CRITERIA RELATING TO THE FATIGUE LIFE OF STEELS SUBJECTED TO ALTERNATING LOADS UNDER CONDITIONS OF UNIAXIAL AND BIAXIAL STATIC STRAIN

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Problems concerned with increasing the fatigue life and reliability of machine parts involve estimating the strength of the latter under conditions resembling as closely as possible those encountered in practice. The majority of vital machine parts operate under a variety of combinations of static and alternating loads, and this fact is inadequately allowed for in existing methods of strength calculation.

An analysis of existing fatigue-strength criteria indicates [1, 2] that these only apply to particular cases of loading. The fundamental possibility of extending criteria satisfied experimentally under conditions of static loading to the case of fatigue loading was demonstrated in [3].

According to [4, 5], a criterion of the following form (1) agrees closely with the results of tests carried out under conditions of static fatigue (creep or "long-term" strength) in a complex state of stress, including the case of transient thermal conditions

$$h_{\Sigma} = \chi\sigma_i + (1-\chi)\sigma_1, \qquad (1)$$

where σ_i is the stress intensity, σ_1 is the maximum (with due allowance for sign) normal stress, and χ is a constant of the material.

In the case of symmetrical and symphase changes of stress, criterion (1) takes the form

$$\chi\sigma_i + (1-\chi)\sigma_1 = \sigma_{-1}, \qquad (2)$$

where σ_1, σ_i, are the peak values of the maximum normal stress and stress intensity;

$$\chi = \frac{1}{\sqrt{3}-1}\left(\frac{\sigma_{-1}}{\tau_{-1}} - 1\right). \qquad (3)$$

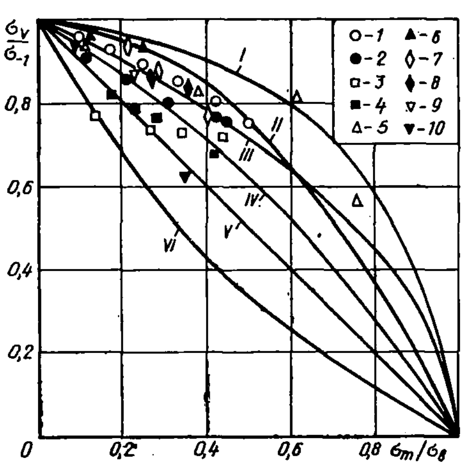

Fig. 1. Fatigue strength of construction materials under asymmetrical loading cycles: 1) chromium–nickel steel; 2) 40Kh steel; 3) St.2; 4) steel 45; 5) mild steel; 6) 1Kh18N9T; 7) 30 KhGSA (σ_B = 75 kg/mm²); 8) 30KhGSA (σ_B = 170 kg/mm²); 9) 22K; 10) ÉI-736; 1), 5) data of Forrest [11]; 2-4) data of Kaplinskii and Gryaznova [12]; 6)-10) author's data.

In order to estimate the reliability of condition (2), we used the results of Gaf and Poledra [6], obtained when testing three types of cast iron subject to simultaneous alternating torsion and bending. We found [3] that Eq. (2) and the condition of D. I. Gol'tsev, which practically coincided with each other, agreed better with experimental data.

An analysis of fatigue-strength criteria for asymmetrical loading cycles showed that most of these were based on strength criteria applicable to static loading (the greatest normal and tangential stresses, the potential energy of deformation, the Coulomb–Moore principle, etc.). Another group of criteria to some extent reflected the characteristics of alternating loads (the relationships of Kinososhvili, Birger, Oding, Heywood,

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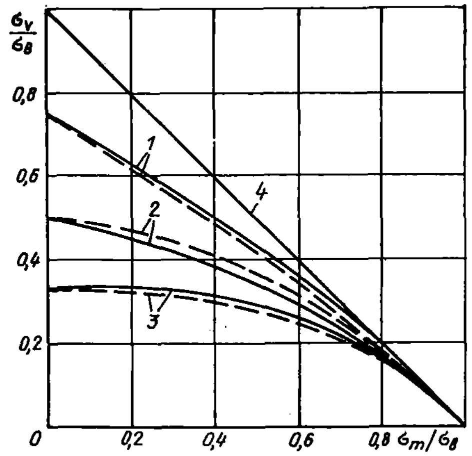

Fig. 2. Diagrams of limiting stresses according to the Heywood condition (continuous lines) and condition (5) (broken lines) 1) A = 1.5; 2) A = 2.0; 3) A = 3.0, A = 1 (line of static rupture).

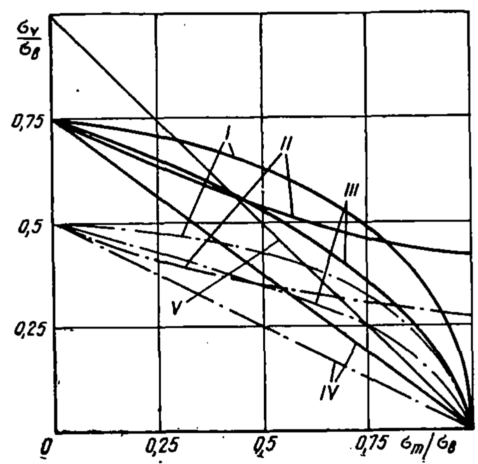

Fig. 3. Diagrams of limiting stresses. I), II), III), IV) results of Marin, Oding, Birger, and Zoderberg respectively; V) line of static rupture.

Sains, Kaplinskii, and others). The advantage of these over the first group lie in their more complete use of the serviceability of the material; however, the use of some of these criteria requires a knowledge of additional constants of the material not always at the disposal of the engineer.

Figure 1 illustrates the results of fatigue tests on a number of construction materials using asymmetrical loading cycles. For the sake of comparison, theoretical curves are also shown, based on the relationships of Goodman (V), Gerber (II), Marin (I), Birger (III), Smith (VI), and Kaplinskii (IV) in relative coordinates σ_v/σ_{-1}, σ_m/σ_B. We see from Fig. 1 that none of the criteria considered satisfactorily describes the behavior of all the materials.*

The majority of the experimental points lie between the parabola of Gerber and the Goodman line; hence the latter may be recommended for calculations in cases in which no additional data regarding the strength in an asymmetrical cycle are available. At the same time, for materials not very sensitive to the asymmetry of the cycle, this leads to a reduction in the use of their carrying capacity.

The behavior of certain materials in asymmetrical cycles is extremely complicated: none of the existing fatigue-strength criteria can describe, for example, the behavior of steels (1), (3), and (5) (see Fig. 1).

Correlation of a great deal of experimental material [8] showed that, if the surface of the metal were free from scratches and residual stresses, the fatigue strength in an asymmetrical cycle could be described to a fair accuracy by the equation

$$\frac{\sigma_v}{\sigma_B} = \left(1 - \frac{\sigma_m}{\sigma_B}\right)[A_0 + \gamma(1 - A_0)], \quad (4)$$

where $A_0 = \sigma_{-1}/\sigma_B$ is the fatigue coefficient, and γ is a function of the mean stress and tensile strength. For smooth steel samples

$$\gamma = \frac{\sigma_m}{3\sigma_B}\left(2 + \frac{\sigma_m}{\sigma_B}\right).$$

Equation (4) agrees closely with experimental data [8]; however, its generalization to the case of multiaxial loading is difficult in view of the complexity of the structure. Hence in order to determine the limiting stresses in the case of an asymmetrical cycle the following simpler equation may be proposed

$$A\frac{\sigma_v}{\sigma_B} + \left(\frac{\sigma_m}{\sigma_B}\right)^{\alpha A} = 1, \quad (5)$$

where $A = 1/A_0 = \sigma_B/\sigma_{-1}$, is a certain coefficient in general depending on the state of the surface and the sensitivity of the metal to stress concentrations; to a first approximation we may take $\alpha = 1$.

The results of a calculation based on this equation (with $\alpha = 1$) are presented in Fig. 2 in the form of a diagram of limiting stresses for three values of A. Diagrams based on Eq. (4) for the same conditions are also shown.

*The relationships proposed by other authors cannot be plotted on a single graph in these coordinates for all materials, as they involve additional material constants.

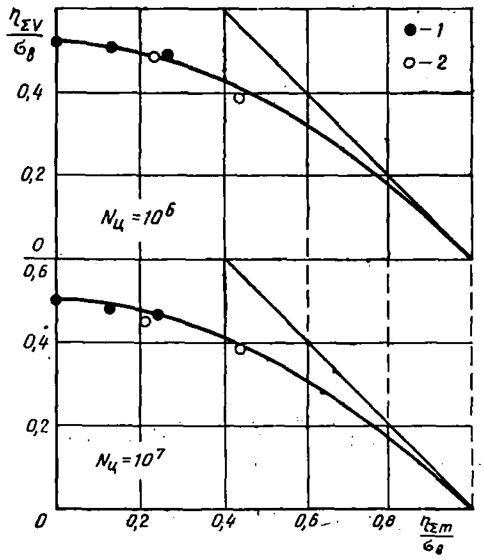

Fig. 4. Diagrams of limiting stresses for 1Kh18N9T steel; 1) for uniaxial elongation (bending with rotation plus elongation); 2) for a complex state of stress (bending with rotation plus internal pressure).

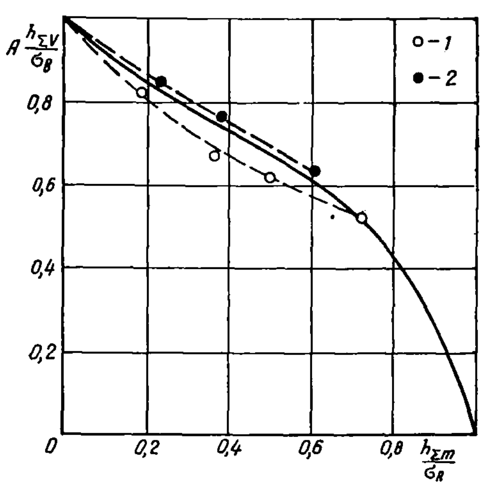

Fig. 5. Diagrams of limiting stresses: 1) 30KhGSA; 2) steel 45.

For comparison, other earlier-proposed approximations for diagrams of limiting stresses with A = 2 (broken lines) and A = 1.5 (continuous lines) are shown in Fig. 3. From Fig. 3 it can be seen that, for large values of the mean stresses, and also for limited fatigue lives, in the case of $\sigma_{-1} \to \sigma_B$, the curves describing the strength conditions of Oding, Birger, and Marin pass outside the region limited by the straight line

$$\frac{\sigma_v}{\sigma_B} + \frac{\sigma_m}{\sigma_B} = 1,$$

i.e., $\sigma_{max} = \sigma_v + \sigma_m$ exceeds the static tensile strength, which is unrealistic for ordinary conditions of testing. Hence in using any criterion it is important to delineate its range of applicability.

Generalizing Eq. (5) to the case of a complex stressed state, we may write

$$A\frac{h_{\Sigma v}}{\sigma_B} \pm \left(\frac{h_{\Sigma m}}{\sigma_B}\right)^{\alpha A} = 1, \qquad (6)$$

where $h_{\Sigma v}$ is the equivalent amplitude of the stresses, $h_{\Sigma m}$ is the equivalent mean stress.

If we base our stress determination on the structure of criterion (1) we obtain

$$\begin{aligned} h_{\Sigma v} &= \chi'\sigma_{iv} + (1-\chi')\,\sigma_{1v}; \\ h_{\Sigma m} &= \chi''\sigma_{im} + (1-\chi'')\,\sigma_{1m}, \end{aligned} \qquad (7)$$

where by analogy with (3)

$$\chi' = \frac{1}{\sqrt{3}-1}\left(\frac{\sigma_{-1}}{\tau_{-1}} - 1\right); \quad \chi'' = \frac{1}{\sqrt{3}-1}\left(\frac{\sigma_B}{\tau_B} - 1\right). \qquad (8)$$

It was mentioned in [3] that $\chi' \approx \chi'' \approx \chi$, and, depending on the particular mechanical characteristics available (τ_{-1}, σ_{-1} or τ_B, σ_B) either of the expressions (8) may be used to determine the parameter χ.

A calculation based on criterion (6) gives excellent agreement for ductile materials. Figure 4 shows the theoretical curve (continuous line) based on criterion (6) and the experimental points obtained for 1Kh18N9T steel. The tests were carried out on thin-walled tubular samples with a combination of alternating bending stresses and steady tensile stresses of the uniaxial and biaxial type. The method of conducting the tests and the experimental apparatus were described in [9].

The tests carried out in this apparatus revealed the great sensitivity of certain materials (30KhGSA, ÉI-435, ÉI-736) to biaxial strain, particularly for comparatively small values of the tensile stresses. None of the criteria considered predicts such a substantial reduction in fatigue strength for biaxial elongation.

Of the existing criteria, that of Birger agrees most closely with experimental data.

If we base our consideration on the structure of criterion (6), this criterion gives an excellent description of the behavior of 30KhGSA steel in the case of biaxial strain for

$$\alpha = \frac{A+1}{A(A-1)}\left(\frac{h_{\Sigma m}}{\sigma_B}\right)^2 + \frac{1}{A}. \qquad (9)$$

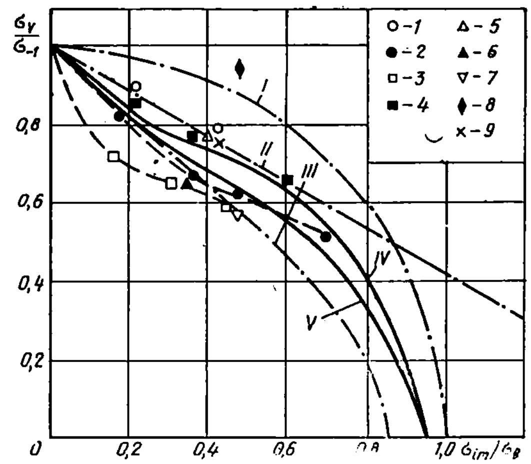

Fig. 6. Fatigue strength of construction materials for a complex state of stress; 1) 1Kh18N9T; 2) 30KhGSA (σ_B = 75 kg/mm²) 3) 30KhGSA (σ_B = 170 kg/mm²) ; 4) steel 45; 5) ÉI-268; 6) ÉI-736; 7) ÉI-435; 8) ÉI-961; 9) 22K.

Figure 5 shows the theoretical curve (continuous line) in coordinates of $h_{\Sigma v}/\sigma_B$, $h_{\Sigma m}/\sigma_B$ based on Eqs. (6) and (9), together with experimental data obtained by the authors for 30KhGSA steel (σ_B = 75 kg/mm²) and steel 45.

In order to construct the theoretical curves (for the case of complex stress here considered) by reference to the various criteria and to superimpose experimental points on these, we must take generalized coordinates. Such coordinates are σ_{im}/σ_B. For our present case $\sigma_{iv}/\sigma_{-1} = \sigma_{bend}/\sigma_{-1} = \sigma_v/\sigma_{-1}$; $\sigma_{im}/\sigma_B = \sqrt{3}\sigma_{oc}/\sigma_B$.

Transforming, for example, the criteria of Zoderberg and Birger, we have in the new coordinates:

Zoderberg

$$\frac{\sigma_v}{\sigma_{-1}} = 1 - \frac{\sigma_{im}}{\sigma_B}\frac{1}{\sqrt{3}}; \qquad (10)$$

Birger

$$\sqrt{\left(\frac{\sigma_v}{\sigma_{-1}}\right)^2 + \frac{1}{12}\left(\frac{\sigma_{im}}{\sigma_B}\right)^2} = 1 - \frac{\sqrt{3}}{2}\frac{\sigma_{im}}{\sigma_B}. \qquad (11)$$

The other criteria may be transformed analogously.

The theoretical curves obtained from the relationships of Marin (I), Zoderberg (II), Birger (III), and the proposed relationship (6) with A = 2 and 3, are presented in Fig. 6, together with experimental results for the complex stressed state in coordinates of σ_v/σ_{-1}, σ_{im}/σ_B.

The proposed relationship (6) is plotted for the cases of A = 2 and A = 3 with $\sigma_{-1}/\tau_{-1} = 1.5$. It should be noted that the general appearance of the theoretical curves plotted on the basis of (6), with α determined from (9), corresponds to the general appearance of the experimental curves presented by Heywood [8] for samples containing stress raisers.

CONCLUSIONS

1. We have analyzed existing criteria relating to fatigue strength for asymmetrical loading cycles under conditions of uniaxial and biaxial strain. We have shown that at present there are no criteria really suitable for describing the behavior of all materials.
2. We have proposed a fatigue-strength criterion which gives an excellent description of the behavior of ductile steels for asymmetrical loading cycles under conditions of uniaxial and biaxial strain.
3. We have shown that the proposed criterion may be modified to describe the behavior of materials sensitive to biaxial strain.

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