

# **Magnetic force microscopy signal of flux line above a semi-infinite type II-superconductor**

## **Theoretical approach**

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We have examined a single flux line in the semi-infinite type-II superconductor. The stray magnetic field of the flux line has been calculated. We have found that the vertical force exerted on a magnetic force microscopy (MFM) tip from the flux line is measurable by currently existing MFM. Two types of magnetic tips were taken into consideration, solid and thin film tips. For example, with a Cobalt film of the thickness of 100 nm and 30 nm on a tip, we found a vertical force of  $4*10^{-10}$  N and  $1.5*10^{-10}$  N, respectively. The lateral force exerted on a tip by the flux line was also calculated. The lateral force must be small enough to prevent the flux line from becoming depinned.

## **1. Introduction**

Superconductors have attracted far more attention and efforts since the discovery of high transition temperature superconductivity in the  $Ba-La-Cu$  oxide system [1]. Many scientists are currently investigating the properties of new high temperature superconductors (HTSC) using numerous experimental techniques. The electron tunneling technique has been recognized as a very powerful method for probing the superconducting state. With the invention of the STM [2] tunneling has become an experimental method with very high spatial resolution, allowing direct access to local electronic properties. Thus the STM has been applied on  $2 H - NbSe<sub>2</sub>$  superconductor by Hess et al. [3]. They imaged the Abrikosov vortex lattice and they were able to resolve the electronic fine structure of a single vortex. Similar experiments have been performed later on  $2 H - Nb_{1-x}Ta_xSe_2$  by Renner et al. [4]. Berthe et al. [5] investigated  $NbSe<sub>2</sub>$  single crystal by STM tunneling through the ferromagnetic Ni tip. They resolved the Abrikosov flux lattice and did not observed any distortion produced by the ferromagnetic tip.

However, STM experiments on HTSC often are not reproducible and can be very misleading. Degradaded surfaces, Coulomb blockade, Schottky barriers and the theoretically predicted drop in the order parameter near the surface of a HTSC, all detract from easily gaining direct access to the parameters of superconductivity with the STM's advantage of high spatial resolution. We therefore propose the magnetic force microscopy (MFM) to investigate HTSC sample below the transition temperature, especially to observe the flux line structure. Magnetic force microscopy (MFM) is a method developed on the basis of atomic force microscopy [6]. MFM consists of a tiny magnetic tip which, when placed close (typically from 10 to 500 nm) to the magnetic sample, interacts with a stray magnetic field of the sample. The force or the force gradient exerted on the scanned tip is measured as a function of the tip position. The performance of MFM has been tested in applying it to different magnetic surfaces, from hard magnets like FeNdB to soft ones like permalloy. Lateral resolution of 25 to 50 nm with sensitivity of  $10^{-12}$  N has been reported [7-11]. This places MFM among the most useful instruments for investigating small magnetic structures. The high-resolution Bitter pattern technique reveals flux line structure at liquid helium temperature but is unable to detect it at the temperature of liquid nitrogen [12, 13]. Last report from Rice and Moreland [14] showed ring structures observed by TSMFM (special version of magnetic force microscopy) on YBa<sub>2</sub>Cu<sub>3</sub>O<sub>7-8</sub> in liquid nitrogen. Authors believe that the ring structures represent vortex flux bundles.

However, there is still a need to describe the interaction between the ferromagnetic tip and a single flux line. This letter shows that MFM is theoretically capable of detecting the stray magnetic field of the single flux line. We show that the force which acts on the MFM tip from the single flux line is strong enough to be measured. Additionally we calculated lateral forces acting between the MFM tip and the flux line. Then these forces have to be compared to the pinning forces. We believe that this is the first such approach in the MFM field.

### **2. Results**

We consider the single flux line embedded in a semiinfinite superconducting medium and directed perpendicularly to its surface. The flux line produces the magnetic field above the surface. To find this field we have used in our calculations the results of Clem  $[15]$  who found the z-component of the magnetic field of the flux line in a type II superconductor. The following formula (from  $[15]$  describes the z-component of the magnetic field produced by the single flux line:

$$
b_z = \left(\frac{\varphi_0}{2\pi\lambda\zeta_v}\right)\frac{K_0\left(\frac{R}{\lambda}\right)}{K_1\left(\frac{\zeta_v}{\lambda}\right)}
$$
(1)

 $\lambda = 1$ 

where  $K_n(x)$  is a modified Bessel function,  $\lambda$  is a London penetration depth,  $\xi_v$  is a variational core radius parameter, R is defined as  $R = \sqrt{\rho^2 + \xi_v^2}$ , where  $\rho$  is the radial coordinate,  $\varphi_0 = \hbar c/2e$ .

A detailed derivation of the above formula can be found in Clem's paper [15]. We assume that the space above the superconductor is free of charges and currents. Then the scalar magnetic potential  $\phi$  satisfies Laplace's equation:

$$
\Delta \phi = 0. \tag{2}
$$

Using cylindrical coordinates we can find the solution of the above equation and then the components of the magnetic field. They are:

$$
H_z(r, z) = \int_0^\infty C(k) k e^{-kz} J_0(kr) \, \mathrm{d}k \tag{3}
$$

$$
H_r(r, z) = \int\limits_0^\infty C(k) k e^{-kz} J_1(kr) \, \mathrm{d}k \tag{4}
$$

where  $J_n(x)$  are Bessel functions.

At the surface of the superconductor  $(z=0)$  the zcomponent of the magnetic field (4) has to satisfy solution (1). Using orthogonality relations of Bessel functions we can find coefficients  $C(k)$ . Finally the components of the magnetic field above the superconductor with the only one flux line are:

$$
H_z(r, z) = \frac{\hbar}{e \lambda^2 K_1(\xi)} \int_0^\infty \frac{K_1(\xi)(1 + k^2)}{k + \sqrt{1 + k^2}} k e^{-kz} J_0(kr) \, \mathrm{d}k \quad (5)
$$

$$
H_r(r, z) = \frac{\hbar}{e \lambda^2 K_1(\xi)} \int_0^\infty \frac{K_1(\xi) \sqrt{1 + k^2}}{k + \sqrt{1 + k^2}} k e^{-kz} J_1(kr) \, \mathrm{d}k \quad (6)
$$

where  $\zeta = \overline{\zeta}/\lambda$ ,  $z = \overline{z}/\lambda$  and  $r = \overline{r}/\lambda$  are dimensionless. Figure 1 shows a 3-dimensional image of both components of the stray magnetic field of the flux line.

In MFM the tiny magnetic tip integrated with a cantilever is placed close to the magnetic sample's surface. It interacts with the stray magnetic field which emerges from the sample. In our case the stray field is produced by the single flux line. The tip is attracted or repelled



Fig. 1. The components  $H_z$  and  $H_t$ , of the stray magnetic field of the single flux line



Fig. 2. Models of the solid and thin-film tips used in the calculations

depending on its magnetic vector's direction. Then the cantilever where the tip is mounted bends up or down, this deflection is detected, and the force exerted on the lever can be calculated. We calculate this force using (5). We consider both solid and thin-film tips. We assume that the tip has the shape of pyramid and it is magnetised along the axis of symmetry. Tips which we use in our laboratory have such a shape. Figure 2 presents the model of the solid and thin-film tips. We use the same mathematical technique as before to find the force which acts on such tips  $[16, 17]$ . Generally the force components on the tip magnetised along z we write as:

$$
F_z = \int\limits_V M_z \frac{\partial H_z}{\partial z} \, \mathrm{d}V \tag{7}
$$

$$
F_r = \int\limits_V M_z \frac{\partial H_z}{\partial r} dV
$$
 (8)

where the integration is carried out over a volume of the tip and  $H<sub>z</sub>$  describes the z-component of the stray magnetic field of the flux line.  $F_z$  is the force causing the deflection of the cantilever in experiment and  $F<sub>r</sub>$  is the force in the lateral direction. It is important to know  $F<sub>r</sub>$  since it might be possible for a vortex to become depinned when the tip is scanning over it, Combining (5) and (7), we write finally the vertical force  $F_z$  as:

$$
F_z = -\frac{\hbar M_z}{e\lambda^3 K_1(\xi)} \int_0^\infty dk \frac{K_1(\xi/\sqrt{1+k^2})}{\sqrt{1+k^2}} k^2 \int_0^L dz' e^{-k(z+z')}
$$
  

$$
\int_{z'\tan\alpha}^{z'\tan\alpha} dx' \int_{-z'\tan\alpha}^{z'\tan\alpha} dy' J_0(k/\sqrt{(x+x')^2 + (y+y')^2}) (9)
$$

where L is the length of the pyramidal tip and  $\alpha$  is the angle between the tip's side and the tip's axis of symmetry (Fig. 2). We have assumed the non-disturbing character of the tip-flux line interaction and we have neglected a force due to induced diamagnetic currents [18],

Using the above formula we have also been able to find the force on non-magnetic tips coated with a magnetic thin film mentioned above. We have used the fact that the force on the thin-film tip can be described by the sum of two forces: both for the solid tips with different magnetic poles on the surface. Sizes of these two solid tips can be adjusted in such a way that the first can be embedded into the second until the distance between surfaces of both tips equals the expected thickness of a thin film.

We have considered the following tip dimensions  $L = 2.1$  µm,  $R_u = 1.6$  µm,  $R_d = 100$  nm with  $M = 1422$  Gsmagnetisation of Cobalt, the material used in our laboratory. The penetration depth  $\lambda = 200$  nm and  $\xi = 1.14$ . The Eq. (9) has been then evaluated numerically.

Figures 3 and 4 present the force components  $F_z$  and  $F<sub>r</sub>$  respectively acting on the solid and the thin-film tips with a film thickness of 30 nm and 100 nm. The scan height is taken as  $z = 25$  nm. Depending on the relative orientation of the magnetisation inside the tip and the flux through the vortex the force acting on the tip is either attractive or repulsive. Obviously the vertical force shows the same rotational symmetry as the stray field of the flux line (Fig. 1) and decreases with increasing distance from the center of the flux line. The maximum force in z-direction is more than  $1.4*10^{-10}$  N for the 30 nm thick film tip, respectively more than  $4 \times 10^{-10}$  N for the 100 nm thick film tip, both well in the range of MFM sensitivity. The lateral force has its maximum some distance away from the center of the flux line. The maximum lateral force reaches a maximum of  $5*10^{-11}$  N at 30 nm film thickness, respectively of  $1.5*10^{-10}$  N at 100 nm. It should be kept as small as possible preventing the flux lines from being depinned.



Fig. 3. The vertical force  $F_z$  which acts on the solid (solid line) and the thin film (dashed line) tips vs. the lateral distance  $x$ , at  $z = 25$  nm



Fig. 4. The lateral force  $F_r$ , which acts on the solid (solid line) and the thin film (dashed line) tips vs. the lateral distance x, at  $z = 25$  nm

Then the lateral force we have calculated has to be compared to the local pinning force. However, it is very difficult to estimate such a local pinning force.

Thin film tips should give smaller magnetic stray field than the solid tips. Therefore the tip effect on the flux line is also smaller.

Figure 5 shows two curves which represent the vertical force  $F<sub>z</sub>$  which acts on two thin film tips of different film thicknesses. The force has been calculated at  $x=0$ . The vertical force  $F<sub>z</sub>$  decreases with z (tip-sample distance) through the exponential dependence as expected from (9). As the distance between tip and flux line reaches 200 nm it should be possible to detect the force by MFM. The increase in force is almost of the same order as the increase in thickness of the magnetic film.

In conclusion, we have presented a preliminary approach toward the theory of using MFM to scan a flux



**Fig. 5.** The vertical force *F<sub>r</sub>* which acts on the thin-film tips for two different thin film ticknesses, plotted vs. tip-flux line distance z, at  $x=0$ 

line structure in type II superconductors. We have calculated the stray magnetic field produced by the single flux line and the force which is exerted on a typical MFM tip while scanning this single flux line. Maximum force of more than  $10^{-9}$  N and  $10^{-10}$  N are found for the solid and the thin-film cobalt tips, respectively. We have also calculated the lateral forces acting on a flux. The lateral force must be small enough to prevent the flux line from becoming depinned. The sign of the vertical as well as of the lateral force depends on the relative orientation of the magnetisation inside the tip and the flux through the vortex.

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