of $SU(3)$ invariance have been computed by Golowich and Holstein [18] and the changes in the usual analyses caused by such terms are found to be at the one percent level and hence can be reasonably neglected. However, a large $SU(4)$ symmetry breaking might induce significant baryon matrix elements of the parity-violating charm-changing weak Hamiltonian. It is the purpose of this paper to calculate these transition matrix elements and explore their effects on nonleptonic charmed-baryon decays.

II. Calculation

We start with the improved soft-meson theorem[1] for the decay

$$\begin{aligned}&\langle M_\alpha(q)B|H_w|B_c\rangle\\&=-\frac{i\sqrt{2}}{f_\alpha}\langle B|[Q_\alpha^5,H_w]|B_c\rangle+P(q)+R(q)\end{aligned}\tag{1}$$

where $f_\pi=0.94\,m_\pi$ and $f_K=1.26f_\pi$, $P(q)$ is the ground-state baryon pole term, and $R(q)$ contains contributions which vanish in the soft-meson limit, such as meson-pole contributions (or equivalently, factorizable contributions) and excited baryon-pole terms. The contribution $R(q)$ is of vital importance since the final meson in charmed-baryon decays can be far away from being "soft". Writing*

$$\langle M_\alpha(q)B|H_w|B_c\rangle=i\bar{u}_B(A+\gamma_5B)u_{B_c}\tag{2}$$

where A and B are the s- and p-wave amplitudes respectively, we have (apart from meson-pole contributions)

$$\begin{aligned}A&=A^{\rm com}+A^{\rm pole}+A^{\rm pole(*)}\\&=-\frac{\sqrt{2}}{f_\alpha}\langle B|[Q_\alpha^5,H^{pv}]|B_c\rangle\\&\quad-\frac{\sqrt{2}}{f_\alpha}(B_c-B)\left\{\frac{g_A^{BB'}b^{B'B_c}}{B_c+B'}-\frac{b^{BB'}g_A^{B'B_c}}{B+B'}\right\}\\&\quad+\frac{\sqrt{2}}{f_\alpha}(B_c-B)\left\{\frac{g_A^{BB^*}b^{B^*B_c}}{B_c-B^*}+\frac{b^{BB^*}g_A^{B^*B_c}}{B-B^*}\right\}\end{aligned}\tag{3}$$

$$\begin{aligned}B&=B^{\rm com}+B^{\rm pole}+B^{\rm pole(*)}\\&=-\frac{\sqrt{2}}{f_\alpha}\langle B|[Q_\alpha^5,H^{pc}]|B_c\rangle\\&\quad-\frac{\sqrt{2}}{f_\alpha}(B_c+B)\left\{\frac{g_A^{BB'}a^{B'B_c}}{B_c-B'}+\frac{a^{BB'}g_A^{B'B_c}}{B-B'}\right\}\\&\quad-\frac{\sqrt{2}}{f_\alpha}(B_c+B^*)\left\{\frac{g_A^{BB^*}a^{B^*B_c}}{B_c-B^*}+\frac{a^{BB^*}g_A^{B^*B_c}}{B-B^*}\right\}\end{aligned}\tag{4}$$

where $(*)$ denotes contributions from $\frac{1}{2}^-$ low-lying baryon resonances for s-wave and from the first excited $\frac{1}{2}^+$ baryon poles for p-wave, $a^{BB'}(b^{BB'})$ are baryon matrix elements of the parity-conserving (-violating) Hamiltonian, g_V and g_A are vector and axial-vector form factors defined in (A.15). We note that $A^{\rm pole(*)}$ is opposite in sign to the commutator term $A^{\rm com}$, which reduces the s-wave amplitude [10]. Furthermore, the $A^{\rm pole}$, $A^{\rm pole(*)}$ and $B^{\rm pole(*)}$ terms all vanish in the soft-meson limit; the only pole term which survives in the soft-meson limit is the ground-state $\frac{1}{2}^+$ pole contribution to the p-wave amplitude.

The relevant QCD corrected effective weak Hamiltonian for Cabibbo favored charmed-baryon decays is

$$\begin{aligned}H_w&=\frac{G_F}{2\sqrt{2}}\cos^2\theta_C(c_-O_-+c_+O_+)\\O_\pm&=\bar{s}\gamma_\mu(1+\gamma_5)c\bar{u}\gamma_\mu(1+\gamma_5)d\\&\quad\pm\bar{s}\gamma_\mu(1+\gamma_5)d\bar{u}\gamma_\mu(1+\gamma_5)c\end{aligned}\tag{5}$$

where θ_C is the Cabibbo angle, and $c_-=1.80\sim2.10$ for $\Lambda_{\rm QCD}=250\ {\rm MeV}\sim500\ {\rm MeV}$. The penguin operator does not contribute to Cabibbo favored channels. Since O_+ is symmetric in color indices it also does not contribute.

It is known that if the meson-meson transition amplitude $\langle M_1|H_w|M_2\rangle$ is evaluated in the vacuum intermediate state approximation (i.e., the vacuum insertion), the vector meson- and pseudoscalar meson-pole amplitudes are then related to the factorizable s- and p-wave amplitudes respectively through PCAC, VMD, and the Goldberger-Treiman relation. Nevertheless, the calculation of meson-pole contribution is in some sense more complete and reliable if the weak transition element $\langle M_1|H_w|M_2\rangle$, which contains contributions from all intermediate states, can be taken directly from the experimental data or

* Here $\gamma_5=-\begin{pmatrix}0&I\\I&0\end{pmatrix}$. To write down the general formula for s- and p-wave amplitudes, it proves to be convenient to adopt this representation for γ_5

reasonably evaluated in the quark model, for instance using the MIT bag model. The well-known example is the matrix element $\langle\pi|H_w|K\rangle$ which cannot be fully explained by the vacuum insertion and QCD corrected Hamiltonian and hence has to be taken from the experimental amplitude for $K\to 2\pi$ decays. For charmed-baryon decays, there are F or D pole contributions (depending on the final-state light meson) for the p-wave amplitude and F^* or D^* pole terms for the s-wave amplitude. In the absence of experimental information on the transition $\langle\pi|H_w|F\rangle$... etc., we rely on the factorization method with vacuum insertion. The factorizable contributions are given by [16, 17]

$$A^{\text{fact}}=-\frac{1}{3\sqrt{2}}G_F\cos^2\theta_C f_\alpha(2c_+\pm c_-)(B_c-B)g_V^{B_cB}$$

$$B^{\text{fact}}=\frac{1}{3\sqrt{2}}G_F\cos^2\theta_C f_\alpha(2c_+\pm c_-)(B_c+B)g_A^{B_cB} \quad (6)$$

where the $+(-)$ sign is for $\pi^+(\bar{K}^0)$ emission.

For numerical estimates we use the following bag parameters [19]

$$m_u=m_d=0,\quad m_s=0.279\text{ GeV},\quad m_c=1.551\text{ GeV}$$
$$\omega_u=2.0428,\quad \omega_s=2.8523,\quad \omega_c=8.2965,$$
$$R=5\text{ GeV}^{-1}. \quad (7)$$

The values of the four-quark overlap integrals defined in (A.11) and (A.13) are*

$$a=1.82\times10^{-3}\text{ GeV}^3,\quad \tilde{a}=1.12\times10^{-3}\text{ GeV}^2$$
$$b=0.41\times10^{-3}\text{ GeV}^3,\quad \tilde{b}=-1.54\times10^{-3}\text{ GeV}^2 \quad (8)$$
$$c=0.36\times10^{-3}\text{ GeV}^3,\quad \tilde{c}=-3.61\times10^{-3}\text{ GeV}^2.$$

The relevant parity-conserving baryon matrix elements are (see Appendix for details)

$$a^{\Sigma^+\Lambda_c^+}=-\frac{1}{\sqrt{3}}(3a-b+4c),\quad a^{\Lambda^0\Sigma_c^0}=-a^{\Sigma^+\Lambda_c^+}$$

$$a^{\Sigma^0\Sigma_c^0}=\frac{1}{3}(9a+b+8c),\quad a^{\Xi^0\Xi_c^0}=\frac{1}{3}(9a-b+10c) \quad (9)$$

$$a^{\Xi^0\Lambda^0}=\frac{1}{\sqrt{3}}(3a+b+2c)$$

where $\Lambda^0(csd)$ and $\Xi_c^0(csd)$ are $SU(3)$ $\bar{3}$ and 6 states, respectively. Our result for $a^{\Xi^0\Xi_c^0}$ disagrees with [16, 17], but satisfies the U-spin relation

$$a^{\Xi^0\Xi_c^0}=\frac{1}{2}a^{\Sigma^0\Sigma_c^0}+\frac{\sqrt{3}}{2}a^{\Lambda^0\Sigma_c^0}. \quad (10)$$

The corresponding parity-violating baryon transition elements are computed using (A.14)

$$\underline{b}^{\Sigma^+\Lambda_c^+}=-\frac{1}{3\sqrt{6}}(\tilde{a}+2\tilde{b}-2\tilde{c}),$$

$$\underline{b}^{\Lambda^0\Sigma_c^0}=\frac{1}{3\sqrt{6}}(\tilde{a}-2\tilde{b}+2\tilde{c})$$

$$\underline{b}^{\Sigma^0\Sigma_c^0}=-\frac{1}{3\sqrt{2}}\tilde{a},\quad \underline{b}^{\Xi^0\Xi_c^0}=-\frac{1}{3\sqrt{2}}(\tilde{b}-\tilde{c})$$

$$\underline{b}^{\Xi^0\Lambda^0}=\frac{1}{3\sqrt{6}}(2\tilde{a}+\tilde{b}-\tilde{c}) \quad (11)$$

where $b^{BB'}=-\frac{8m_Bm_{B'}}{m_B+m_{B'}}\underline{b}^{BB'}$.

The vector and axial-vector form factors are

$$g_V^{\Lambda\Lambda_c^+}=0.95,\quad g_V^{P\Lambda_c^+}=\sqrt{\tfrac{3}{2}}(0.88)$$
$$g_A^{\Lambda\Lambda_c^+}=0.86,\quad g_A^{P\Lambda_c^+}=\sqrt{\tfrac{3}{2}}(0.77)$$
$$g_A^{\Sigma^+\Lambda}=-g_A^{\Lambda_c^+\Sigma_c^0}=\tfrac{1}{2}g_A^{\Sigma^+\Sigma^0}=\sqrt{\tfrac{2}{3}}(0.65)$$
$$g_A^{P\Sigma^+}=\frac{1}{2\sqrt{6}}g_A^{\Sigma^+\Delta^{++}}=-\frac{1}{5}g_A^{\Sigma^+\Xi^0}$$
$$=\frac{1}{\sqrt{3}}g_A^{\Lambda_c^+\Xi_c^0}=-\frac{1}{3}(0.71). \quad (12)$$

In this paper we consider the following Cabibbo favored Λ_c^+ decay modes: $\Lambda_c^+\to\Lambda\pi^+$, $p\bar{K}^0$, $\Delta^{++}K^-$, $\Sigma^0\pi^+$ and Ξ^0K^+. Among these channels, only the first two decays receive factorizable contributions. The process $\Lambda_c^+\to\Delta^{++}K^-$ (also $\Lambda_c^+\to\Xi^0K^+$) is of particular interest since it receives pole contributions alone given by*

$$A^{\text{pole}}=\frac{1}{f_K}g_A^{\Sigma^+\Delta^{++}}\frac{b^{\Sigma^+\Lambda_c^+}}{\Lambda_c^+-\Sigma^+}$$

$$B^{\text{pole}}=-\frac{1}{f_K}g_A^{\Sigma^+\Delta^{++}}\frac{a^{\Sigma^+\Lambda_c^+}}{\Lambda_c^+-\Sigma^+}. \quad (13)$$

The W-exchange diagram is the only mechanism responsible for $\Lambda_c^+\to\Delta^{++}K^-$ decay. Unlike charmed-meson decays, the W-exchange in charmed-baryon decays is neither helicity nor color suppressed since

* Although we use the same values of bag parameters as in [16], our numerical results for the overlap integrals a, b, c, and for form factors are slightly different from theirs

* Since the Goldberger-Treiman relation for $g_A^{\Sigma^+\Delta^{++}}$ is poorly satisfied, the p-wave amplitude is not written in terms of the pseudoscalar coupling constant $g_{\Sigma^+\Delta^{++}K^-}$

Table 1. Numerical values of Λ_c^+ decay amplitudes in units of $c_- G_F \cos^2\theta_C \times 10^2$. Branching ratios for various reactions and experimental values are given in the last two columns

Decay mode	$A^{\rm com}$	$A^{\rm pole}$	$A^{\rm fact}$	$A^{\rm total}$	$B^{\rm com}$	$B^{\rm pole}$	$B^{\rm fact}$	$B^{\rm total}$	Br(%)	Expt(%)
$\Lambda_c^+ \to \Lambda\pi^+$	0	−0.34	−5.87	−6.21	0	0.87	15.46	16.33	8.2	0.6 ±0.5
$\Lambda_c^+ \to p\bar{K}^0$	2.35	0.09	1.49	3.93	−0.99	−1.64	−3.10	−5.73	2.2	1.1 ±0.7
$\Lambda_c^+ \to \Delta^{++}K^-$	0	0.33	0	0.33	0	2.49	0	2.49	0.72	0.45±0.27
$\Lambda_c^+ \to \Sigma^0\pi^+$	−4.09	−0.31	0	−4.40	1.72	−1.79	0	−0.07	2.3	
$\Lambda_c^+ \to \Xi^0 K^+$	0.03	−0.40	0	−0.37	0.10	4.31	0	4.41	0.12	

a diquark subsystem with spin zero can be present inside a baryon. Hence, W-exchange diagrams can be important for decays of charmed baryons, and a measurement of the decay rate of $\Lambda_c^+ \to \Delta^{++}K^-$ provides a direct test of such a picture [20]. The role of the W-exchange process in the inclusive nonleptonic decay of charmed baryon has been discussed by many authors [21]. Since the decay mode $\Lambda_c^+ \to \Delta^{++}K^-$ involves a Rarita-Schwinger particle Δ^{++}, its decay rate is

$$\Gamma = \frac{q^3}{12\pi} \cdot \left\{ |A|^2 \frac{(\Lambda_c - \Delta^{++})^2 - K^2}{(\Delta^{++})^2} + |B|^2 \frac{(\Lambda_c + \Delta^{++})^2 - K^2}{(\Delta^{++})^2} \right\} \quad (14)$$

where q is the center-of-mass momentum of the decay particles.

Results of calculations of s- and p-wave amplitudes and branching ratios for various decay modes are shown in Table 1, where use of $c_- = 1.960$, $c_+ = 0.714$, and $\Gamma_{\rm tot} = 4.348 \times 10^{12}\,{\rm s}^{-1}$ has been made*. It is clear from the table that contributions from the baryon matrix elements of the parity-violating Hamiltonian are in general not important except in the s-wave amplitude of $\Lambda_c^+ \to \Xi^0 K^+$, $\Delta^{++}K^-$ and the p-wave of $\Lambda_c^+ \to \Sigma^0\pi^+$ (Even so, the partial decay rate is essentially unaffected). This can be understood as follows: The ratio $b^{BB'}/a^{BB'}$ ranges from 0.1 to 0.4 for various charmed-baryon decays. (For hyperon decays, it is typically of order 0.1 or less [18].) Furthermore, for both s- and p-wave amplitudes, contributions from the parity-violating baryon matrix elements relative to that from the parity-conserving ones are suppressed by the factor $(m_{B_c} - m_B)/(m_{B_c} + m_B)$. Consequently, the effects of the baryon transition elements from the parity-violating Hamiltonian on charmed-baryon nonleptonic decays are in general not significant. The exceptional case occurs if there are some large cancellations among parity-conserving terms, for example, the s-wave commutator term in $\Lambda_c^+ \to \Xi^0 K^+$, and the p-wave pole term in $\Lambda_c^+ \to \Sigma^0\pi^+$.

Although the predicted branching ratio* for $\Lambda_c^+ \to p\bar{K}^0$ and $\Delta^{++}K^-$ is consistent with experiment**, the calculated branching ratio for $\Lambda_c^+ \to \Lambda\pi^+$ is still too large by a factor of about 10. As pointed out by Ebert and Kallies [17], the commutator term vanishes and the factorizable term seems to be overestimated. This might be cured by the corrections due to the $\frac{1}{2}^-$ poles of $(70, 1^-)$ or $(168, 1^-)$ multiplet for the s-waves, but a huge cancellation between $A^{(\rm fact)}$ and $A^{\rm pole(*)}$ is required, which thus does not seem very promising. An independent way of checking this factorizable term is through the calculation of meson-pole term for which, unfortunately, we do not have reliable estimates at the present time. Anyway, the decay rate of $\Lambda_c^+ \to \Lambda\pi^+$ still remains an open problem.

III. Conclusion

We have calculated the baryon-baryon matrix elements of the parity-violating charm-changing weak Hamiltonian. Contributions to the s- and p-wave amplitudes of Cabibbo-favored nonleptonic decays of charmed baryon due to such terms are estimated and found to be small in general. However for some reactions such as the s-wave of $\Lambda_c^+ \to \Xi^0 K^+$, $\Delta^{++}K^-$ and the p-wave of $\Lambda_c^+ \to \Sigma^0\pi^+$ they could overwhelm or be comparable to contributions arising from pari-

* Both [16] and [17] use $c_- = 1.96$ and $c_+ = 0.64$

* As in [17], we also have different relative signs of the pole terms in the p-wave amplitudes of $\Lambda_c^+ \to \Xi^0 K^+$, when compared with [16]

** The branching ratios of the two-body decays given in the Particle Data Group [22] are determined by the measurement of the decay rate of $\Lambda_c^+ \to pK^-\pi^+$, which itself could be underestimated [16] ty-conserving baryon matrix elements. The large discrepancy between theory and experiment for $\Lambda_c^+ \to \Lambda\pi^+$ decay is not accounted for by such parity-violating transition matrices and a reliable estimate of the meson-pole contribution is needed to check if the factorizable term is overestimated.

It is also expected that the low-lying $\frac{1}{2}^-$ baryon poles (for s-wave) and excited $\frac{1}{2}^+$ poles (for p-wave) would play the same important role as in hyperon decays and this should be investigated in future study.

Appendix

In this Appendix we give the general expressions for the baryon matrix elements $a^{BB'}$ and $b^{BB'}$ of the parity-conserving (pc) and parity-violating (pv) weak Hamiltonian respectively

$$\langle B(p)|H^{pc}|B'(p')\rangle = a^{BB'}\bar{u}_B(p)u_{B'}(p')$$
$$\langle B(p)|H^{pv}|B'(p')\rangle = b^{BB'}\bar{u}_B(p)\gamma_5 u_{B'}(p') \tag{A.1}$$

within the context of the MIT static bag model. In this model the static baryon bag state $|B\rangle_{\text{bag}}$ is not a momentum eigenstate, and the baryon bag matrix element $\langle B|H(x)|B'\rangle_{\text{bag}}$ is related to $\langle B(p)|H(0)|B(p')\rangle$ by the following relation [23].

$$\langle B|H(x)|B'\rangle_{\text{bag}} = \int d^3p\, \frac{m_B}{E_p}\chi^*(p)e^{-ipx} \cdot \int d^3p'\, \frac{m_{B'}}{E'_{p'}}\chi(p')e^{ip'\cdot x}\langle B(p)|H(0)|B'(p')\rangle \tag{A.2}$$

where the momentum wave function $\chi(p)$ is subject to the renormalization condition

$$1 = \int d^3p\, \frac{m}{E_p}|\chi(p)|^2(2\pi)^3. \tag{A.3}$$

Approximating $E \simeq m$ and using $\chi_B(p) = \chi_{B'}(p)$ we find that

$$\int d^3x \langle B|H(\mathbf{x})|B'\rangle_{\text{bag}} \cong \langle B(p)|H(0)|B'(p)\rangle. \tag{A.4}$$

Therefore the parity-conserving baryon matrix element is given by

$$a^{BB'} \cong \int d^3x \langle B|H^{pc}(\mathbf{x})|B'\rangle_{\text{bag}}. \tag{A.5}$$

Unlike $a^{BB'}$, $b^{BB'}$ is not simply related to the bag matrix element of the parity-violating Hamiltonian integrated over space; such an integral vanishes due to the parity argument. As pointed out in [18], one may consider instead the following quantity

$$F(q) = \int d^3x\, e^{-i\mathbf{q}\cdot\mathbf{x}}\langle B|H^{pv}(\mathbf{x})|B'\rangle_{\text{bag}}. \tag{A.6}$$

Again in the nonrelativistic approximation and to the lowest order in q such that $\chi_{B'}(p') \approx \chi_{B'}(p) \approx \chi_B(p)$, it turns out that [18]

$$F(q) \cong -b^{BB'}\frac{m_B + m_{B'}}{4m_B m_{B'}}\chi_B^+ \boldsymbol{\sigma}\cdot\mathbf{q}\chi_{B'} \tag{A.7}$$

where χ^+ and χ are the two-component baryon spinors.

In the MIT bag model the quark spatial wave function is given by [19]

$$q(r) = \frac{N(x)}{(4\pi R^3)^{\frac{1}{2}}}\begin{pmatrix} ij_0(xr/R)\chi \\ -\varepsilon^{\frac{1}{2}}j_1(xr/R)\boldsymbol{\sigma}\cdot\hat{r}\chi \end{pmatrix} \equiv \begin{pmatrix} iu(r)\chi \\ v(r)\boldsymbol{\sigma}\cdot\hat{r}\chi \end{pmatrix} \tag{A.8}$$

where $x = (\omega^2 - m^2R^2)^{1/2}$ for a quark of mass m existing within a bag of radius R in mode ω, $\varepsilon = (\omega - mR)/(\omega + mR)$, and $N^2(x) = x^4/([2\omega(\omega-1) + mR] \cdot \sin^2 x)$. In terms of the large and small components of the quark wave function $u(r)$ and $v(r)$ respectively, the matrix elements of the two-quark vector operator $V_\mu(x) = \bar{q}'\gamma_\mu q$ and the axial-vector operator $A_\mu(x) = \bar{q}'\gamma_\mu\gamma_5 q$ are given by [15]

$$\langle q'|V_0|q\rangle = u'u + v'v$$
$$\langle q'|A_0|q\rangle = -i(u'v - v'u)\boldsymbol{\sigma}\cdot\hat{r}$$
$$\langle q'|\mathbf{V}|q\rangle = i(u'v + v'u)\boldsymbol{\sigma}\times\hat{r} - (u'v - uv')\hat{r}$$
$$\langle q'|\mathbf{A}|q\rangle = i(u'u - v'v)\boldsymbol{\sigma} + 2iv'v\hat{r}\boldsymbol{\sigma}\cdot\hat{r}. \tag{A.9}$$

The four-quark operator $O_-(x) = (\bar{s}c)(\bar{u}d) - (\bar{s}d)(\bar{u}c)$, where $(\bar{q}'q) = \bar{q}'\gamma_\mu(1+\gamma_5)q$, can be written as $O_-(x) = 6[(\bar{s}c)_1(\bar{u}d)_2 - (\bar{s}d)_1(\bar{u}c)_2]$, where the subscript i indicates that the quark operator acts only on the i^{th} quark in the baryon wave function. It is then straightforward to show that

$$\int d^3r\langle q'_1q'_2|(\bar{s}c)_1(\bar{u}d)_2^{pc}|q_1q_2\rangle = (a+b) - \tfrac{1}{3}(3a - b + 4c)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2$$
$$\int d^3r\langle q'_1q'_2|(\bar{s}d)_1(\bar{u}c)_2^{pc}|q_1q_2\rangle = (a - b + 2c) - \tfrac{1}{3}(3a + b + 2c)\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2 \tag{A.10}$$

where use of $\langle\boldsymbol{\sigma}_1\cdot\hat{r}\boldsymbol{\sigma}_2\cdot\hat{r}\rangle = \frac{1}{3}\langle\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\rangle$ has been made for s-wave baryon states, and a, b, c are four-quark overlap integrals given by [16]

$$a = \int d^3r(u_u^2u_cu_s + v_u^2v_cv_s)$$
$$b = \int d^3r(u_u^2v_cv_s + v_u^3u_cu_s)$$
$$c = \int d^3r\,u_uv_u(u_cv_s + v_c\,u_s). \tag{A.11}$$

To determine the parity-violating baryon matrix elements we expand $F(q)$ in powers of q and retain the lowest order nontrivial term, $F(q) \approx -i\int d^3r\mathbf{q}\cdot\mathbf{r}\langle B|H^{pv}|B'\rangle_{\text{bag}}$. Hence the relevant transition matrices are

$$\int d^3r\langle q_1' q_2'|\mathbf{q}\cdot\mathbf{r}(\bar{s}c)_1(\bar{u}d)_2^{pv}|q_1 q_2\rangle = -\frac{i}{3}[\tilde{a}(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\mathbf{q}-i(\tilde{b}-\tilde{c})(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2)\cdot\mathbf{q}]$$

$$\int d^3r\langle q_1' q_2'|\mathbf{q}\cdot\mathbf{r}(\bar{s}d)_1(\bar{u}c)_2^{pv}|q_1 q_2\rangle = -\frac{i}{3}\tilde{a}[(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\cdot\mathbf{q}+i(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2)\cdot\mathbf{q}] \quad (A.12)$$

where

$$\tilde{a}=\int r d^3r(u_u^2+v_u^2)(u_s v_c - v_s u_c)$$
$$\tilde{b}=\int r d^3r(u_u^2-v_u^2)(u_s v_c + v_s u_c)$$
$$\tilde{c}=\int r d^3r(u_s u_c - v_s v_c)2u_u v_u. \quad (A.13)$$

It is easily seen that in the limit of $SU(4)$ symmetry, all parity-violating matrix elements vanish, as they should be.

The flavor-spin wave function of the baryon in the bag model is the same as that in the nonrelativistic $SU(6)$ model. To compute $b^{BB'}$ it is convenient to consider the spin-up state for both initial and final baryons. From (A.7) and (A.12) the baryon-baryon matrix element of the parity-violating part of O_- becomes

$$b^{BB'}=8\frac{m_B m_{B'}}{m_B+m_{B'}}\langle B\uparrow|\tilde{O}|B'\uparrow\rangle_{\text{flover-spin}} \quad (A.14)$$

$$\tilde{O}=d_{1s}^+ b_{1c} d_{2u}^+ b_{2d}[\tilde{a}(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)_z - i(\tilde{b}-\tilde{c})(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2)_z] - d_{1s}^+ b_{1d} d_{2u}^+ b_{2c}\tilde{a}[(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)_z + i(\boldsymbol{\sigma}_1\times\boldsymbol{\sigma}_2)_z]$$

where b_{1c} is a charm quark destruction operator acting on the first quark in the baryon wave function. Similarly, $a^{BB'}$ can be easily calculated from (A.5) and (A.10).

Finally, the vector and axial-vector form factors $g_{V,A}$ at zero momentum transfer

$$\langle B(p)|V_\mu|B'(p)\rangle = g_V^{BB'}\bar{u}_B(p)\gamma_\mu u_{B'}(p)$$
$$\langle B(p)|A_\mu|B'(p)\rangle = g_A^{BB'}\bar{u}_B(p)\gamma_\mu\gamma_5 u_{B'}(p) \quad (A.15)$$

are determined in the bag model by virtue of (A.5) and are given by

$$g_V^{BB'}=\int d^3r(u'u+v'v)\langle B\uparrow|d_q^+ b_{q'}|B'\uparrow\rangle_{\text{flavor-spin}}$$
$$g_A^{BB'}=\int d^3r(u'u-\tfrac{1}{3}v'v)\langle B\uparrow|d_q^+ b_{q'}\sigma_z|B'\uparrow\rangle_{\text{flavor-spin}} \quad (A.16)$$

where the nonrelativistic approximation $\bar{u}_B\gamma\gamma_5 u_{B'}\approx\chi_B^+\boldsymbol{\sigma}\chi_{B'}$ is used.

Noted Added in Proof. After this work was completed, we became aware of a paper by D. Ebert and W. Kallies (Yad. Fiz. **40**, 1250 (1984)), in which a similar calculation was performed.

References

1. R.E. Marshak, Riazzudin, C.P. Ryan: Theory of weak interactions in particle physics. New York: Wiley 1969
2. J. Finjord, M.K. Gaillard: Phys. Rev. **D22**, 778 (1980), and references therein
3. M.D. Scadron, L.R. Thebaud: Phys. Rev. **D8**, 2190 (1973)
4. C. Schmid: Phys. Lett. **66B**, 353 (1977); A. Le Yaouanc, L. Oliver, O. Pene, J.C. Raynal ibid. **72B**, 53 (1977); Y. Hara, K. Hikosaka: Prog. Theor. Phys. **66**, 2206 (1981)
5. J. Katz, S. Tatur: Phys. Rev. **D14**, 2247 (1976)
6. J.F. Donoghue, E. Golowich, B.R. Holstein: Phys. Rev. **D12**, 2875 (1975); J.F. Donoghue, E. Golowich, W.A. Ponce, B.R. Holstein: ibid. **D21**, 186 (1980); **D23**, 1213 (1981)
7. H. Galić, D. Tadić, J. Trampetić: Nucl. Phys. **B152**, 306 (1979); D. Tadić, J. Trampetić: Phys. Rev. **D23**, 144 (1981)
8. C. Itzykson, M. Jacob: Nuovo Cimento **48A**, 655 (1967); M. Gronau: Phys. Rev. **D5**, 118 (1972)
9. A.I. Vainshtein, V.I. Zakharov, M.A. Shifman: Zh. Eksp. Teor. Fiz. **72**, 1275 (1977) [Sov. Phys. JETP **45**, 670 (1977)]
10. A. Le Yaouanc, O. Pene, J.C. Raynal, L. Oliver: Nucl. Phys. **B149**, 321 (1979)
11. T.N. Pham: Phys. Rev. Lett. **53**, 326 (1984)
12. M. Milosević, D. Tadić, J. Trampetić: Nucl. Phys. **B207**, 461 (1982)
13. M. Bonvin: Nucl. Phys. **B238**, 241 (1984)
14. J.G. Körner, G. Kramer, J. Willrodt: Phys. Lett. **78B**, 492 (1978); Z. Phys. C – Particles and Fields **2**, 117 (1979)
15. J.F. Donoghue, B.R. Holstein: Phys. Rev. **D29**, 489 (1984)
16. B. Guberina, D. Tadić, J. Trampetić: Z. Phys. C – Particles and Fields **13**, 251 (1982)
17. D. Ebert, W. Kallies: Phys. Lett. **131B**, 183 (1983)
18. E. Golowich, B.R. Holstein: Phys. Rev. **D26**, 182 (1982)
19. A. Chodos, R.L. Jaffe, K. Johnson, C.B. Thorn: Phys. Rev. **D10**, 2599 (1974); T. De Grand, R.L. Jaffe, K. Johnson, J. Kiskis: ibid. **D12**, 2060 (1975)
20. I.I. Bigi: Z. Phys. C – Particles and Fields **9**, 197 (1981)
21. V. Barger, J.P. Leveille, P.M. Stevenson: Phys. Rev. Lett. **44**, 226 (1980); R. Rückl: Phys. Lett. **120B**, 449 (1983); T. Okazaki: Phys. Rev. **D30**, 677 (1984); I. Bediaga et al.: CBPF preprint, CBPF-NF-035/84 (1984)
22. Particle Data Group: Rev. Mod. Phys. **56**, S1 (1984)
23. J.F. Donoghue, K. Johnson: Phys. Rev. **D21**, 1975 (1980)