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ENDURANCE OF STRUCTURAL MATERIALS WITH NONPROPORTIONAL PATHS OF LOW CYCLE LOADING

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Stress analysis of structural elements operating under conditions of cyclic processes of complex loading are among the problems that have received little attention so far. The existing methods of predicting the load bearing capacity and endurance of structural materials under low cycle loading are based on criteria and equations of state that were experimentally confirmed for simple loading [1-3]. At present there are available experimental works [4-7] from which it follows that disregarding the complexities of the process of loading a structural element may lead to substantial errors in evaluating its endurance.

In connection with the increasing power of modern machinery and mechanisms, and also the more stringent requirements regarding their reliability and service life, the problem of complex low cycle loading becomes more topical every year, especially in branches of production such as the aircraft industry and power generating machinery.

The problem of evaluating the endurance of structural elements is solved in several stages. Specifically, with the use of some calculation theory the state of stress and strain of the element is determined and its most highly stressed zones are found. Then we study in them the kinetics of the process of elastoplastic deformation, and proceeding from this we choose the criterion of strength. The accuracy of the calculation depends equally on the substantiation of the chosen equations of state characterizing the cycle-by-cycle correlation between the running values of stresses and strains and on the correctness of the adopted criterion of fatigue strength of the materials.

When the criteria of low cycle fatigue had been classified according to the nature of the parameters of damage used in them, it was decided to divide them into strength, strain, and energy parameters. We substantiated experimentally the position that in loading, involving large plastic deformation, the decisive influence on the process of fatigue is exerted by plastic deformations and by the plasticity of the material; consequently, the basic parameter of damage is the amplitude of plastic deformation, or in consequence of the insignificant elastic component, the amplitude of full deformation. The deformation treatment presupposes that failure occurs when one-sidedly accumulated or cyclic elastoplastic deformation or their sum attains its limit value. A known shortcoming of the hypothesis of low cycle strength in the strength treatment is that it is applicable solely under conditions of fatigue failure. In the solution of problems of strength, in particular of fatigue strength, the most general is the approach based on the energy concept of failure which reduces to the assertion that damage is associated with a certain proportion of irreversible work of cyclic deformation [8].

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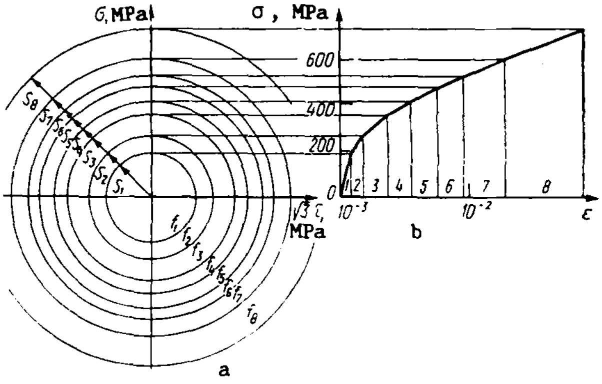

Fig. 1. Field of the moduli of plasticity (a) and approximation of the cyclic stress—strain curve in uniaxial loading of steel Kh18N10T (sections 7 and 8 were extrapolated) (b).

In most investigations dealing with the elaboration of criteria of strength in low cycle loading under conditions of a complex state of stress, up to now the concept of "equivalent" stresses and strains is used; it presupposes the reduction of multiaxial loading, including complex one, to "equivalent" uniaxial loading. As a rule, we deal with a relation of the type of the Coffin—Manson relation

$$\varepsilon^* \ (\text{or} \quad \sigma^*) = c \cdot N^m, \tag{1}$$

where ε^* and σ^* are the equivalent amplitudes of strain and stress, respectively; c and m are constants of the material; N is the number of cycles to failure.

However, criteria of this kind do not take into account the effect of the history of loading, or else they correspond to a special kind of it [4] because they all are formulated in the form of some algebraic equations. Novozhilov and Rybakina believe that the correlation between the limit number of cycles and stresses (strains) under complex loading has to be plotted by using differential dependences and not algebraic ones [9].

From among similar suggestions we pick two approaches which in our opinion are most expedient in the scheme of practical realization. One of them is the energy approach to the calculation of low cycle fatigue with multiaxial loadings [10]. It is assumed that cumulation of damage does not manifest itself in the behavior of material prior to failure, and in the calculation of endurance we may proceed from the steady state which, as a rule, is attained after a brief transient stage. It is postulated that endurance up to the instant of the nucleation of macrocracks depends on the work of plastic deformation per cycle W_{cycle}:

$$N = \Phi(W_{\text{cycle}}), \tag{2}$$

where

$$W_{\text{cycle}} = \int_{\text{cycle}} \sigma_{ij} d\varepsilon^p_{ij}; \tag{3}$$

σ_{ij} and $d\varepsilon^p_{ij}$ are the stress tensor and the tensor of increase of plastic deformation, respectively.

To determine the magnitude of W_{cycle} it is suggested to use the theory of plastic flow and the law of strengthening of Mrous [11] based on the piecewise linear approximation of the cyclic strain diagram when a family of loading surfaces is introduced with kinematic strengthening corresponding to the points of the break in the diagram.

The second effective method of describing low cycle fatigue is the approach connected with the use of the kinetic equations of damageability [9, 12-15]. When we follow this approach, the state of damage of the material is determined by the parameter Ω which is dependent on the entire preceding history of deformation and is equal to zero in the undeformed state and equal to unity in the state after failure. It was established that failure occurs when the path of plastic deformation has attained a length inversely proportional to the intensity of the amplitude of plastic deformation $\varepsilon^p{}_i$ [9]; in differential form this looks as follows:

$$\frac{d\Omega}{dL} = K \cdot \varepsilon^p_i, \tag{4}$$

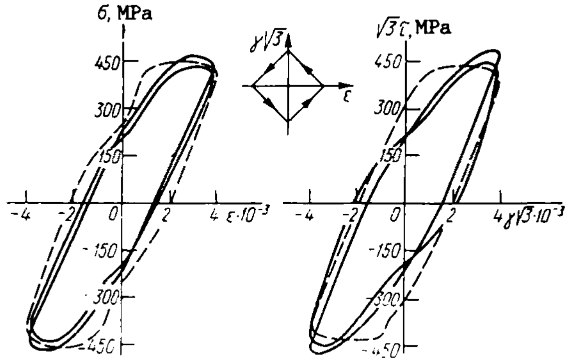

Fig. 2. Experimental (solid lines) and theoretical (dashed lines) cyclic strain diagrams of steel Kh18H10T with $\Delta\varepsilon_i$ = 0.8%, ω_e = 45°, θ = 90°.

where $L=\int\sqrt{\frac{2}{3}d\varepsilon^p_{ij}\cdot d\varepsilon^p_{ij}}$ is the length of the path of plastic deformation; K is a constant of the material; εP_i is the intensity of the amplitude of plastic deformation.

To develop the given concept, it was assumed that the decisive factor of failure are microstresses [12], as a result of which Eq. (4) is transformed in the following manner:

$$\frac{d\Omega}{dL}=\lambda\cdot P, \tag{5}$$

where $P=\sqrt{\frac{3}{2}\rho_{ij}\cdot\rho_{ij}}$ is the intensity of the residual microstresses; ρ_{ij} is the deviator of residual microstresses; λ is a constant of the material.

Movchan [13] provided the energy treatment of the criterion under consideration. For the case of unidimensional deformation by tension—compression it was shown that within the framework of the presented model the energy of translational strengthening is responsible for failure. This result once again confirms the generality of the energy approach to problems of the mechanics of deformed solids.

Equations (4) and (5) do not express the fact that cumulation of damage is of an approximate nature and cannot be described accurately by scalar relations only. This circumstance was noted by a number of authors [14, 15]. The variants for taking the orientation of the process of damage cumulation into account suggested by them reduce, respectively, to the equation

$$\frac{d\Omega}{d\varepsilon^p_{ij}}=\Phi_{ij} \tag{6}$$

or

$$\frac{d\Omega}{dL}=F_{ij}, \tag{7}$$

where Ω_{ij}, Φ_{ij}, F_{ij} are symmetric tensors of second rank the first of which was called the tensor of damage to the material [15], and the other two are bound to be specified by the function of microstresses.

Equations (5) and (6) were verified for the case of axisymmetric loading of conical thin-walled shells in thermal cycling with the use of the theory of plastic flow and creep under conditions of translationally isotropic strengthening [14]. A comparison of the theoretical and experimental data showed that Eq. (5) yielded too low a number of cycles to failure whereas the results of the calculation obtained with the criterion of the "work of the microstresses" (Eq. (6)) agreed well with the experiment.

On the basis of experimental investigations carried out with austenitic steel Kh18N10T under conditions of the state of plane stress with the simultaneous actions of an axial force P and a torque M_{tor} [17] we show in the present work the applicability of the above-described approaches to the processes of simple and complex loading. The tests were carried out at room temperature. In the regime of rigid loading with the components ε and γ (ε is axial deformation, γ is torsional deformation) we realized a sawtooth symmetric cycle with a frequency close

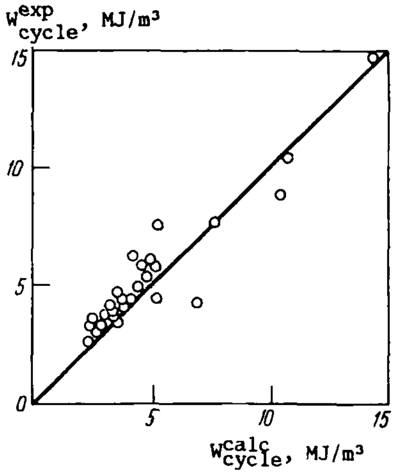

Fig. 3. Correlation between the experimental and the theoretical values of the energy of plastic strain per cycle.

to 1/60 sec^{-1}. The test equipment and the methods of carrying out the experiment were described earlier [16, 17].

To obtain an analytical description of the dependence of the endurance of steel Kh18N10T on the change of shape of the path of deformation, we used the methods of planning multifactor experiments. In constructing the mathematical model of the process we used the regular plan $3^3 \times 2^1/27$ with the power of 3. As independent factors x^{H}_1, x^{H}_2, x^{H}_3 determining the shape and dimensions of the path of deformation we adopted: $x^{H}_1 = \Delta\varepsilon_i = \sqrt{(\Delta\varepsilon)^2 + \frac{1}{3}(\Delta\gamma)^2}$ is the double amplitude of strain intensity; $x^{H}_2 = \omega_e = \text{arc tg}\,(\Delta\gamma/\sqrt{3}\Delta\varepsilon)$ is the angle of the kind of state of strain; $x^{H}_3 = \theta$ is the angle of phase shift between axial and torsional deformation. The fourth independent factor x^{H}_4 characterizing the supply of the material was used simultaneously as block factor with randomization of the experiments within the blocks.

The program of the investigations made it possible to include in the examination instead of the separate deformation paths, like in most works concerned with complex loading, a whole family of piecewise broken paths plotted in the plane of full axial deformations and torsional deformations. Then the paths of proportional loading are a subregion of the investigated set (for $\theta = 0$).

As a result of the application of the standard procedures of regression analysis we constructed the regression equation expressing the change of the logarithm of the number of cycles to nucleation of a macrocrack in dependence on the amplitude of the strain intensity (%), the angle of the kind of state of strain (deg) and the angle of phase shift (deg):

$$\ln N = 9{,}865817 - 3{,}4150985\cdot\Delta\varepsilon_i + 1{,}710087\ 10^{-2}\cdot\omega_e - 6{,}367276\cdot10^{-5}\cdot\omega_e^2 - 1{,}5838587\cdot10^{-2}\cdot\theta + 8{,}407713\cdot10^{-5}\cdot\theta^2 + 1{,}330333\cdot10^{-2}\cdot\Delta\varepsilon_i\cdot\theta + 3{,}1328702\cdot10^{-5}\cdot\omega_e^2\cdot\theta - 2{,}4018197\cdot10^{-3}\cdot\omega_e\cdot\theta + 1{,}371823\cdot10^{-5}\cdot\omega_e\cdot\theta^2 - 2{,}0400045\cdot10^{-7}\cdot\omega_e^2\cdot\theta^2. \quad (8)$$

Verification of Eq. (8) as to information content and adequacy showed that the hypotheses on the correspondence of the model to the experiment and its information content are not discarded at a significance level $q = 0.05$.

On the basis of the experimental results obtained in uniaxial tension—compression tests, and using the least squares method, we constructed an equation of the Coffin—Manson type. The amplitude of strain intensity ε_i served as equivalent parameter:

$$N = C\cdot\varepsilon_i^m, \quad (9)$$

where $C = 37.9$; $m = -4.09$.

As noted before, endurance may be treated as a function of the work of plastic deformation per cycle [10]. Concretizing the form of this function, we suggested to approximate it by the exponential dependence

$$N = A\cdot W^{\alpha}_{cycle}, \quad (10)$$

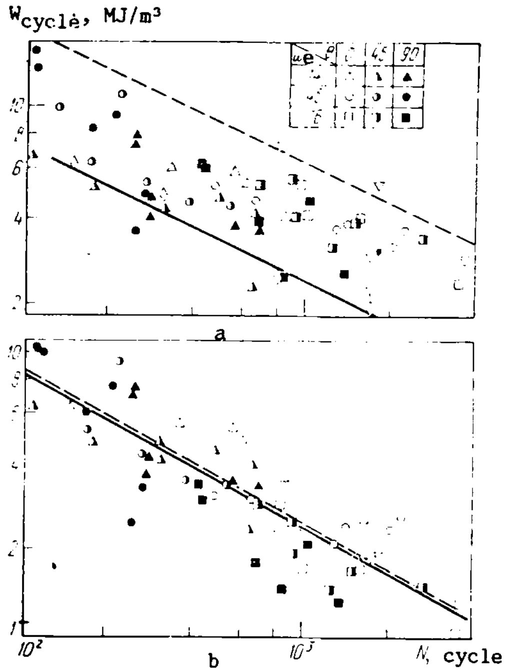

Fig. 4. Dependence of the work of plastic strain per cycle on the number of cycles to failure with the weight coefficient not taken (a) and taken into account (b). (Solid lines: tension—compression; dashed lines: torsion.)

where A and α are constants of the material determined under uniaxial cyclic loading. For steel Kh18N10T we obtained the following values from tension—compression tests: A = 5365; A = —1.913. Here, W_{cycle} has the dimensionality MJ/m³.

Figure 1 presents the approximation of the curve of cyclic strengthening of steel Kh18N10T in tension—compression (sections 7, 8 were extrapolated) and the field of surface plasticity in the unloaded state corresponding to it which was used in the calculation of plastic energy per cycle by the method of [10].

A comparison of the experimental and theoretical cyclic strain diagrams and also of their areas for all the used loading paths yielded quite good coincidence. The standard error S_{st} in the calculation of W_{cycle}, determined as the total area of the plastic hysteresis loops in tension—compression and torsion, amounted to 0.04314. As illustration Fig. 2 shows the stabilized curves of cyclic strain for the case $\Delta\varepsilon_i$ = 0.8%, ω_e = 45°, and θ = 90°, and Fig. 3 shows the graph of the correlation between the experimental and the theoretical values of the work of plastic strain per cycle.

The dependence of the number of cycles to failure on the energy per cycle for the loading regimes under consideration in log—log coordinates is presented in Fig. 4a. It can be seen that the results obtained for different paths of complex and proportional loading lie in a band between the practically parallel straight lines corresponding to tension—compression (the lower straight line) and reversed torsion. Thus Eq. (10) for cyclic loading with complex state of stress, even when the cyclic strain curves are ideally described, yields too low a value of endurance when the calculation is based on data obtained under conditions of tension—compression, and too high a value when the results of pure torsion are used. The latter is more dangerous because it may lead to premature failure of structural elements.

Garud suggested that the plastic strain energy of shear be calculated with a view to the weight coefficient b [10] which is determined by reducing the graph of endurance in pure torsion to the straight line of tension—compression, and then the expression for the total energy per cycle assumes the following form:

$$W_{cycle} = \int_{cycle} \sigma d\varepsilon + b \int_{cycle} \tau \cdot d\gamma, \qquad (11)$$

where σ is the normal, τ the tangential stresses; dε, dγ are the increments of axial and shear strain, respectively.

For steel Kh18N10T we obtained b = 1/2. The dependences ln N—ln W_{cycle} plotted with a view to the weight coefficient b = 1/2 are shown in Fig. 4b. The results of the calculation of endurance with the aid of the given method for the experimentally realized loading paths are presented in Table 1 and in Fig. 5. In all cases under consideration we obtained good coincidence of the theoretical and experimental values of endurance.

TABLE 1. Comparison of the Theoretical and Experimental Values of Endurance

Expt. No.	Loading parameters			No. of cycles to failure				
	$\Delta\varepsilon_i$	ω_e	θ	expt.	Eq. (8)	Eq. (9)	Eq. (10)	Eq. (16)
1	0,8	14	0	1645	1386	1608	919	898
2	1,2	45	0	565	524	306	517	406
3	1	76	0	1291	1417	645	1861	1540
4	1,2	14	45	143	165	306	259	274
5	1	45	45	221	225	645	161	212
6	0,8	76	45	2094	1845	1608	2686	2240
7	1	14	90	276	312	645	415	435
8	0,8	45	90	259	245	1608	225	309
9	1,2	76	90	437	406	306	764	621
10	1	14	0	730	849	645	496	487
11	0,8	45	0	2163	2490	1608	1583	1254
12	1,2	76	0	886	868	306	1124	921
13	0,8	14	45	655	618	1608	796	857
14	1,2	45	45	176	156	306	85	116
15	1	76	45	1079	1274	645	1390	1091
16	1,2	14	90	248	243	306	250	260
17	1	45	90	191	191	645	104	148
18	0,8	76	90	1102	1195	1608	2484	2246
19	1,2	14	0	458	429	306	304	293
20	1	45	0	1360	1258	645	847	679
21	0,8	76	0	3586	3402	1608	3608	2877
22	1	14	45	403	352	645	429	461
23	0,8	45	45	462	479	1608	348	462
24	1,2	76	45	801	726	306	810	609
25	0,8	14	90	627	591	1608	772	806
26	1,2	45	90	116	122	306	56	84
27	1	76	90	855	768	645	1279	1089
$S_{st}=\frac{1}{27}\left[\sum_{i=1}^{27}\left(\frac{N_i^{theo}-N_i^{exp}}{N_i^{exp}}\right)^2\right]^{1/2}$					0,0190	0,2785	0,0857	0,0748

The applicability of the approach based on the kinetic equations of damage to the calculation of the endurance of structural materials operating under conditions of complex low cycle loading was verified for different variants of the criterion of microstresses which are represented by Eqs. (5)-(7). An analysis of the results of the calculation carried out by procedures suggested in these equations did not reveal any substantial difference between them; this has possibly to do with the specifics of the symmetric load cycle. We will describe the method of calculation on the example of Eq. (5), for the other two cases the calculation procedures are practically analogous.

The microstress tensor is calculated according to the multisurface theory of plastic flow, a variant of which was dealt with in [10]. Similarly to [18], the components of the microstress tensor are identified with the coordinates of the center of the region of elastic deformations (Fig. 1).

It is known that in real materials the characteristics of strength depend on the kind of state of stress of the material. To take the effect of the kind of state of stress on the limit value of the measure of damage Ω into account, Novozhilov suggested replacing the equality $\Omega = 1$ by the following equation [15]:

$$\Omega=\left(\frac{S}{S_1}-1\right)^n, \tag{12}$$

where S_1 is the largest reduced stress; S is the breaking strength; n is a constant of the material.

However, the application of Eq. (12) is greatly hindered by the determination of the breaking strength S, i.e., the normal tensile strength destroying the material without preliminary plastic deformations; this is carried out in tests of cubic tension. In the suggested method of calculation we retained the equality $\Omega = 1$; the effect of the kind of state of stress on the magnitude of damage Ω was taken into account by the introduction of the modified tensor of microstresses

$$P_{ij}^{*}=\begin{vmatrix} P_{11} & kP_{12} & kP_{13} \\ kP_{21} & P_{22} & kP_{23} \\ kP_{31} & kP_{32} & P_{33}\end{vmatrix}, \tag{13}$$

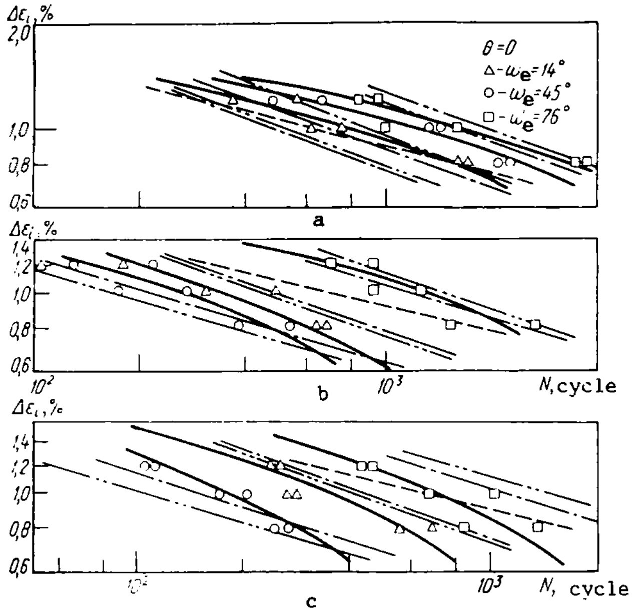

Fig. 5. Experimental and theoretical curves of endurance of steel Kh18N10T in dependence on the kind of state of strain and phase shift. (Solid lines: statistical model; dashed lines: Coffin—Manson equation; dot—dash lines: criterion of microstresses; double-dot—dash lines: energy approach.)

where k is the weight coefficient determined from uniaxial tension—compression and torsion tests, k = 0.25.

Calculation of these regimes of uniaxial loading showed that λ is a linear function of the double amplitude of the strain intensity. For Eq. (5)

$$\lambda = \lambda(\Delta\varepsilon_i) = 0{,}00133\cdot\Delta\varepsilon_i. \tag{14}$$

When we substitute (13) and (14) into Eq. (5), integrate it in a cycle and sum by cycles, we obtain the following dependence:

$$\sum_{n=1}^{N}\int\limits_{L_n}\lambda(\Delta\varepsilon_i)\cdot\left(\frac{3}{2}\rho^{*}_{ij}\cdot\rho^{*}_{ij}\right)^{1/2}dL = 1, \tag{15}$$

where ρ^*_{ij} is the deviator of the modified tensor of microstresses; L_n is the length of the deformation path of the n-th cycle. For cyclically stable materials, which steel Kh18N10T, in particular, at room temperature is, Eq. (15) is transformed to the form

$$N\int\limits_{\text{cycle}}\lambda(\Delta\varepsilon_i)\cdot\left(\frac{3}{2}\rho^{*}_{ij}\cdot\rho^{*}_{ij}\right)^{1/2}dL = 1. \tag{16}$$

Thus it is possible to suggest a procedure of calculating the endurance of materials based on the concept of plastic damage realized in the form of Eqs. (5)-(7). The initial data are the standard mechanical characteristics of the material, the low cycle fatigue curves in tension—compression, and also the curve of cyclic strengthening. First we carry out a piecewise linear approximation of the cyclic diagram of strengthening and determine the field of surfaces of plastic flow whose parameters, jointly with the mechanical characteristics of the material, are entered into the program for calculating the damage per cycle Ω_{cycle}. The surfaces of plastic flow are described either by Mises' equation or by Trusk's equation. The calculation of Ω_{cycle} is carried out according to the multisurface theory of plastic flow [10]. The function λ is determined by "tuning" of the calculation program to the regime of tension—compression (λ is calculated for several levels of $\Delta\varepsilon_i$, and on this basis the dependence λ—$\Delta\varepsilon_i$ is plotted). The value of the endurance curve of pure torsion makes it possible to calculate the weight coefficient k. After that the program is prepared for the calculation of endurance for arbitrary loading paths (within a cycle).

The nature of the coincidence of the theoretical and experimental data can be judged from the results presented in Table 1. It can be seen from the table that for the energy approach and the concept of plastic damage to the material, the standard errors of the methods are practically equal to each other, amounting to about 8%, whereas for an equation of the Coffin–Manson type they amount to 30%.

The good description of endurance under complex low cycle loading was attained in the first place on account of the high degree of adequacy of the equations of the multisurface theory of plastic flow to the experimental data.

Some authors [19-21] analyzed the applicability of some equations of state to the description of cyclic plasticity. Thus the following three formulations of the theory of plastic flow with kinematic strengthening [19] were described:

1. transfer of Mises' yield surface in accordance with Prager's condition of strengthening;
2. transfer of Trusk's yield surface in accordance with Ziegler's condition of strengthening;
3. transfer of Trusk's yield surface in accordance with the calculation of Mrous' condition of strengthening within Trusk's steady limit surface.

The authors note that only in the last formulation, i.e., the multisurface formulation of the theory of plastic flow, is there good agreement between the experimental and the theoretical data. The authors of [20] examined the applicability of the single-surface theory of plasticity with kinematic and isotropically kinematic strengthening to the description of cyclic plasticity when there are out-of-phase strain cycles. As a result of the investigations it was concluded that both models are inapplicable to processes of complex cyclic loading. It follows from another publication [21] that for nonproportional cycling Mrous' model of kinematic strengthening yields more accurate results than Ziegler's model.

Thus it may be concluded that the single-surface theories of plastic flow are inapplicable in nonproportional cyclic loading whereas Mrous' model, which relates to the multisurface theories of plasticity, yields good agreement between theoretical and experimental results. Further development of the methods of determining the endurance of structural materials operating under conditions of nonproportional low cycle loading is closely bound up with the use of more developed multisurface models of plastic flow [22-24].

Conclusions

1. As a result of experimental investigations it was shown that the application of equations of the Coffin–Manson type under conditions of nonproportional cyclic loading may lead to substantial errors in the evaluation of the endurance of a material. For some regimes of complex loading the number of cycles to failure came out four or five times higher when calculated by Eq. (9).

2. The energy approach and the concept of damage cumulation show good agreement between theoretical and experimental results, and they may therefore be recommended for the stress analysis of structures operating under conditions of complex low cycle loading.

3. The multisurface theory of plastic flow in the form suggested by Garud [10] made it possible to describe qualitatively and with a sufficient degree of accuracy also quantitatively the cyclic strain diagrams of steel Kh18N10T with nonproportional loading paths.

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