RESONANCE VIBRATIONS OF TURBINE ROTORS WITH AN ANNULAR SHROUD.

REPORT NO. 1. THEORETICAL MODEL

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The reliable operation of shrouded blading rings in turbines is assured by proper tightening of the flanges of the blade assembly. This condition can be met by preliminary twisting of the blades during construction and as a result of the centrifugal forces developed during service. Under these circumstances, all of the flanges form a closed ring.

However, in reality, the need for machining tolerances and wear of the contact surfaces of the flanges during operation may result in appreciable scatter of contact conditions for the flanges. In this case, either mutual slip of the flanges may be completely absent or gaps may develop between individual pairs of flanges. Also, individual blades have different natural frequencies, i.e., the rotor blades may be mistuned.

These factors cause the rotor to deviate from strict rotational symmetry, which is usually taken into account in turbine design. As a result, there is some scatter of the resonance amplitudes of the blades [1, 2]. It is therefore important, in designing highly reliable turbines, to establish the effect of the character of asymmetry of the blading ring of the rotor on laws governing the formation of the resonance-amplitude scatter of the blades.

In light of the complexity of experimentally determining these laws, analytical investigation becomes particularly important. One step in performing such an investigation is choosing a theoretical model. The model chosen should reflect the basic properties of the object of investigation.

Figure 1a present a diagram of a theoretical model of a turbine rotor. It is assumed that the disk is rigid and that the blades are connected to each other through the disk. Each blade is modeled by a subsystem with one degree of freedom. The natural frequency of this subsystem $p_j = \sqrt{k_j/M_j}$ ($j = 1, 2, \ldots, N$) corresponds to a certain bending mode of blade vibration, where k_j and M_j are the corrected stiffness coefficient and mass of the j-th blade.

We will examine a blade coupling made by means of a flanged shroud. Since the contact surfaces of the flanges may undergo relative displacements during cyclic deformation of the blades, energy will be lost to friction. This and the fact that the flanges possess a certain compliance make it necessary in the general case to consider the elastic–dissipative coupling of the blades through the shroud.

The two types of shroud coupling most often realized under service conditions are analyzed when studying the vibrations of shrouded rotors [1, 3, 4]. These couplings are as follows:

1) guaranteed flange tightness in which the contact surfaces do not undergo mutual displacement;

2) guaranteed flange tightness in which the mutual displacements of the contact surfaces are developed in character and the relative energy dissipation due to friction depends only slightly on the amplitude of the displacements.

Blade coupling is characterized by a continuous elastic zone in the first case, while friction losses may occur in the joints between the flanges in the second case. In each of these states, the system is close to linear. These features also dictate the choice of theoretical model for an element of the blade coupling: a model with a viscously damped absorber. Here, as noted above, allowance should be made for the probability of the development of a third state of blade interaction — the presence of gaps between individual blades of the ring at the flanges.

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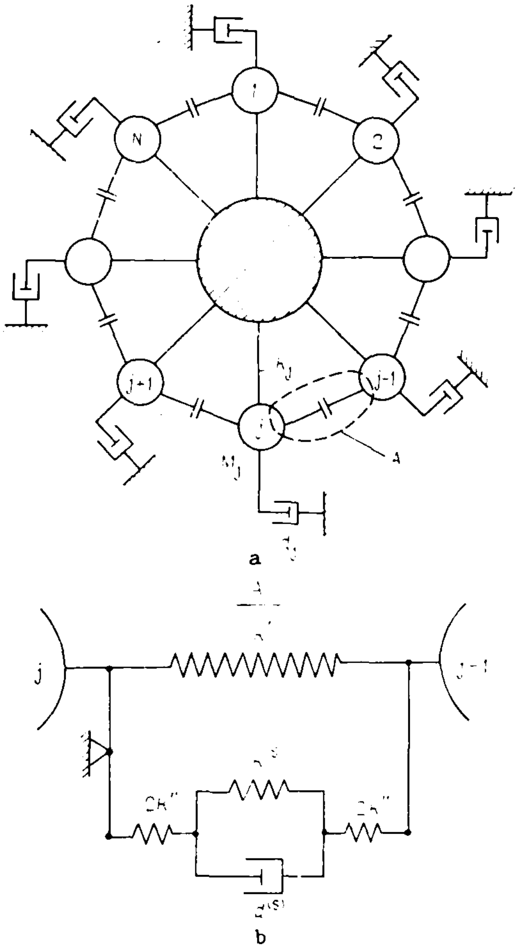

Fig. 1. Diagram of theoretical model of a turbine blade with an annular shroud coupling of the blades (a) and model of an elastic-dissipative element of the blade coupling (b).

The model proposed for an elastic-dissipative element of the coupling of adjacent blades (Fig. 1b) makes it possible to describe potential states of blade interaction.

First State. The shroud is represented as an elastic coupling between blades ($d^{(s)}_{j,j\pm1}\equiv0$). We will examine cophasal and antiphase vibrations of the blades as limiting (maximum and minimum phase difference) cases of their combined vibration. Since the stiffness of the given elastic coupling is different during such vibrations, the shroud element between the j-th and (j + 1)-th blades is modeled by springs with stiffness coefficients $k'_{j,j+1}$ and $2k''_{j,j+1}$. Given the same amplitude, the elastic coupling factor in such a model is equal to $k''_{j,j+1}$ in the case of cophasal vibration and $k'_{j,j+1}$ in the case of antiphase vibration. In the general case, the stiffness of the elastic coupling depends on the ratio of the amplitudes of vibration of the subsystems.

Thus, the potential strain energy of the springs is determined by the expression

$$\Pi^{(1)}_{j,j+1} = \frac{k'_{j,j+1}(x_j - x_{j+1})^2}{2} + \frac{k''_{j,j+1}(x_j - x_{j+1})^2}{2}, \tag{1}$$

where x_j is the displacement of the mass of the j-th subsystem.

Second State. In this state, along with the elastic coupling of the blades, we need to consider energy losses due to friction in the mutual displacement of the contact surfaces of the flanges.

Let us examine certain features of the deformation of the blades in the presence of such displacements. A theoretical model of paired blades with a scarf joint was described in [5].

The contact surfaces of the joint move in opposite directions during cophasal vibrations of the blades and in the same direction during antiphase vibrations. Since the amount of energy dissipated is determined by the mutual slip of the contact surfaces, this amount will be greater, the greater their relative displacement. Energy dissipation reaches a maximum during cophasal vibration of the blades. Since the shroud flange is made together with the tip of the blade, these laws also hold for displacements of the contact surfaces of the flanges.

In modeling the elastic coupling of blades for the state being discussed, it is necessary to note the following. Given the same amplitude of antiphase vibration of adjacent blades, the stiffness of the coupling remains the same as in the first case, i.e., the stiffness coefficient of the elastic coupling is equal to $k'_{j,j+1}$. In the case of cophasal vibrations, the degree of elastic coupling of the blades decreases due to the displacement of the contact surfaces of the shroud flanges, i.e., the degree of coupling should be less than $k''_{j,j+1}$. Thus, we introduce an additional spring with the stiffness coefficient $k^{(s)}_{j,j+1}$ into the model.

Proceeding on the basis of the above, we use the following expressions to determine the potential energy and the dissipative function of the elastic-dissipative element between the j-th and (j + 1)-th subsystems, which characterizes the second state:

$$\Pi^{(2)}_{j,j+1} = \frac{k'_{j,j+1}(x_j - x_{j+1})^2}{2} + \frac{\bar{k}^{(s)}_{j,j+1}(x_j + x_{j+1})^2}{2}; \quad (2)$$

$$F_{j,j+1} = \frac{d^{(s)}_{j,j+1}(\dot{x}_j + \dot{x}_{j+1})^2}{2}, \quad (3)$$

where

$$\bar{k}^{(s)}_{j,j+1} = \frac{k''_{j,j+1} \cdot k^{(s)}_{j,j+1}}{k''_{j,j+1} + k^{(s)}_{j,j+1}}.$$

In the absence of displacement of the contact surfaces of the shroud flanges, i.e., with $d^{(s)}_{j,j+1} \to \infty$, the elastic coupling factor $\bar{k}^{(s)}_{j,j+1} \to k''_{j,j+1}$. Thus, we automatically obtain the first state.

Third State. In the presence of a gap between the j-th and (j + 1)-th blades at the flanges

$$k'_{j,j+1} = k''_{j,j+1} = k^{(s)}_{j,j+1} = d^{(s)}_{j,j+1} \equiv 0.$$

It is known that circumferential nonuniformity of the gas flow is one of the main sources of excitation of resonance vibrations in turbine rotors. We therefore represent the generating force acting on the j-th subsystem in the form

$$P_j = P_0 \cos[\nu t + (j-1)\varphi_{m_e}], \quad (4)$$

where $\varphi_{m_e} = \frac{2\pi m_e}{N}$; m_e is the number of the excitation harmonic; P_0 and ν are the amplitude and frequency of the generating force.

With allowance for Eqs. (1)-(4), the equations of motion of the system will have the form:

$$M_j\ddot{x}_j + d^{(s)}_{j,j-1}\dot{x}_{j-1} + (d_j + d^{(s)}_{j,j-1} + d^{(s)}_{j,j+1})\dot{x}_j + d^{(s)}_{j,j+1}\dot{x}_{j+1} - (k'_{j,j-1} - \bar{k}^{(s)}_{j,j-1})x_{j-1} +$$
$$+ (k_j + k'_{j,j-1} + \bar{k}^{(s)}_{j,j-1} + k'_{j,j+1} + \bar{k}^{(s)}_{j,j+1})x_j -$$
$$-(k'_{j,j+1} - \bar{k}^{(s)}_{j,j+1})x_{j+1} = P_0\cos(\nu t + (j-1)\varphi_{m_e}],$$
$$j = 1, 2, \ldots, N, \quad (5)$$

where d_j is the corrected coefficient of viscous friction, reflecting the energy losses in the material of the blade tip and in the scarf joint.

Equations (5) are easily solved by the method of complex amplitudes.

The proposed theoretical model makes it possible to describe the dynamic behavior of the investigated design of turbine rotor more completely and more accurately than the discrete models proposed in other studies. For example, in [6] the shroud was regarded as a solid

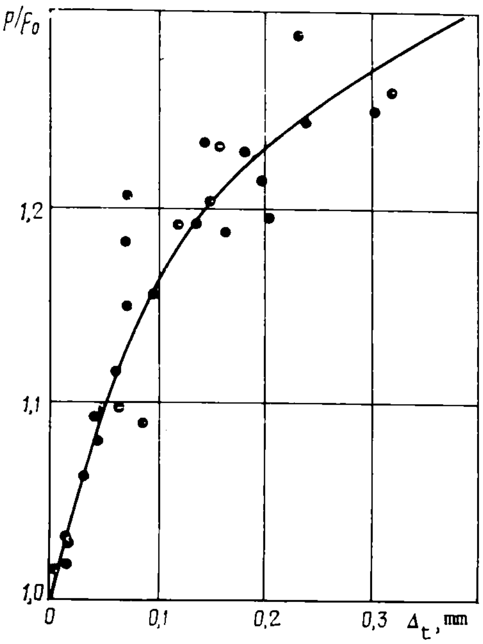

Fig. 2. Theoretical (line) and experimental (points) dependence of the natural frequency of cophasal vibrations of paired blades on the tightness at the flanges.

elastic ring, and the model did not consider possible displacements of flange contact surfaces. In the theoretical model proposed by N. K. Nevedomskii in [7], energy dissipation in the shroud was considered in such a way that it reached a maximum during antiphase vibrations of the blades. This situation is inconsistent with the energy dissipation mechanism associated with the given type of flanged shrouding.

Analytical Description of Parameters of a Blade-Coupling Element. The degree of coupling of the blades depends appreciably on the tightness Δ_t at the flanges or on the magnitude of the normal force F_t, which in turn is linearly dependent on Δ_t. This means that the stiffness coefficients $k'_{j,j\pm1}$, $k''_{j,j\pm1}$, and $k^{(s)}_{j,j\pm1}$ and the coefficient of viscous friction $d^{(s)}_{j,j\pm1}$ are functions of the tightness Δ_t.

Considering that an increase in the tightness of the flanges is accompanied by an increase in the stiffness of the system (this is manifest in an increase in the resonance frequency of cophasal vibration of paired blades), we will describe the conditions of elastic coupling of the blades through relations expressing a power dependence of the stiffness coefficients of the elastic coupling on tightness:

$$k'(\Delta_t) = \varkappa\Delta_t^k; \tag{6}$$

$$k''(\Delta_t) = \chi\Delta_t^k, \tag{7}$$

$$k^{(s)}(\Delta_t) = \gamma\Delta_t^k. \tag{8}$$

Given these relations, the natural frequency of cophasal vibration of a tuned ($k_1 = k_2 = k_0$, $M_1 = M_2 = M_0$, $d_1 = d_2 = d_0$) model of a blade pair is determined by the expression

$$p = p_0\sqrt{1 + 2\frac{\tilde{k}^{(s)}}{k_0}} = p_0\sqrt{1 + 2\frac{\eta\Delta_t^k}{k_0}}, \tag{9}$$

where

$$p_0 = \sqrt{k_0/M_0}; \qquad \eta = \frac{\chi\cdot\gamma}{\chi+\gamma}.$$

The parameters k and η can be determined from experimental data on the resonance frequency of pairs of full-size blades with different tightnesses Δ_t obtained in a study of the damping capacity and vibrational loading of pair-shrouded turbine blades [8].

Figure 2 shows the analytical dependence of the natural frequency of the paired-blade model on the tightness at the flanges, determined from Eq. (9) with k = 0.5 and $\eta/k_0 = 0.56$.

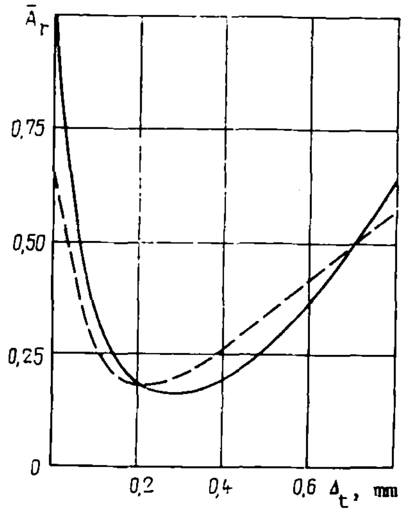

Fig. 3. Theoretical (solid line) and experimental (dashed line) dependence of the relative resonance amplitude of cophasal vibration of a tuned pair of blades on the tightness at the flanges.

Also shown are values of the natural frequency of cophasal vibration of paired blades in one rotor. Despite the experimental scatter due to the unavoidable mistuning of the blade frequencies, a comparison of the results shows the correctness of the chosen analytical dependence of the coefficients of the elastic coupling on the flange tightness.

The analytical and experimental studies established that the level of damping for the blades depends considerably on flange tightness. For each design there is an optimum tightness that will ensure maximum energy dissipation at the given vibration amplitude; i.e., the dependence of the damping characteristic and, accordingly, the maximum amplitude of resonance vibration of paired blades on tightness has an extremum.

Proceeding on this basis, we choose to express the dependence of the coefficient of viscous friction $d^{(s)}$ on tightness Δ_t in the form

$$d^{(s)} = c_1 e^{-(c_2\Delta_t)^2}\left(e^{2c_2\Delta_t} - 1\right), \tag{10}$$

where the coefficients c_1 and c_2 are determined by the parameters of the investigated blades and their mode of vibration.

In the case of cophasal vibration, the maximum relative resonance amplitude $\bar{A}_r$ of vibration of a tuned pair of blades is described by the expression

$$\bar{A}_r = \frac{A_r}{A_{0r}} = \frac{1}{\frac{p}{p_0}\left(1 + 2\frac{d^{(s)}}{d_0}\right)} = \frac{1}{\sqrt{1 + 2\frac{\eta}{k_0}\Delta_t^k}\cdot\left[1 + 2\frac{c_1}{d_0}e^{-(c_2\Delta_t)^2}\left(e^{2c_2\Delta_t} - 1\right)\right]}, \tag{11}$$

where $A_{0r} = P_0/p_0 d_0$ is the resonance amplitude of vibration of the isolated subsystem.

Choosing the values $c_1/d_0 = 1.61$ and $c_2 = 3.6$ as an example, we obtain a relation $\bar{A}_r(\Delta_t)$ which is close to the experimental value (Fig. 3) for a full-size pair of blades [8].

To determine the effect of asymmetry on resonance vibrations of the system being examined, it is necessary to first determine the resonance characteristics in the symmetrical case. For such a system, all subsystems are identical:

$$k_j = k_0; \quad M_j = M_0; \quad d_j = d_0.$$

The coupling conditions between the subsystems are also identical:

$$k'_{j,j\pm1} = k'_0; \quad k''_{j,j\pm1} = k''_0;$$
$$k^{(s)}_{j,j\pm1} = k^{(s)}_0; \qquad d^{(s)}_{j,j\pm1} = d^{(s)}_0.$$

The free vibrations of this system are determined by the following differential equations:

$$M_0\ddot{x}_j + (k_0 + 2k_0' + 2\bar{k}_0^{(s)})\, x_j - (k_0' - \bar{k}_0^{(s)})\, x_{j-1} + $$
$$+ (k_0' - \bar{k}_0^{(s)})\, x_{j+1} = 0, \quad j = 1, 2, \ldots, N. \tag{12}$$

All of the subsystems vibrate with the same amplitudes, but with a phase shift $\varphi_m = \frac{2\pi m}{N}$, where m is the number of strain waves about the circumference of the system. We seek the solution of Eqs. (12) in the form

$$x_j = A_0 e^{i[\nu t + (j-1)\varphi_m]}. \tag{13}$$

Having inserted solution (13) into system (12), we find the natural frequencies of the system:

$$\bar{p}_m = \frac{p_m}{p_0} = \sqrt{1 + 2\frac{k_0'}{k_0}(1 - \cos\varphi_m) + 2\frac{\bar{k}_0^{(s)}}{k_0}(1 + \cos\varphi_m)},$$

$$m = \begin{cases} 0, 1, \ldots, N/2 & \text{with N even;} \\ 0, 1, \ldots, (N-1)/2 & \text{with N odd.} \end{cases} \tag{14}$$

It should be noted that the natural frequencies p_m are double frequencies for $0 < m < N/2$.

Resonance of the system will occur under the condition that $p_m = \nu$. Then after inserting Eq. (13) into (5), we find the resonance amplitude of the vibrations

$$A_{0r}^{(m)} = \frac{P_0}{p_m d_0 \left[1 + 2\frac{d_0^{(s)}}{d_0}(1 + \cos\varphi_m)\right]} \tag{15}$$

or the relative resonance amplitude

$$\bar{A}_{0r}^{(m)} = \frac{A_{0r}^{(m)}}{A_{0r}} = \frac{1}{\bar{p}_m \left[1 + 2\frac{d_0^{(s)}}{d_0}(1 + \cos\varphi_m)\right]}. \tag{16}$$

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