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THERMAL ELASTICITY OF A HOLLOW SPHERE WITH TEMPERATURE-DEPENDENT THERMAL CONDUCTIVITY

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Consider a hollow sphere free of external loading, the inner surface of which ($r = R_1$) is subjected to heating by a heat flux of constant density q, while the outer surface ($r = R_2$) is maintained at a constant temperature T_0.

As was indicated earlier [1], the temperature dependence of Young's modulus E, Poisson's ratio ν, and the coefficient of thermal expansion α_t is immaterial for many crystalline dielectrics, and these quantities can be considered constant. The coefficient of thermal conductivity λ_t depends heavily on temperature.

To determine the steady temperature field t(r), the radial displacement u(r), and the temperature – stress components σ_{rr}, $\sigma_{\varphi\varphi} = \sigma_{\theta\theta}$, we have in this case the following relationships:

$$\frac{1}{r^2} \cdot \frac{d}{dr}\left[r^2 \lambda_t(t) \frac{dt}{dr}\right] = 0; \tag{1}$$

$$\sigma_{rr} = \frac{E}{(1+\nu)(1-2\nu)}\left[(1-\nu)\frac{du}{dr} + 2\nu\frac{u}{r} - \alpha_t(1+\nu)t(r)\right];$$

$$\sigma_{\varphi\varphi} = \sigma_{\theta\theta} = \frac{E}{(1+\nu)(1-2\nu)}\left[\frac{u}{r} + \nu\frac{du}{dr} - \alpha_t(1+\nu)t(r)\right]; \tag{2}$$

$$\frac{d}{dr}\left[\frac{1}{r^2}\frac{d}{dr}(r^2 u)\right] = \frac{\alpha_t(1+\nu)}{1-\nu}\frac{d}{dr}[t(r)], \tag{3}$$

as well as boundary conditions

$$q = \left[-\lambda_t(t)\frac{dt}{dr}\right]_{r=R_1}; \quad T_0 = t|_{r=R_2}; \tag{4}$$

$$\sigma_{rr}|_{r=R_1} = 0; \quad \sigma_{rr}|_{r=R_2} = 0. \tag{5}$$

If the law governing the change in the coefficient of thermal conductivity with temperature takes the form [1]

$$\lambda_t(t) = \frac{k}{t}; \quad k = \text{const}, \tag{6}$$

one obtains from Eq. (1) with consideration given to Eq. (6) and Eq. (4) this expression for determination of the steady temperature field:

$$t(r) = T_0 e^{\frac{\alpha}{R_2}\frac{R_2 - r}{r}}, \tag{7}$$

where

$$\alpha = \frac{qR_1^2}{k}.$$

Integrating Eq. (3) with consideration given to Eq. (7), we have

$$u = \frac{\alpha_t(1+\nu)}{(1-\nu)r^2}\Theta(r) + \frac{Cr}{3} + \frac{D}{r^2}. \tag{8}$$

Here

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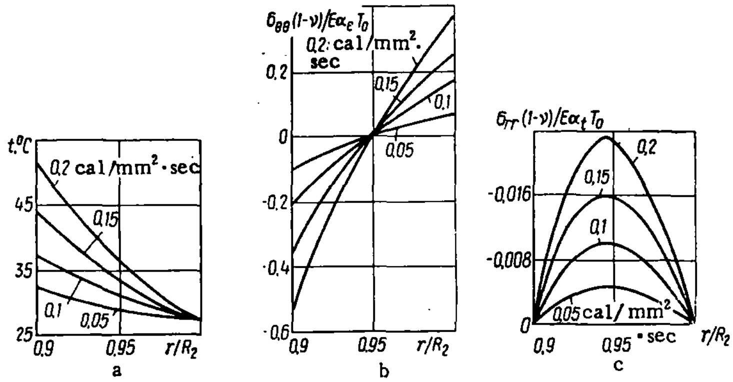

Fig. 1. Distribution of temperature (a), and circumferential (b) and radial (c) stresses as function of r/R_2 with outer surface at temperature of 27°C and with internal surface heated by heat flux.

$$\Theta(r) = \frac{1}{6} T_0 e^{-\frac{\alpha}{R_2}} \left\{ e^{\frac{\alpha}{r}} [2r^3 + r\alpha(r+\alpha)] - \alpha^3 \mathrm{E}i\left(\frac{\alpha}{r}\right) \right\},$$

where $\mathrm{E}i(\alpha/r)$ is an integral exponential function.

Solving Eq. (8) for boundary conditions (5), we can determine the constants of integration

$$C = \frac{6\alpha_t(1-2\nu)}{(1-\nu)(R_2^3-R_1^3)}[\Theta(R_2)-\Theta(R_1)];$$
$$D = \frac{\alpha_t(1+\nu)}{(1-\nu)(R_2^3-R_1^3)}[R_1^3\Theta(R_2)-R_2^3\Theta(R_1)]. \qquad (9)$$

Substituting relationship (8) in Eq. (2) with consideration given to Eq. (9), one obtains the desired temperature stresses

$$\sigma_{rr} = \frac{2E\alpha_t}{1-\nu}\left[\frac{\Theta(R_2)-\Theta(R_1)}{R_2^3-R_1^3} - \frac{R_1^3\Theta(R_2)-R_2^3\Theta(R_1)}{r^3(R_2^3-R_1^3)} - \frac{\Theta(r)}{r^3}\right];$$
$$\sigma_{\varphi\varphi} = \sigma_{\theta\theta} = \frac{\alpha_t E}{1-\nu}\left\{\frac{2[\Theta(R_2)-\Theta(R_1)]}{R_2^3-R_1^3} + \frac{R_1^3\Theta(R_2)-R_2^3\Theta(R_1)}{r^3(R_2^3-R_1^3)} + \frac{\Theta(r)}{r^3} - t(r)\right\}. \qquad (10)$$

The temperature field and field-induced temperature stresses in a sphere made of 98% polycrystalline aluminum oxide Al_2O_3 with $k = 2.8$ cal/mm · sec · deg were computed from Eqs. (7) and (10). The results of the computations are presented in Fig. 1 from which it follows that the maximum temperature is observed on the inner surface of the sphere. Radial stresses reach a maximum value when $r/R_2 = 0.945$, and circumferential stresses are maximum on the inner surface of the sphere; in this case, the latter exceed the maximum radial stresses by a factor of ≈20.

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