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AN INTEGRAL EQUATION FORMULATION OF PLATE BENDING PROBLEMS

by

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SUMMARY

The mathematical theory of thin elastic plates loaded by transverse forces leads to biharmonic boundary value problems. These may be formulated in terms of singular integral equations, which can be solved numerically to a tolerable accuracy for any shape of boundary by digital computer programs. Particular attention is devoted to clamped and simply-supported rectangular plates. Our results indicate support for the generally accepted treatment of such plates and for the intuitive picture of deflection behaviour at a corner.

1. *Introduction*

It has been demonstrated in a recent paper [1] that some biharmonic boundary-value problems related to two-dimensional elastostatics may be solved numerically by an adaption of Jaswon's integral equation formulation [2]. We demonstrate in the present paper that this adaption also works well for certain biharmonic problems related to the theory of thin plates loaded by transverse forces. The relevant field quantity in plate theory is the transverse deflection w, analogous mathematically to Airy's stress function χ, and second derivatives of w yield moment components just as second derivatives of χ yield stress components. An attractive feature of plate theory is that w has an immediate physical significance, and of course it can be computed to a higher accuracy than moment or stress components. However plate theory offers some special difficulties which do not arise in elastostatics. First, it is necessary to compute moments at the boundary it they have not been prescribed thereon, and this requires the differentiation of simple layer potentials at a source point on the boundary. Secondly, in the case of polygonal boundaries, complications appear at corners owing to their infinite curvatures. Finally, as regards free boundaries, higher derivative conditions enter which are not well adapted either to theoretical or numerical analysis. These difficulties make an independent treatment of plate problems necessary.

Three distinct problems are considered in this paper: clamped rectangular plates of various dimensions subject to transverse loading; the simply-supported square plate subject to uniform transverse loading; the partly clamped, partly simply-supported square plate subject to uniform transverse loading. All our results are in excellent agreement with the approximate analytic solutions quoted by Timoshenko and Woinowsky-Krieger [3]. Our results for clamped rectangular plates also agree well with those recently obtained by Morley [4] on the basis of variational principles.

As regards simply-supported rectangular plates subject to uniform transverse loading, the problem may in effect be reduced from biharmonic to harmonic function theory by omitting the boundary term $\rho^{-1} \frac{\partial w}{\partial n}$ (ρ^{-1} is the

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curvature and $\frac{\partial}{\partial n}$ denotes normal derivative), an extensively employed step apparently first pointed out in the literature by Marcus [5]. This term is indeed zero along a straight line since $\rho^{-1} = 0$, but could possibly become finite or even infinite at a corner owing to the behaviour of ρ^{-1} there. No systematic analytical or numerical investigation of $\rho^{-1} \frac{\partial w}{\partial n}$ omission effects seems to be available. Our procedure here is to round off each corner by a circular arc of radius ρ_o as described elsewhere [1], solve the complete biharmonic problem numerically retaining $\rho_o^{-1} \frac{\partial w}{\partial n}$ along the arcs, and examine numerical behaviour as $\rho_o \to 0$. We find the results for a square plate to be almost indistinguishable from those obtained by solving the reduced problem numerically. We also find that $\rho_o^{-1} \frac{\partial w}{\partial n}$ increases,whilst $\frac{\partial w}{\partial n}$ decreases, as $\rho_o^{-1} \to 0$, so implying (as will be explained later) support for Timoshenko's intuitive picture [3] of deflection behaviour near a corner [6]. A similar, though simpler, reduction occurs on omitting $\rho^{-1} \frac{\partial w}{\partial n}$ from the boundary conditions of a simply-supported plate subject to a uniform thermal moment [7]. This problem has been treated on the same lines as the preceding, yielding similar conclusions. Further problems now under investigation are the partly clamped, partly free rectangle, and the clamped ellipse subject to a concentrated transverse load.

Three main conclusions emerge from this paper. First, the integral equation method rapidly provides a reliable overall picture of the deflection and moment distribution, though finer details are probably best supplied by more sophisticated analytical techniques such as the polar coordinate transformations of Morley [8] or of Williams [9], or the λ-method of Quinlan [10]. Secondly, its numerical results lend support to the widely accepted omission of $\rho^{-1} \frac{\partial w}{\partial n}$ for rectangular plates, so enabling the analytical treatments to be correspondingly simplified. Thirdly, it appears that the corners of a simply-supported square plate behave theoretically as expected on intuitive physical grounds.

The rest of the paper divides into three main sections: thin plate theory, integral equation formulation, numerical results and comparisons.

2. *Thin Plate Theory*

The transverse deflection of a thin plate under a uniform load k per unit area satisfies the equation

$$\nabla^2 (\nabla^2 w) \equiv \nabla^4 w = k/D, \tag{1}$$

where D is the flexural rigidity. With w known, the moment components at any point x, y are determined from

$$M_{xx} = -D\left[\frac{\partial^2 w}{\partial x^2} + \nu \frac{\partial^2 w}{\partial y^2}\right], \quad M_{yy} = -D\left[\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2}\right],$$

$$M_{xy} = -M_{yx} = D(1-\nu)\frac{\partial^2 w}{\partial x \partial y}, \tag{2}$$

using the notations and conventions of Timoshenko [3]. These formulae can immediately be adapted to the boundary, L, by identifying $x \equiv n$, $y \equiv t$, where n, t denote the (inward) normal and tangential boundary variables as indicated in Fig. 1. Accordingly we write

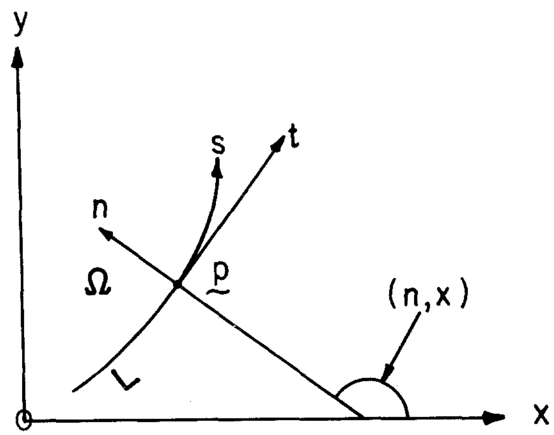

Fig.1. Orientation of tangent and normal at a point **p** of the boundary curve L enclosing the domain Ω.

$$M_{nn} = - D\left[\frac{\partial^2 w}{\partial n^2} + \nu \frac{\partial^2 w}{\partial t^2}\right], \quad M_{tt} = - D\left[\frac{\partial^2 w}{\partial t^2} + \nu \frac{\partial^2 w}{\partial n^2}\right],$$

$$M_{nt} = - D(1-\nu) \frac{\partial^2 w}{\partial n \partial t}, \tag{3}$$

noting that M_{tt} does not enter into any boundary conditions since it refers entirely to the material just inside L. Although the t-direction lies at right angles to the n-direction, it proves more convenient to work with the arc variable s rather than with t, necessitating the derivative transformations

$$\frac{\partial w}{\partial s} = \frac{\partial w}{\partial t}, \quad \frac{\partial^2 w}{\partial s^2} = \frac{\partial^2 w}{\partial t^2} + \frac{1}{\rho}\frac{\partial w}{\partial n},$$

$$\frac{\partial^2 w}{\partial n \partial s} = \frac{\partial^2 w}{\partial n \partial t}, \quad \frac{\partial^2 w}{\partial s \partial n} = \frac{\partial^2 w}{\partial n \partial s} - \frac{1}{\rho}\frac{\partial w}{\partial s}. \tag{4}$$

For a straight line $\rho^{-1} = 0$ and s has then exactly the same significance as t. Substituting from (4) into (3) yields the general boundary formulae

$$M_{nn} = - D\left[\frac{\partial^2 w}{\partial n^2} + \nu\left(\frac{\partial^2 w}{\partial s^2} - \frac{1}{\rho}\frac{\partial w}{\partial n}\right)\right],$$

$$M_{nt} = - D(1-\nu)\frac{\partial^2 w}{\partial n \partial s} = - D(1-\nu)\left[\frac{\partial^2 w}{\partial s \partial n} + \frac{1}{\rho}\frac{\partial w}{\partial s}\right]. \tag{5}$$

The conditions w = 0 holds for both clamped and simply supported boundaries. This implies

$$\frac{\partial w}{\partial s} = \frac{\partial^2 w}{\partial s^2} = 0 \text{ on L}, \tag{6}$$

so that (5) becomes

$$
\begin{aligned}
M_{nn} &= -D\left[\frac{\partial^2 w}{\partial n^2} - \frac{\nu}{\rho}\frac{\partial w}{\partial n}\right] = -D\left[\nabla^2 w + \frac{1-\nu}{\rho}\frac{\partial w}{\partial n}\right], \\
M_{nt} &= -D(1-\nu)\frac{\partial^2 w}{\partial s \partial n} = -D(1-\nu)\frac{\partial}{\partial s}\left(\frac{\partial w}{\partial n}\right),
\end{aligned} \tag{7}
$$

where

$$\nabla^2 w = \frac{\partial^2 w}{\partial n^2} + \frac{\partial^2 w}{\partial t^2} = \frac{\partial^2 w}{\partial n^2} + \frac{\partial^2 w}{\partial s^2} - \frac{1}{\rho}\frac{\partial w}{\partial n}.$$

On a clamped boundary $\frac{\partial w}{\partial n} = 0$, so that (7) becomes

$$M_{nn} = -D\nabla^2 w, \quad M_{nt} = -D(1-\nu)\frac{\partial}{\partial s}\left(\frac{\partial w}{\partial n}\right) = 0. \tag{8}$$

On a simply-supported boundary $M_{nn} = 0$, but no simplifications appear in (7). To summarize, the prescribed conditions on a clamped boundary are

$$w = \frac{\partial w}{\partial n} = 0, \tag{9}$$

with M_{nn} to be determined from (8); the prescribed conditions on a simply-supported boundary are

$$w = \nabla^2 w + \frac{1-\nu}{\rho}\frac{\partial w}{\partial n} = 0, \tag{10}$$

with M_{nt} to be determined by (7).

Condition (10) simplifies to $w = \nabla^2 w = 0$ on omitting $\rho^{-1}\frac{\partial w}{\partial n}$ along a polygonal boundary. Accordingly, writing $\nabla^2 w \equiv M$, we see from (1) that $\nabla^2 M = k/D$ throughout the plate domain Λ coupled with $M = 0$ on L. This is a classical Dirichlet problem for M. With M known in Λ, we may determine w by solving the second Dirichlet problem: $\nabla^2 w = M$ in Λ subject to $w = 0$ on L. For the simply-supported polygonal plate subject to a uniform thermal moment, w satisfies $\nabla^2(\nabla^2 w) = 0$ in Λ under the boundary conditions

$$w = 0, \quad \nabla^2 w = K \tag{11}$$

where K is a constant. Since $\nabla^2 w$ is now a harmonic function in Λ, and since $\nabla^2 w = K$ on L, it follows that $\nabla^2 w = K$ throughout Λ. Coupling this equation with $w = 0$ on L defines a Dirichlet problem for w. Exact solutions of these relatively simple Dirichlet problems do not seem to be available for rectangular domains, but reliable approximate solutions can be readily obtained by an integral equation method [12].

3. *Integral Equation Formulation*

Equation (1) admits the particular integral

$$W_1 = \frac{k}{48D}(x^4 + y^4), \tag{12}$$

and its general solution can hence be written

$$w = W + W_1 \tag{13}$$

where W satisfies $\nabla^4 W = 0$.

Throughout any compact domain Λ we may adopt the representation*

$$W = r^2\phi + \psi; \quad \nabla^2\phi = \nabla^2\psi = 0 \tag{14}$$

where $r^2 = x^2 + y^2$, or equivalent representations such as $x\phi + \psi$ or $y\phi + \psi$. Since W is prescribed on L, i.e. $W = -W_1$ so making $w = 0$ in accordance with (9) and (10), equation (14) may be regarded as a linear functional relation coupling ϕ, ψ on L. Also, since $W'(\equiv \frac{\partial W}{\partial n})$, or $\nabla^2 W$, or $\nabla^2 W + \frac{1-\nu}{\rho} W'$, is prescribed on L, there exists a second linear functional relation

$$W' = (r^2\phi)' + \psi' = 2rr'\phi + r^2\phi' + \psi' \tag{15}$$

or

$$\nabla^2 W = 4\left(x\frac{\partial\phi}{\partial x} + y\frac{\partial\phi}{\partial y} + \phi\right), \tag{16}$$

or

$$\nabla^2 W + \frac{1-\nu}{\rho} W' = 4\left(x\frac{\partial\phi}{\partial x} + y\frac{\partial\phi}{\partial y} + \phi\right) + \frac{1-\nu}{\rho}(2rr'\phi + r^2\phi' + \psi') \tag{17}$$

coupling the derivatives of ϕ, ψ on L. Relation (14), together with (15), (16) or (17), in principle suffices to determine ϕ, ψ on L, whence they can be continued into Λ, so continuing W into Λ, and thereby continuing w into Λ. A practicable method of carrying out this program is to identify ϕ, ψ as potentials generated by continuous simple source distributions on L, with densities to be determined. Thus we write

$$\phi(\mathbf{P}) = \int \log|\mathbf{P}-\mathbf{q}|\,\sigma(\mathbf{q})dq \tag{18}$$

where $\mathbf{q}$ is a vector variable defining source points on L, dq denotes the arc differential at $\mathbf{q}$ directed so as to keep Λ on the left, $\sigma(\mathbf{q})$ is a source density at $\mathbf{q}$ to be determined and $\mathbf{P}$ is a vector variable defining points within Λ. This potentials remains continuous as $\mathbf{P}$ approaches any point $\mathbf{p}$ of L, and so on L we may write.

$$\phi(\mathbf{p}) = \int \log|\mathbf{p}-\mathbf{q}|\,\sigma(\mathbf{q})dq \tag{19}$$

where $\phi(\mathbf{P}) \to \phi(\mathbf{p})$ as $\mathbf{P} \to \mathbf{p}$. It is a known result [11] that

$$\phi'(\mathbf{p}) = \int \log'|\mathbf{p}-\mathbf{q}|\,\sigma(\mathbf{q})dq + \pi\sigma(\mathbf{p}) \tag{20}$$

where $\log'|\mathbf{p}-\mathbf{q}|$ signifies the inward normal derivative of $\log|\mathbf{p}-\mathbf{q}|$ at $\mathbf{p}$ keeping $\mathbf{q}$ fixed. Writing

$$\psi(\mathbf{p}) = \int \log|\mathbf{p}-\mathbf{q}|\,\mu(\mathbf{q})dq, \quad \psi'(\mathbf{p}) = \int \log'|\mathbf{p}-\mathbf{q}|\,\mu(\mathbf{q})dq + \pi\mu(\mathbf{p}) \tag{21}$$

where $\mu(\mathbf{q})$ is a second source density at $\mathbf{q}$ to be determined, and substituting (19), (20), (21) into (14) and (15), we arrive at two coupled linear integral equations for σ, μ in the clamped plate problem. With these known, ϕ and ψ, and hence also W, and therefore also w, can be generated throughout Λ.

Derivatives of ϕ at any interior point $\mathbf{P} \equiv x(\mathbf{P}), y(\mathbf{P})$ may be generated by the formulae

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* This representation is discussed more fully in [1].

$$\frac{\partial\phi}{\partial x} = \frac{\partial}{\partial x}\int \log|\mathbf{P}-\mathbf{q}|\sigma(\mathbf{q})dq = \int \log_x|\mathbf{P}-\mathbf{q}|\sigma(\mathbf{q})dq \tag{22}$$

etc., where

$$\log_x|\mathbf{P}-\mathbf{q}| = \frac{x(\mathbf{P})-x(\mathbf{q})}{|\mathbf{P}-\mathbf{q}|^2}, \quad \log_{xy}|\mathbf{P}-\mathbf{q}| = \frac{-2[x(\mathbf{P})-x(\mathbf{q})][y(\mathbf{P})-y(\mathbf{q})]}{|\mathbf{P}-\mathbf{q}|^4}$$

$$\log_{xx}|\mathbf{P}-\mathbf{q}| = \frac{1}{|\mathbf{P}-\mathbf{q}|^2} - \frac{2[x(\mathbf{P})-x(\mathbf{q})]^2}{|\mathbf{P}-\mathbf{q}|^4}, \quad \text{etc.}$$

However difficulties arise on L owing to discontinuities exemplified by (20). By contrast to the normal derivative, the tangential derivative of a simple layer potential remain continuous [11] at L, a property symbolised by writing

$$\frac{\partial}{\partial t}\int \log|\mathbf{p}-\mathbf{q}|\ \sigma(\mathbf{q})dq = \int \log_t|\mathbf{p}-\mathbf{q}|\ \sigma(\mathbf{q})dq \tag{23}$$

where $\log_t|\mathbf{p}-\mathbf{q}|$ signifies the tangential derivative of $\log|\mathbf{p}-\mathbf{q}|$ at $\mathbf{p}$ keeping $\mathbf{q}$ fixed. Accordingly, since

$$\frac{\partial}{\partial x} = \frac{\partial}{\partial n}\cdot\frac{dn}{dx} + \frac{\partial}{\partial t}\cdot\frac{dt}{dx},$$

it follows that

$$\begin{aligned}\frac{\partial}{\partial x}\int \log|\mathbf{p}-\mathbf{q}|\ \sigma(\mathbf{q})dq &= \left[\int \log'|\mathbf{p}-\mathbf{q}|\ \sigma(\mathbf{q})dq + \pi\sigma(\mathbf{p})\right]\frac{dn}{dx} \\ &\quad + \left[\int \log_t|\mathbf{p}-\mathbf{q}|\ \sigma(\mathbf{q})dq\right]\frac{dt}{dx} \\ &= \int \log_x|\mathbf{p}-\mathbf{q}|\ \sigma(\mathbf{q})dq + \pi\sigma(\mathbf{p})\frac{dn}{dx},\end{aligned} \tag{24}$$

and similarly

$$\frac{\partial}{\partial y}\int \log|\mathbf{p}-\mathbf{q}|\ \sigma(\mathbf{q})dq = \int \log_y|\mathbf{p}-\mathbf{q}|\ \sigma(\mathbf{q})dq + \pi\sigma(\mathbf{p})\frac{dn}{dy} \tag{25}$$

where

$$\frac{dn}{dx} = \cos(n,x), \quad \frac{dn}{dy} = \cos(n,y) \quad \text{etc.},$$

as exhibited in Fig. 1. It will be noted from (15), (16) and (17) that second derivatives on L are not required. Substituting (24), (25) as well as (19), (20), (21) into (14) and (17) yields two coupled linear integral equations for σ, μ in the simply-supported plate problem for any domain. If (17) is replaced by the simplified condition (16) for rectangular plates, this immediately gives a linear integral equation for σ independently of μ, thereby determining ϕ independently of ψ; with ϕ known on L, ψ is then determined at once on L from (14). This is the counterpart of the Marcus reduction according to our formulation.

Successful techniques for the numerical solutions of boundary integral equations have been developed by Maiti [6] and by Symm [12, 13], and these suffice for all the formulations of the present paper. The rounding of corners is fully described in [1] and exemplified in Fig. 2. Problems involving the second derivatives of simple source potentials on L will be treated in a subsequent paper.

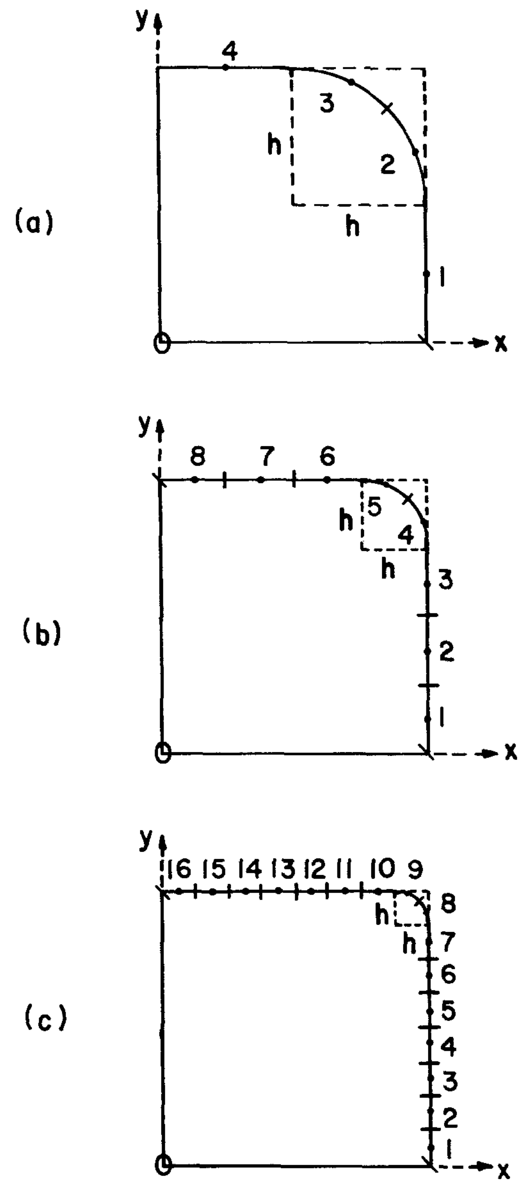

Fig. 2. Quadrant of a square: successive subdivisions and corner roundings.

4. Numerical Results and Comparisons

For a rectangular clamped plate of dimensions a/b = 2.0, we round off the corners and choose n (number of nodal points) = 24 initially as exhibited in Fig. 3. The effective number of nodal points reduces from n to n/4 owing to symmetry. Numerical solutions were achieved by a digital computer program for n = 24, 48, 96 so that numerical conditioning could be examined. The problem was also solved for a/b = 1.5 taking n = 20, 40, 80 and for a/b = 1.0 taking n = 16, 32, 64. All tabulated results refer to the final value of n in each case. Table 1 provides the central deflection, and some important bending moments, as computed by us, and they are seen to be in excellent agreement with the results quoted by Timoshenko

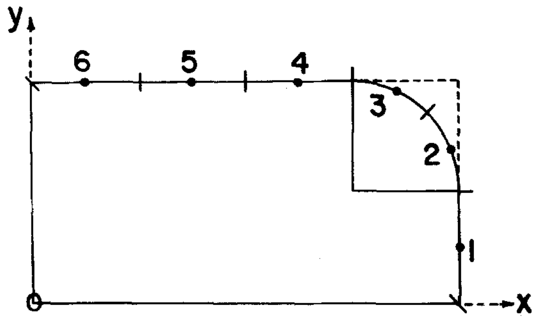

Fig.3. Quadrant of a rectangle (a/b = 2): initial subdivision and corner rounding.

et al [3] which also appear in Table 1. These comparisons demonstrate

Table 1. Clamped rectangle: central deflection and some important bending moments, computed from integral equation solution (first row) and from solution quoted by Timoshenko et al (second row).

a/b	w(0,0)	$M_{xx}(0,0)$	$M_{xx}(a,0)$	$M_{yy}(0,0)$	$M_{yy}(0,b)$
1.0	$0.0202kb^4/D$ $0.0202kb^4/D$	$0.0916kb^2$ $0.0924kb^2$	$-0.2042kb^2$ $-0.2052kb^2$	$0.0916kb^2$ $0.0924kb^2$	$-0.2042kb^2$ $-0.2052kb^2$
1.5	$0.0351kb^4/D$ $0.0352kb^4/D$	$0.0811kb^2$ $0.0812kb^2$	$-0.2268kb^2$ $-0.2280kb^2$	$0.1471kb^2$ $0.1473kb^2$	$-0.3020kb^2$ $-0.3028kb^2$
2.0	$0.0405kb^4/D$ $0.0406kb^4/D$	$0.0631kb^2$ $0.0632kb^2$	$-0.2270kb^2$ $-0.2284kb^2$	$0.1646kb^2$ $0.1648kb^2$	$-0.3312kb^2$ $-0.3316kb^2$

that the integral equation method works as well as any other for clamped plate problems.

The simply-supported square plate has been treated utilising (10), with rounded-off corners, taking n = 16, 32, 64. The variation of ρ^{-1} along an edge is exhibited in Fig. 4, it being noted that ρ_o = h (interval length) at any stage. Our numerical solution yields $\frac{\partial w}{\partial n}$ at the midpoint of the circular arc, and hence also $\rho_o^{-1} \frac{\partial w}{\partial n}$ at that point, as provided in Table 2. It will be seen that $\frac{\partial w}{\partial n}$ decreases as ρ_o decrease, thereby supporting the conjecture [3] that $\frac{\partial w}{\partial n}$ = 0 at a corner. It will also been that $\rho_o^{-1} \frac{\partial w}{\partial n}$ increases as ρ_o decreases, implying from M_{nn} = 0 that $\frac{\partial^2 w}{\partial n^2}$ increases as ρ_o decreases, i.e. that the deflected surface has an appreciable curvature in the n-direction at a corner point contrasting with its zero curvature at other edge points. This supports the physically based view [3] that simply-supported plates have a tendency to ride up at the corners, so requiring reactions of opposite sign to those elsewhere along the edge in order to maintain w = 0. Computations were also performed on the basis of w = $\nabla^2 w$ = 0 everywhere on L, including the arcs, with results almost indistinguishable from the preceding, though somewhat closer to those quoted by Timoshenko et al [3] relying on the same simplification (Table 3). We may infer that ignoring the corner anomaly has no significant effect on the solution of simply-supported plate problems.

The square plate with one pair of opposite edges clamped, and the other pair simply-supported, has also been treated by our formulation. On the

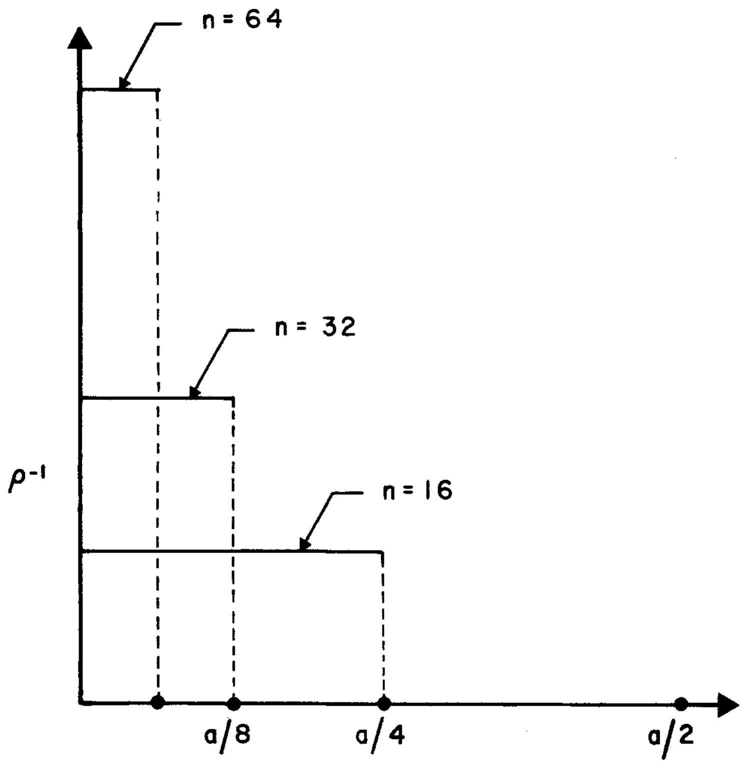

Fig. 4. Variation of curvature along half side of square for successive subdivisions. The step function denoted n = 16, 32, 64 correspond respectively with (a), (b), (c) of Fig. 2.

Table 2. Simply-supported square: numerical behaviour at the corner nodal point for successively decreasing interval lengths.

h	$\frac{\partial w}{\partial n}$	$h^{-1} \frac{\partial w}{\partial n}$
0.2500	0.0095k/D	0.0381k/D
0.1250	0.0053k/D	0.0427k/D
0.0625	0.0028k/D	0.0452k/D

basis of $\rho^{-1} = 0$ everywhere along the edges, we find results (Table 4) indistinguishable from those previously quoted [3].

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Table 3. Simply-supported square: deflection and bending moments along lines of symmetry radiating from the centre, computed from integral equation solution of biharmonic problem (first row), from integral solution of reduced problem (second row), and from solution of reduced problem quoted by Timoshenko et al (third row). The Greek symbols denote standard numerical factors.

	x = 0 y = 0	0.2 0	0.4 0	0.5 0	0 0.2	0 0.4	0 0.5	0.2 0.2	0.4 0.4
$w = \alpha k/D$	0.0042 0.0042 0.0041	0.0035 0.0034 -	0.0014 0.0014 -	0.0000	0.0035 0.0034 -	0.0014 0.0014 -	0.0000	0.0029 0.0028 -	0.0005 0.0005 -
$M_{xx} = \beta k$	0.0491 0.0487 0.0479	0.0438 0.0433 0.0424	0.0220 0.0216 0.0209	0.0000	0.0409 0.0406 0.0400	0.0170 0.0170 0.0168	-	0.0370 0.0365 -	0.0146 0.0114 -
$M_{yy} = \gamma k$	0.0491 0.0487 0.0479	0.0409 0.0406 0.0400	0.0170 0.0170 0.0168	-	0.0438 0.0433 0.0424	0.0220 0.0216 0.0209	0.0000	0.0370 0.0365 -	0.0146 0.0114 -

Table 4. Partly clamped, partly simply-supported, square: central deflection and some important bending moments, computed from integral equation solution (first row) and from solution quoted by Timoshenko et al (second row).

$w(0,0)$	$M_{xx}(0,0)$	$M_{yy}(0,0)$	$M_{yy}(0,a)$
0.0019k/D	0.0244k	0.0333k	-0.0699k
0.0019k/D	0.0244k	0.0333k	-0.0697k

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