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Natural convection in rectangular enclosures with partial heating and cooling

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Abstract The flow and heat transfer in partially heated and partially cooled cavities were numerically analyzed. Using the control volume approach, a computer program based on SIMPLE algorithm was developed. A square enclosure with variable size heater and cooler on the vertical walls was considered. Computations were carried out to investigate the effects of heater and cooler size on the heat transfer rate. It was observed that for a given cooler size, the mean Nusselt number decreases with increasing heater size. On the other hand, for a given heater size, the mean Nusselt number increases with increasing cooler size. For all Rayleigh numbers considered, the same behavior was observed.

Natürliche Konvektion in geschlossenen Räumen mit partieller Heizung und Kühlung der Wände

Zusammenfassung Es wurde ein numerisches Modell zur Analyse des Strömungs- und Wärmeübergangsverhaltens in teilweise beheizten und gekühlten Hohlräumen entwickelt und unter Verwendung des Kontrollvolumenprinzips und des Algorithmus „SIMPLE" als Computer-Programm formuliert. Der Hohlraum ist rechteckig und die variablen Heiz- und Kühlflächen befinden sich auf gegenüberliegenden Vertikalseiten. Hauptziel der Berechnungen war es, den Einfluß der variablen Heiz- und Kühlflächen auf den Wärmeübergang zu ermitteln. Für eine bestimmte Kühlergröße zeigte sich eine Abnahme der gemittelten Nußelt-Zahl mit zunehmender Heizfläche. Andererseits – bei gegebener Heizfläche – stieg die Nußelt-Zahl mit der Kühlfläche an. Dieses Verhalten wurde bei allen untersuchten Rayleighzahlen gefunden.

Nomenclature

g	gravitational acceleration
H	height of cavity
k	thermal conductivity of fluid
l_C	cooler size
l_H	heater size
$\overline{Nu}$	mean Nusselt number
Nu_y	local Nusselt number
P	pressure
Pr	Prandtl number
Ra	Rayleigh number ($Ra = g\beta H^3 (T_H - T_C)/(\alpha\nu)$)
T	temperature
T_C	temperature of cold surface
T_H	temperature of hot surface
T_R	reference temperature ($T_R = (T_C + T_H)/2$)
u	velocity component in x-direction
x	horizontal axis
v	velocity component in y-direction
y	vertical axis

Greek symbols

α	thermal diffusivity
β	thermal expansion coefficient
ρ	density of fluid
Ψ	stream function
ν	kinematic viscosity

1 Introduction

Natural convection flows play an important role in many engineering applications. Some examples are solar collectors, building heating, insulation with double pane window, cooling of electronic equipments, cooling of nuclear reactors, etc. Due to these wide applications, natural convection problems have been studied by different researchers. Natural convection problems also have been a subject for the studies whose objective was to develop numerical methods for solution of partial differential equations governing the conservation of mass, momentum and energy. Problems involving natural convection can broadly be classified into two types: Enclosures heated from side and enclosures heated from below.

Natural convection in enclosures is a complex phenomenon. For confined natural convection, a boundary layer forms near the walls. The region exterior to it will form a circulating core. Since the core region is encircled by the boundary layer, the boundary layer region can not be considered to be independent of the core. Therefore, the boundary layer and the core are closely coupled to each other. This coupling constitutes the main difficulty in obtaining analytic solutions. Hence, internal natural convection phenomena are mostly analyzed by numerical or experimental techniques.

Many researches have been performed on the natural convection in enclosures, which are heated from side or heated from below. Natural convection in enclosures with isothermal vertical walls has been studied from different point of views. Analytical methods have been developed to investigate the effects of aspect ratio on the heat transfer rate (wall-to-wall). Gill [1] proposed a

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boundary layer type method to investigate the heat transfer mechanism. He envisioned a boundary layer type flow regime in which the fluid motion is confined to layers near the vertical surfaces, leaving the fluid in the core stagnant and vertically stratified. It should be pointed out that the core is not truly stagnant and stratified. The overall Nusselt number predicted by Gill shows good agreement with the experimental results for cavities with high Rayleigh number. Bejan [2] extended Gill's theory to account for the effects of end walls. With this modification, the theory predicted better results. Cormack et al. [3] and Bejan and Tien [4] constructed models for shallow cavities. A more complete review of the analytical solution methods for natural convection in enclosures was given by Bejan [5]. In parallel with the analytical studies, numerical and experimental studies have also been carried out [6, 7]. Novak and Nowak [8] numerically analyzed the natural convection in rectangular enclosure to investigate effect of the distance between the vertical walls on the heat transfer rate. As a result of this study, they proposed an optimum gap width for double pane windows. Vertical cavities with isothermal vertical walls have also been studied, among others, by Dropkin and Sommerscale [9], Eckert and Carlson [10], Emery and Chu [11], Korpela et al. [12] and Lee and Korpela [13].

Natural convection in differentially heated corner region has been studied by Kimura and Bejan [14], November and Nansteel [15]. Such circumstances arises for example when solar radiation passing through a large window is incident on the floor which receives the radiation attains a temperature relatively greater than that of the cool window surfaces. Circular cylinder cavities heated from below was studied by Greenspan and Schultz [16], Torrance and Rockett [17].

In most of the studies found in the literature, whole vertical walls were considered to be isothermal or whole wall is exposed to a heat flux. However, in many engineering applications, heating and/or cooling take place over a narrow segment of the vertical walls. In such cases, the size and location of the heater and cooler play an important role on the fluid flow and heat transfer mechanisms. Hence, optimum heater and cooler size and location should be determined for better utilization of such systems. Chu et al. [18] studied effect of heater size and location on the heat transfer rate. In this study, as one of the vertical walls was partially heated, the whole opposite vertical wall was kept at a lower temperature.

In the present study, a square enclosure was considered. An isothermal heating element was located on the left vertical wall. An isothermal cooling element was placed on the right vertical wall. Top and bottom horizontal walls were assumed to be insulated (see Fig. 1). Parts of the vertical walls other than the heater and cooler were also insulated. The size of the heating and cooling elements were considered to be variable. A numerical model was developed to predict flow and temperature fields in the fluid in the enclosure. Computations were performed for different size heaters and coolers, and for different Rayleigh numbers. Using the predicted temperature fields, the mean Nusselt number was calculated. The variation of the mean Nusselt number with the heater and cooler size was investigated. From the predicted velocity fields, values of stream function over the flow domain were also calculated.

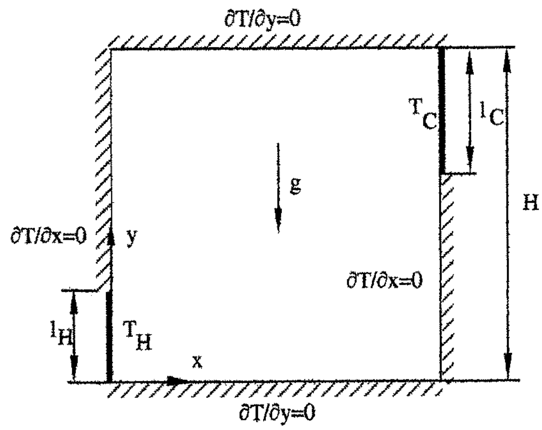

Fig. 1. Geometry and coordinate system of the problem

2 Mathematical formulation

Analysis of the problem under consideration involves solution of fluid motion and energy equations in the Cartesian coordinates with appropriate boundary conditions.

Fluid flow equations

Geometry of the problem and the coordinate system are shown in Fig. 1. The height of the enclosure is denoted by H. Dimension of the enclosure, and the heater and cooler perpendicular to the plane of the diagram are assumed to be long. Hence, the problem can be considered to be two-dimensional. Then, adapting the Boussinesq approximation, the steady state equations of fluid motion are written as,

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial P}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right) \tag{2}$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial P}{\partial y}+g\beta(T-T_R)+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right) \tag{3}$$

where u and v are the velocity components in the x and y direction, respectively. P is the pressure.

The energy equation is written as

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \tag{4}$$

In Eq. (3), β and T_R represent the coefficient of thermal expansion and reference temperature, respectively. Average of the hot and the cold surface temperatures are taken to be the reference temperature.

Equations (1), (2), (3) and (4) are subject to the following boundary conditions:

On the left wall ($x=0$)

$$u=0 \quad \text{for} \quad 0 \le y \le H \tag{5a}$$
$$v=0 \quad \text{for} \quad 0 \le y \le H \tag{5b}$$
$$T=T_H \quad \text{for} \quad 0 \le y \le l_H \tag{5c}$$
$$\partial T/\partial x=0 \quad \text{for} \quad l_H < y \le H \tag{5d}$$

On the right wall ($x=H$)

$$u=0 \quad \text{for} \quad 0 \le y \le H \tag{6a}$$
$$v=0 \quad \text{for} \quad 0 \le y \le H \tag{6b}$$
$$T=T_C \quad \text{for} \quad H-l_C \le y \le H \tag{6c}$$
$$\partial T/\partial x=0 \quad \text{for} \quad 0 \le y < l_C \tag{6d}$$

On the upper wall ($y=H$)

$$u(x,H)=0 \tag{7a}$$
$$v(x,H)=0 \tag{7b}$$
$$\partial T/\partial y=0 \tag{7c}$$

On the bottom wall ($y=0$)

$$u(x,0)=0 \tag{8a}$$
$$v(x,0)=0 \tag{8b}$$
$$\partial T/\partial y=0 \tag{8c}$$

Equations (1), (2), (3) and (4) are solved together with the relevant boundary conditions to determine the velocity and temperature distributions in the enclosure. Using the velocity field, values of the streamlines over the flow domain are calculated. Local Nusselt number is calculated from the temperature distribution as,

$$Nu_y=-\frac{H}{\Delta T}\left(\frac{\partial T}{\partial x}\right)_{x=0} \tag{9}$$

From the local Nusselt number, the mean Nusselt number can be calculated as,

$$\overline{Nu}=\frac{l}{l_H}\int_0^{l_H} Nu_y \, dy \tag{10}$$

3
Numerical solution procedure

To transform the differential equations into algebraic forms, they were integrated over finite control volumes. The control volumes for velocity components were staggered relative to those of the scalar variables. An upwind scheme was employed to approximate the convection and diffusion terms. Using SIMPLE (19) algorithm, a computer program was developed. For the solution of the algebraic equations, Gauss-Seidel point by point iteration technique was employed. For all the grid points, u, v, and P were solved. The computations were terminated when the continuity for each control volume was satisfied within the magnitude of error less than 10^{-3}. In order to obtain grid independent solutions, the computations were carried out with different grid sizes. It was found that the grid size of 32×32 (x, y) is reasonable for accuracy and computational economy.

4
Results and discussion

All the results presented in this section are for $Pr=0.7$ and a partially heated and cooled square cavity. Computations are carried out for three different Rayleigh numbers, i.e. $Ra=10^5$, 5×10^5 and 2×10^4. Top and bottom surfaces are insulated. The sizes of the heater and cooler are taken to be 1/4, 2/4, 3/4 and 4/4 of the cavity height. For all the cases, the heater is located on the left vertical wall from the bottom. The cooling element is placed on the right vertical wall starting from the top. Parts of the vertical walls other than heater and cooler are insulated. The computations are carried out for different combinations of heater and cooler sizes, i.e. the heater size is kept at a predetermined value and the cooler size is varied and vise versa. With similar combinations, 48 cases are simulated.

In order to obtain results independent from the number of grid points, a grid independence study was performed. For a selected case ($Ra=10^5$, $l_H/H=0.25$ and $l_C/H=0.25$), computations were repeated with mesh systems of (28×28), (32×32) and (48×48) (x, y). To choose a mesh system of optimum size, the mean Nusselt number was calculated and plotted against the number of grids in Fig. 2. As seen in this figure, no significant change is observed in the Nusselt number for mesh systems of size greater than (32×32). It was assumed that this case is more sensible to the grid number than the other cases, and hence it can be argued that the grid size satisfactory for this case will be satisfactory for all other cases. Therefore, for all the following computations, the mesh system of size (32×32) was used.

To investigate the effects of the heater size on the flow and heat transfer, the cooler size (i.e. l_C/H) was kept constant and the heater size was varied (i.e., $l_H/H=0.25$, 0.50, 0.75 and 1.0), and computations were repeated. For the case of $l_H/H=0.25$, the streamlines and isotherms are illustrated in Figs. 3 and 4, respectively. In Fig. 3, it is seen that flow patterns do not change much

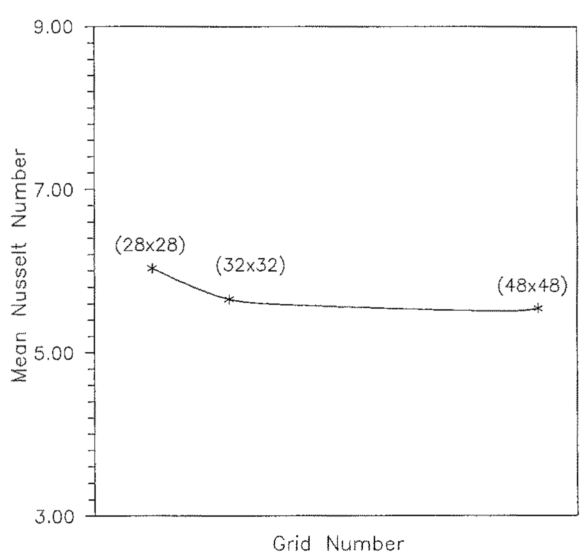

Fig. 2. Variation of Nusselt number with grid number ($Ra=10^5$, $l_H/H=0.25$, $l_C/H=0.25$)

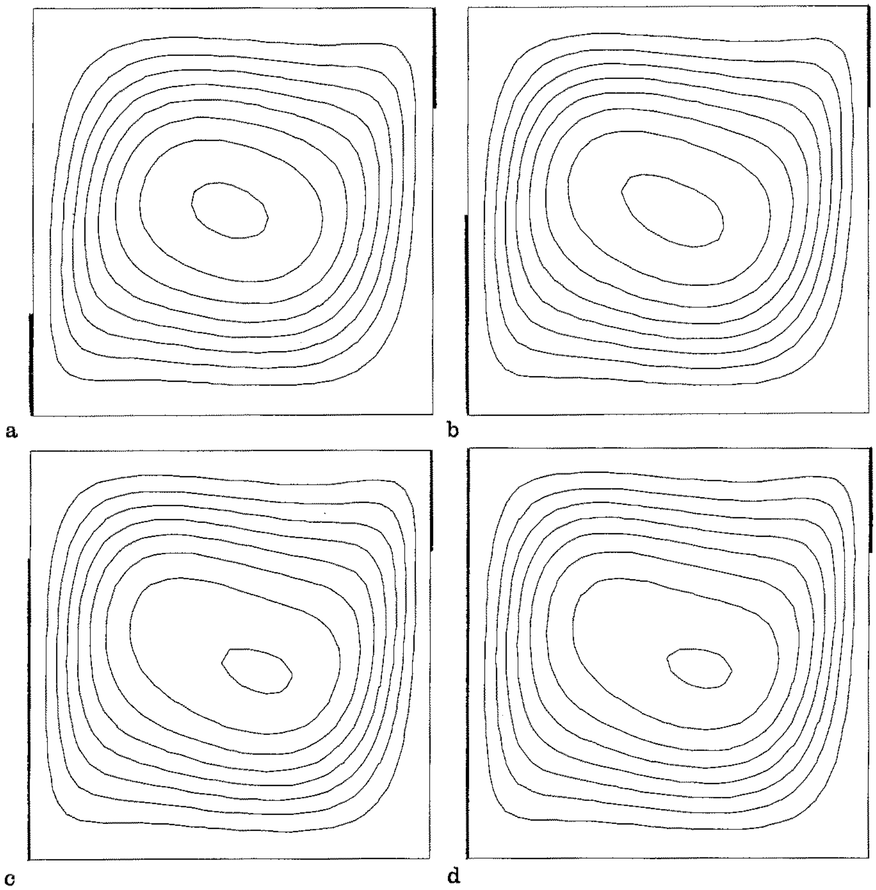

Fig. 3 a – d. Streamline contours ($Ra = 10^5$, $l_C/H = 0.25$). **a** $l_H/H = 0.25$; **b** $l_H/H = 0.50$; **c** $l_H/H = 0.75$; **d** $l_H/H = 1.0$

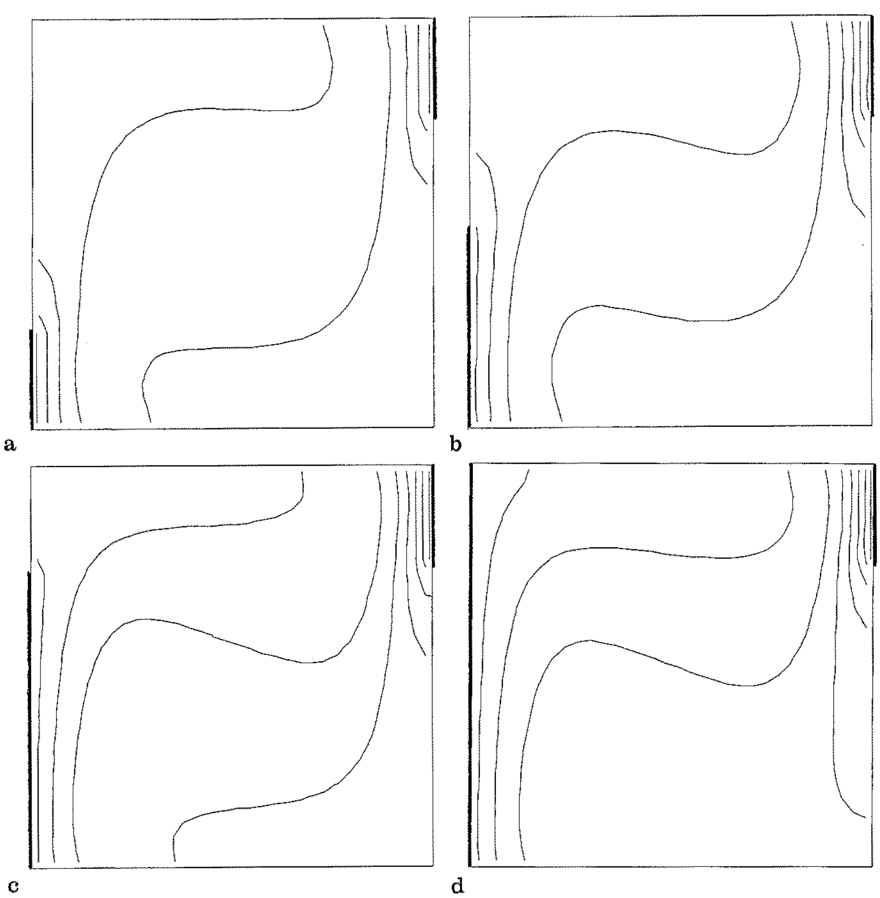

Fig. 4 a – d. Isotherm contours ($Ra = 10^5$, $l_C/H = 0.25$). **a** $l_H/H = 0.25$; **b** $l_H/H = 0.50$; **c** $l_H/H = 0.75$; **d** $l_H/H = 1.0$

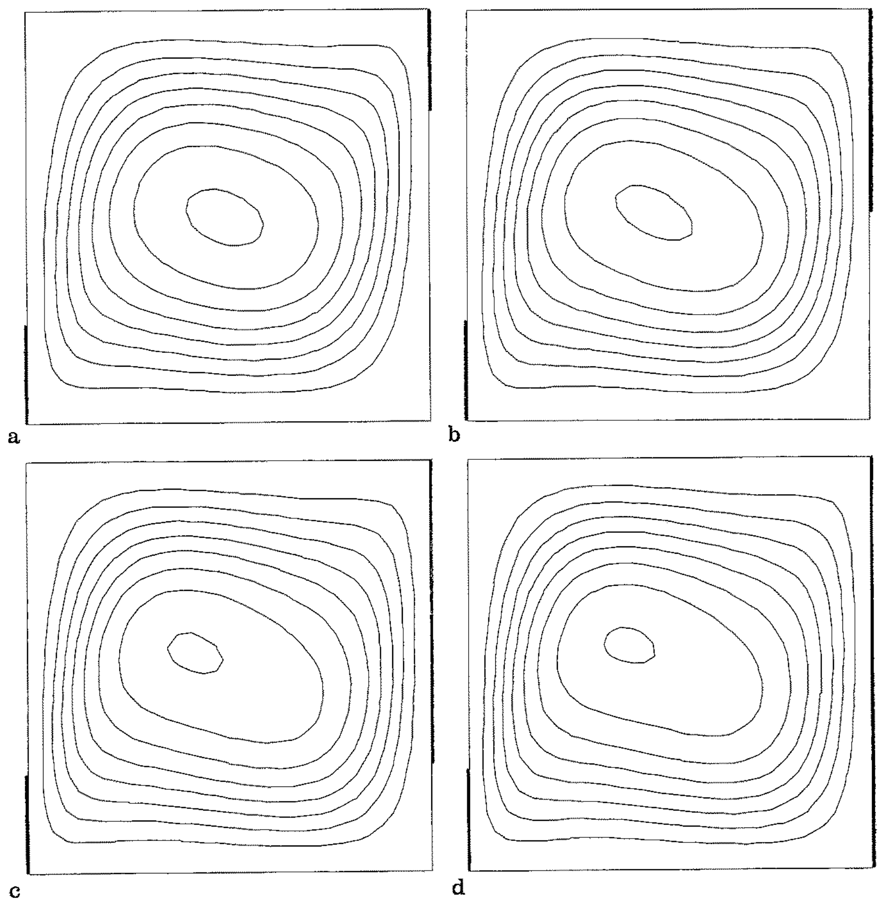

Fig. 5 a–d. Streamline contours ($Ra = 10^5$, $l_H/H = 0.25$). **a** $l_C/H = 0.25$; **b** $l_C/H = 0.50$; **c** $l_C/H = 0.75$; **d** $l_C/H = 1.0$

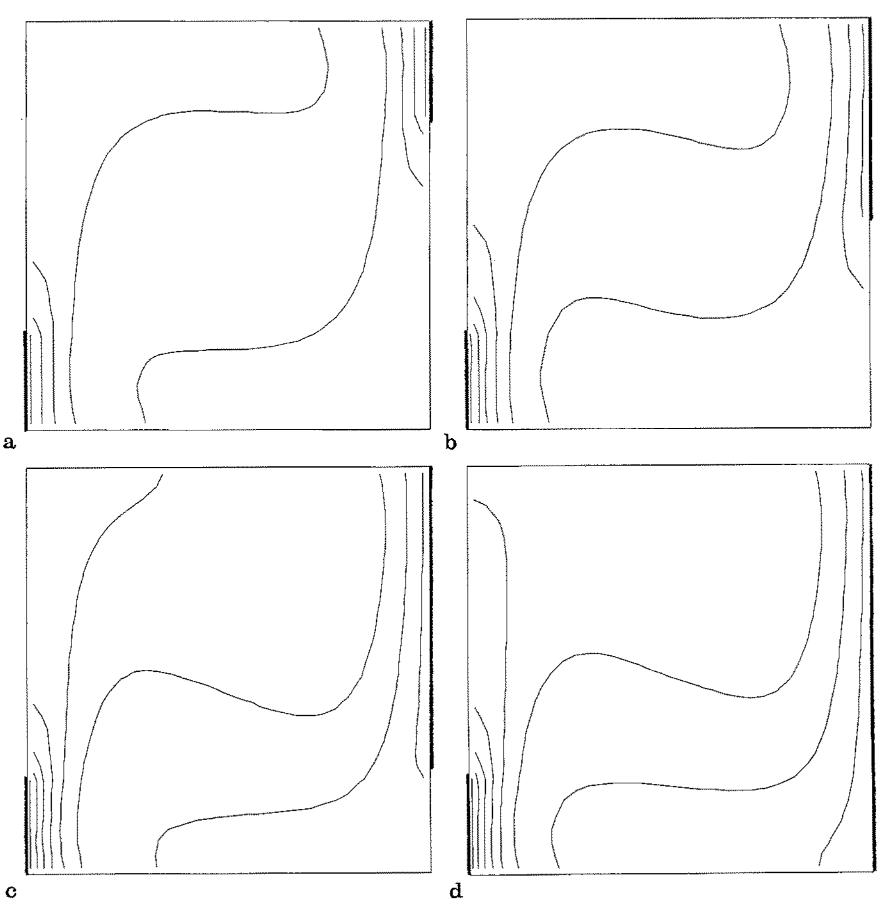

Fig. 6 a–d. Isotherm contours ($Ra = 10^5$, $l_H/H = 0.25$). **a** $l_C/H = 0.25$; **b** $l_C/H = 0.50$; **c** $l_C/H = 0.75$; **d** $l_C/H = 1.0$

with the varying heater size. It was observed that the amount of fluid circulating in the cavity is almost the same for four cases. The isotherms with higher values move toward the cold wall, as the heater size increases as seen in Fig. 4. This implies that overall temperature of the fluid in the enclosure increases with the increasing heater size. (Similar graphics are obtained for all cooler sizes, but not presented here.)

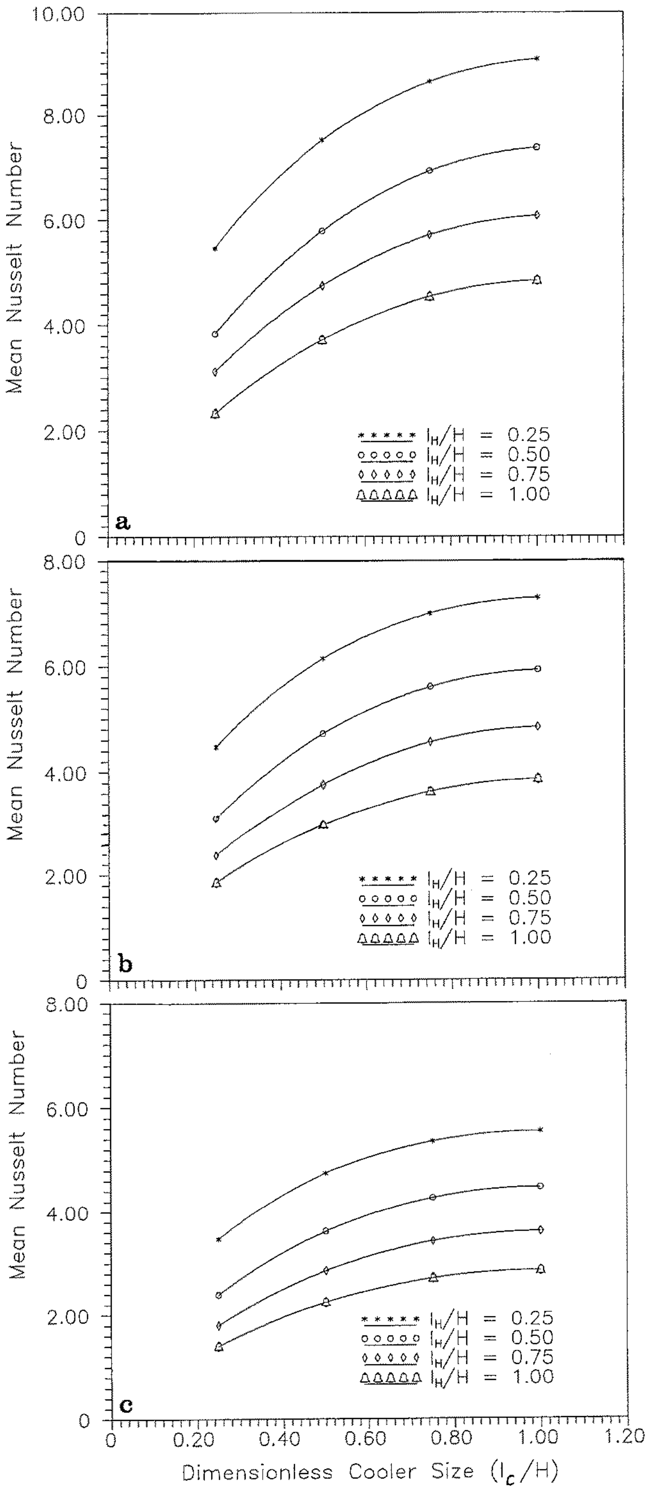

Fig. 7 a – c. Variation of the mean Nusselt number with heater and cooler size. **a** $Ra = 10^5$; **b** $Ra = 5\times10^4$; **c** $Ra = 2\times10^4$

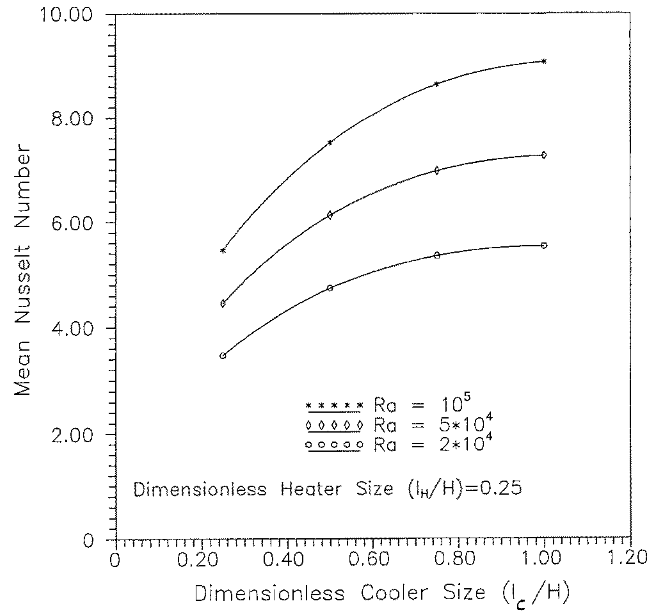

Fig. 8. Variation of the mean Nusselt number with cooler size and Rayleigh number

To analyze the effects of the cooler size on the flow and heat transfer, the computations were performed by taking the heater size constant and varying the cooler size ($l_C/H = 0.25$, 0.5, 0.75 and 1.0). For the case of $l_H/H = 0.25$, streamlines and isotherms are shown in Figs. 5 and 6. As seen in Fig. 5, the flow patterns obtained with different cooler sizes are similar. The rates of the fluid circulation are very close for all the cases. As the cooler size increases, the temperature curves with lower values move closer to the vertical wall on which the heater is located. Therefore, the bulk temperature of the fluid and the temperature in the fluid layer near the heater decrease.

The variation of Nusselt number with the heater and cooler size were plotted in Figs. 7a, 7b, and 7c for Rayleigh numbers 10^5, 5×10^4 and 2×10^4, respectively. Analysis of these figures reveals that for all Rayleigh numbers, the mean Nusselt number decreases with increasing heater size. The decrease in the mean Nusselt number with increasing heater size can be attributed to the fact that the temperature difference between the hot wall and the fluid decreases with increasing heater size, as the circulation rate remains the same. As a result of this, less heat is transfered from the heater to the fluid. Hence, the mean Nusselt number decreases with increasing heater size.

From Figs. 7a, 7b, and 7c, it is also observed that as the cooler size increases, the mean Nusselt number also increases. When the heater size is kept constant, the amount of heat transfered through the cooler increases with increasing cooler size. In consequence, the average temperature of the fluid in the enclosure drops. Hence, the temperature difference between the heater and the fluid increases. As a result, heat transfer rate from the hot surface to the fluid increases. Variation of the Nusselt number with the Rayleigh number is depicted in Fig. 8. As seen in this figure, for all the cases Nusselt number increases with increasing Rayleigh number.

5
Conclusions

A numerical model was employed to analyze the flow and heat transfer in partially heated and partially cooled cavities. Using the control volume approach, a computer program based on SIMPLE algorithm was developed. The effect of heater and cooler size on the heat transfer rate was investigated. It was observed that for a given cooler size, the mean Nusselt number decreases with increasing heater size. On the other hand, for a given heater size, Nusselt number increases with increasing cooler size. The same behavior was observed for all Rayleigh numbers.

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