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Computer Calculation of Two Target Trajectory with "Centre of Arc-Target" Stereotaxic Equipment

By

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With 7 Figures

In a previous paper (*Peluso* and *Gybels*, in press) we have developed the general principles of a method which makes it possible in stereotaxic surgery to reach target via a second, predefined point. This allows us to use exactly the same trajectory in a given group of patients, to make two lesions at a distance from each other with a single electrode penetration, and to obtain easily physiological information from a predefined structure which is not the target. In this paper, we will be concerned with the problem how to direct the electrode in order to follow a two target trajectory when using stereotaxic equipment which is constructed in such a way that the target coincides with the centre of the arc which carries the electrode holder. The U.T.E.C. stereotaxic instrument (*Dereymaeker* and *De Dobbeleer* 1959), which we have been using the last ten years and many other apparatus are based on this principle.

Referring to Fig. 1, we will have to calculate two angles δ and ν; the value of the two angles will of course depend on the coordinates of the chosen point and target. These coordinates can only be measured on the lateral and A.P. radiographs, while δ and ν are measured on the electrode carrier arc. This arc's position will change from patient to patient while the direction of the electrode with respect to the brain will have to be constant for a chosen point and target. As we are using polar projection one knows that lengths measured on the X-ray pictures are magnified with respect to the real lengths. So we are obliged to introduce two different coordinate systems: one system on the pictures, named x'', y'' and z'' (Fig. 2), which is the same as the system of *Talairach* et al. (1957) and another system in the patient's brain, named u'', v'' and w'' (Fig. 3). In the mathematics to be developed we will transform (x'', y'', z'') into (x', y', z') by a translation over x_0'', y_0'' and z_0'' and then into (x, y, z) by a rotation over α degrees (Fig. 2). The latter coordinate system has two horizontal axes: x and z, and one vertical: y. Of course the same holds for (u'', v'', w''), (u', v', w') and (u, v, w) Fig. 3. In referring to the two above mentioned

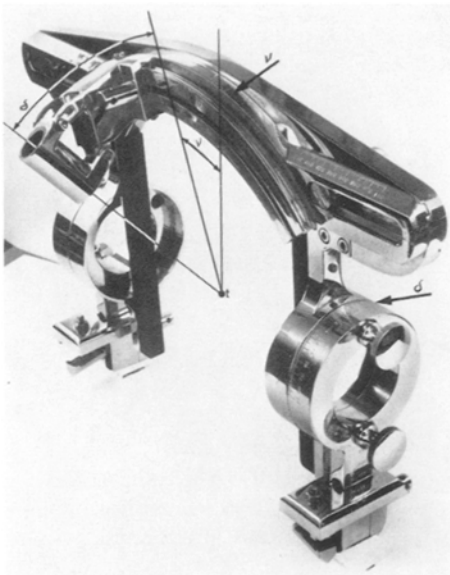

Fig. 1. The electrodeholder arc. The two possible movements of the electrode are shown.

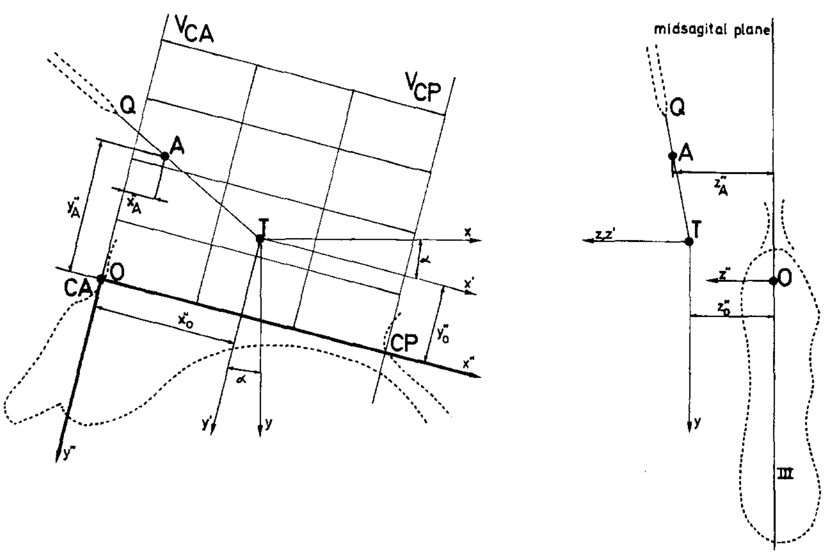

Fig. 2. Lateral and A.P. X-ray pictures. x is horizontal, y is vertical and z is perpendicular to the plane (x, y). Only the necessary axes are drawn. T is target and A is the second point to be reached.

different coordinate systems we also will call A and T the chosen points on the X-ray films and a and t the real points in the patients' brain (Fig. 4). Since x''_Q, y''_Q, z''_Q, x_0'', y_0'' and z_0'' are measured on the X-ray pictures the coordinates of the electrode tip P and the target t can be found as follows (Fig. 3):

$$u''_P = \frac{b}{c} \cdot x''_Q \qquad u_0'' = \frac{b}{c} \cdot x_0''$$

$$v''_P = \frac{b}{c} \cdot y''_Q \qquad \text{and} \qquad v_0'' = \frac{b}{c} \cdot y_0'' \tag{1}$$

$$w''_P = \frac{e}{f} \cdot z''_Q \qquad w_0'' = \frac{e}{f} \cdot z_0''.$$

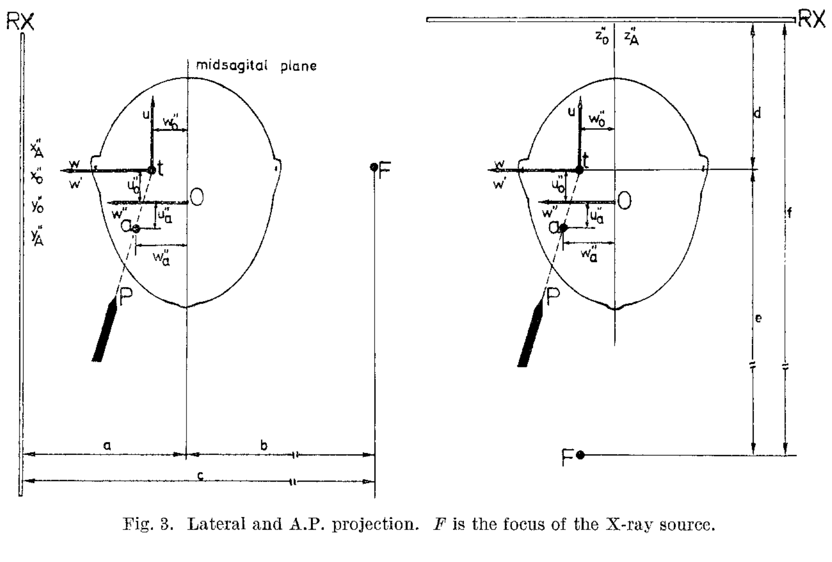

Fig. 3. Lateral and A.P. projection. F is the focus of the X-ray source.

The mathematical development may now start. Referring to Fig. 2, 3 and 4, the above mentioned translation may be written as:

$$\begin{aligned} u'_P &= u''_P - u_0'' \\ v'_P &= v''_P - v_0'' \\ w'_P &= w''_P - w_0''. \end{aligned} \tag{2}$$

and the rotation around the w axis:

$$\begin{aligned} u_P &= u'_P \cos\alpha + v'_P \sin\alpha \\ v_P &= -u'_P \sin\alpha + v'_P \cos\alpha \\ w_P &= w'_P. \end{aligned} \tag{3}$$

The electrode point P will change while the electrode penetrates the brain. Also referring to Fig. 5 we may write:

$$u_P = -p \sin\beta \cos\gamma$$
$$v_P = p \cos\beta \qquad (4)$$
$$w_P = p \sin\beta \sin\gamma.$$

The statements (1), (2) and (3) give us permanent information about the situation of the electrode point, from each distance from target, when it is

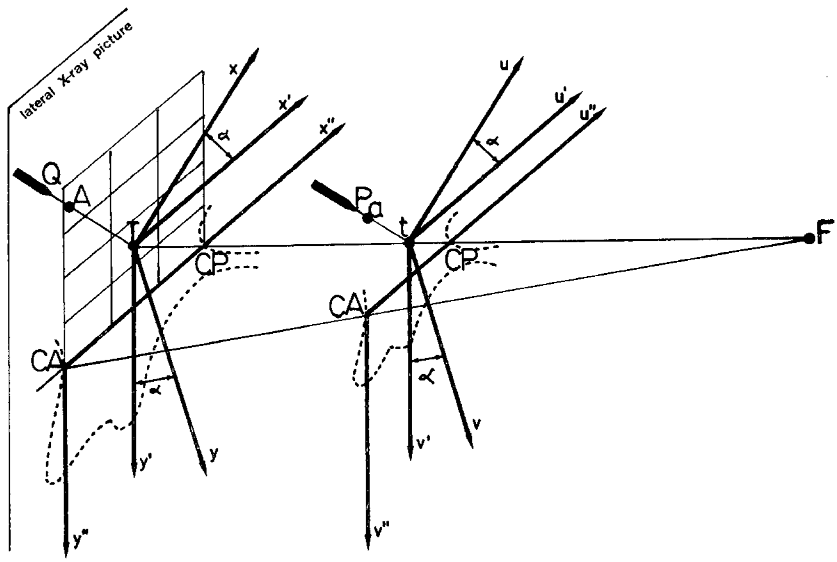

Fig. 4. Representation of the different sets of coordinate systems. (x, y, z), (x', y', z'), and (x'', y'', z'') on the X-ray picture and (u, v, w), (u', v', w') and (u'', v'', w'') in brain of patient. F is the focus of the X-ray source.

in the brain. But until now we only are concerned with a special case of point P, namely point a. Thus we interchange P by a but for simplicity we omit writing "a" as index.

So, referring to Fig. 6, we may write that:

$$\tan\delta = \frac{DP'}{PP'} \qquad (5)$$

δ being the inclination around the horizontal axis w.
Further:

$$DP' = tP' \cdot \sin\gamma' = -p \sin\beta \sin\gamma'$$

or

$$DP' = -p \sin\beta \cos\gamma$$

because

$$\gamma' = \frac{\pi}{2} - \gamma.$$

Also
$$PP' = -p \cos \beta. \tag{6}$$
So statement (5) gives:
$$\tan \delta = \frac{\sin \beta \cos \gamma}{\cos \beta} = \tan \beta \cos \gamma \tag{7}$$

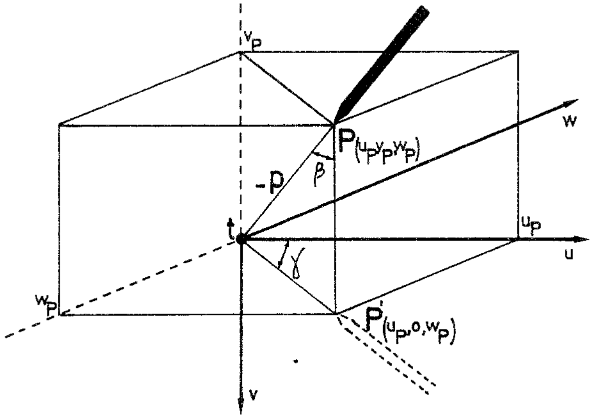

Fig. 5. Representation of the angles β and γ. *P* is the electrode tip and *p* a parameter going from — 30 mm to + 10 mm.

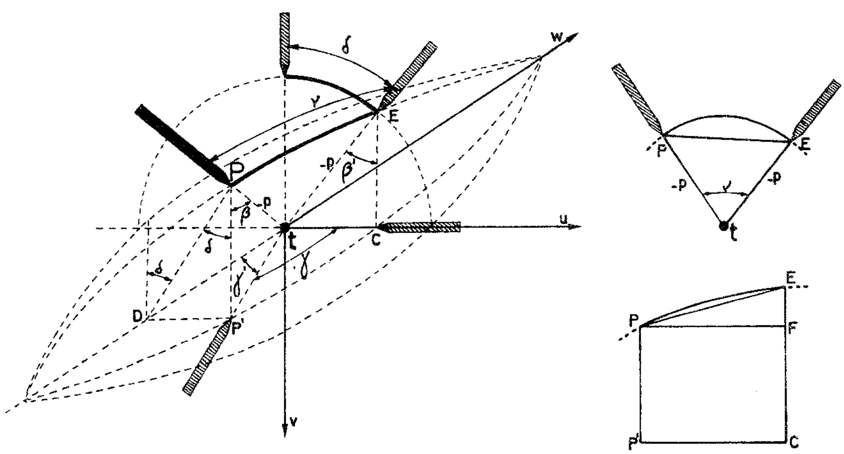

Fig. 6. In this figures the possible movements of the electrode are shown: on the A.P. circle δ is measured and on the sagittal circle ν is measured.

or
$$\boxed{\delta = \arctan\,(\tan \beta \cos \gamma)}\,. \tag{8}$$

The calculation of the inclination around the axis *u*, or the A.P. axis, thus the angle ν, is longer but not more difficult. Applying trigonometric rules in Fig. 6

$$PE^2 = p^2 + p^2 - 2\,p^2 \cos \nu$$

there follows:

$$\cos \nu = 1 - \frac{PE^2}{2p^2}. \tag{9}$$

Because PE is unknown the development is not finished yet.
From Fig. 6 we may also write:

$$PE^2 = EF^2 + PF^2$$

$$EF = EC - FC = EC - PP' = p\,(\cos \beta - \cos \beta')$$

since $\beta' = \delta$.

Thus

$$EF^2 = p^2\,(\cos \beta - \cos \delta)^2. \tag{10}$$

The same trigonometric rules applied in Fig. 6 give:

$$P'C^2 = P't^2 + tC^2 - 2P't \cdot tC \cdot \cos \gamma = PF^2$$

or

$$PF^2 = p^2 \sin^2 \beta + p^2 \sin^2 \delta - 2\,p^2 \sin \beta \sin \delta \cos \gamma \tag{11}$$

and

$$PE^2 = p^2\,(\cos \beta - \cos \delta)^2 + p^2\,(\sin^2 \beta + \sin^2 \delta) - 2\,p^2 \sin \beta \sin \delta \cos \gamma.$$

Since

$$\sin^2 \varphi + \cos^2 \varphi = 1$$

there follows:

$$PE^2 = -2\,p^2 \cos \beta \cos \delta + 2\,p^2 - 2\,p^2 \sin \beta \sin \delta \cos \gamma$$

and finally because of (9)

$$\cos \nu = (1 + \tan \beta \tan \delta \cos \gamma) \cos \beta \cos \delta$$

or

$$\cos \nu = (1 + \tan^2 \delta) \cos \beta \cos \delta \tag{12}$$

because of (7).

Thus

$$\boxed{\nu = \text{arc cos}\,[(1 + \tan^2 \delta) \cos \beta \cos \delta]}. \tag{13}$$

The solution of our problem is only complete if β and γ are known. This is not difficult.

Indeed, referring to Fig. 5 and statement (4) we find

$$\tan \gamma = -\frac{w}{u} \tag{14}$$

or

$$\gamma = \arctan \left(-\frac{w}{u}\right)$$

and

$$\tan \beta = \frac{-u}{v \cos \gamma}$$

or

$$\beta = \arctan \left(\frac{-u}{v \cos \gamma}\right). \tag{15}$$

The distance between t and P in position a is:

$$\boxed{p_a = \frac{v}{\cos\beta}} \,. \tag{16}$$

Finally, we want to point out that all calculations are processed by a computer, supplied with a program written in Fortran IV (*Peluso* and

P3 CALCULATION OF TWO TARGET TRAJECTORY

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XO	YO	ZO	X2	Y2	Z2	CAP	A	B	D	E
12.9	-2.9	-19.7	10.3	-25.7	-22.5	27.0	210	1170	320	1120

ALPHA	BETA	GAMMA	DELTA	NU	P2	CAPP
5 0	13 14	150 27	-11 34	6 29	-19.3	114.5

##########################

P	P1	U25	V25	W22	U2	V2	W2	X2	Y2	Z2
-30.00	30.00	36.92	-160.20	-18.71	7.38	-32.04	-18.71	8.85	-38.40	-24.06
≈ ↓										
-2.00	2.00	52.83	-22.00	-15.55	10.57	-4.40	-15.55	12.63	-5.26	-19.99
-1.50	1.50	53.12	-19.54	-15.49	10.62	-3.91	-15.49	12.70	-4.67	-19.92
-1.00	1.00	53.40	-17.07	-15.44	10.68	-3.41	-15.44	12.77	-4.08	-19.85
-0.50	0.50	53.68	-14.60	-15.38	10.74	-2.92	-15.38	12.83	-3.49	-19.77
0.00	0.00	53.97	-12.13	-15.32	10.79	-2.43	-15.32	12.90	-2.90	-19.70
0.50	0.50	54.25	-9.66	-15.27	10.85	-1.93	-15.27	12.97	-2.31	-19.63
1.00	1.00	54.54	-7.20	-15.21	10.91	-1.44	-15.21	13.03	-1.72	-19.55
1.50	1.50	54.82	-4.73	-15.15	10.96	-0.95	-15.15	13.10	-1.13	-19.48
2.00	2.00	55.11	-2.26	-15.10	11.02	-0.45	-15.10	13.17	-0.54	-19.41
≈ ↓										
10.00	10.00	59.65	37.22	-14.19	11.93	7.44	-14.19	14.24	8.89	-18.25
P	P1	U25	V25	W22	U2	V2	W2	X2	Y2	Z2

Fig. 7. Computed coordinates of electrode tip during a frontal penetration. (*X 0, Y 0, Z 0*), (*X 2, Y 2, Z 2*) and (*U 2, V 2, W 2*) correspond to (x_0'', y_0'', z_0''), (x_Q'', y_Q'', z_Q'') and (u_P'', v_P'', w_P'') of the text. CAP, CAPP and (*U 25, V 25, W 22*), which are not mentioned in the paper, are the coordinates of the electrode tip, during the penetration related to the projection of sagittal anatomical slides.

Gybels, in press). Only eleven variables x_0'', y_0'', z_0'', x_A'', y_A'', z_A'', a, b, d, e and α have to be measured and punched on one card. As results are given the four angles β, γ, δ, and ν, the distance p_a between target and the chosen point a and the coordinates u_P'', v_P'', w_P'', x_P'', y_P'', z_P'' for p going from 30 mm before target to 10 mm behind target in steps of 0.5 mm (Fig. 7). The results are immediately available.

Acknowledgments

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Summary

Computer Calculation of Two Target Trajectory with „Centre of Arc-Target" Stereotaxic Equipment

A method is described permitting computer calculation of a two target trajectory with "centre of arc = target" stereotaxic equipment.

Zusammenfassung

Computerberechnung einer Zweipunkte-Elektrodenführung mit einer stereotaktischen Einrichtung, die den Zielpunkt als Mittelpunkt eines Bogens ermittelt

Es wird eine Methode beschrieben, die eine Computerberechnung einer Zweipunkte-Elektrodenführung ermöglicht, unter Verwendung einer stereotaktischen Einrichtung, die den Zielpunkt als Mittelpunkt eines Bogens ermittelt.

Résumé

Calcul par ordinateur d'une trajectoire à 2 cibles avec équipement stéréotaxique dans lequel le centre de l'arc — la cible

Une méthode est décrite permettant le calcul par ordinateur d'une trajectoire à 2 cibles utilisant un appareil stéréotaxique où le centre de l'arc coincide avec la cible.

Riassunto

Calcolo con computer di una traiettoria a due targets con attrezzatura stereotassica con target-centro dell'arco

Gli autori descrivono un metodo che permette di calcolare con un computer una traiettoria a due targets con attrezzatura stereotassica della quale target é il centro dell'arco.

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