Z. Phys. C – Particles and Fields 42, 671–678 (1989)

Zeitschrift für Physik C Particles and Fields

Formfactor effects in exclusive D and B decays

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Received 3 August 1988; in revised form 3 January 1989

Abstract. Recent experimental results on the semi-leptonic $D \to K^*$ transition seem to be in conflict with quark model expectations. Motivated by this finding we reinvestigate the predictions for exclusive D and B decays in the relativistic quark model approach. Some of the invariant formfactors relevant for the transition matrix elements $D \to K^*$ and $B \to D^*$ depend strongly on an explicit quarkmass-dependent integral over the meson wave functions. The dependence of decay rates and spectra in semi-leptonic D and B transitions on this integral is analysed and discussed in detail. Furthermore, we discuss how the predictions of the relativistic quark model for semi-leptonic D and B decays can be tested through measurements of the polarization of the produced vector meson K^* and D^*, respectively. Some remarks on exclusive non-leptonic two-body decays of the heavy mesons are also presented. Finally the theoretical uncertainties for the determination of the K–M matrix element $|V_{ub}|$ from exclusive semi-leptonic decays are discussed.

1 Introduction

The determination of the so far unknown Kobayashi–Maskawa (K–M) matrix element $|V_{ub}|$ from experiment relies on theoretical models for the decay of B mesons [1]. The light quarks produced in the decay of the b quark have to combine with the spectator quark to form colour singlets. This is clearly a non-perturbative process and cannot be calculated from first principles so far. Therefore one has to rely on phenomenological approaches. Fortunately, the theoretical uncertainties can be minimized by studying semi-leptonic transitions.

It is known from experiment [2, 3] that the semi-leptonic decays of heavy mesons are dominated by few exclusive decay channels. The hadronic system near the endpoint of the lepton spectrum—which is especially important for the determination of the $b \to c$ and $b \to u$ transition strengths—must consist of the lowest possible mass states for kinematical reasons. Therefore it seems appropriate to study *exclusive* semi-leptonic transitions. Many theoretical models have been suggested for their description [4–14]. In this paper we investigate in detail the predictions of the relativistic quark model developed in [6] for semi-leptonic D and B decays. We will first describe this model and discuss its uncertainties and assumptions. In the following section we will study polarization effects in exclusive semi-leptonic heavy meson decays. The polarization of the final vectormeson is rather sensitive to the choice of the parameter of the model [15]. Therefore the measurement of rates and decay spectra can provide helpful informations. We will then extend our investigation to exclusive non-leptonic decays of B mesons. Finally we will summarize and discuss our results.

II Description of the model

In an exclusive treatment the decay distributions are given in terms of matrix elements of the weak currents between initial and final meson states:

$$d\Gamma(M \to X + l\nu) \equiv d\Gamma_{SL}(M \to X) = \frac{1}{2m_M}\frac{1}{(2\pi)^5}\,\delta^4(K_M - K_X - K_l - K_\nu) \cdot |A_{SL}(M \to X)|^2 \frac{d^3K_X}{2E_X}\frac{d^3K_l}{2E_l}\frac{d^3K_\nu}{2E_\nu} \tag{1}$$

where

$$A_{SL}(M \to X) = \frac{G_F}{\sqrt{2}} V_{Qq} L^\mu H_\mu \tag{2}$$

and

* Supported by the Deutsche Forschungsgemeinschaft (DFG)

$$L^{\mu} = \bar{u}(K_2)\gamma^{\mu}(1-\gamma_5)v(K_1)$$
$$H_{\mu} = \langle X | J_{\mu}(0) | M \rangle \tag{3}$$

$J_{\mu}(0)$ is the weak $V-A$ current and $K_1 = K_l(K_\nu)$, $K_2 = K_\nu(K_l)$ if the decaying quark is a $c(b)$ quark. V_{Qq} is the appropriate K–M mixing matrix element for the $Q \to q$ transition. M and X denote the initial and final meson, respectively. In the following we will consider the transitions involving a pseudoscalar ($X=P$) or a vectormeson ($X=V$) and a lepton pair in the final state.

From Lorentz invariance one finds the decomposition of the hadronic matrix element in terms of unknown formfactors:

$$\begin{aligned}&\langle P | J_{\mu}(0) | M \rangle \\ &= \left\{ (K_M + K_P)_{\mu} - \frac{(m_M^2 - m_P^2)}{q^2} q_{\mu} \right\} F_1(q^2) \\ &+ \frac{m_M^2 - m_P^2}{q^2} q_{\mu} F_0(q^2)\end{aligned} \tag{4}$$

where $q_{\mu} = (K_M - K_P)_{\mu}$ and $F_1(0) = F_0(0)$, and

$$\begin{aligned}&\langle V | J_{\mu}(0) | M \rangle \\ &= \varepsilon_{\mu\nu\alpha\beta} K_M^{\nu} K_V^{\alpha} \varepsilon^{*\beta} \frac{2}{m_M + m_V} V(q^2) \\ &+ i \left\{ \varepsilon_{\mu}^{*}(m_M + m_V) A_1(q^2) \right. \\ &- \frac{\varepsilon^{*}\cdot q}{m_M + m_V} (K_M + K_V)_{\mu} A_2(q^2) \\ &\left. - \frac{\varepsilon^{*}\cdot q}{q^2} q_{\mu} 2m_V A_3(q^2) \right\} \\ &+ i \frac{\varepsilon^{*}\cdot q}{q^2} q_{\mu} 2m_V A_0(q^2)\end{aligned} \tag{5}$$

where A_1, A_2 and A_3 are related by

$$A_3(q^2) = \frac{m_M + m_V}{2m_V} A_1(q^2) - \frac{m_M - m_V}{2m_V} A_2(q^2) \tag{6}$$

$A_3(0) = A_0(0)$ and ε_{μ}^{*} denotes the polarization vector of the outgoing vectormeson.

Various approaches have been suggested to estimate the invariant formfactors [4–14]. Here we will employ the model of [6] where monopole type formfactors have been assumed for the q^2 dependence*.

$$F_1(q^2) \simeq \frac{h_1}{1 - q^2/m_{1^-}^2}, \quad F_0(q^2) \simeq \frac{h_0}{1 - q^2/m_{0^+}^2}$$

and

$$\begin{aligned}&A_i(q^2) \simeq \frac{h_{Ai}}{1 - q^2/m_{1^+}^2}, \quad i = 1,2,3 \\ &A_0(q^2) \simeq \frac{h_{A_3}}{1 - q^2/m_{0^-}^2}, \quad V(q^2) \simeq \frac{h_V}{1 - q^2/m_{1^-}^2}.\end{aligned} \tag{7}$$

The initial and final mesons have been described as relativistic bound states of quark–antiquark pairs $(Q_1\bar{q}_2)$ and $(q_1\bar{q}_2)$, respectively, in the infinite momentum frame to fix the formfactors at $q^2 = 0$ (overlap factors):

$$\begin{aligned}|\mathbf{K}, m; J, J_z\rangle &= \sqrt{2}(2\pi)^{3/2} \sum_{s_1 s_2} \int d^3k_1\, d^3k_2 \\ &\cdot \delta^3(\mathbf{K} - \mathbf{k}_1 - \mathbf{k}_2) \varphi_m^{J,J_z}(\mathbf{k}_{1T}, x; s_1, s_2) \\ &\cdot a_1^{+}(\mathbf{k}_1, s_1) b_2^{+}(\mathbf{k}_2, s_2) | 0 \rangle\end{aligned} \tag{8}$$

where $K^{\mu} = (K^0, 0, 0, K)$ with $K \to \infty$ and

$$x = \frac{k_{1z}}{K} \quad \mathbf{k}_{1T} = (k_{1x}, k_{1y}). \tag{9}$$

The normalization used here is

$$\{a(\mathbf{k}, s), a^{+}(\mathbf{k}', s')\} = \delta_{ss'} \delta^3(\mathbf{k} - \mathbf{k}') \tag{10}$$

and

$$\sum_{s_1, s_2} \int d^2k_T\, dx\, |\varphi_m^{J,J_z}(\mathbf{k}_T, x, s_1, s_2)|^2 = 1$$

giving

$$\langle \mathbf{K}' | \mathbf{K} \rangle = 2K^0 (2\pi)^3 \delta^3(\mathbf{K} - \mathbf{K}'). \tag{11}$$

The formfactors at $q^2 = 0$ (overlap factors) are obtained by expressing the current $J_{\mu}(0)$ in terms of annihilation and creation operators and sandwiching the appropriate current component between initial and final meson states [17]. Doing this one encounters one difficulty, however. It is well known [18] that employing time ordered perturbation theory in the infinite momentum frame ($K \to \infty$) the hadrons can be regarded as the sum of free constituents moving in the same direction. Configurations with constituents moving opposite to the hadron (for example $x \leqq 0$ in (9)) are suppressed by powers of $1/K$. Care must be taken, however, when one considers matrix elements of currents, since the vertex with a current contributes extra powers of K. One is led to the distinction between 'good' and 'bad' current components: 'Good' current components are characterized by the fact that their matrix elements between hadron states are of the order K as $K \to \infty$, whereas matrix elements of 'bad' current components are of the order $(m/K)\cdot K$, where m is some mass. 'Good' currents are suppressed by a factor $1/K$, when they turn a line moving along K into one moving in the opposite direction. Thus all lines move in the same direction in matrix elements involving 'good' currents only, i.e. so-called 'Z-diagrams' are suppressed. The distinction between 'good' and 'bad' also holds for the operators obtained by taking the space-integrals of currents. 'Good' operators give rise to sum rules with improved convergence properties [19].

* In [12] monopole and dipole formfactors have been used according to the power counting rules of QCD [16]

It is easy to show, that the 'good' current components are J^0 and J^3, whereas the transverse current components J^1 and J^2 are 'bad'. The space integrals of J^0 and J^3 are generators of an $SU(4)_W$ symmetry acting on two flavours and two spin states in the formal limit of an exact collinear symmetry combining spin and flavour. The overlap factors h_1 and h_{A_3}—determined by matrix elements of J^0 and J^3—are given by the overlap of the wavefunctions of the initial and final meson:

$$h_1 = \int_0^1 d^2k_T\, dx\, \varphi_P^*(\mathbf{k}_T, x)\varphi_M(\mathbf{k}_T, x)$$
$$h_{A_3} = \int_0^1 d^2k_T\, dx\, \varphi_V^{*1,0}(\mathbf{k}_T, x)\sigma^3\varphi_M(\mathbf{k}_T, x) \qquad (12)$$

where the Pauli matrix σ^3 acts on the spin indices of the decaying quark Q_1. It is evident from the argument above and also from (12) that $h_{A_3} = h_1 = 1$ in the limit of an exact $SU(4)_W$ symmetry.

The space integrals of the transverse current components J^1 and J^2, however, are not generators of a $SU(4)_W$ symmetry ('bad' operators). The matrix elements of J^1 and J^2 contain explicitly the masses of the non-spectator quarks Q_1 and q_1. The overlap factors h_V and h_{A_1} are estimated by sandwiching J^1 and J^2. The result is the following:

$$h_V = \frac{m_{Q_1} - m_{q_1}}{m_M - m_V} J_M$$
$$h_{A_1} = \frac{m_{Q_1} + m_{q_1}}{m_M + m_V} J_M \qquad (13)$$

where

$$J_M = \sqrt{2}\int d^2k_T \int_0^1 \frac{dx}{x} \varphi_V^{*1,-1}(\mathbf{k}_T, x) i\sigma^2\varphi_M(\mathbf{k}_T, x). \qquad (14)$$

Finally, h_{A_2} is given by:

$$h_{A_2} = \frac{m_M + m_V}{m_M - m_V} h_{A_1} - \frac{2m_V}{m_M - m_V} h_{A_3} \qquad (15)$$

In [6] the solution of a relativistic scalar harmonic oscillator potential [20, 21] has been employed for the orbital part of the wavefunction:

$$\varphi_m(\mathbf{k}_T, x) = N_m\sqrt{x(1-x)}\exp\{-\mathbf{k}_T^2/2\omega^2\}$$
$$\cdot\exp\left\{-\frac{m^2}{2\omega^2}\left(x - \frac{1}{2} - \frac{m_{Q_1}^2 - m_{q_1}^2}{2m^2}\right)^2\right\} \qquad (16)$$

where x denotes the momentum fraction of the decaying quark (see (9)) and N_m is a normalization factor. This ansatz for the wavefunction depends—besides the quark masses—only on one free parameter ω which determines the average transverse quark momentum:

$$\langle \mathbf{k}_T^2 \rangle = \omega^2. \qquad (17)$$

Using (12) to (16) the formfactors and therefore the widths and spectra can now be calculated once ω has been fixed. Most of the experimental data have been quite well reproduced with $\omega = 0.4$ GeV and the rate predicted for the $B \to D^* e\nu$ transition has been confirmed recently by the ARGUS collaboration [22].

III Semi-leptonic D and B decays

The approach described in the preceeding section uses the quark model at $q^2 = 0$ in order to fix the normalization of the formfactors. A similar model has been employed by Körner and Schuler [12]. However, they choose a common wavefunction overlap factor for all formfactors. The approach of [7], which has been improved by Altomari and Wolfenstein [10], uses the non-relativistic quark model. The normalization of the formfactors is fixed therefore at maximal q^2, where both mesons are at rest.

One of the predictions common to all these models is the inequality

$$R_M \equiv \frac{\Gamma_{SL}(M \to V)}{\Gamma_{SL}(M \to P)} \gtrsim 1 \qquad (18)$$

for both D and B meson decays, which is a reminiscence of the spectator quark model [23]. However, this ratio has been determined by the MARK III [3] and the Tagged Photon Spectrometer [24] collaborations to be:

$$R_D = \frac{\Gamma_{SL}(D \to K^*)}{\Gamma_{SL}(D \to K)} \approx 0.5 - 0.6 \qquad (19)$$

for D meson decays in contradiction to the theoretical expectation. On the other hand theory and experiment agree for $\Gamma_{SL}(M \to P)$ [25]. $\Gamma_{SL}(M \to P)$ and $\Gamma_{SL}(M \to V)$ are determined by the overlap factors h_1 and h_V, h_{A_1}, h_{A_2}, respectively, in our relativistic approach (see (4) to (7)). h_V and h_{A_1} are calculated by taking matrix elements of 'bad' operators. Therefore, they are not as reliable as the overlap factors determined by matrix elements of 'good' operators. Thus we will treat the overlap integral J_M in (13) as an additional free parameter. We keep, however, the overlap factors h_{A_3} and h_1 fixed. This implies that the total rates for $M \to P + l\nu_l$ are fixed as well.

In Tables 1 and 2 and Figs. 1 and 2 we present our results for the ratios R_B and R_D, respectively, as a function of J_M/J_M^0, where J_M^0 is calculated using (14) ($J_D^0 \simeq 1.08$, $J_B^0 \simeq 0.72$). Figures 1 and 2 show a strong dependence of $\Gamma_{SL}(M \to V)$ on the choice of the parameter J_M. In order to fix this parameter and at the same time test the predictions of the model experimental informations are needed. In the case of semi-leptonic $D \to K$ and $D \to K^*$ transitions the result for R_D from the Tagged Photon Spectrometer collaboration leads to $J_D/J_D^0 \lesssim 0.7$. From the measurement of $Br_{SL}(B \to D^*)$ by the ARGUS collaboration [22] we conclude that $0.5 \lesssim J_B/J_B^0 \lesssim 1.3$. Here we assumed that more than 80% of the inclusive semi-leptonic

Table 1. Overlap factors, the ratio R_B, the widths for semi-leptonic B decays to transversely and longitudinally polarized D^* mesons and α_B for various values of J_B/J_B^0. The widths are given in units of $10^{12}|V_{cb}|^2\,s^{-1}$. We used $\Gamma_{SL}(B\to D)=8.08\cdot10^{12}|V_{cb}|^2\,s^{-1}$ to calculate R_B

J_B/J_B^0	h_v	h_{A_1}	h_{A_2}	R_B	D^*_{trans}	D^*_{long}	α_B
1.3	0.92	0.85	1.12	4.05	17.89	14.80	0.66
1.2	0.85	0.78	0.98	3.53	15.06	13.46	0.78
1.1	0.78	0.72	0.83	3.13	12.84	12.43	0.94
1.0	0.71	0.65	0.69	2.69	10.48	11.24	1.14
0.9	0.63	0.59	0.54	2.33	8.62	10.18	1.36
0.8	0.56	0.52	0.40	1.96	6.71	9.15	1.72
0.7	0.49	0.46	0.25	1.67	5.24	8.23	2.14
0.6	0.42	0.39	0.10	1.38	3.78	7.34	2.88
0.5	0.35	0.33	−0.04	1.16	2.70	6.68	3.94

D^*_{transv} is an abbreviation for $\Gamma_{SL}(B\to D^*_{\text{transv}})$ D^*_{long} is an abbreviation for $\Gamma_{SL}(B\to D^*_{\text{long}})$

Table 2. Overlap factors, the ratio R_D, the widths for semi-leptonic D decays to transversely and longitudinally polarized K^* mesons and α_D for various values of J_D/J_D^0. The widths are given in units of $10^{10}\,s^{-1}$. We used $\Gamma_{SL}(D\to K)=8.26\cdot10^{10}\,s^{-1}$ to calculate R_D

J_D/J_D^0	h_v	h_{A_1}	h_{A_2}	R_D	K^*_{transv}	K^*_{long}	α_D
1.3	1.65	1.14	1.89	1.74	8.35	6.00	0.44
1.2	1.57	1.05	1.64	1.52	7.17	5.44	0.52
1.1	1.40	0.97	1.39	1.35	6.04	5.07	0.68
1.0	1.27	0.88	1.15	1.15	4.97	4.54	0.91
0.9	1.14	0.79	0.90	0.98	4.01	4.07	1.03
0.8	1.01	0.70	0.65	0.82	3.15	3.63	1.30
0.7	0.89	0.62	0.40	0.71	2.47	3.36	1.72
0.6	0.76	0.53	0.15	0.58	1.80	2.98	2.31
0.5	0.63	0.44	−0.10	0.47	1.24	2.64	3.26

K^*_{transv} is an abbreviation for $\Gamma_{SL}(D\to K^*_{\text{transv}})$ K^*_{long} is an abbreviation for $\Gamma_{SL}(D\to K^*_{\text{long}})$

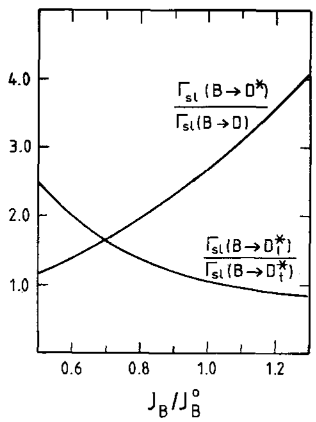

Fig. 1. Plot of $\Gamma_{SL}(B\to D^*)/\Gamma_{SL}(B\to D)$ and $\Gamma_{SL}(B\to D^*_{\text{long}})/\Gamma_{SL}(B\to D^*_{\text{transv}})$ as functions of J_B/J_B^0. We used $\Gamma_{SL}(B\to D)=8.08\cdot10^{12}|V_{cb}|^2\,s^{-1}$

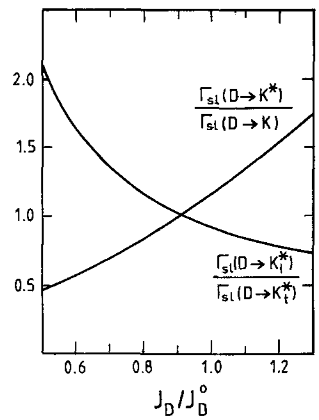

Fig. 2. Plots of $\Gamma_{SL}(D\to K^*)/\Gamma_{SL}(D\to K)$ and $\Gamma_{SL}(D\to K^*_{\text{long}})/\Gamma_{SL}(D\to K^*_{\text{transv}})$ as functions of J_D/J_D^0. We used $\Gamma_{SL}(D\to K)=8.26\cdot10^{10}\,s^{-1}$

width is saturated by the D and D^* decay channels. This result is independent of the value taken for the weak mixing matrix element $|V_{cb}|$.

Additional information can be obtained by studying the polarization dependence of the vectormeson V on the parameter J_M. In the rest frame of the decaying meson ($\mathbf{K}_M=0$) the squared matrix elements for the production of transversely and longitudinally polarized vectormesons take a simple form [12]:

$$|A_{SL}(M\to V_{\text{transv}})|^2 = \frac{G_F^2}{2}|V_{Qq}|^2 2q^2\{(1-z)^2|H_-|^2+(1+z)^2|H_+|^2\} \tag{20}$$

and

$$|A_{SL}(M\to V_{\text{long}})|^2 = \frac{G_F^2}{2}|V_{Qq}|^2 4q^2\{(1-z^2)|H_0|^2\} \tag{21}$$

where the helicity amplitudes $H_\pm$ and H_0 of the vectormeson V are given by:

$$H_\pm(q^2) = (m_M + m_V)A_1(q^2) \mp \eta 2 \frac{m_M K}{m_M + m_V} V(q^2) \quad (22)$$

$$H_0(q^2) = \frac{1}{2m_V} \frac{1}{\sqrt{q^2}} \cdot \left[(m_M^2 - m_V^2 - q^2)(m_M + m_V)A_1(q^2) - 4 \frac{m_M^2 K^2}{m_M + m_V} A_2(q^2) \right] \quad (23)$$

with

$$K = \frac{1}{2m_M} [(m_M^2 - m_V^2 - q^2)^2 - 4m_V^2 q^2]^{1/2}$$

and

$$z = \frac{1}{2m_M} \frac{1}{K} (m_M^2 - m_V^2 + q^2 - 4m_M E_l). \quad (24)$$

In (22) $\eta = 1$ for b and $\bar{b}$ quark decays and $\eta = -1$ for c and $\bar{c}$ quark decays. K is the momentum of the vectormeson in the rest frame of the decaying meson and z can be identified as the cosine of the angle between the vectormeson and the charged lepton in the $(l\nu)$ rest system. The zero-lepton-mass approximation has been used in deriving (20) to (24). The energy of the charged lepton is denoted by E_l.

The results for the ratios $\Gamma_{SL}(M \to V_{\text{long}})/\Gamma_{SL}(M \to V_{\text{transv}})$ are also shown in Figs. 1 and 2 and Tables 1 and 2 for semi-leptonic B and D decays, respectively. It is clear from these figures that the experimental determination of $\Gamma_{SL}(M \to V_{\text{long}})/\Gamma_{SL}(M \to V_{\text{transv}})$ as well as R_M will be a decisive test of our model due to the strong dependence of these ratios on the parameter J_M. The production rates for transversely and longitudinally polarized vectormesons are both affected by variation of J_M. It is interesting to note that longitudinally polarized vectormesons are favoured for a wide range of J_M for semi-leptonic B as well as D decays.

In the non-relativistic quark model approach of [7] the helicity amplitude $H_0(q^2)$ is not well known since the determination of the formfactor $A_2(q^2)$—equal to a_+ in the notation of [7]—requires knowledge of contributions proportional to $\mathbf{K}_V^2/m_V^2$ [10], which are neglected in the non-relativistic limit. A_2 at maximal q^2 is therefore treated as a free parameter in this approach. The production rate for longitudinally polarized vectormesons depends strongly on the value choosen for this parameter, whereas the rate for transversely polarized V's is fixed.

The polarization of the vectormeson V can be determined by measuring the angular distribution of the strong decay $V \to P\pi$. The angular distribution in the V rest frame is proportional to $\cos^2 \Theta^*$ for longitudinally polarized V and $\sin^2 \Theta^*$ for trans-

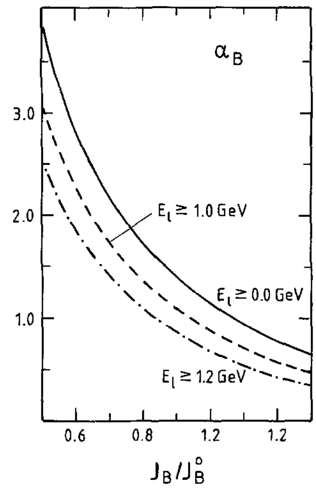

Fig. 3. Plot of $\alpha_B = 2\Gamma_{SL}(B \to D^*_{\text{long}})/\Gamma_{SL}(B \to D^*_{\text{transv}}) - 1$ versus J_B/J_B^0 for various lower cut-offs of the charged lepton energy

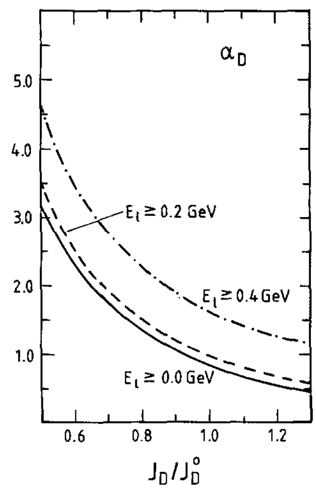

Fig. 4. Plot of $\alpha_D = 2\Gamma_{SL}(D \to K^*_{\text{long}})/\Gamma_{SL}(D \to K^*_{\text{transv}}) - 1$ versus J_D/J_D^0 for various lower cut-offs of the charged lepton energy

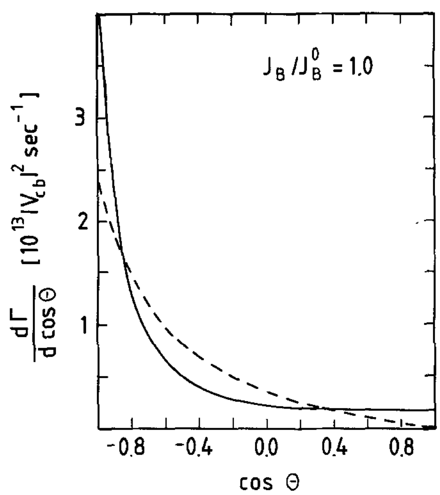

Fig. 5. $d\Gamma_{SL}(B \to D^*_{\text{transv}})/d\cos\Theta$ (straight line) and $d\Gamma_{SL}(B \to D^*_{\text{long}})/d\cos\Theta$ (dashed line) versus $\cos\Theta$ for $J_B/J_B^0 = 1$

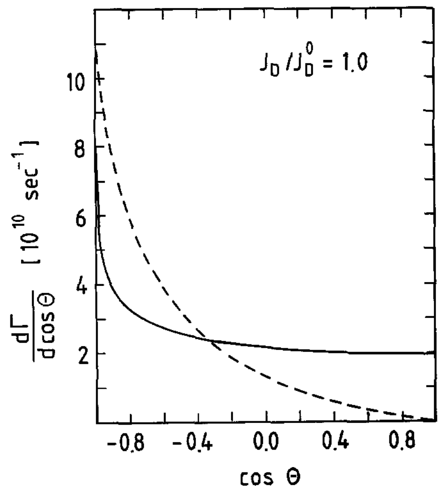

Fig. 6. $d\Gamma_{SL}(D\to K^*_{\text{transv}})/d\cos\Theta$ (straight line) and $d\Gamma_{SL}(D\to K^*_{\text{long}})/d\cos\Theta$ (dashed line) versus $\cos\Theta$ for $J_D/J_D^0=1$

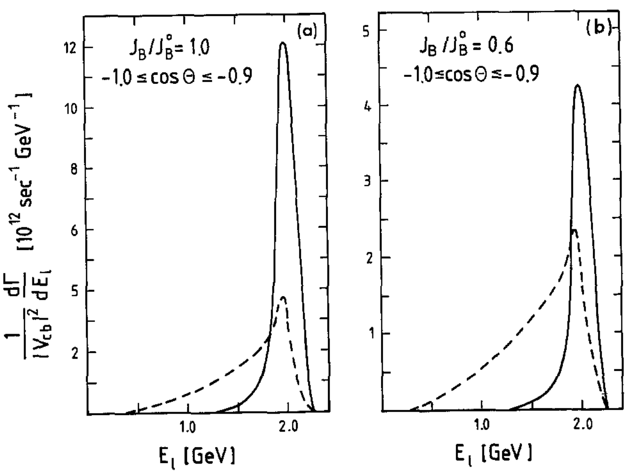

Fig. 7 a, b. $d\Gamma_{SL}(B\to D^*_{\text{transv}})/dE_l$ (straight lines) and $d\Gamma_{SL}(B\to D^*_{\text{long}})/dE_l$ (dashed lines) for $J_B/J_B^0=1.0$ (**a**) and $J_B/J_B^0=0.6$ (**b**). $\cos\Theta$ has been integrated over the range $-1.0\leqq\cos\Theta\leqq-0.9$

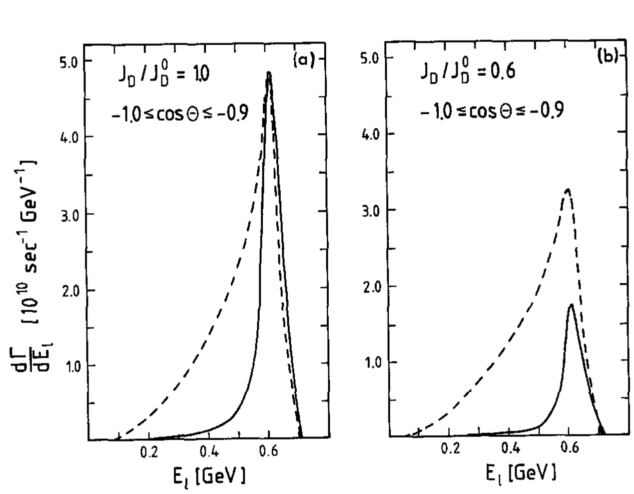

Fig. 8 a, b. $d\Gamma_{SL}(D\to K^*_{\text{transv}})/dE_l$ (straight lines) and $d\Gamma_{SL}(D\to K^*_{\text{long}})/dE_l$ (dashed lines) for $J_D/J_D^0=1.0$ (**a**) and $J_D/J_D^0=0.6$ (**b**). $\cos\Theta$ has been integrated over the range $-1.0\leqq\cos\Theta\leqq-0.9$

versely polarized V, respectively, which follows from angular momentum conservation. Θ^* is the angle between the π meson and the momentum direction of the vectormeson V. The total decay distribution can therefore be parametrized by:

$$\frac{d\Gamma_{SL}(M\to V)}{d\cos\Theta^*}\sim(1+\alpha_M\cos^2\Theta^*) \tag{25}$$

where α_M is given by:

$$\alpha_M=2\frac{\Gamma_{SL}(M\to V_{\text{long}})}{\Gamma_{SL}(M\to V_{\text{transv}})}-1. \tag{26}$$

The determination of α_M strongly depends on the experimental cuts. We therefore present the predictions for α_M for various cut-offs of the lepton energy in Figs. 3 and 4 as a function of J_M/J_M^0. As a consequence of the $V-A$ structure of the weak current α_B decreases and α_D increases with increasing lepton energy cut-off.

A first measurement of the parameter α_B has been performed by the ARGUS [27] collaboration. They use for the lepton energy cut-off $E_e>1.2$ GeV. Their result is:

$$\text{ARGUS: } \alpha_B=0.9\pm1.1. \tag{27}$$

The result of the ARGUS collaboration is in accordance with the expectation form our model but unfortunately the error is still too large to determine the parameter J_B.

The dependence of α_M on the lepton energy cut-off already indicates that the relative rates for production of longitudinally and transversely polarized vectormesons are not constant over the allowed phase space. In Figs. 5 and 6 we have plotted $d\Gamma_{SL}(M\to V_{\text{transv}})/d\cos\Theta$ (straight lines) and $d\Gamma_{SL}(M\to V_{\text{long}})/d\cos\Theta$ (dashed lines) for B and D meson decays, respectively. Θ is the angle between the charged lepton and the vectormeson in the restframe of the decaying meson ($\mathbf{K}_M=0$). The energy distributions of the charged lepton also show a characteristic behaviour for leptons from the decay to a transversely or longitudinally polarized vectormeson in the final state. This is especially the case near the edge of the phase space ($\cos\Theta=-1$) where the production of transversely polarized V's must vanish for $E_e<(m_M-m_V)/2$. In Figs. 7 and 8 we present our results for $d\Gamma_{SL}(M\to V_{\text{transv}})/dE_e$ (straight lines) and $d\Gamma_{SL}(M\to V_{\text{long}})/dE_e$ (dashed lines) for B and D meson decays, respectively, and two choices of the parameter J_M. $\cos\Theta$ has been integrated over the range $-1.0\leqq\cos\Theta\leqq-0.9$ in these figures.

IV Some remarks on exclusive non-leptonic decays

The analysis of non-leptonic two-body decays of D and B mesons can be of additional help in order to fix the parameter J_M introduced in the previous section. The predictions of our relativistic quark model

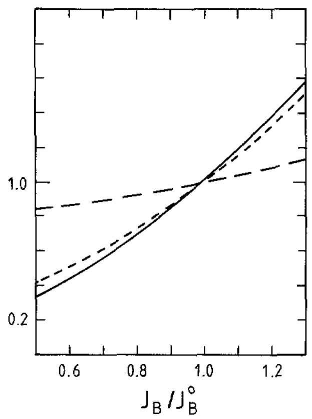

Fig. 9. Normalized total rates for $\bar{B}^0 \to \bar{K}^{*0} J/\Psi$ (——), $\bar{B}^0 \to D^{*+}\rho^-$ (– –) and $\bar{B}^0 \to D^{*+}D^{*-}$, $\bar{B}^0 \to \rho^+\rho^-$ (- - -) as functions of J_B/J_B^0. The predicted rates for $J_B/J_B^0 = 1$ are $\Gamma(\bar{B}^0 \to \bar{K}^{*0} J/\psi) \simeq 14.5\ a_2^2 |V_{cb}|^2\ 10^{12}\,\mathrm{s}^{-1}$, $\Gamma(\bar{B}^0 \to D^{*+}\rho^-) \simeq 3.9\ a_1^2 |V_{cb}|^2\ 10^{12}\,\mathrm{s}^{-1}$, $\Gamma(\bar{B}^0 \to D^{*+}D_s^{*-}) \simeq 6.8\ a_1^2 |V_{cb}|^2\ 10^{12}\,\mathrm{s}^{-1}$ and $\Gamma(\bar{B}^0 \to \rho^+\rho^-) \simeq 1.2\ a_1^2\ 10^{12} |V_{ub}|^2\,\mathrm{s}^{-1}$

concerning non-leptonic two-body decay modes of the heavy mesons have been presented in detail in [28]. Here we will discuss only the modifications of those predictions due to the introduction of the parameter J_M.

Two-body decay modes of D and B mesons to two pseudoscalar mesons or a pseudoscalar and a vectormeson in the final state will not be affected by the variation of J_M since the relevant formfactors at $q^2 = 0$ have been calculated from sandwiching 'good' operators only. Note that in the transitions to P and V mesons—in the rest frame of the decaying meson—only longitudinally polarized vectormesons can be produced. However, D and B transitions to two vectormesons depend strongly on the variation of J_M. In the rest frame of the decaying meson one can introduce helicity amplitudes similar to those which have been employed for semi-leptonic transitions. These amplitudes are no more functions of q^2 due to the two body kinematics. Instead one can test the helicity amplitudes and thus the invariant formfactors contributing at fixed momentum transfer. In the factorization approximation one of the produced vectormesons will be created through the weak current out of the vacuum. Thus one finds in general for the total widths in terms of helicity amplitudes:

$$
\begin{aligned}
&\Gamma(M \to V_1 V_2) \\
&= \frac{G_F^2}{2} |V_{Qq_1}|^2 |V_{q_3q_4}|^2 \frac{K}{16\pi} \frac{1}{m_M^3} \Big\{ a_1^2 (f_{V_1} m_{V_1})^2 \\
&\cdot \{ |H_+^{V_2}(m_{V_1}^2)|^2 + |H_-^{V_2}(m_{V_1}^2)|^2 + |H_0^{V_2}(m_{V_1}^2)|^2 \} \\
&+ a_2^2 (f_{V_2} m_{V_2})^2 \{ |H_+^{V_1}(m_{V_2}^2)|^2 \\
&+ |H_-^{V_1}(m_{V_2}^2)|^2 + |H_0^{V_1}(m_{V_2}^2)|^2 \} \\
&+ 2 (f_{V_1} m_{V_1} f_{V_2} m_{V_2}) a_1 \cdot a_2 \\
&\cdot \{ H_+^{V_1}(m_{V_2}^2) H_+^{V_2}(m_{V_1}^2) + H_-^{V_1}(m_{V_2}^2) H_-^{V_2}(m_{V_1}^2) \\
&+ H_0^{V_1}(m_{V_2}^2) H_0^{V_2}(m_{V_1}^2) \} \Big\}
\end{aligned}
\tag{28}
$$

f_{V_i} are the vectormeson decay constants, a_1 and a_2 denote the effective QCD coefficients*.

$$
H_\pm^{V_i}(m_{V_j}^2) = (m_M + m_{V_i}) A_1(m_{V_j}^2) \mp \eta_M \frac{2 m_M K}{m_M - m_{V_i}} V(m_{V_j}^2)
\tag{29}
$$

and

$$
\begin{aligned}
H_0^{V_i}(m_{V_j}^2) &= \frac{1}{2 m_{V_i}} \frac{1}{m_{V_j}} \\
&\cdot \Big[(m_M^2 - m_{V_i}^2 - m_{V_j}^2)(m_M + m_{V_i}) A_1(m_{V_j}) \\
&- \frac{4 m_M^2 K^2}{m_M + m_{V_i}} A_2(m_{V_j}^2) \Big]
\end{aligned}
\tag{30}
$$

$i \neq j \in [1,2]$ and $K = 1/2m_M [(m_M^2 - m_{V_i}^2 - m_{V_j}^2)^2 - 4 m_{V_i}^2 m_{V_j}^2]^{1/2}$. V_1 and V_2 are created out of the vacuum in the first and second term in (28), respectively. The third term gives the interference contribution. In Fig. 9 we present the widths of a few vector–vector decay channels of the B meson as functions of the parameter J_B. Of special interest is the dependence of the decay width $\bar{B}^0 \to \bar{K}^{*0} J/\psi$ on J_B. This decay has been measured by the ARGUS collaboration, they found $Br(\bar{B}^0 \to \bar{K}^{*0} J/\psi) = 0.33 \pm 0.18\%$ which translates—using $\tau_B = 1.2$ ps – into $\Gamma(\bar{B}^0 \to \bar{K}^{*0} J/\psi) = (0.28 \pm 0.15) 10^{10}\,\mathrm{s}^{-1}$ [29]. The theoretical decay width for $\bar{B}^0 \to \bar{K}^{*0} J/\psi$ is proportional to the square of the effective QCD parameter $a_2 \approx c_2(m_b)$. Assuming that $|a_2| = 0.3$ we find $0.5 < J_B < 1.2$ in agreement with our estimation from semi-leptonic B decays. However, due to the large experimental and theoretical uncertainties these estimations provide only a kind of consistency test so far.

V Summary

In this paper we have studied the formfactor dependence of exclusive D and B meson decays employing the relativistic quark model of ref. [6]. In this approach the formfactors are normalized at $q^2 = 0$ and the q^2 dependence is assumed to be of monopole type. However, some of the theoretical normalizations are uncertain since they involve hadronic matrix elements of 'bad' operators. In order to take into account this uncertainty we have introduced an additional free parameter J_M and studied in detail the dependence of the exclusive semi-leptonic decays $D \to K^* + l\nu$ and $B \to D^* + l\nu$ on these parameters. The transitions into a pseudoscalar meson are not affected since

* The dependence of non-leptonic decay amplitudes on the effective QCD parameter $a_1 \approx c_1(\mu)$, $a_2 \approx c_2(\mu)$ have been discussed in detail in [28]

the corresponding amplitudes involve only matrix elements of 'good' operators. The ratios $R_M = \Gamma_{Sl}(M \to V)/\Gamma_{Sl}(M \to P)$ and $\Gamma_{Sl}(M \to V_{\text{long}})/\Gamma_{Sl}(M \to V_{\text{transv}})$ show a strong dependence on the choice of J_M.

The experimental result $R_D \simeq 0.5$ can be explained in this approach for a reasonable range of the parameter J_D. From $R_D \simeq 0.5$ we conclude that $\Gamma_{Sl}(D \to K^*_{\text{long}})/\Gamma_{Sl}(D \to K^*_{\text{transv}}) \gtrsim 1.4$. A measurement of K^* polarization in semi-leptonic D decays can therefore give valuable information on the structure of weak hadronic matrix elements.

The D^* polarization in semi-leptonic B decays has already been determined by the ARGUS collaboration. This result together with the observed branching ratio for the semi-leptonic $B \to D^*$ transition can well be explained within our approach. Contrary to D decays we predict $R_B \gtrsim 1$ for a wide range of the parameter J_B. Unfortunately, the experimental errors are still too large to place stringent restrictions on the range allowed for J_B.

The rates of semi-leptonic decays involving $b \to u$ transitions of course also depend on the choice for the corresponding parameter $J_{B(u)}$. Varying again $J_{B(u)}/J_B^0$ we find

$$\Gamma_{SL}(\bar{B}^0 \to \rho^+) \simeq \begin{cases} 9{:}10^{12}|V_{ub}|^2\,\text{s}^{-1}, & \text{for} \quad J_{B(u)}/J^0_{B(u)} = 0.5 \\ 26{:}10^{12}|V_{ub}|^2\,\text{s}^{-1}, & \text{for} \quad J_{B(u)}/J^0_{B(u)} = 1.0 \\ 40{:}10^{12}|V_{ub}|^2\,\text{s}^{-1}, & \text{for} \quad J_{B(u)}/J^0_{B(u)} = 1.3 \end{cases} \tag{31}$$

For comparison we predict

$$\Gamma_{SL}(\bar{B}^0 \to \pi^+) \simeq 7 \cdot 10^{12}|V_{ub}|^2\,\text{s}^{-1}. \tag{32}$$

In order to reduce the theoretical uncertainties in the determination of $|V_{ub}|$ it is therefore desirable to determine the rate for the $B \to \pi l \nu$ transition alone.

In hadronic decays only the $M \to VV$ modes are affected by a variation of J_M. We have concentrated on $B \to VV$ decay modes, for example $\bar{B}^0 \to \bar{K}^{*0} J/\psi$, since the energy release in $D \to VV$ decay modes, like $D \to K^* \rho$, is very small. Therefore the factorization assumption is questionable for $D \to VV$ decays. In addition the interpretation of experimental results for exclusive D decays is complicated due to possible final state interaction effects.

Up to now we have only discussed uncertainties due to the normalization of the form-factors. Of course the q^2 dependence is also unknown, although nearest pole dominance seems to be a reasonable assumption. However, the uncertainty from the normalization is probably by far larger. The only decay channels where the q^2 dependence becomes important are the $B \to \pi + l\nu$ and $B \to \rho + l\nu$ transitions due to the proximity of the B^* pole: $m_{B^*} - m_B \simeq 50\,\text{MeV}$. This causes a strong variation in the formfactor across the kinematically allowed region $0 \leqq q^2 \leqq (m_B - m_{\pi,\rho})^2$. Again the measurement of $\Gamma_{Sl}(B \to \pi)$ is more favourable from the point of view of theoretical uncertainties since the rate is dominated by contributions from the small q^2 region in phase space. Employing, for example, constant formfactors but keeping the normalizations fixed reduces the rates for the $B \to \pi$ and $B \to \rho$ channels by factors 2 and 2.5, respectively. We therefore estimate the theoretical uncertainty for $|V_{ub}|$ determined from a measurement of $\Gamma_{Sl}(B \to \pi)$ to be less than 50%.

Acknowledgement. We like to thank H. Kolanoski, D. Wegener, R. Morrison and B. Stech for helpful informations and discussions. We are also grateful to the Deutsche Forschungsgemeinschaft for financial support.

References

1. For reviews and many references see, for example: I.I. Bigi: On charm and beauty decays—a theorist's perspective. Talk presented at the International Symposium on Production and Decay of Heavy Flavours, Stanford (1987), SLAC-PUB-4455; E.H. Throndike, R.A. Poling: Decays of the b-quark, UR-1022, submitted to Phys. Rep. (1987); M. Wirbel: Prog. Part. Nucl. Phys. 21 (1988) 33
2. See, for example Thorndike and Poling: [1]
3. D. Hitlin: Weak decays of charmed quarks. An experimental review. Lectures presented at the Charm Physics Symposium/ Workshop, Beijing (1987), CALT-68-1463
4. A. Ali: Z. Phys. C—Particles and Fields 1 (1979) 25
5. A. Ali, J.G. Körner, G. Kramer, J. Willrodt: Z. Phys. C—Particles and Fields 1 (1979) 269
6. M. Wirbel, B. Stech, M. Bauer: Z. Phys. C—Particles and Fields 29 (1985) 637
7. B. Grinstein, M.B. Wise, N. Isgur: Phys. Rev. Lett. 56 (1986) 298; CALTECH preprint CALT-68-1311 (1986)
8. F. Schöberl, H. Pietschmann: Europhys. Lett. 2 (1986) 583
9. S. Nussinov, W. Wetzel: Phys. Rev. D36 (1987) 130
10. T. Altomari, L. Wolfenstein: Phys. Rev. Lett. 58, 1583 (1987); Carnegie–Mellon preprint CMU-HEP 86-17 (1986)
11. B. Grinstein, M.B. Wise: Phys. Lett. 197B (1987) 249
12. J.G. Körner, G.A. Schuler: Z. Phys. C—Particles and Fields 38 (1988) 511; Errakum: Z. Phys. C—Particles and Fields 41 (1989) 690
13. M. Suzuki: Phys. Rev. D37 (1988) 239
14. C.A. Dominguez, N. Paver: DESY preprint DESY 88-035; DESY 88-063 (1988)
15. This has first been noted by Altomari, Wolfenstein: [10]
16. S.J. Brodsky, G.P. Lepage: Phys. Rev. D22 (1980) 2157
17. D. Fakirov, B. Stech: Nucl. Phys. B133 (1978) 315
18. R.P. Feynman, see, e.g.: high energy collisions, C.N. Yang et al. (eds.) New York 1969; J.D. Bjorken, E.A. Paschos: Phys. Rev. 185 (1969) 1975; S.D. Drell, D. Levy, T.M. Yan: Phys. Rev. 187 (1969) 2159
19. S. Fubini, G. Segré D. Walecka: Ann. Phys. 39 (1966) 381; R. Dashen, M. Gell-Mann: in proceedings of the 3rd Coral Gables Conference on Symmetry Principles at High Energy: Freeman 1969; see also V. de Alfaro et al.: currents in hadron physics San Francisco: Amsterdam 1973
20. P. Droz Vincent: Phys. Rev. D19 (1979) 702
21. H. Sazdjian: Phys. Rev. D33 (1986) 3401
22. H. Albrecht et al. ARGUS Coll.: Phys. Lett. 197B (1987) 452
23. M. Suzuki: Phys. Lett. 155B (1985) 112
24. J.C. Anjos et al. Tagged photon spectrometer Coll.: Fermilab preprint Fermilab-Pub 88/143-E (1988)
25. See, for example M. Wirbel: ref. 1
26. G.B. Word: Cornell University preprint CLNS 87/113 (1987)
27. D. Wegener: Talk presented at the Ringberg Workshop. Hadronic matrix elements and weak decays; Munich (1988)
28. M. Bauer, B. Stech M. Wirbel: Z. Phys. C—Particles and Fields 34 (1987) 103
29. H. Albrecht et al. ARGUS Coll.: Phys. Lett. 199B (1987) 451