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Dissipative processes in an expanding massive gluon gas

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Abstract. The temperature dependence of the kinetic coefficients is obtained in the nonperturbative region with the help of Green-Kubo-type formulae in the model of massive gluon gas motivated by numerical results from simulations of lattice QCD. The entropy production rate is estimated using scaling hydrodynamics. It is shown that the increase in the viscosity coefficients leads to entropy generation in heavy ion collision processes which could be big, especially for temperatures close to the critical one.

The forthcoming projects for ultrarelativistic heavy ion collisions at RHIC and LHC require comprehensive estimates of the space-time development of these reactions. It is commonly believed that several stages are involved and it is natural to consider the preequilibrium one separately as most of the entropy is created at this stage [1, 2]. The subsequent hydrodynamic expansion has often been considered as isentropic, but it is clear that the evolution of this stage will be complicated by dissipative processes generating entropy, and by the possible phase transition (or transitions) [3–5]. The aim of the present study is to explore the entropy generation at this hydrodynamic stage. We understand that the freeze-out stage, where the system is made up of free-streaming final particles, could also add to the entropy [6, 7].

In order to estimate properly the dissipative effects as well as the dynamics [8] of the QCD phase transition we need to know the behaviour of the kinetic coefficients (KC) over a wide range of temperatures, including the ones close to the phase transition point where non-perturbative effects are dominant. Many previous calculations extrapolated the asymptotic behaviour, found perturbatively, to the critical region [9–11]. However, a recent estimate of the shear viscosity coefficient, using a model for the contribution of the nonperturbative region, has demonstrated that the behaviour in the critical region is very different from the standard T^3 one and the amount of entropy generated in the region close to this temperature appears to be substantial [12, 13].

In those calculations we exploited the so-called momentum "cut-off model" motivated by an analysis of the numerical results of lattice Yang-Mills field thermodynamics [14, 15], This model provides a good fit to the data, but it is of course very much ad hoc and does not explain why the low-momentum modes are removed. Here we deal with an ideal gas where the effect of interactions in the plasma is provided by the temperature dependence of the effective mass [16]. In a sense it is rather similar to the "cut-off model" as now the medium (plasma) properties suppress low momentum excitations since for $M(T)$ increasing $\exp(-\sqrt{M^2-p^2/T})\to\exp(-M/T)$ at small momenta $p\to 0$, but it has more advantages as was argued in a recent analysis [17], the most important of them is that it leads to a very good description of high precision $SU(2)$ pure gluodynamics lattice data. As to the KC calculations, this model is more involved since we need to deal with massive scalar $\lambda\phi^4$ theory. This latter approximation is also in line with the model of "massive" gluons where these waves are nevertheless considered to be only transversal [18]. Combining our calculations of KC's in $\lambda\phi^4$-theory with the temperature dependence of the gluon mass extracted from lattice Monte Carlo data, in particular for the mass gap in pure gluodynamics [19], we are able to estimate the entropy generated based on linear hydrodynamics and to explore the applicability of this approach to the evolution of gluon systems.

As our basic starting point we take the hydrodynamical equation including a viscosity term as

$$\frac{d\varepsilon}{d\tau}+\frac{\varepsilon+p}{\tau}-\frac{\chi}{\tau^2}=0, \tag{1}$$

with Bjorken initial conditions [20]. We fix the equation of state ($p=p(\tau_0)$ and $\varepsilon=\varepsilon(\tau_0)$ are the initial pressure and energy density respectively), and, taking the initial conditions at a time $\tau_0\sim 1$ fm. The dissipative term in (1) contains the factor $\chi=(4/3\eta_s+\eta_v)$ with η_s and η_v as transport coefficients of shear and bulk viscosities. Note that the inequality $\tau>\chi/(\varepsilon+p)$ must hold, otherwise we would be

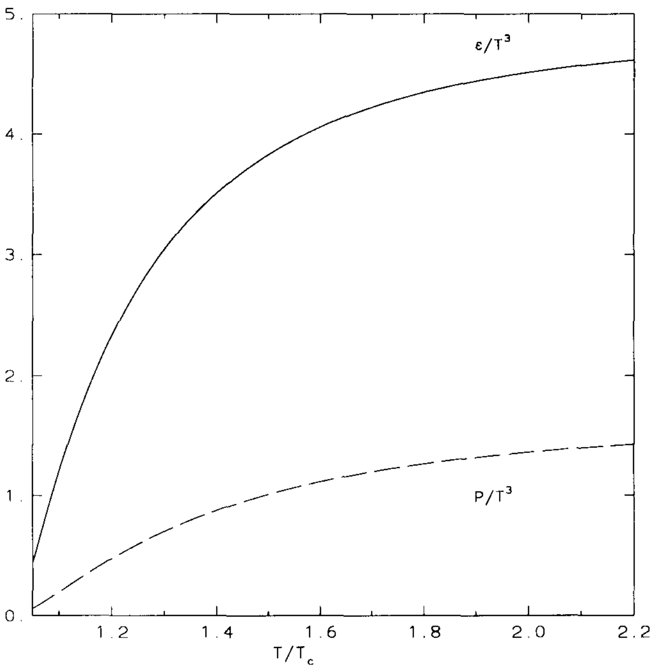

Fig. 1. Dependence of the scaled energy density and pressure on the temperature

dealing with the unrealistic case of gluon gas contraction (see [5, 11]).

The total entropy of the system is defined as [5, 10]

$$S=\int d\sigma^\mu s^\mu=\int dy\, s(\tau)\tau, \tag{2}$$

where $s^\mu = s u^\mu$, $s(\tau)=[\varepsilon(\tau)+p(\tau)]/T$ is the local entropy density and y is the hydrodynamic rapidity ($\tanh y = x/t$). Equations (1) and (2) give us a simple formula to estimate the entropy production in an expanding gluon gas

$$\frac{dS}{dy}=\int \frac{d\tau\,\chi(\tau)}{\tau T(\tau)}, \tag{3}$$

here $\chi(\tau)=\chi[T(\tau)]$, $T(\tau)$ being the solution of (1). In order to solve (2) we need the temperature dependence of the KC's and the equation of state in the whole temperature interval, including the nonperturbative critical region.

For pressure and energy density calculations we use the standard expressions [21]

$$p(T)=\frac{g}{6\pi^2}\int_0^\infty dk\,\frac{k^4}{E(k)}\,n(E(k)), \tag{4}$$

$$\varepsilon(T)=\frac{g}{2\pi^2}\int_0^\infty dk\,k^2 E(k) n(E(k)), \tag{5}$$

where $E(k)=\sqrt{k^2+m_g^2}$ is the relativistic energy; $n(x)=[\exp(\beta x)-1]^{-1}$ is Bose's distribution function; β is the inverse temperature and g is the degeneracy factor. In the ultrarelativistic case ($T \gg m_g$) it leads to the Stephan-Boltzmann (SB) law (see Fig. 1). However, if the temperature is close to T_c, the gluon mass m_g is increasing, (at least for SU(2)-gluodynamics) and becomes too large to be negligible and Bose's distribution function may be replaced by Boltzmann's ($n(x)=\exp(-\beta x)$). Then (6), (7) may be integrated and written in the following form

$$p(T)=\frac{gT^4}{6\pi^2}\int_\alpha^\infty dz[z^2-\alpha^2]^{3/2}e^{-z}=\frac{gT^4}{2\pi^2}\alpha^2 K_2(\alpha). \tag{6}$$

$$\varepsilon(T)=\frac{gT^4}{2\pi^2}\int_\alpha^\infty dz\,z^2\sqrt{z^2-\alpha^2}\,e^{-z}=\frac{gT^4}{2\pi^2}\alpha^2(3K_2(\alpha)+\alpha K_1(\alpha)), \tag{7}$$

where $K_i(\alpha)$ are modified Bessel function and $\alpha \equiv m_g \beta$. The asymptotic expansion of the modified Bessel functions is given by $K(\alpha)\sim\exp(-\alpha)\sqrt{\pi/2\alpha}$ for large α, i.e. for $T \sim T_c$, both pressure and energy become small close to T_c. In the case when $\alpha \to 0$ we recover the Stephan–Boltzmann law. This temperature dependence can fit the SU(3) Monte–Carlo lattice data if one uses a constant gluon mass [18] or if one includes a finite jump in mass around T_c. As to SU(2)-gluodynamics, for which a very elaborated Monte Carlo analysis exists [17], we used the following parametrization

$$m_g = m_0 T_c \left(\frac{T_c}{T-T_c}\right)^q,$$

with $m_0 = 1.83$ and $q = 0.4$. The difference reflects our understanding in behaviours of first- and second order phase transitions as seen in lattice Monte–Carlo simulations [19]. To calculate the KC's of shear and bulk viscosities we use well-known relations obtained within a formalism based on Kubo-type formulae for the $\lambda\phi^4$-thermofield theory [10, 11] (the analogous expression for η_s was also obtained for the case of vector fields [22])

$$\eta_s=\frac{\beta}{15} I_{2,1}, \tag{8}$$

$$\eta_v=\frac{\beta}{9}\{I_{2,1}-6c_s^2 I_{1,0}+9c_s^4 I_{0,-1}\}, \tag{9}$$

where $c_s^2=\partial p/\partial\varepsilon$ is the square of sound velocity, and the integrals $I_{m,n}$ are defined as

$$I_{m,n}=2\int \frac{d^3\tilde{p}\,p^{2m}}{E^{2n}(p)\Gamma(\mathbf{p})}\,n(p)[1+n(p)], \tag{10}$$

here $\Gamma(\mathbf{p})$ is the damping rate of quasiparticle excitation (it is assumed that $\Gamma\beta \ll 1$). In the scalar theory one obtains in one-loop approximation

$$\Gamma(\mathbf{p})=\frac{\lambda^2(2\pi)^4}{24E(\mathbf{p})n(\mathbf{p})}\int d^3\tilde{p}_1 d^3\tilde{p}_2 d^3\tilde{p}_3 \delta(p+p_1-p_2-p_3) \times (1+n_1)n_2 n_3 \tag{11}$$

with the following notations $d^3\tilde{p}=[2(2\pi)^3 E(\mathbf{p})]^{-1} d^3 p$, in both (10) and (11), $n_i=[\exp(\beta E(\mathbf{p}_i))-1]^{-1}$.

The gluon gas viscosities may be obtained from (8)–(11) by the standard procedure of changing [11, 23]

$$\lambda^2 \to c^* 32\pi^2\alpha_s^2 \ln\alpha_s^{-1}, \quad c^*=20\div 60, \tag{12}$$

where (for the value of c^*, see also [24])

$$\alpha_s=6\pi[11/2N\ln(M^2/\Lambda^2)]^{-1}, \tag{13}$$

and N is a number of colours, $M^2=\frac{4}{3}\langle p^2\rangle$ and $\langle p^2\rangle$ is the thermodynamically averaged squared momentum of the gluon field [21]. The degeneracy factor is absent in the final result, since the numerator of (11) must contain it as well as the damping rate in the denominator. This phenomenological estimate can be justified by the fact that to lowest order in the interaction the cross sections for gluons and scalar particles have similar momentum dependences. This procedure (13) is fair only for small coupling constant. The model under consideration brings us to the following temperature dependence for the M^2 factor

$$M^2=\frac{3}{4}\frac{\int d^3p\,p^2 n(E(p))}{\int d^3p\,n(E(p))}=4T^2\alpha\frac{K_3(\alpha)}{K_2(\alpha)}. \tag{14}$$

In the ultrarelativistic case where $\alpha\to 0$ we obtain the conventional result $M\sim 4T$. When $T\to T_c$, and $\alpha\to\infty$ we have $M^2\sim 4T^2\alpha$. This means that the coupling constant α_s remains small even when the temperature is close to T_c. This lucky fact has been met already in the so-called cut-off model [14, 15] that interprets the Monte–Carlo data as well as the present model does. It can be explained by the fact that both models take the contribution of long wavelength excitations away.

$$\Gamma(\mathbf{p})=\frac{\lambda^2(2\pi)^{-4}}{192E(p)n(E(p))}\int d^2p_1\frac{(1+n_1)}{E(\mathbf{p}_1)}I_1, \tag{15}$$

where

$$I_1=2\pi\beta^{-1}\int_{\alpha}^{\beta\Omega} dy\exp(-\beta\Omega)\Theta(z_0(y)-1)\Theta(Z_0(y)+1),$$

$$z_0(y)=\frac{K^2-\Omega^2+2\Omega y}{2K\sqrt{y^2-m_g^2}}. \tag{16}$$

where

$$\Omega=\sqrt{p_1^2+m_g^2}+\sqrt{p^2+m_g^2} \tag{17}$$

and $K=\sqrt{p^2+p_1^2+2pp_1\cos(\theta)}$, θ being the angle between $\mathbf{p}$ and $\mathbf{p}_1$. Here we need to solve the inequality

$$-1\le z_0(y)\le 1, \tag{18}$$

It leads us to

$$y_-\le y\le y_+;\quad y_\pm=\frac{\Omega}{2}\pm\frac{K\Lambda}{2};\quad \Lambda=\sqrt{1-\frac{4m_g^2}{\Omega^2-K^2}}, \tag{19}$$

with the values of Ω and K satisfying the inequality

$$\Omega^2-K^2>4m_g^2. \tag{20}$$

After analysis of (19) and (20) we obtain for (15)

$$\Gamma(\mathbf{p})=\frac{\lambda^2(2\pi)^{-3}}{\beta^2 192E(p)}\int_0^\infty dx\frac{x^2[1+\exp(-\sqrt{x^2+\alpha^2})]}{\exp(-\sqrt{x^2+\alpha^2})\sqrt{x^2+\alpha^2}}$$
$$\times\int_0^\pi d\theta\sin(\theta)K\Lambda. \tag{21}$$

In order to get an approximate analytical form we can calculate the angular integral in (21) for the $p=0$ case. This gives a minimal value for the damping rate and a maximum one for the kinetic coefficients. Here we also take into account that the main contribution in (8)–(10) is connected with long wavelength excitations. Thus, in this approximation the damping rate looks as

$$\Gamma(p)=\frac{\lambda^2(2\pi)^{-3}\beta^{-2}}{96E(p)}\int_\alpha^\infty dz(z-\alpha)\sqrt{z^2-\alpha^2}(e^{-z}+e^{-2z}). \tag{22}$$

It is evident that the momentum dependence is very simple in this equation. This allows us to represent the integral $I_{m,n}$ from (10) in the following form

$$I_{m,n}=\frac{T^{2(n-m)-2}}{2\pi^2\tilde{\Gamma}(T)}J_{m,n}, \tag{23}$$

with

$$J_{m,n}=\int_\alpha^\infty dz\,z^{2-2n}(z^2-\alpha^2)^{\frac{2m+1}{2}}[\exp(-z)+\exp(-2z)]. \tag{24}$$

After integration we obtain, for some specific cases

$$J_{2,1}=\frac{15}{8}[(2\alpha)^3K_3(\alpha)+\alpha^3K_3(2\alpha)], \tag{25}$$

$$J_{1,0}=3\alpha^3\left[\frac{5}{8}K_3(2\alpha)+5K_3(\alpha)+\frac{\alpha}{4}K_2(2\alpha)+\alpha K_2(\alpha)\right], \tag{26}$$

$$J_{0,-1}=\alpha^2\left[60K_2(\alpha)+\frac{15}{4}K_2(2\alpha)+27\alpha K_1(\alpha)+\frac{27}{8}K_1(2\alpha)\right.$$
$$\left.+6\alpha^2K_0(\alpha)+\frac{3}{2}\alpha^2K_0(2\alpha)+\alpha^3K_1(\alpha)+\frac{\alpha^3}{2}K_1(2\alpha)\right]. \tag{27}$$

The result for the damping increment can be written as

$$\tilde{\Gamma}(T)=E(p)T^2\Gamma(p)$$
$$=\frac{\lambda^2\pi^{-3}}{32^9}\alpha^2(T)[2K_2(\alpha)-2K_1(\alpha)+K_2(2\alpha)-K_1(2\alpha)]. \tag{28}$$

For the calculation of the bulk viscosity we also need to know the expression for the velocity of sound. With the help of (6) and (7) we obtain

$$c_s^2=\frac{1}{3+\Delta}, \tag{29}$$

with

$$\Delta=\alpha\frac{4K_1(\alpha)+T\frac{d\alpha}{dT}[K_2(\alpha)-2K_0(\alpha)]}{4K_2(\alpha)-T\frac{d\alpha}{dT}K_1(\alpha)}. \tag{30}$$

Here we see that, for α small, $\Delta\sim\alpha$, and the sound velocity, c_s vanishes. In the $\alpha\to 0$ case (29) gives the value of $c_s^2=1/3$, $\Delta\to 0$ (see Fig. 2).

Equation (24–28) together with (8, 9, 10, 12–14) determine the temperature dependence of the KC of the gluon

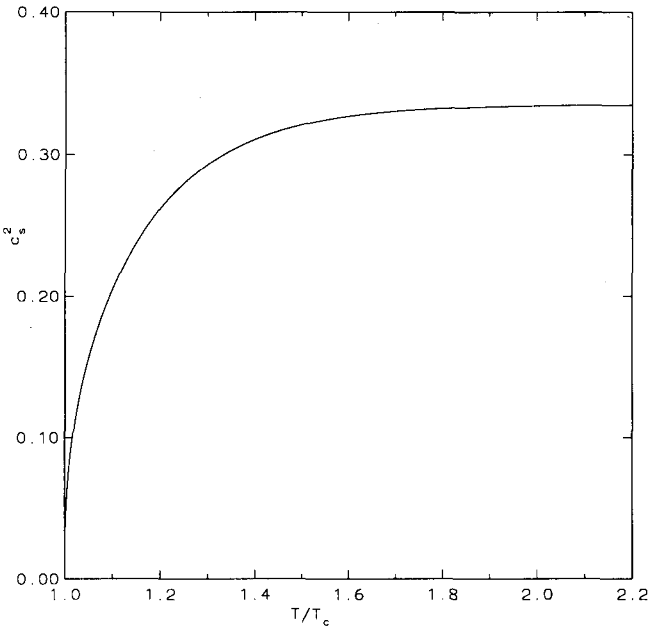

Fig. 2. Dependence of the speed of sound on the temperature

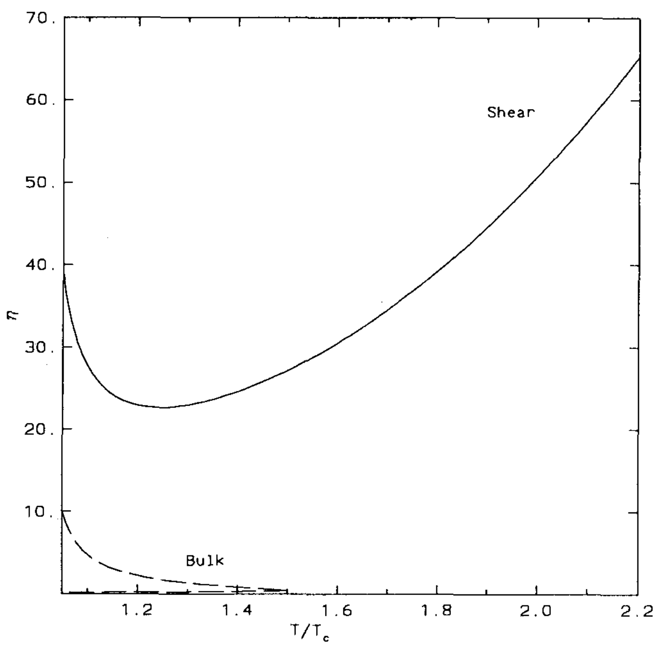

Fig. 3. Dependence of the shear and bulk viscosities on the temperature

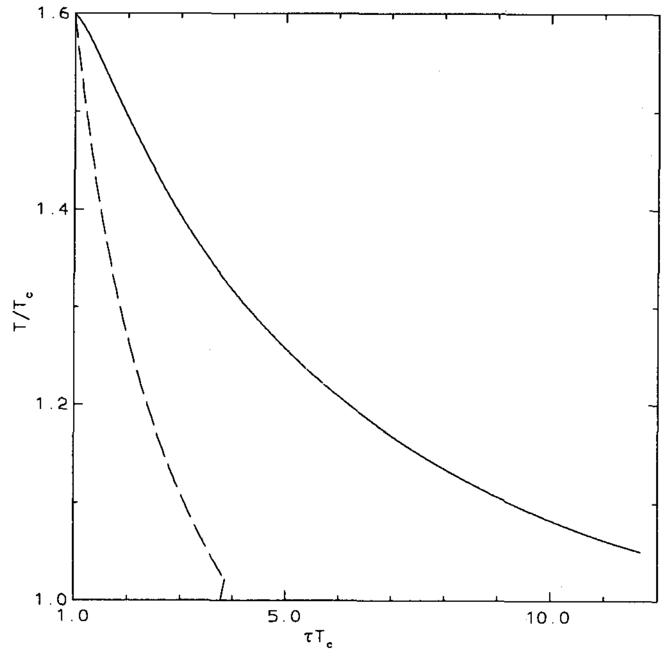

Fig. 4. Evolution of the temperature with proper time

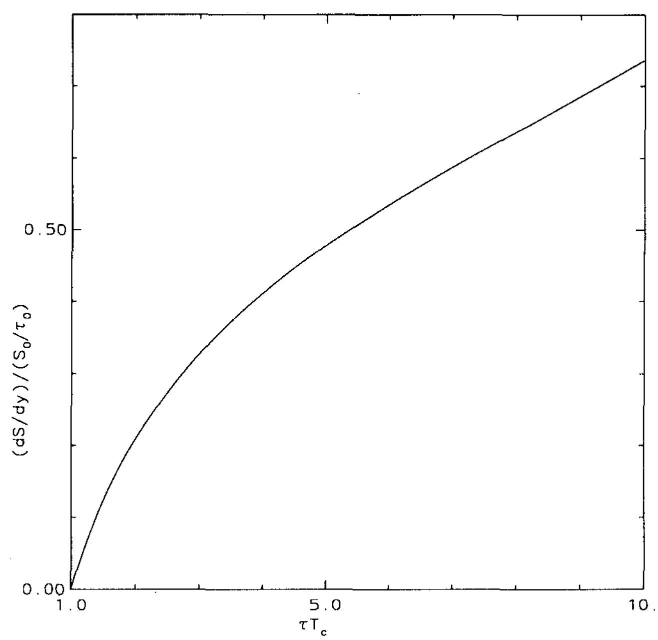

Fig. 5. Entropy generation as a function of proper time

gas. The asymptotic behaviour $\alpha \to 0$ gives an over-estimate, in analogy with [11], (see comment below (21)). The temperature dependences of the KC are presented in Fig. 3. It is evident that the KC increase considerably in the temperature region close to T_c. This leads to a large deviation of the solution of (1) from the scaling one, and to a critical delay for the evolution of the system near T_c. This is clearly seen in Fig. 4, the full line takes into account the dissipative terms while the dashed line does not.

The rate of entropy production as a function of proper time is depicted in Fig. 5. It is evident that the entropy increases rapidly. Analogous calculations for entropy production rate based on the T^3-dependency of KC leads to a 20 per cent increase in the cooling process of the gluon gas.

The authors are aware of the fact that the obtained results are model dependent. The increase of the kinetic coefficients makes the application of linear hydrodynamics for the description of quark gluon gas questionable.

Evidently, taking into account other dissipative mechanisms will lead to finite values for the KC and diminish somehow the entropy production. However the results obtained indicate that a realistic picture of the evolution of the system under consideration can differ a lot from the scaling one, especially in the phase transition region.

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