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Shock Waves

Formation of shock waves in gas-liquid foams

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Abstract. The purpose of the present investigation is to analyze the phenomenon of shock wave formation in gas-liquid foams and to explain the qualitative differences which are found when comparing results from shock tube experiments performed with foams and bubbly liquids. It is well known that oscillatory pressure waves in bubbly liquids may reach an amplitude twice as large as that of the original pressure impulse. However, experiments showed that pressure disturbances in foams always attenuate without significant change in the wave pressure profile. In the present study this behavior is explained by analyzing shock wave formation using the Burgers equation which is derived from the conservation laws for a bubbly liquid. It is shown that the parameter of non linearity in the Burgers equation describing wave propagation in bubbly liquids is about 40 times higher than in foams. At the same time coefficient of bulk viscosity of a foam is about 10^3 times greater than that of a bubbly liquid. This explains why in shock tube experiments with foams shock waves are not detected while they are easily observed when bubbly liquids are used under similar conditions.

1 Introduction

The main difference between gas-liquid foams and the well known and investigated bubbly liquids is in the high (close to unity) gas content which characterizes gas-liquid foams. Foams are composed of a large number of gas bubbles separated by thin liquid films and have a cellular quasi-ordered structure. The structure of foams can be well described by a polyhedral model whereby an elementary foam cell represents an irregular polyhedron (see Bikerman (1973), §41, §42 and Kraynik (1988)). The liquid film junction regions called Plateau borders, determine the boundaries of polyhedral gas bubbles. Foams with a liquid content which does not exceed 0.1 are of particular interest. In such foams the thickness of the liquid film is much less than the radius of the gas bubbles. Stability of such a delicate structure is achieved due to the presence of surfactants in the foam-forming liquid. Absence of this small amount of surfactants (about 1%) will lead to rupture of thin liquid films and a rapid bubble coagulation. Due to the very small amount of stabilizing surfactants they have practically no effect on the properties of the foam-forming liquid.

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Propagation of pressure disturbances in foams can be described by the method developed for analyzing bubbly liquid flows (see, e.g., Miksis and Ting (1991), Nakoryakov et al. (1993)). However, it is essential to take into account the creeping liquid flow in the capillary network formed by the Plateau borders. This flow is caused by changes in the volume of gas bubbles resulting from disturbances induced by the incident pressure wave. The latter process is complicated by other phenomena, e.g., viscous effects at the gas-liquid interface, inertial effects, surface tension, etc.

It is well known that oscillatory pressure waves in bubbly liquids may reach an amplitude twice as large as that of the original pressure impulse (see, e.g., Nakoryakov et al. (1993)). However, experiments showed that pressure disturbances in foams always attenuate without significant change in the wave's pressure profile. Shock waves are formed due to interaction of two competing effects: nonlinearity and dissipation (and dispersion). As it will be shown later the parameter of nonlinearity is much higher in bubbly liquids than in foams while the dissipation in foams is significantly higher than that found in a bubbly liquid. Therefore, shock waves in bubbly liquids can be observed in shock tubes of $\sim$ 1 m length. The latter is impossible in foams since the length of shock wave formation in foams and the initial amplitute of the incident pressure disturbance which results in shock wave formation, are considerably higher than in the bubbly liquid case.

The purpose of the present investigation is to study the process of shock wave formation in gas-liquid foams and to explain the qualitative differences found while comparing results from shock tube experiments with foams with those obtained with bubbly liquids.

2 Derivation of conservation equations for gas-liquid foams

In the present investigation the cellular model of the multiphase media (see, e.g., Miksis and Ting (1991)) is employed whereby the foam is modeled by a regular cellular structure. Each cell consists of a spherical gas bubble with radius R surrounded by a thin liquid film with outer radius R_*. The foam density ρ_f may be expressed as:

$$\rho_f = \rho_\ell(1 - \beta) + \beta\rho_g \tag{1}$$

where ρ_ℓ and ρ_g are the densities of liquid and gas, respectively and β is the gas volume fraction in the foam.

The incident pressure wave, which travels through the foam, causes deformation of each of the foam cells and thereby induces a creeping fluid flow in the channels between Plateau borders. The network of these channels can be viewed as a kind of porous medium. The fluid filtration flow in this porous medium causes dissipation and damping of the incident pressure wave. Since the fluid slip velocity at the gas-liquid interface is zero, due to the presence of surfactants [Bikerman (1973)], in the following we assume that gas and liquid flows have the same velocity. We further assume that the entire foam structure is not damaged by the incident wave, i.e., there is no annihilation or formation of cells. Then, since $\rho_\ell \gg \rho_g$ and liquid phase is considered incompressible, the velocity of sound in the foam c_f, is given by the following relation [Nakoryakov el al. (1993)]:

$$c_f^2 = \frac{\gamma P_f}{\rho_f \beta} \tag{2}$$

where γ is the specific heats ratio in the gaseous phase and P_f is pressure in the foam. In the derivation of expression (2) the liquid phase is considered incompressible. Consider a variation of expression (2) with respect to parameters P, ρ, β. Then, after some algebra, using basic relations from the theory of homogeneous mixtures (see, e.g., Miksis and Ting), we arrive at the following relation for speed of sound in foams:

$$c_f = c_0\left(1 + \frac{\delta P}{P_0}\,\frac{1}{2}\,\frac{\gamma + 1 - 2\beta}{\gamma}\right) \tag{3}$$

where subscript 0 indicates the undisturbed state. In the following index f is omitted from the equations.

The propagation of a pressure wave in foam is governed by the following system of mass and momentum conservation equations written in a homogeneous mixture approximation:

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(u\rho) = 0 \tag{4}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = -\frac{1}{\rho}\,\frac{\partial P}{\partial x} \tag{5}$$

where u is velocity. Note that conservation equations (4), (5) are the averaged conservation laws where the averaging is performed over the foam volume. In the cellular model of foam which is used in this investigation the averaging in the conservation equations is carried over a cell with a characteristic size much smaller than a wave length.

In order to close the above system of conservation equations it is necessary to determine an equation of state for the foam, i.e., a relation between its density and pressure $\rho = \rho(P)$. In the case of shock wave propagation in bubbly liquids such closure is achieved by using the Rayleigh equation [see, e.g., Miksis and Ting (1991)] which describes single bubble dynamics in an infinite fluid. This approximation is valid for bubbly liquids since the bubble radius is much smaller than the mean spacing between bubbles. In a foam, air bubbles occupy almost the entire volume with only a thin liquid layer separating the bubbles. Therefore the approach based upon the use of the Rayleigh equation is not valid. In order to derive an equation of state for foams we apply the cellular model of foams, whereby the foam is viewed as a periodic structure of spherical cells with radius R_*. Note that the gas volume fraction in foam, in this model, is $\beta = \left(\frac{R}{R_*}\right)^3$. Recently similar cellular model was used by Amon and Denson (1984, 1986) to describe dynamics of polymeric foam growth.

Due to the fluid inertia and the gas compressibility, the gas bubble in a foam cell experiences radially symmetrical oscillations. These oscillations cause a creeping fluid flow in the network of channels between the Plateau borders.

At the gas-liquid interface a condition of balance between the viscous shear stresses and pressures at both sides of the interface yields:

$$P_\ell(R) = P_g - 4\mu\frac{1}{R}\,\frac{dR}{dt} \tag{6}$$

where $R(t)$ – radius of the gas bubble, μ – the dynamic viscosity of the liquid, P_ℓ and P_g – pressures in liquid and gaseous phases, respectively.

The liquid pressure and liquid flow rate velocity V in the network of channels between the Plateau borders, are related by Darcy's equation [Dullien (1992)]:

$$-\frac{\partial P_\ell}{\partial r} = \frac{\mu V}{k} \tag{7}$$

where k is the permeability of the foam. The integral mass balance equation reads:

$$4\pi r^2 V = 4\pi R^2 \frac{dR}{dt}\,\alpha \tag{8}$$

where $\alpha = 1 - \beta$ is a surface porosity, i.e., fraction of a unit cross-section of foam which is occupied by liquid. Note that the presence of surface porosity, α in the right-hand side of equation (8) is due to wall deformation of gas bubbles caused by squeezing liquid through the channels between Plateau borders. It is essential that Darcy's filtration flow described by equation (7) does not involve net flow through the foam. This expression describes a local flow in the vicinity of an oscillating bubble with zero volume average flow. Propagation of shock waves through foams is associated with compressibility of gas bubbles. The attenuation of shock waves occurs due to local friction during filtration flow through the Plateau borders and stretching/oscillations of liquid films. Note also that superficial velocity of fluid given by equation (8) varies as r^{-2}. It can be assumed therefore that the fluid velocity vanishes at the cell boundary, i.e., $V(R_*) = 0$.

The latter condition is in compliance with the condition of periodicity at the cell boundary $\frac{\partial P}{\partial r}|_{r=R_*} = 0$ and Darcy's equation (7).

Integrating equation (7) from R to R_* and using equation (8) yields

$$-P(R_*) + P_\ell(R) = (1-\beta)(1-\beta^{1/3})\frac{\mu}{k}R\frac{\mathrm{d}R}{\mathrm{d}t} \quad (9)$$

where $P = P(R_*)$ is a cell averaged pressure in a foam.

Using the condition at the gas-liquid interface, Eq. (6), we find that

$$4\mu\left[1+\frac{1}{4}\frac{(1-\beta)(1-\beta^{1/3})R_0^2}{k}\right]\frac{1}{R}\frac{\mathrm{d}R}{\mathrm{d}t} = P_g - P \quad (10)$$

where R_0 is the average equilibrium size of the gas bubble.

It should be noted that the inertial term in Rayleigh equation for bubbly liquids is much higher (by a factor $\propto \frac{R}{\delta}$, where $\delta = R_* - R$ is the width of a liquid film) than a viscous term and causes wave dispersion and amplification of pressure in the oscillating wave front [Karpman (1974)]. In order to take into account the inertial effects during wave propagation in foams, the Darcy's equation (7) must be replaced by the momentum conservation equation which takes into account pressure drop during fluid filtration:

$$\frac{\partial(\rho_\ell\varepsilon U)}{\partial t} + \frac{\partial(\rho_\ell\varepsilon U^2)}{\partial r} = -\varepsilon\frac{\partial P_\ell}{\partial r} - \frac{\mu\varepsilon^2 U}{k} \quad (11)$$

where $U = V/\varepsilon$ is velocity of fluid in channels between the Plateau borders and $\varepsilon = 1-\beta$. The nonstationary and inertial effects in momentum conservation equation (11) with Darcy's friction term can be neglected since the Reynolds number of the filtration flow in channels between the Plateau borders, $\mathrm{Re} = \frac{\rho_\ell U k^{1/2}}{\mu}$, is very small [for details see, e.g., Dullien (1992), §3.2].

Since in the adopted model the number of bubbles per unit mass of foam remains constant and $\rho_\ell \gg \rho_g$ and the liquid phase is considered incompressible,

$$\frac{1}{R}\frac{\mathrm{d}R}{\mathrm{d}t} = -\frac{1}{3\rho\beta}\frac{\partial\rho}{\partial t}\,. \quad (12)$$

Combining equations (10) and (12), results in the nonlocal equation of state for foam [see, e.g., Karpman (1974), §9], $P = P(\rho,\dot\rho)$, which can be written as follows:

$$\delta P = \left(\frac{\partial P}{\partial\rho}\right)_{S,\dot\rho}\delta\rho + \left(\frac{\partial P}{\partial\dot\rho}\right)_{S,\rho}\delta\dot\rho = c^2\delta\rho + \nu_b\delta\dot\rho \quad (13)$$

where ν_b is the bulk viscosity of the foam:

$$\nu_b = \frac{4\mu}{3\rho\beta}\left[1+\frac{1}{4}\frac{(1-\beta)(1-\beta^{1/3})R_0^2}{k}\right] \quad (14)$$

For further analysis it is essential to estimate the bulk viscosity, given by equation (14), for gas-liquid foams ($\beta \sim 0.9$) and for bubbly liquids ($b \sim 0.1$). For estimating the permeability of foam we use the Carman-Kozeny equation [Dullien (1992), p. 243]:

$$k = \frac{R_0^2(1-\beta)^3}{720\beta^2} \quad (15)$$

Then, from equation (14) one obtains $\nu_b(\beta = 0.9)/\nu_b(\beta = 0.1) \sim 10^3$, i.e., the bulk viscosity of foam is 10^3 times greater than that of a bubbly liquid. In the following it is shown that this large difference which exists between the two bulk viscosities is the reason for the strong attenuation of pressure waves in foams.

3 Derivation of Burgers equation

Based on equation (12) the equation of state for foams, in the form of $P = P(\rho,\dot\rho)$, was derived. Using the regular definition for the speed of sound, i.e.,

$$c^2 = \left(\frac{\partial P}{\partial\rho}\right)_{S,\dot\rho} \quad (16)$$

the conservation equations (4), (5) can be written as follows:

$$\frac{\partial\rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) = 0 \quad (17)$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \frac{c}{\rho^2}\frac{\partial\rho}{\partial x} = \left(\frac{\partial P}{\partial\dot\rho}\right)_\rho\frac{\partial^2 u}{\partial x^2} \quad (18)$$

Equations (17), (18) have a solution in the form of a quasi-simple wave [Karpman (1974), §13]:

$$\rho(x,t) = \rho(u) + \psi(x,t) \quad (19)$$

where

$$\frac{\mathrm{d}\rho}{\mathrm{d}u} = \frac{\rho(u)}{c(u)} \quad (20)$$

and

$$\frac{\mathrm{d}P}{\mathrm{d}u} = c(u)\rho(u)\,. \quad (21)$$

Function $\psi(x,t)$ is the solution to the follwing linear wave equation:

$$\frac{\partial\psi}{\partial t} + c_0\frac{\partial\psi}{\partial x} = 0$$

Substituting (20) and (21) into equations (17), (18) results, after some algebra, in the following equation for quasi-simple waves:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + c\frac{\partial u}{\partial x} = \frac{1}{2}\left(\frac{\partial P}{\partial\dot\rho}\right)_\rho\frac{\partial^2 u}{\partial x^2} \quad (22)$$

Since in a simple wave $\Delta P = c\rho\Delta u$ equations (2) and (3) yield:

$$c = c_0 + \frac{\gamma+1-2\beta}{2\beta}u \quad (23)$$

where c_0 is the sound velocity in an undisturbed foam.

Combining equations (14), (22) and (23) results at the Burgers equation which describes wave propagation in a gas-liquid foam:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + (c_0 + \Gamma u)\frac{\partial u}{\partial x} = \frac{\nu_b}{2}\frac{\partial^2 u}{\partial x^2} \quad (24)$$

where ν_b is a bulk viscosity of the foam and Γ is a nonlinearity parameter defined by the following expression:

$$\Gamma = \frac{\gamma + 1 - 2\beta}{2\beta} \tag{25}$$

Burgers equation (Eq. 24) describes pressure wave propagation, with steepening profile, in foams, i.e., the formation of a shock wave. The shock wave velocity is given by the following expression [Karpman (1974), §15]:

$$D = c_0 + \frac{\Gamma + 1}{2} u \cong c_0 + \frac{\gamma + 1}{4\gamma} \frac{\Delta P}{P_0} \tag{26}$$

which can be derived from the Hugoniot relations for small pressure difference ΔP. The shock wave width is given by the following relation:

$$\delta = \frac{2\gamma \nu_b P_0}{c \Delta P (\gamma + 1)} \tag{27}$$

where ΔP is the pressure jump across the shock wave.

Shock waves are formed due to interaction between two competing mechanisms: non linearity and dissipation. The speed of sound c, in foams and in bubbly liquids, is determined by the following expression [see, e.g., Nakoryakov el al. (1993)]:

$$\frac{1}{c_f^2} = \frac{(1 - \phi)\rho_f}{\rho_\ell^2 c_\ell^2} + \frac{\phi \rho_f}{\rho_g^2 c_g^2} \tag{28}$$

where $\phi = \rho_g \beta / \rho$ is the gas mass fraction. In the case when $\beta \sim 0.1$ and $\beta \sim 0.9$, i.e., in bubbly liquids and foams, respectively, the sound velocities are practically equal. However, the non linearity parameter Γ, is about 40 times higher in a bubbly liquid than in a foam. It is of interest to note that when heat transfer is taken into account the parameter γ is reduced and the nonlinearity parameter Γ, for foams, becomes even smaller than that of a pure gas. Therefore, under such circumstances the length required for shock waves formation in foams is 40 times longer than in a bubbly liquid. At the same time the bulk viscosity of a foam is about 10^3 times greater than that of a bubbly liquid. This results in a very wide shock wave front as can be seen from equation (27).

The above analysis explains why in shock tube experiments with foams shock waves are not detected while they are easily observed when bubbly liquids are used under similar conditions.

In contrast to a pure gas and/or bubbly liquid cases, strong shock waves cannot propagate in gas-liquid foams. Therefore, propagation of strong shock waves in foams is accompanied by their destruction. The latter is the reason why foams are widely used for attenuation of strong pressure waves (Krasinski [1992)].

4 Conclusions

The propagation of strong pressure waves in gas-liquid foams is analyzed. It is shown that propagation of pressure waves and their steepening in foams are governed by the Burgers equation similarly to a bubbly liquid. However, the parameter of nonlinearity which determines the length of formation of a shock wave, is significantly higher in foams than in a bubbly liquid. The dissipation coefficient (bulk viscosity) of foams is about 10^3 times higher than that of a bubbly liquid. The latter is caused by high viscous losses during fluid filtration through the porous structure formed by the gas bubbles. The present analysis explains why shock waves are not formed in foams while they are observed in bubbly liquids under similar experimental conditions.

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