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Viscoplastic properties of asphaltic bitumen

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With 3 figures and 3 tables

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Introduction

This paper deals with the experimental and theoretical investigation of the rheological behaviour of the asphaltic bitumen represented by the relation between shear stress and shear-strain rate.

All bituminous surfacing materials used in the construction and maintenance of the highway system, change their properties with the time in one way or another under the combined action of weather and traffic. The viscosity of the binder varies with temperature. The binder becomes softer in warm weather and harder in cold weather. The traffic forces reduce the life of the binder, which affects the degree of the resistance to deformation.

These effects lead to the investigation of the rheological behaviour of asphaltic bitumen and to a modest improvement in traditional materials.

On base of test results, the author has obtained the non-linear rheological relation of the second degree, which is theoretically explicated by a complex rheological model formed by an infinity of rheological groups consisting of *Newton*ian dashpots and *Saint-Venant* friction elements in a parallel connection. Other rheological models for asphaltic bitumen have been presented by *M. Reiner* his the book (1).

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Experimental investigation

The tests are concerned with the rheological properties of the asphaltic bitumen A 80, with particular attention to the effect of load and temperature on the viscosity.

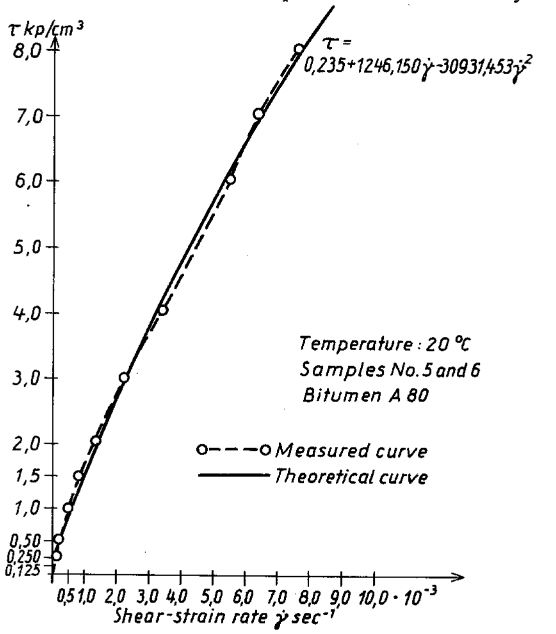

Fig. 1. Relation between $\dot{\gamma}$ and τ at a temperature 20 °C

Table 1. The measured values of strain rate at 20 °C for samples 5 and 6

Measurement by *Höppler* consistometer — Temperature: 20 °C
Samples: No. 5 and 6 A 80 — Ball: $D = 16$ mm, $F = 2$ cm²

Load G kp	τ kp/cm²	Distance s_1 cm	Distance s_2 cm	$s = \frac{s_1+s_2}{2}$	Time t_1 sec	Time t_2 sec	$t = \frac{t_1+t_2}{2}$	Shear strain rate $\dot{\gamma} = s/t$
0.25	0.125	0.002	0.0025	0.002	30	30	30	$0.066 \cdot 10^{-3}$
0.50	0.250	0.004	0.004	0.004	30	30	30	$0.132 \cdot 10^{-3}$
1.00	0.500	0.008	0.009	0.008	30	30	30	$0.264 \cdot 10^{-3}$
2.00	1.000	0.016	0.016	0.016	30	30	30	$0.528 \cdot 10^{-3}$
3.00	1.500	0.024	0.026	0.025	30	30	30	$0.833 \cdot 10^{-3}$
4.00	2.000	0.039	0.041	0.040	30	30	30	$1.330 \cdot 10^{-3}$
6.00	3.000	0.070	0.066	0.068	30	30	30	$2.260 \cdot 10^{-3}$
8.00	4.000	0.102	0.106	0.104	30	30	30	$3.460 \cdot 10^{-3}$
10.00	5.000	0.130	0.130	0.130	30	30	30	$4.330 \cdot 10^{-3}$
12.00	6.000	0.162	0.168	0.165	30	30	30	$5.500 \cdot 10^{-3}$
14.00	7.000	0.185	0.195	0.190	30	30	30	$6.330 \cdot 10^{-3}$
16.00	8.000	0.220	0.240	0.230	30	30	30	$7.660 \cdot 10^{-3}$

Various samples of the asphaltic bitumen A 80, have been tested by means of the *Höppler* consistometer at various temperatures 10 °C, 20 °C and 30 °C. From these tests, the relations between the shear stress and shear-strain rate have been determined (see table 1).

In the most general case, the author has obtained the curve, which is shown in fig. 1.

The experimental curve has been replaced by means of the procedure of least squares by the parabola of the second degree, which is expressed by the following relation:

$$\tau = \tau_p + \eta \dot{\gamma} + \chi \dot{\gamma}^2 . \qquad [1]$$

Using the procedure of least squares, the author has determined the equilibrated values of τ_p^*, η^* and χ^* from the following set of equations:

$$\tau_p^* = \frac{(S_2 S_4 - S_3^2) \sum_{i=1}^{n} \tau_i + (S_2 S_3 - S_1 S_4) \sum_{i=1}^{n} \dot{\gamma}_i \tau_i + (S_1 S_2 - n S_3) \sum_{i=1}^{n} \dot{\gamma}_i^2 \tau_i}{\Delta}, \qquad [2]$$

$$\eta^* = \frac{(S_2 S_3 - S_1 S_4) \sum_{i=1}^{n} \tau_i + (n S_4 - S_2^2) \sum_{i=1}^{n} \dot{\gamma}_i \tau_i + (S_1 S_2 - n S_3) \sum_{i=1}^{n} \dot{\gamma}_i^2 \tau_i}{\Delta}, \qquad [3]$$

$$\chi^* = \frac{(S_1 S_3 - S_2^2) \sum_{i=1}^{n} \tau_i + (S_1 S_2 - n S_3) \sum_{i=1}^{n} \dot{\gamma}_i \tau_i + (n S_2 - S_1^2) \sum_{i=1}^{n} \dot{\gamma}_i^2 \tau_i}{\Delta}, \qquad [4]$$

where

$$\Delta = n(S_2 S_3 - S_3^2) + S_1(S_2 S_3 - S_1 S_4) + S_2(S_1 S_3 - S_2^2) . \qquad [5]$$

Taking the measured values from table 1, we may perform the corresponding calculations by the procedure of least squares as shown in tables 2 and 3. According to eq. [5], we find from the sums in table 2 the numerical value of $\Delta = 280207{,}786 \cdot 10^{-18}$.

By the preceding mathematical method of least squares, the author has determined the values of the equilibrated yield limit in simple shear $\tau_p^* = 0{,}235$ kp/cm², the equilibrated coefficient of shear viscosity $\eta^* = 1246{,}158$ kpsec/cm² and the equilibrated coefficient of shear viscosity of the second order $\chi^* = -30931{,}458$ kpsec²/cm².

Table 2. Calculated values

i	$\dot{\gamma}_i$	$\dot{\gamma}_i^2$	$\dot{\gamma}_i^3$	$\dot{\gamma}_i^4$	τ_i	$\dot{\gamma}_i \tau_i$	$\dot{\gamma}_i^2 \tau_i$
1	$0.066 \cdot 10^{-3}$	$0.004 \cdot 10^{-6}$	$0.00026 \cdot 10^{-9}$	$0.00002 \cdot 10^{-12}$	0.125	$0.008 \cdot 10^{-3}$	$0.001 \cdot 10^{-6}$
2	$0.132 \cdot 10^{-3}$	$0.017 \cdot 10^{-6}$	$0.0023 \cdot 10^{-9}$	$0.0003 \cdot 10^{-12}$	0.250	$0.033 \cdot 10^{-3}$	$0.004 \cdot 10^{-6}$
3	$0.264 \cdot 10^{-3}$	$0.070 \cdot 10^{-6}$	$0.018 \cdot 10^{-9}$	$0.005 \cdot 10^{-12}$	0.50	$0.132 \cdot 10^{-3}$	$0.035 \cdot 10^{-6}$
4	$0.528 \cdot 10^{-3}$	$0.279 \cdot 10^{-6}$	$0.147 \cdot 10^{-9}$	$0.078 \cdot 10^{-12}$	1.00	$0.528 \cdot 10^{-3}$	$0.279 \cdot 10^{-6}$
5	$0.833 \cdot 10^{-3}$	$0.694 \cdot 10^{-6}$	$0.578 \cdot 10^{-9}$	$0.481 \cdot 10^{-12}$	1.50	$1.250 \cdot 10^{-3}$	$1.041 \cdot 10^{-6}$
6	$1.330 \cdot 10^{-3}$	$1.769 \cdot 10^{-6}$	$2.353 \cdot 10^{-9}$	$3.129 \cdot 10^{-12}$	2.00	$2.660 \cdot 10^{-3}$	$3.538 \cdot 10^{-6}$
7	$2.260 \cdot 10^{-3}$	$5.108 \cdot 10^{-6}$	$11.544 \cdot 10^{-9}$	$26.089 \cdot 10^{-12}$	3.00	$6.780 \cdot 10^{-3}$	$15.324 \cdot 10^{-6}$
8	$3.460 \cdot 10^{-3}$	$11.972 \cdot 10^{-6}$	$41.423 \cdot 10^{-9}$	$143.324 \cdot 10^{-12}$	4.00	$13.840 \cdot 10^{-3}$	$47.888 \cdot 10^{-6}$
9	$4.330 \cdot 10^{-3}$	$18.749 \cdot 10^{-6}$	$81.183 \cdot 10^{-9}$	$351.522 \cdot 10^{-12}$	5.00	$21.650 \cdot 10^{-3}$	$93.745 \cdot 10^{-6}$
10	$5.500 \cdot 10^{-3}$	$30.250 \cdot 10^{-6}$	$166.375 \cdot 10^{-9}$	$915.063 \cdot 10^{-12}$	6.00	$33.000 \cdot 10^{-3}$	$181.500 \cdot 10^{-6}$
11	$6.330 \cdot 10^{-3}$	$40.069 \cdot 10^{-6}$	$253.637 \cdot 10^{-9}$	$1605.522 \cdot 10^{-12}$	7.00	$44.310 \cdot 10^{-3}$	$280.483 \cdot 10^{-6}$
12	$7.660 \cdot 10^{-3}$	$58.676 \cdot 10^{-6}$	$449.458 \cdot 10^{-9}$	$3442.848 \cdot 10^{-12}$	8.00	$61.280 \cdot 10^{-3}$	$469.408 \cdot 10^{-6}$
Σ	$32.693 \cdot 10^{-3}$	$167.657 \cdot 10^{-6}$	$1006.719 \cdot 10^{-9}$	$6488.061 \cdot 10^{-12}$	38.375	$185.471 \cdot 10^{-3}$	$1093.246 \cdot 10^{-6}$
	S_1	S_2	S_3	S_4			

Table 3. Theoretical values of shear stress

1	2	3	4	5
$\dot{\gamma}$	$\dot{\gamma}^2$	$\eta^* \dot{\gamma}$	$\chi^* \dot{\gamma}^2$	$\tau_p + 3 + 4 = \tau$
$0.0 \cdot 10^{-3}$	–	–	–	0.235
$0.5 \cdot 10^{-3}$	$0.25 \cdot 10^{-6}$	$623.079 \cdot 10^{-3}$	$-7732.863 \cdot 10^{-6}$	0.851
$1.0 \cdot 10^{-3}$	$1.00 \cdot 10^{-6}$	$1246.158 \cdot 10^{-3}$	$-30931.453 \cdot 10^{-6}$	1.451
$2.0 \cdot 10^{-3}$	$4.00 \cdot 10^{-6}$	$2492.316 \cdot 10^{-3}$	$-123725.812 \cdot 10^{-6}$	2.604
$4.0 \cdot 10^{-3}$	$16.00 \cdot 10^{-6}$	$4984.632 \cdot 10^{-3}$	$-494903.248 \cdot 10^{-6}$	4.730
$6.0 \cdot 10^{-3}$	$36.00 \cdot 10^{-6}$	$7476.948 \cdot 10^{-3}$	$-1113532.308 \cdot 10^{-6}$	6.598
$8.0 \cdot 10^{-3}$	$64.00 \cdot 10^{-6}$	$9969.264 \cdot 10^{-3}$	$-1979612.992 \cdot 10^{-6}$	8.225

Introducing the calculated values of τ_p^*, η^* and χ^* into eq. [1], we get the relation:

$$\tau = 0{,}235 + 1246{,}158\,\dot{\gamma} - 30931{,}453\,\dot{\gamma}^2, \qquad [6]$$

which is the rheological equation of the empirical curve.

Theoretical considerations

The rheological behaviour of the asphaltic bitumen represented by the curve in fig. 1 may be characterized by a theoretical rheological model of a complex viscoplastic body, which is shown in fig. 2. This model consists of many *Newton*ian dashpots and *Saint-Venant* friction elements.

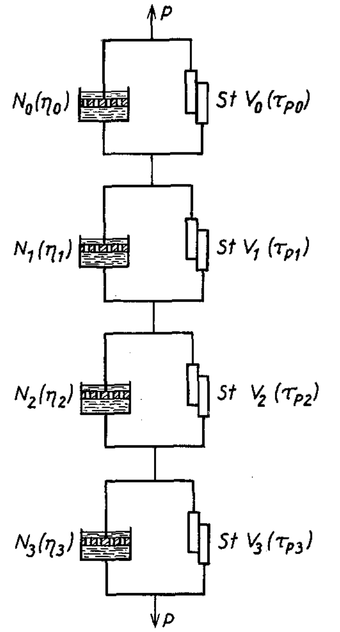

Fig. 2. Rheological model of a complex viscoplastic body

Such a model with many groups of *Newton*ian and *Saint-Venant* elements in parallel connection gives a fraction line with linear sections as shown in fig. 3.

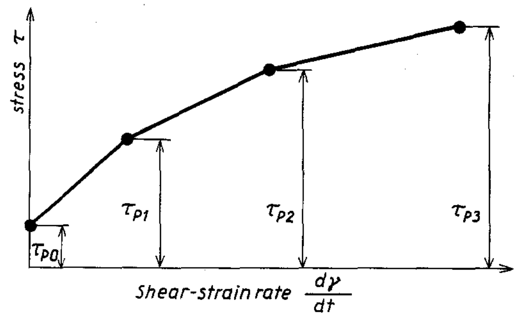

Fig. 3. Strain rate-stress diagram with linear sections

Denoting by $\tau_{P0}, \tau_{P1}, \tau_{P2}, \tau_{P3}$, etc. the individual yield limits, we may express the rheological behaviour corresponding to individual linear sections of the strain rate-stress diagram by the following equations:

$$\frac{d\gamma}{dt} = \frac{\tau - \tau_{P0}}{\eta_0}, \qquad [7]$$

$$\frac{d\gamma}{dt} = \frac{\tau - \tau_{P0}}{\eta_0} + \frac{\tau - \tau_{P1}}{\eta_1}, \qquad [8]$$

$$\frac{d\gamma}{dt} = \frac{\tau - \tau_{P0}}{\eta_0} + \frac{\tau - \tau_{P1}}{\eta_1} + \frac{\tau - \tau_{P2}}{\eta_2}, \qquad [9]$$

$$\frac{d\gamma}{dt} = \frac{\tau - \tau_{P0}}{\eta_0} + \frac{\tau - \tau_{P1}}{\eta_1} + \frac{\tau - \tau_{P2}}{\eta_2} + \frac{\tau - \tau_{P3}}{\eta_3}, \qquad [10]$$

where $\eta_0, \eta_1, \eta_2, \eta_3$ are the coefficients of shear viscosity in the individual sections, which correspond to the *Newton*ian dashpots in the rheological model shown in fig. 2.

For the n-th section, we have:

$$\frac{d\gamma}{dt} = \sum_{k=0}^{n} \frac{\tau - \tau_{Pk}}{\eta_k}. \qquad [11]$$

From the preceding equations, we may express the shear stress values in the individual sections. We find

$$\tau = \eta_0 \frac{d\gamma}{dt} + \tau_{P0}, \qquad [12]$$

$$\tau = \frac{\frac{d\gamma}{dt} + \frac{\tau_{P0}}{\eta_0} + \frac{\tau_{P1}}{\eta_1}}{\frac{1}{\eta_0} + \frac{1}{\eta_1}}, \qquad [13]$$

$$\tau = \frac{\frac{d\gamma}{dt} + \frac{\tau_{P0}}{\eta_0} + \frac{\tau_{P1}}{\eta_1} + \frac{\tau_{P2}}{\eta_2}}{\frac{1}{\eta_0} + \frac{1}{\eta_1} + \frac{1}{\eta_2}}, \qquad [14]$$

$$\tau = \frac{\frac{d\gamma}{dt} + \frac{\tau_{P0}}{\eta_0} + \frac{\tau_{P1}}{\eta_1} + \frac{\tau_{P2}}{\eta_2} + \frac{\tau_{P3}}{\eta_3}}{\frac{1}{\eta_0} + \frac{1}{\eta_1} + \frac{1}{\eta_2} + \frac{1}{\eta_3}}. \qquad [15]$$

For the n-th section, we obtain:

$$\tau = \frac{\frac{d\gamma}{dt} + \sum_{k=0}^{n} \frac{\tau_{Pk}}{\eta_k}}{\sum_{k=0}^{n} \frac{1}{\eta_k}}. \qquad [16]$$

If the number of groups, consisting of *Newtonian* dashpot and a *Saint-Venant* friction element, tends to the infinity, the fraction line representing the relationship between shear stress and strain rate becomes a continuous curve, which may be expressed the most simply by eq. [1].

Such considerations are in a certain manner analogous to those made for complex viscoelastic solids in papers (2) and (3).

Conclusions

In the present paper, various relationships between stress and strain rate have been derived. They are interpreted by rheological models, which may express the mechanical behaviour of the tested material.

The experimental curves are compared with theoretical relationships corresponding to various presented rheological models. From these models, the theoretical rheological equations have been derived, which may express the behaviour of bitumens exhibiting viscous and plastic properties.

Some experimental curves are evaluated by the procedure of least squares and the empirical equations of the first and second degree between stress and strain rate are found. These equations contain the equilibrated yield stresses, the coefficient of viscosity and the second-order coefficient of viscosity. The influence of temperature on various samples has been observed.

In the most general case, the bitumen exhibits viscous and plastic properties, which may be represented by rheological models consisting of viscous and plastic elements.

Summary

The paper deals with the theoretical and experimental investigation of the rheological behaviour of the asphaltic bitumen.

The results of rheological tests are evaluated by the procedure of least squares and various empirical equations between shear stress and shear-strain rate are found.

The experimental curves are compared with theoretical relationships derived on base of rheological models consisting of many groups of plastic and viscous elements.

Resumé

La présente contribution contient les résultats de recherches théoriques et expérimentales du bitume.

Les résultats des essais rhéologiques sont représentés par les équations empiriques dérivées par le procédé des carrés minimum, qui expriment les relations entre la contrainte de cisaillement et le glissement.

Les courbes expérimentales sont comparées avec les relations théoriques dérivées sur la base de modèles rhéologiques formés d'un nombre élevé des groups d'éléments plastiques et visqueux.

References

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