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Weak Interaction Effects in Positronium

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Abstract. Within the context of the Weinberg-Salam (standard) model we study possible effects of weak interactions in positronium (Ps), such as parity mixing and weak decays of Ps states. As expected, weak interaction amplitudes in Ps turn out to be extremely small, their magnitude being characterized by $G \cdot m_e^2 \simeq 3 \cdot 10^{-12}$ where G is Fermi's constant and m_e the electron mass. We show that the standard model forbids parity-violating correlations in a large class of Ps reactions and decays due to CP conservation in the lepton sector. We then consider situations in which parity-odd effects in Ps will occur in the standard model and may even be large enough to be observable. Beyond the context of the standard model we discuss the decay of orthopositronium into a photon and the hypothetical axion under the assumption that the mass of the axion is smaller than twice the mass of the electron.

1. Introduction

Positronium (Ps) is one of the fundamental bound-state systems provided to us by nature. It consists of leptons only and allows precision tests of quantum electrodynamics practically free from complications due to strong interactions. Furthermore, as a fermion-antifermion bound-state Ps has simple properties under C and CP transformations where C denotes the charge conjugation and P the parity operation. This makes Ps a potentially interesting place to study the violation of C and CP invariance. (For recent reviews on Ps cf. [1, 2].)

In this article we will consider weak interaction effects in Ps. The motivation for our investigation arose from plans of a group of experimentalists to make high-precision Ps studies with a crystal ball detector built mainly for nuclear physics experiments [3].

Today we have a theory of weak and electromagnetic interactions, the Salam-Weinberg or standard model [4, 5], which is in excellent agreement with experiment. However, the standard model is not established beyond doubt and it should always be interesting to check a model as completely as possible. We have for instance plenty of experimental information on the weak interaction of electrons with other leptons or quarks. But very little is known about the weak interaction of electrons with themselves, which is relevant for Ps and for Bhabba scattering $e^+e^- \to e^+e^-$. Experiments at PETRA are only beginning to see weak effects in Bhabba scattering [6]. It was pointed out a long time ago that "diagonal" and "nondiagonal" weak interactions may behave quite differently [7]. In the modern context models with more than one Z^0 boson are tested by comparing "diagonal" and "nondiagonal" processes (see for instance [8]).

In Sect. 2 of this article we will give estimates of weak effects in Ps expected in the standard model. Numerically these effects turn out to be extremely small, since invariably they involve Fermi's constant G multiplied by m_e^2, the electron mass squared:

$$G \cdot m_e^2 \simeq 3.04 \cdot 10^{-12}. \tag{1.1}$$

There are then two possible attitudes. We can either say that it is not worthwhile to look for weak effects in Ps since they are predicted to be so small or we say that Ps is an ideal place to test the standard model since any sizeable weak interaction effect found by experiments would force us to revise the model. We must leave it to the reader to choose one point or the other.

In Sect. 3 we will indeed go beyond the context of the standard model with one Higgs doublet and consider the decay of orthopositronium into a photon and the hypothetical axion a [9, 10],

$$\mathrm{Ps}(^3S_1) \to \gamma + a. \tag{1.2}$$

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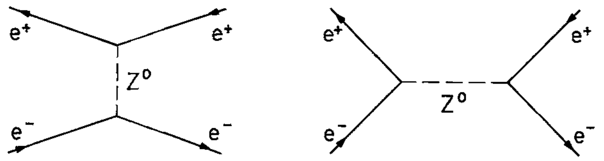

Fig. 1. Diagrams for the weak interaction between electrons due to exchange of the heavy boson Z^0

A search for axions in this decay has been suggested previously by Mikaelian [11]. We find, however, that his calculation contains an error.

For the convenience of the reader we collect a few formulae for Ps in Appendix A. Appendix B deals with the calculation of parity mixing between Ps states. In Appendix C we show that CP conservation excludes parity-violating correlations in a large class of Ps decays and reactions. In Appendix D we discuss some correlations in the decay of polarized orthopositronium into 3 photons which might be of interest for experimentalists. We are not aware of any such discussion in the literature on Ps.

2. Predictions of the Standard Model for Weak Interaction Effects in Positronium

In this section we will first discuss the mixing of Ps states with different parity due to weak interactions. In the standard model with 3 charged leptons (e, μ, τ), 3 massless neutrinos, and one Higgs doublet, CP invariance holds in the lepton sector. The standard model predicts, therefore, no mixing of singlet and triplet Ps states which have opposite CP eigenvalues (cf. Table 2 of Appendix A). The fact that the Ps states are eigenstates of CP also makes it difficult to obtain weak-electromagnetic interference effects. This will be discussed below in detail. A search for CP violating effects in Ps would in principle be very interesting, since this could for instance tell us something about grand unified theories. In practice, however, we expect such effects to be astronomically small since standard weak effects are already very small.

Further topics of this section will be weak decays of Ps and T-violating correlations, where T denotes the time reversal operation.

2.1. Parity Mixing of Ps *States*

In the context of the standard model the weak interactions which are relevant for parity mixing of Ps states are generated by Z^0 exchange* (Fig. 1). Since in Ps the momentum transfers are much smaller in magnitude than the mass of the Z^0, $m_Z \simeq 90\,\text{GeV}$, these interactions can be described by the effective low-energy Lagrangian

$$\mathscr{L}_{\text{eff}}(x) = -\frac{G}{4\sqrt{2}}\bar{e}(x)\gamma^\mu(g_V-\gamma_5) \cdot e(x)\bar{e}(x)\gamma_\mu(g_V-\gamma_5)e(x)$$
$$g_V = 1-4\sin^2\Theta_W \simeq 0.08\,, \tag{2.1}$$

where $e(x)$ denotes the electron field, G the Fermi coupling constant, and Θ_W the weak angle. We have used $\sin^2\Theta_W \simeq 0.23$ (cf. for instance [13]). Our conventions concerning γ-matrices follow [14].

The parity conserving part of $\mathscr{L}_{\text{eff}}$ (2.1) leads to shifts of the energy levels of Ps which are, however, too small to be observable. The parity violating part of $\mathscr{L}_{\text{eff}}$ (2.1) leads to a parity violating contribution H_{PV} to the Hamiltonian,

$$H_{PV} = -\frac{G}{2\sqrt{2}} g_V \int d^3x (\bar{e}(x)\,\gamma^\mu e(x))(\bar{e}(x)\gamma_\mu\gamma_5 e(x)) \tag{2.2}$$

which mixes states of opposite parity. In the standard model H_{PV} (2.2) commutes with the CP operator. Therefore, singlet and triplet Ps states which are eigenstates of CP with eigenvalues -1 and $+1$ are not mixed with each other (cf. Table 2 of Appendix A). As shown in Appendix B, only triplet Ps states with total angular momentum $j=1$ can be parity mixed, if we neglect radiative corrections. We find for instance the following admixture to the triplet ground state 1^3S_1 from the nearest $^3P_1$ state 2^3P_1 (cf. Appendix B)

$$|1^3S_1\rangle \to |1^3\tilde{S}_1\rangle \simeq |1^3S_1\rangle + i\delta_{2,1}|2^3P_1\rangle$$
$$\delta_{2,1} = \frac{\alpha^2 G m_e^2 g_V}{6\pi\sqrt{2}}\left\{1 + i\alpha^3\frac{2^{10}}{3^9}\right\}. \tag{2.3}$$

Numerically this is extremely small

$$\delta_{2,1} \simeq 4.9\cdot 10^{-19}\{1+i2.02\cdot 10^{-8}\}\,. \tag{2.4}$$

For the 2^3S_1 state the admixture of the nearest $^3P_1$ state, again the 2^3P_1, is somewhat larger due to a smaller energy denominator in (B.5) of Appendix B.

$$|2^3S_1\rangle \to |2^3\tilde{S}_1\rangle \simeq 2^3S_1\rangle + i\delta_{2,2}|2^3P_1\rangle$$
$$\delta_{2,2} = -\frac{3Gm_e^2 g_V}{14\pi}\left\{1 - i\alpha\frac{2^{12}}{7\cdot 3^7}\right\}. \tag{2.5}$$

Numerically we find here

$$\delta_{2,2} \simeq -1.66\cdot 10^{-14}\{1-i1.95\cdot 10^{-3}\}\,. \tag{2.6}$$

Certainly it will not be easy to detect such tiny admixtures.

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* Higgs boson exchange amplitudes are suppressed by a factor of order (m_e^2/m_H^2) compared to those of Fig. 1. They can be savely neglected since we expect $m_H \gtrsim 10\,\text{GeV}$ [12]

2.2. *The Static Electric and Magnetic Dipole Moments of Triplet* Ps *States*

Let us recall the definition of the electric and magnetic dipole operators $\mathbf{D}$ and $\boldsymbol{\mu}$ and their transformation properties under the operations P, C, CP, and time reversal T (see Table 1).

$$\mathbf{D}=\int d^3x\mathbf{x}j_0(\mathbf{x}), \tag{2.7}$$

$$\boldsymbol{\mu}=\tfrac{1}{2}\int d^3x\mathbf{x}\times\mathbf{j}(\mathbf{x}), \tag{2.8}$$

where j_0 and $\mathbf{j}$ are the time and space components of the electromagnetic current operator

$$j^\mu(x)=-e\bar{e}(x)\gamma^\mu e(x). \tag{2.9}$$

We denote the *positron* charge by e $(e>0)$.

Table 1. Transformation properties of $\mathbf{D}$ and $\boldsymbol{\mu}$ under space inversion (P), charge conjugation (C), and time reversal (T)

A	PAP^{-1}	CAC^{-1}	$(CP)A(CP)^{-1}$	$(TAT^{-1})^\dagger$
$\mathbf{D}$	$-\mathbf{D}$	$-\mathbf{D}$	$\mathbf{D}$	$\mathbf{D}$
$\boldsymbol{\mu}$	$\boldsymbol{\mu}$	$-\boldsymbol{\mu}$	$-\boldsymbol{\mu}$	$-\boldsymbol{\mu}$

Table 1 shows that $\mathbf{D}$ and $\boldsymbol{\mu}$ commute and anticommute with the CP operator, respectively. Since the standard model respects CP invariance in the lepton sector, the Ps states are CP eigenstates. Defining the dipole moments by the ordinary expectation values, it follows that Ps states can have a static electric dipole moment but not a static magnetic dipole moment. Time reversal invariance does allow such an electric dipole moment for unstable states [15].

Consider now the states $1^3\tilde{S}_1$ (2.3) and $2^3\tilde{S}_1$ (2.5) with density matrix as shown in (A.4). We find for the expectation value of the electric dipole operator

$$\langle\mathbf{D}\rangle_{1^3\tilde{S}_1}=\mathbf{s}D(1^3\tilde{S}_1), \tag{2.10}$$

$$\begin{aligned} D(1^3\tilde{S}_1)&\simeq -er_b\frac{2^8}{3^5}\operatorname{Im}\delta_{2,1}\\ &=-er_b\alpha^5 Gm_e^2 g_V\frac{\sqrt{2}2^{16}}{\pi 3^{15}}\\ &\simeq -e1.1\cdot 10^{-34}\,\text{cm}, \end{aligned} \tag{2.11}$$

$$\langle\mathbf{D}\rangle_{2^3\tilde{S}_1}=\mathbf{s}D(2^3\tilde{S}_1), \tag{2.12}$$

$$\begin{aligned} D(2^3\tilde{S}_1)&\simeq er_b 3\sqrt{2}\operatorname{Im}\delta_{2,2}\\ &=er_b\alpha Gm_e^2 g_V\frac{\sqrt{2}2^{11}}{\pi 7^2\cdot 3^5}\\ &\simeq e1.5\cdot 10^{-24}\,\text{cm}, \end{aligned} \tag{2.13}$$

where r_b is the Bohr radius of Ps (cf. Appendix A).

When Ps is placed in a uniform external electric field, these electric dipole moments will not produce a linear Stark effect*. They are, however, relevant for the rate at which the external field delivers angular momentum to the decay products of Ps [16, 17].

2.3. *Parity Violating Interference Effects in* Ps

We will now discuss the possibility of observing interference effects between parity violating and parity conserving amplitudes. We will see that such effects are much harder to observe than in normal atoms (cf. [18] for a review) since Ps is a particle-antiparticle bound state.

We will first show that CP conservation in the lepton sector as is true for instance in the standard model which we are considering predicts *no parity violating correlations* for the following types of reactions:

$$\text{Ps}\to n\gamma, \tag{2.14}$$

$$\text{Ps}\to\text{Ps}'+n\gamma, \tag{2.15}$$

$$n\gamma+\text{Ps}\to\text{Ps}'+n'\gamma. \tag{2.16}$$

To prove our assertion we consider first the reaction (2.14) in the rest system of the decaying Ps state. This is an eigenstate of CP (cf. Table 2). The $n\gamma$ system on the right hand side of (2.14) must then also be in an eigenstate of CP and, since its C quantum number is fixed to be $(-1)^n$, it must be in an eigenstate of the parity operator P. This implies the absence of any parity-odd correlation.

The argument for the reactions (2.15) and (2.16) is similar and we give the details in Appendix C. We note as a corollary that CP conservation excludes optical activity for Ps. Indeed, as a special case of (2.16) we find equality for the forward scattering amplitudes of right- and left-circularly polarized photons on any spin-averaged Ps state. This implies that Ps-vapour will have the same refractive index for right- and left-circularly polarized light.

Parity violating correlations can occur when we look for a final state where the particles are not eigenstates of the charge conjugation operator C. An example is the break-up reaction

$$\gamma+2^3\tilde{S}_1\to e^+ + e^-. \tag{2.17}$$

However, the P-violating effects for this reaction are of order $\delta_{2,2}\simeq 10^{-14}$ and we could find no kinematic situation where they would be enhanced considerably.

An interesting possibility for observing parity-violating interference effects occurs for Ps in a static external magnetic field $\mathbf{B}$. The interaction Lagrangian for such an external field is given by

$$L=\tfrac{1}{2}\int d^3x j(\mathbf{x})(\mathbf{B}\times\mathbf{x}), \tag{2.18}$$

* We thank C. Bouchiat for pointing this out to us

where $\mathbf{j}$ is the current operator (2.9). This conserves parity P and can, therefore, not induce parity-odd effects by itself. However, charge conjugation invariance is violated by the Lagrangian (2.18), and this opens up the possibility of observing parity-odd interference terms due to weak interactions in reactions of the type (2.14)–(2.16).

As an example we will discuss the absorption of light by the 1^3S_1 ground state leading to the 2^3S_1 state in a magnetic field. For a weak field B pointing in the z-direction, which also serves as quantization axis, we have the following energy eigenstates where the parity mixing of the 2^3S_1 state (2.5) is taken into account:

$$|1^3\tilde{\tilde{S}}_1(m=\pm 1)\rangle = |1^3S_1(m=\pm 1)\rangle$$
$$|1^3\tilde{\tilde{S}}_1(m=0)\rangle = |1^3S_1(m=0)\rangle + \frac{eB}{m_e(E_{1^3S_1}-E_{1^1S_0})}|1^1S_0\rangle, \qquad (2.19)$$

$$|2^3\tilde{\tilde{S}}_1(m=\pm 1)\rangle = |2^3S_1(m=\pm 1)\rangle + i\delta_{2,2}|2^3P_1(m=\pm 1)\rangle$$
$$|2^3\tilde{\tilde{S}}_1(m=0)\rangle = |2^3S_1(m=0)\rangle + i\delta_{2,2}|2^3P_1(m=0)\rangle. + \frac{eB}{m_e(E_{2^3S_1}-E_{2^1S_0})}|2^1S_0\rangle. \qquad (2.20)$$

In the transition

$$1^3\tilde{\tilde{S}}_1(m=\pm 1)+\gamma \to 2^3\tilde{\tilde{S}}_1(m=0) \qquad (2.21)$$

we will then have two contributions: A magnetic M1 amplitude leading to the 2^1S_0 component of the $2^3S_1(m=0)$ state and an electric E1 amplitude leading to the opposite parity admixture 2^3P_1

$$\begin{aligned} &\mathrm{M1:}\ A(1^3S_1+\gamma\to 2^1S_0)\\ &\mathrm{E1:}\ A(1^3S_1+\gamma\to 2^3P_1). \end{aligned} \qquad (2.22)$$

These two amplitudes can interfere and produce parity-odd effects, for instance a different absorption probability for right- and left-circularly polarized photons. Such an asymmetry (As) will be proportional to the following factors

$$\mathrm{As} \propto \delta_{2,2}\frac{m_e(E_{2^3S_1}-E_{2^1S_0})}{eB}\cdot\frac{\mathrm{E1}}{\mathrm{M1}}. \qquad (2.23)$$

The M1 transition from the 1^3S_1 to the 2^1S_0 state is highly forbidden. In the nonrelativistic approximation this amplitude vanishes. We expect that in analogy to atomic hydrogen (cf. p. 285 of [19]) the ratio of M1 to E1 amplitudes should be of order α^3. Inserting this and the value of the energy difference from [1] we find

$$\mathrm{As} \propto \delta_{2,2}\sqrt{\frac{1}{4\pi\alpha^3}\frac{7}{24Br_b^2}} \simeq 4\cdot 10^{-4}/B(\mathrm{Gauss}). \qquad (2.24)$$

Therefore, parity-odd correlations should be quite sizeable for weak magnetic fields of the order of 10^{-3} Gauss. Such a weak magnetic field does not appreciably change the energy denominators in (2.19) and (2.20). For strong magnetic fields the energy levels of the 2^3S_1 and 2^3P_1 states never cross [20], thus there is no chance to enhance parity-violating effects in Ps in the same way as in hydrogen [18].

Another transition where we expect parity-odd effects of similar magnitude is

$$2^3\tilde{\tilde{S}}_1(m=0)+\gamma \to 3^3\tilde{\tilde{D}}_2(m=\pm 1), \qquad (2.25)$$

where we denote again with a double tilde the states perturbed by the magnetic field and by the parity-violating Hamiltonian (2.2).

Recently, Doppler-free two-photon absorption has become an interesting tool of atomic spectroscopy and Ps experiments using this technique are already planned [21]. We have therefore investigated the possibility of observing parity-odd effects in two-photon transitions. Most promising seems to us the 2γ transition leading from the 1^3S_1 ground state to the 2^3P_1 state in a weak magnetic field. The 2^2P_1 state perturbed by the weak interactions and the magnetic field has the following structure:

$$\begin{aligned} |2^3P_1(m)\rangle = {} & |2^3P_1(m)\rangle + i\delta_{2,2}|2^3S_1(m)\rangle \\ & + m\frac{eB}{\sqrt{2}m_e(E_{2^3P_1}-E_{2^1P_1})}|2^1P_1(m)\rangle \\ & m=\pm 1, 0. \end{aligned} \qquad (2.26)$$

In the 2γ transition

$$\gamma(\mathbf{k},\boldsymbol{\varepsilon}_1)+\gamma(-\mathbf{k},\boldsymbol{\varepsilon}_2)+1^3\tilde{\tilde{S}}_1(m)\to 2^3\tilde{\tilde{P}}_1(m) \qquad (2.27)$$

we have again two contributions. An E1–M1 amplitude leading from the 1^3S_1 to the 2^1P_1 components* and an E1–E1 amplitude leading from the 1^3S_1 to the opposite parity admixture 2^3S_1.

$$\begin{aligned} &\mathrm{E1\text{–}M1:}\ A(1^3S_1+2\gamma\to 2^1P_1)\\ &\mathrm{E1\text{–}E1:}\ A(1^3S_1+2\gamma\to 2^3S_1). \end{aligned} \qquad (2.28)$$

These two amplitudes can interfere and lead to P-violating asymmetries, for instance when comparing the absorption of two right-circularly and two left-circularly photons on the state $1^3S_1(m=1)$. We estimate the magnitude of these P-violating asymmetries to be of the order given in (2.24), since we have found by explicit calculation that the E1–M1 amplitude of (2.28) vanishes in the non-relativistic approximation as does the M1 amplitude of (2.22).

* For the case we consider in (2.27) there is no two-photon amplitude between the 1^1S_0 and the 2^3P_1 components of our states due to the Landau-Yang theorem [22]

The Ps used in an experiment will hardly be at rest in the apparatus and one might fear that the motional Stark effect which also mixes S and P states will completely overwhelm all weak interaction effects. However, the Lagrangian (2.18) is strictly parity-conserving. Therefore, the motional Stark effect cannot produce any P-odd effects as long as the density matrix of the initial state (γ+Ps) is parity symmetric. In some cases this may be simply achieved by flipping the magnetic field in successive data-taking runs.

The parity-violating interference terms discussed in this section are probably the best place to see a weak interaction effect predicted by the standard model in Ps. Even if the experiments we propose look rather difficult, they may be worthwhile to perform. They offer the possibility of measuring the weak angle Θ_W in a *purely leptonic* reaction at low energies. The only other purely leptonic reaction suitable for this purpose is neutrino-electron scattering which is not particularly easy to measure either. Once the Z^0 and $W^{\pm}$ bosons are found and their masses determined, precision low-energy measurements of $\sin^2\Theta_W$ and of the Fermi constant G will allow us to test the higher-order corrections in the Weinberg-Salam theory and to make predictions for the mass of the Higgs boson [23].

2.4. Weak Decays of Positronium

In this section we will examine weak decay modes of Ps which are forbidden by the selection rules of QED. Our estimates indicate that all these decay modes have astronomically small branching ratios. Thus there is hardly any chance to observe them experimentally if the standard model is correct.

We first examine the decays

$$1^1S_0 \to 3\gamma, \tag{2.29}$$

$$1^3S_1 \to 4\gamma. \tag{2.30}$$

These decays* which violate C-invariance can occur within the standard model via C and P violating neutral-current interactions as illustrated by a sample of diagrams in Fig. 2a and b. For the respective branching ratios we estimate

$$\frac{\Gamma(1^1S_0\to 3\gamma)}{\Gamma(1^1S_0\to 2\gamma)} \approx \frac{\Gamma(1^3S_1\to 4\gamma)}{\Gamma(1^3S_1\to 3\gamma)} \approx \alpha(Gm_e^2 g_V)^2 \approx 10^{-27}. \tag{2.31}$$

The present experimental upper limits are [24]

$$\frac{\Gamma(1^1S_0\to 3\gamma)}{\Gamma(1^1S_0\to 2\gamma)} \leqq 2.8\cdot 10^{-6}, \tag{2.32}$$

$$\frac{\Gamma(1^3S_1\to 4\gamma)}{\Gamma(1^3S_1\to 3\gamma)} \leqq 8\cdot 10^{-6} \tag{2.33}$$

* Recall that $1^3S_1\to 2\gamma$ is forbidden by Bose symmetry [22]

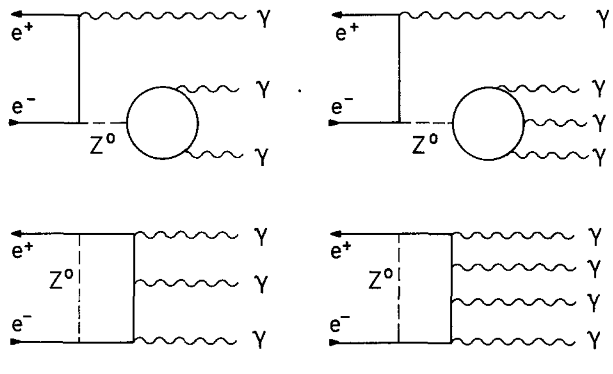

Fig. 2a and b. Sample diagrams for the weak decays $^1S_0\to 3\gamma$ **a** and $^3S_1\to 4\gamma$ **b**

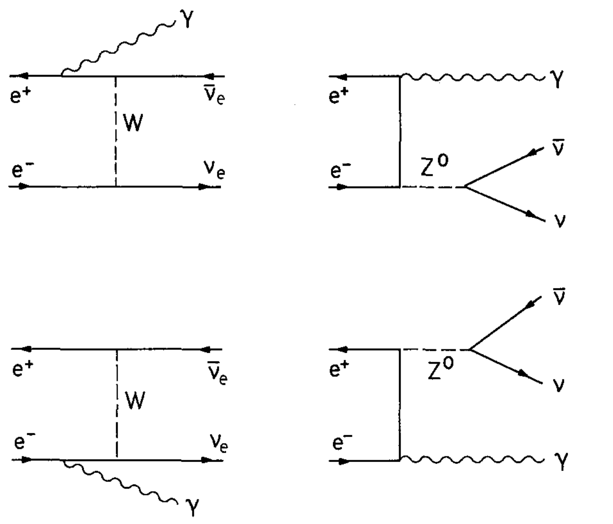

Fig. 3a and b. Diagrams for the decay Ps$\to\nu\bar{\nu}\gamma$ proceeding through W-exchange **a** and Z^0-exchange **b**

which are about 20 orders of magnitude larger than the theoretical estimates (2.31)!

Next we examine the decay

$$\mathrm{Ps}\to\nu\bar{\nu}\gamma. \tag{2.34}$$

The interest in this decay, which proceeds via the diagrams of Fig. 3, lies in the fact that the diagrams of Fig. 3b give a contribution to the decay rate proportional to the number n_ν of neutrino species with mass less than m_e. The decay (2.34) could be observed by detecting single photons having the characteristic momentum distribution of a three-body decay. In this way we could determine how many different types of very light neutrinos exist. However, we estimate the probability for this decay occuring to be exceedingly small. For the triplet Ps ground state we estimate the branching ratio as follows:

$$\frac{\Gamma(1^3S_1\to\nu\bar{\nu}\gamma)}{\Gamma(1^3S_1\to 3\gamma)} \approx \left(\frac{Gm_e^2}{\alpha}\right)^2 \simeq 10^{-19}. \tag{2.35}$$

2.5. T-*Violating Correlations in* Ps *Annihilation*

For completeness we discuss here briefly time reversal (T) violating correlations in the annihilation of polarized triplet Ps into 3 photons

$$n^3S_1 \to 3\gamma. \tag{2.36}$$

We assume that the initial Ps state is described by a density matrix as shown in (A.4). Let **n** be the normal to the decay plane spanned by the momenta of the three photons [cf. Appendix D, (D.3)]. A time-reversal-invariance violating interaction will lead to a nonvanishing correlation

$$\langle \mathbf{s}\cdot\mathbf{n}\rangle \neq 0. \tag{2.37}$$

However, also final state interactions make this correlation nonzero, of order α

$$\langle \mathbf{s}\cdot\mathbf{n}\rangle = O(\alpha). \tag{2.38}$$

The prediction of the standard model where T-invariance holds in the lepton sector is that *only* final state interactions will contribute to the expectation value $\langle \mathbf{s}\cdot\mathbf{n}\rangle$.

3. Axion Emission in Orthopositronium Decay

It was suggested a couple of years ago that one way to avoid P-, CP-, and T-violations caused by instantons in the standard model of strong interactions, quantum chromodynamics (QCD), is to impose an additional global chiral U(1) symmetry [usually called $U(1)_{PQ}$] on the Lagrangian describing strong, weak, and electromagnetic interactions [25]. It was subsequently pointed out that this leads to a very light neutral pseudoscalar boson, the so-called axion a [9, 10]. However, such a particle appeared to be ruled out experimentally. Various reactor and beam-dump experiments provided experimental limits on the production and detection of axions which were well below the estimates based on QCD and the simplest weak interaction model having an additional $U(1)_{PQ}$ symmetry [26]. Yet the theoretical interpretation of these experiments is rather uncertain. Since then quite a few models have been devised to make the axion's appearance much more elusive or to avoid it altogether [27].

Very recently interest in axions has been revived by results of an experimental group which could indicate the existence of such a particle [28]. If the axion mass m_a is less than twice the electron mass, $m_a < 2m_e$, the decay of Ps into an axion and a photon is possible and offers a possibility to search for axions [11].

We will, therefore, investigate here the reaction

$$n^3S_1 \to \gamma + a. \tag{3.1}$$

The experimental signal for this reaction is the monochromatic γ-ray whose energy E_γ determines the mass of the axion

$$E_\gamma = m_e\left(1 - \frac{m_a^2}{4m_e^2}\right). \tag{3.2}$$

Our calculations are based on the simplest extension of the standard model allowing for axions as discussed in [9]. The model has the same particle content and couplings as the standard model except for the Higgs sector. Instead of one there are two Higgs doublets, which are necessary to implement the $U(1)_{PQ}$ symmetry. In this model the coupling of the axion field $a(x)$ to electrons is given by

$$\mathrm{L}_{a,e} = ic_e 2^{1/4} G^{1/2} m_e a(x)\bar{e}(x)\gamma_5 e(x), \tag{3.3}$$

where c_e is a parameter not determined by the model. The factor c_e can, however, be related to the axion mass m_a. With some approximations this relation looks as follows [9]:

$$c_e = \tan\alpha \text{ or } \cot\alpha$$
$$m_a \simeq (23\,\mathrm{keV})\frac{f}{\sin 2\alpha}, \tag{3.4}$$

where α $\left(0 \leqq \alpha \leqq \frac{\pi}{2}\right)$ is a mixing angle and f is the number of quark flavors which we take as $f=6$. The two options for c_e (3.4) arise since one or the other of the two Higgs doublets can couple to the electron and produce the electron mass through the Higgs mechanism. Finally we recall that for $m_a < 2m_e$ the axion will decay almost exclusively into two photons.

It is now a simple matter to calculate the rate for the reaction (3.1). The contributing diagrams are shown in Fig. 4. We find

$$\Gamma(n^3S_1 \to \gamma + a) = \frac{2\sqrt{2}}{3}\alpha |c_e|^2 G\left(1 - \frac{m_a^2}{4m_e^2}\right)|\Psi_n(0)|^2, \tag{3.5}$$

where $\Psi_n(0)$ is the wave function at the origin. This leads to the branching ratio for the ground state ($n=1$) decay

$$\frac{\Gamma(1^3S_1 \to \gamma + a)}{\Gamma(1^3S_1 \to 3\gamma)} = \frac{3|c_e|^2 G m_e^2\left(1 - \frac{m_a^2}{4m_e^2}\right)}{4\sqrt{2}(\pi^2 - 9)\alpha}$$
$$= 3.48\cdot 10^{-8}|c_e|^2\left(1 - \frac{m_a^2}{4m_e^2}\right). \tag{3.6}$$

Our result disagrees with that of [11] by a factor 2.

In Fig. 5 we have plotted this branching ratio as a function of $x = m_a/2m_e$ using the naive estimate for m_a

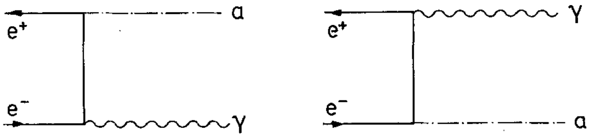

Fig. 4. Diagrams for the decay of ortho-Ps into an axion and a photon

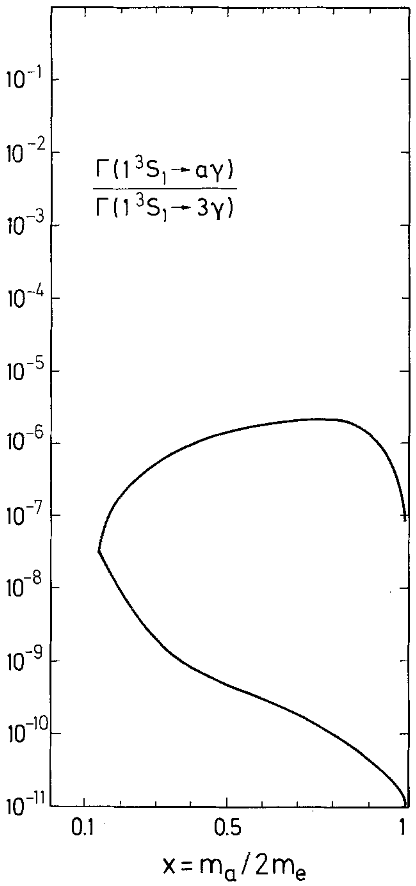

Fig. 5. The branching ratio of the triplet Ps groundstate into axion and photon. The two branches are explained in the text

quoted above (3.4) with six quark flavors. We have then a minimal value for x

$$x=\frac{m_a}{2m_e}=\frac{x_0}{\sin 2\alpha}\geqq x_0 \qquad x_0=0.135. \tag{3.7}$$

A given value for x leads to two solutions for α in the range $0\leqq\alpha\leqq\frac{\pi}{2}$ and thus to two solutions for $\tan\alpha$ and $\cot\alpha$, respectively.

$$\tan\alpha=\frac{x\mp\sqrt{x^2-x_0^2}}{x_0} \qquad \cot\alpha=\frac{x\pm\sqrt{x^2-x_0^2}}{x_0}. \tag{3.8}$$

Therefore, in this model the axion mass m_a determines the axion-electron coupling c_e only up to a two-fold ambiguity no matter which one of the two possibilities for c_e, $\tan\alpha$ or $\cot\alpha$ is chosen (3.4). This leads to the two branches for the curve shown in Fig. 5. We should emphasize that these curves are based on the estimate for the axion mass (3.4) taken from [9]. This estimate is subject to considerable uncertainty. For example, instanton effects may change the mass by a large factor (cf. [29]).

We will now consider the decay of polarized Ps into axion and photon

$$1^3S_1(\boldsymbol{\lambda})\to\gamma(\mathbf{k},\boldsymbol{\varepsilon})+\mathrm{a}(-\mathbf{k}), \tag{3.9}$$

where we have indicated polarization and momentum vectors in the rest system of the decaying Ps state. Observation of polarizations offers the interesting possibility of checking the γ_5 coupling of the axion to electrons (3.3). Let us assume that the axion has spin O. For a pseudoscalar coupling as in (3.3) the amplitude can only be of the form

$$\boldsymbol{\varepsilon}^*\cdot(\mathbf{k}\times\boldsymbol{\lambda}). \tag{3.10}$$

For a scalar coupling the amplitude must be of the form

$$\boldsymbol{\varepsilon}\cdot\boldsymbol{\lambda}. \tag{3.11}$$

Both amplitudes lead to the same angular distribution of the photon if the photon polarization is not observed.

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\Omega_k}(1^3S_1(\boldsymbol{\lambda})\to\gamma(\mathbf{k})+a(-\mathbf{k}))=\frac{3}{8\pi}(1-|\boldsymbol{\lambda}\cdot\hat{\mathbf{k}}|^2) \qquad \hat{\mathbf{k}}=\mathbf{k}/|\mathbf{k}|. \tag{3.12}$$

For the spherical base states of Ps with magnetic quantum numbers $m=0$, ± 1 this leads to

$$\frac{8\pi}{3}\frac{1}{\Gamma}\frac{d\Gamma}{d\Omega_k}(1^3S_1(m=0)\to\gamma+a)=1-\cos^2\vartheta_k, \tag{3.13}$$

$$\frac{8\pi}{3}\frac{1}{\Gamma}\frac{d\Gamma}{d\Omega_k}(1^3S_1(m=\pm 1)\to\gamma+a)=\frac{1}{2}(1+\cos^2\vartheta_k), \tag{3.14}$$

where ϑ_k denotes the angle between the momentum vector of the photon and the spin-quantization axis. The vanishing of the right-hand side of (3.13) for $\vartheta_k=0,\pi$ is dictated by angular momentum conservation.

A distinction between the pseudoscalar and scalar couplings is, therefore, only possible if also the polarization of the emitted photon is observed. For a pseudoscalar amplitude (3.10) the polarization vectors of Ps and γ will be preferentially orthogonal for a scalar amplitude (3.11) preferentially parallel.

Finally we note that axion exchange in singlet and triplet Ps and singlet Ps annihilation into a virtual axion and subsequent recreation would induce small shifts of the Ps energy levels. We estimate that this will give a contribution to the hyperfine splitting $E_{1^3S_1} - E_{1^1S_0}$ of order $Gm_e^2/\alpha \sim 10^{-10}$, well below current theoretical and experimental uncertainties [2]. This conclusion does, however, not hold if the axion mass is very close to the mass of an $n\ {}^1S_0$ Ps state. Concerning bounds on the coupling and mass of the axion derived from other experiments*, for example from the measurement of $(g-2)$ of the myon [30], it seems that the axion, if it exists, must have a mass less than 300 keV.

4. Summary

Not surprisingly most of the weak interaction effects in Ps which are predicted by the Weinberg-Salam model are extremely small. Moreover we have shown that CP-invariance which holds for the lepton sector in the standard model forbids parity-violating correlations in a large class of Ps reactions and decays. We have given two examples where parity-odd effects will nevertheless occur within the standard model: (1) The break-up reaction triplet $-\mathrm{Ps} + \gamma \to e^+ + e^-$. (2) Optical transitions in Ps in an external magnetic field. For magnetic fields of the order of 10^{-3} Gauss we expect sizeable effects in some transitions. We point out that these P-violating effects are of interest in connection with higher order corrections in the standard model.

Even more interesting seems to us the possibility of observing the decay of orthopositronium into a photon and an axion which can occur if $m_a < 2m_e$. Observation of this decay mode would determine the mass of the axion and the strength of the axion-electron coupling. Moreover, by measuring the polarization of the photons one could deduce the parity of the axion. We expect the branching ratio of this decay mode to be 10^{-9} or larger if $m_a \lesssim 400$ keV. Measurement of such branching fractions appear to be possible [3]. In this way further evidence or counter-evidence for the existence of a neutral particle with mass around 300 keV [28] might be obtained.

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Note added: After completion of this manuscript we received preprints by C. Carboni [CERNI-EP|81-20 (1981)], and P. Fayet and M. Mezard [Ecole Normale Paris preprint LPTENS 81/2 (1981)], where the decay $\mathrm{Ps} \to a + \gamma$ is also considered. Their results for the branching ratio agree with our Eq. (3.6).

* We thank P. Fayet for a correspondence on this point (cf. also [31])

Appendix A

Properties of Positronium States in Pure QED

The C, P, and CP properties of Ps states are shown in Table 2 where n is the principal quantum number, l the orbital angular momentum, j the total angular momentum, and s the total spin. It follows in particular that all Ps states are eigenstates of CP with eigenvalue -1 for singlet states $(s=0)$ and $+1$ for triplet states $(s=1)$.

Table 2. Transformation properties of Ps states

Ps-state	C	P	CP
$n^{2s+1}(l)_j$	$(-1)^{l+s}$	$(-1)^{l+1}$	$(-1)^{s+1}$

The Bohr radius of the Ps system is given by

$$r_b = \frac{2}{m_e \alpha} \simeq 1.06 \cdot 10^{-8}\,\mathrm{cm}. \tag{A.1}$$

The values of the S-wave functions at the origin are

$$\psi_n(0) = \frac{1}{\sqrt{\pi}} (n r_b)^{-3/2}. \tag{A.2}$$

For the radial part of the P-wave function we have

$$\left.\frac{\partial}{\partial r} \varphi_n(r)\right|_{r=0} = \frac{2}{3}\left(1 - \frac{1}{n^2}\right)^{1/2} (n r_b)^{-3/2} r_b^{-1}. \tag{A.3}$$

A general Ps-state having $j=1$ is described by a density matrix

$$\begin{aligned} \varrho_{ij} &= \frac{1}{3}\delta_{ij} + \frac{1}{2i}\varepsilon_{ijk}s_k - s_{ij} \\ s_{ij} &= s_{ji}, \quad s_{ii} = 0 \\ (i,j &= 1,2,3). \end{aligned} \tag{A.4}$$

For a pure state with polarization vector $\boldsymbol{\lambda}$ we have for instance

$$\begin{aligned} \varrho_{ij} &= \lambda_i \lambda_j^* \\ \mathbf{s} &= i\boldsymbol{\lambda} \times \boldsymbol{\lambda}^*. \end{aligned} \tag{A.5}$$

Appendix B

Calculation of Parity Mixing Between Ps States

In this appendix we will calculate the mixing between Ps states of different parity due to the Hamiltonian H_{PV} (2.2). As noted in Sect. 2, triplet and singlet Ps states do not mix with each other since H_{PV} conserves CP. Only states with the same total angular momentum j can be mixed. But for singlet states the parity is fixed by the total angular momentum. Therefore, H_{PV}

cannot induce any parity mixing for singlet states at all.

To discuss the mixing of triplet states, we write the electron fields in H_{PV} (2.2) as sum of the creation part $e_+(x)$ and annihilation part $e_-(x)$

$$e(x)=e_+(x)+e_-(x). \tag{B.1}$$

Retaining only those parts in H_{PV} (2.2) containing two creation and two annihilation operators, we obtain after a Fierz rearrangement

$$\begin{aligned}H_{PV}=&-\frac{Gg_V}{\sqrt{2}}\int d^3x\{(\bar{e}_+(x)\gamma^\mu e_+(x))(\bar{e}_-(x)\gamma_\mu\gamma_5 e_-(x))\\&+(\bar{e}_+(x)\gamma^\mu\gamma_5 e_+(x))(\bar{e}_-(x)\gamma_\mu e_-(x))\}.\end{aligned} \tag{B.2}$$

Sandwiching H_{PV} (B.2) between Ps states we find that only the vacuum can contribute as intermediate state in the product of currents

$$\begin{aligned}\langle \mathrm{Ps}'|H_{PV}|\mathrm{Ps}\rangle =& -\frac{Gg_V}{\sqrt{2}}\int d^3x\\&\cdot\{\langle \mathrm{Ps}'|\bar{e}_+\gamma^\mu e_+|0\rangle\langle 0|\bar{e}_-\gamma_\mu\gamma_5 e_-|\mathrm{Ps}\rangle\\&+\langle \mathrm{Ps}'|\bar{e}_+\gamma^\mu\gamma_5 e_+|0\rangle\langle 0|\bar{e}_-\gamma_\mu e_-|\mathrm{Ps}\rangle\}.\end{aligned} \tag{B.3}$$

The time components of the currents can only mix Ps states with $j=0$, but the only $j=0$ triplet Ps states are the states $^3P_0$, and they have fixed parity (Table 2). The space components of the current will mix only $j=1$ states, i.e., the states $^3S_1$ and $^3D_1$ with $^3P_1$. In the nonrelativistic approximation the wave function of the D state vanishes at the origin, which leads to a vanishing matrix element in (B.3). We are, therefore, left with the mixing of the $^3S_1$ and $^3P_1$ states*. After some algebra we find

$$\begin{aligned}&\langle n'^3P_1,\lambda|H_{PV}|n^3S_1,\lambda\rangle\\&=-i\frac{Gg_V}{m_e\sqrt{\pi}}\Psi_n(0)\frac{\partial}{\partial r}\varphi_{n'}(r)\bigg|_{r=0}.\end{aligned} \tag{B.4}$$

The admixture of $^3P_1$ states to $^3S_1$ states is then calculated as follows, where we denote by $^3\tilde{S}_1$ the perturbed states:

$$\begin{aligned}&|n^3S_1\rangle\to|n^3\tilde{S}_1\rangle=|n^3S_1\rangle+\sum_{n'=2}^{\infty}i\delta_{n',n}|n'^3P_1\rangle\\&i\delta_{n',n}=\frac{\langle n'^3P_1|H_{PV}|n^3S_1\rangle}{E_{n^3S_1}-E_{n'^3P_1}-\frac{i}{2}(\Gamma_{n^3S_1}-\Gamma_{n'^3P_1})},\end{aligned} \tag{B.5}$$

where Γ denotes the total decay rate of the corresponding states. Using (B.4) and the formulae cited in Appendix A (2.3) and (2.5) are easily obtained.

* Note that states with $j>1$ can obtain opposite parity admixtures by radiative corrections to H_{PV}

Appendix C

CP Conservation and Parity-violating Correlations

In this appendix we will prove that CP conservation implies absence of parity-violating correlations in the reactions (2.14)–(2.16). Let us for instance write down the reaction (2.15) in the c.m. system in more detail by indicating momenta and polarizations.

$$\begin{aligned}&^{2s+1}\mathrm{Ps}(j,m)\to{}^{2s'+1}\mathrm{Ps}(j',m',\mathbf{p}')\\&\quad+\gamma(\mathbf{k}_1,\boldsymbol{\varepsilon}_1)+\ldots+\gamma(\mathbf{k}_n,\boldsymbol{\varepsilon}_n)\\&\quad-j\leqq m\leqq j;\quad -j'\leqq m'\leqq j'.\end{aligned} \tag{C.1}$$

The amplitude for this reaction will be of the form

$$\begin{aligned}&\varepsilon^*_{1,\alpha_1}\ldots\varepsilon^*_{n,\alpha_n}\mathscr{A}^{m',m}_{\alpha_1,\ldots,\alpha_n}(\mathbf{k}_1,\ldots,\mathbf{k}_n)\\&(\alpha_i=1,2,3).\end{aligned} \tag{C.2}$$

Let us denote the indices $m', m, \alpha_1, \ldots, \alpha_n$ collectively by η. In the case of CP conservation the amplitude $\mathscr{A}$ must satisfy the relation

$$\mathscr{A}_\eta(\mathbf{k}_i)=(-1)^{s+1}(-1)^{s'+1}\mathscr{A}_\eta(-\mathbf{k}_i). \tag{C.3}$$

By definition a parity-odd correlation is a matrix function

$$C_{\eta,\eta'}(\mathbf{k}_i) \tag{C.4}$$

satisfying the relation

$$C_{\eta,\eta'}(-\mathbf{k}_i)=-C_{\eta,\eta'}(\mathbf{k}_i). \tag{C.5}$$

Due to (C.3) the expectation value of any such correlation must be zero

$$\langle C\rangle=\int\prod_{i=1}^{n}d^3k_i C_{\eta,\eta'}(\mathbf{k}_i)\mathscr{A}^*_\eta(\mathbf{k}_i)\mathscr{A}_{\eta'}(\mathbf{k}_i)=0. \tag{C.6}$$

This and a completely analogous argument for the reaction (2.16) proves our assertion.

Appendix D

The Decay of Polarized $^3S_1$ Ps into three Photons

In this appendix we consider the decay

$$n^3S_1(\lambda)\to\gamma(\mathbf{k}_1,\boldsymbol{\varepsilon}_1)+\gamma(\mathbf{k}_2,\boldsymbol{\varepsilon}_2)+\gamma(\mathbf{k}_3,\boldsymbol{\varepsilon}_3), \tag{D.1}$$

where we have indicated polarization vectors and momenta in the rest system of the decaying Ps state. We will choose the labeling of the photon momenta such that

$$|\mathbf{k}_1|\geqq|\mathbf{k}_2|\geqq|\mathbf{k}_3| \tag{D.2}$$

and define the normal to the 3γ-plane by

$$\mathbf{n}=\frac{\mathbf{k}_1\times\mathbf{k}_2}{|\mathbf{k}_1\times\mathbf{k}_2|} \tag{D.3}$$

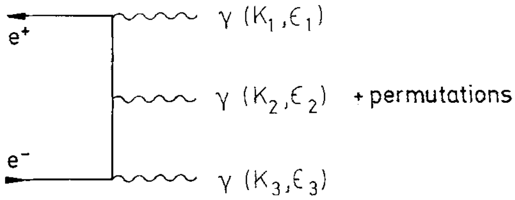

Fig. 6. Diagrams for the decay of Ps into 3 photons

We will calculate the amplitudes for reaction (D.1) for arbitrary polarization states of the photons and give the distribution of the normal $\mathbf{n}$ with respect to the polarization vector of the Ps. Such distributions may be useful for experimentalists when dealing with polarized orthopositronium. To the best of our knowledge these distributions have not been given explicitly before in the literature, although some special cases can be extracted from analogous calculations for heavy quarkonia [32].

We compute the relevant diagrams, shown in Fig. 6, in the nonrelativistic approximation. The decay amplitude for arbitrary photon polarization is then as follows

$$\langle \gamma(\mathbf{k}_1,\boldsymbol{\varepsilon}_1),\gamma(\mathbf{k}_2,\boldsymbol{\varepsilon}_2),\gamma(\mathbf{k}_3,\varepsilon_3)|S|n^3S_1(\boldsymbol{\lambda})\rangle$$

$$= -ie^3\frac{\Psi_n(0)}{\sqrt{24m_e^2}\sqrt{8\omega_1\omega_2\omega_3}}\frac{1}{V^2}(2\pi)^4\delta^{(3)}(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)$$

$$\cdot\delta(2m_e-\omega_1-\omega_2-\omega_3)C(\boldsymbol{\lambda}',\boldsymbol{\varepsilon}_i^*), \tag{D.4}$$

where ω_i are the energies of the photons, $\Psi_n(0)$ is the wave function at the origin (A.2), and V is the normalization volume. The amplitude $C(\boldsymbol{\lambda},\boldsymbol{\varepsilon}_i^*)$ is given by $(\hat{\mathbf{k}}_i=\mathbf{k}_i/|\mathbf{k}_i)$

$$\begin{aligned}C(\boldsymbol{\lambda},\boldsymbol{\varepsilon}_i^*)=4\{&(\boldsymbol{\varepsilon}_1^*\cdot\boldsymbol{\varepsilon}_2^*)(\boldsymbol{\varepsilon}_3^*\cdot\boldsymbol{\lambda})\\ &-(\hat{\mathbf{k}}_1\times\boldsymbol{\varepsilon}_1)\cdot(\hat{\mathbf{k}}_2\times\boldsymbol{\varepsilon}_2^*)(\boldsymbol{\varepsilon}_3^*\cdot\boldsymbol{\lambda})\\ &+(\hat{\mathbf{k}}_1\times\boldsymbol{\varepsilon}_1^*)\cdot\boldsymbol{\varepsilon}_2^*(\hat{\mathbf{k}}_3\times\boldsymbol{\varepsilon}_3^*)\cdot\boldsymbol{\lambda}\\ &+\boldsymbol{\varepsilon}_1^*\cdot(\hat{\mathbf{k}}_2\times\boldsymbol{\varepsilon}_2^*)(\hat{\mathbf{k}}_3\times\boldsymbol{\varepsilon}_3^*)\cdot\boldsymbol{\lambda}\\ &+\text{cyclic perm}\}.\end{aligned} \tag{D.5}$$

We have listed this quantity for a complete set of linearly polarized photon states in Table 3. We define polarization vectors parallel (P) and transverse (T) with respect to the 3γ-plane as follows

$$\begin{aligned}\boldsymbol{\varepsilon}_{i,P}&=\mathbf{n}\times\hat{\mathbf{k}}_i\\ \boldsymbol{\varepsilon}_{i,T}&=\mathbf{n}.\end{aligned} \tag{D.6}$$

Summing over all photon polarizations we find for the correlation between $\mathbf{n}$ and $\boldsymbol{\lambda}$:

$$\frac{1}{\Gamma}\frac{d\Gamma}{d\Omega_n}=\frac{3}{16\pi}(1+|\mathbf{n}\cdot\boldsymbol{\lambda}|^2), \tag{D.7}$$

where Γ is the total 3γ-decay rate of unpolarized triplet Ps and is given by the well-known Ore-Powell formula [33].

It is worth noting that the angular distribution (D.7) stays the same for arbitrary cuts made in the photon energies, at least if radiative corrections are disregarded as we have done here.

Finally in Table 4 the angular distributions following from (D.7) are listed for the Ps states in the spherical basis with magnetic quantum numbers $m=0$, ± 1.

Table 3. The matrix element $C(\boldsymbol{\lambda},\boldsymbol{\varepsilon}_i^*)$ for a complete set of photon polarizations. The combinations TPT and PTT (PTP and TPP) are obtained by cyclic permutations from TTP (PPT)

$\varepsilon_1,\varepsilon_2,\varepsilon_3$	$C(\boldsymbol{\lambda},\varepsilon_i)$
TTT	$4(\mathbf{n}\cdot\boldsymbol{\lambda})[3-\hat{\mathbf{k}}_1\hat{\mathbf{k}}_2-\hat{\mathbf{k}}_2\hat{\mathbf{k}}_3-\hat{\mathbf{k}}_3\hat{\mathbf{k}}_1]$ $=\frac{16m_e}{\omega_1\omega_2\omega_3}\cdot(\mathbf{n}\cdot\boldsymbol{\lambda})[\omega_1\omega_2+\omega_2\omega_3+\omega_3\omega_1-m_e^2]$
PPP	$-4(\boldsymbol{\lambda}\times\mathbf{n})\cdot[(1-\hat{\mathbf{k}}_2\cdot\hat{\mathbf{k}}_3)\hat{\mathbf{k}}_1+(1-\hat{\mathbf{k}}_3\cdot\hat{\mathbf{k}}_1)\hat{\mathbf{k}}_2+(1-\hat{\mathbf{k}}_1\cdot\hat{\mathbf{k}}_2)\hat{\mathbf{k}}_3]$ $=\frac{8m_e}{\omega_1\omega_2\omega_3}(\boldsymbol{\lambda}\times\mathbf{n})\cdot(\omega_1\mathbf{k}_1+\omega_2\mathbf{k}_2+\omega_3\mathbf{k}_3)$
TTP	$4(\boldsymbol{\lambda}\times\mathbf{n})[(1-\hat{\mathbf{k}}_1\hat{\mathbf{k}}_2)\hat{\mathbf{k}}_3-(1-\hat{\mathbf{k}}_2\hat{\mathbf{k}}_3)\hat{\mathbf{k}}_1-(1-\hat{\mathbf{k}}_1\hat{\mathbf{k}}_3)\hat{\mathbf{k}}_2]$ $=-\frac{8m_e}{\omega_1\omega_2\omega_3}(\lambda\times\mathbf{n})\cdot(\omega_2\mathbf{k}_1+\omega_1\mathbf{k}_2)$
PPT	$4(\mathbf{n}\cdot\boldsymbol{\lambda})[1+\hat{\mathbf{k}}_1\hat{\mathbf{k}}_2-\hat{\mathbf{k}}_2\hat{\mathbf{k}}_3-\hat{\mathbf{k}}_1\hat{\mathbf{k}}_3]$ $=\frac{16m_e}{\omega_1\omega_2\omega_3}(\mathbf{n}\cdot\boldsymbol{\lambda})[-m_e^2+m_e\omega_3+\omega_1\omega_2]$

Table 4. The angular distributions of $\mathbf{n}$ with respect to the spin-quantization axis $\mathbf{e}_3\cdot(\cos\vartheta_n=\mathbf{n}\cdot\mathbf{e}_3)$

m	$\boldsymbol{\lambda}$	$\frac{16\pi}{3}\frac{1}{\Gamma}\frac{d\Gamma}{d\Omega^n}$
+1	$-\frac{1}{\sqrt{2}}(\mathbf{e}_1+i\mathbf{e}_2)$	$\frac{1}{2}(3-\cos^2\vartheta^n)$
0	$\mathbf{e}_3$	$1+\cos^2\vartheta^n$
−1	$\frac{1}{\sqrt{2}}(\mathbf{e}_1-i\mathbf{e}_2)$	$\frac{1}{2}(3-\cos^2\vartheta^n)$

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