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On the Homology Groups of Stein Spaces

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§ **1.** It is well known that if X is a Stein manifold of dimension n, then the singular homology groups $H_k(X, \boldsymbol{Z})$ are zero if $k>n$ and $H_n(X, \boldsymbol{Z})$ is free [1], [2]. Moreover if Y is a Runge domain on X, i.e. an open set which is holomorphically convex with respect to X, then for any abelian coefficient group G, one has $H_k(X \bmod Y, G)=0$ for $k>n$.

We shall prove, using the above results, that *if X is an arbitrary Stein space (with singularities) of dimension n, then $H_k(X, \boldsymbol{Z})$ is* 0 if $k>n$, *and torsionfree* if $k=n$. The proof uses, in addition to the results mentioned above, the following fact:

Any analytic set on a Stein space X has a fundamental system of neighborhoods which are Runge in X.

We shall give two proofs of this fact. The first is based on the following theorem, which asserts, roughly speaking, that any analytic set on a Stein space can "almost" be blown down to a point.

If X is a Stein space of finite dimension, A is an analytic set on X and U is any neighborhood of A, then there exists a holomorphic map $f: X \to \boldsymbol{C}^p$ such that $f^{-1}(0)=A$, $f|X-A$ is injective and $f|X-U$ is proper. Also, if, for example, $X-A$ is a manifold, f can be chosen so that its jacobian has maximal rank on $X-A$.

In the last section, we give a method to find Runge domains (in particular, domains of holomorphy) in $\boldsymbol{C}^n$ with prescribed $H_k(D, \boldsymbol{Z})$ when k is small. The results are the following.

1. *Given a finitely generated abelian group G and integers $k \geqq 1$, $n \geqq k+3$, there is a Runge domain D in $\boldsymbol{C}^n$ with $H_k(D, \boldsymbol{Z}) \approx G$.*

2. *If G is any countable abelian group and $k \geqq 1$, there is a Runge domain in $\boldsymbol{C}^n$ for $n \geqq 2k+3$ such that $H_k(D, \boldsymbol{Z}) \approx G$.*

The theorem on the vanishing of the homology groups in dimension $>n$ of Stein spaces has also been obtained by L. KAUP, Eine topologische Eigenschaft Steinscher Räume, Nachr. Akad. Wiss. Göttingen, Math.-Phys. Kl., 213–224 (1966). Further, K.-J. RAMSPOTT [6] has constructed domains of holomorphy in $\boldsymbol{C}^2$ with a given first Betti group.

§ **2.** We begin with the following result.

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Theorem 1. *Let X be a Stein space of dimension n and A a (closed) analytic subset of X. If U is any neighborhood of A, there exists a holomorphic map $f: X \to C^N$ with the following properties.*

1. $f^{-1}(0)=A$;
2. $f|X-A$ *is injective*;
3. $f|X-U$ *is a proper map into* C^N.

Moreover, if the dimension of the Zariski tangent space to X is bounded on $X-A$, then f can be so chosen as to be a local imbedding at any point of $X-A$.

Remarks. 1. It can be shown that one may always take $N=n(2n+1)$ (if we only require properties 1.–3.). There are some cases in which a smaller value of N can be obtained. For example, if X is a manifold and A is a submanifold of codimension p, we may take $N=p(2n+1)$.

2. If we take for A a single point, Theorem 1 reduces to the imbedding theorem of REMMERT; see [*4*].

3. The presence of the neighborhood U is necessary. It is easy to show that if A is not compact and is nowhere dense, then, for any map f of X into a locally compact space Y on which there is a point y_0 with $f^{-1}(y_0)=A$, the map $f|X-A \to Y-\{y_0\}$ is not proper. Thus, a noncompact analytic set can never be blown down to a point.

For the proof of Theorem 1, we need some lemmas.

Lemma 1. *If A is an analytic set in a Stein space of dimension n, there is a holomorphic map $\varphi: X \to C^{n+1}$ with $\varphi^{-1}(0)=A$.*

This lemma is due to GRAUERT [*3*]. It can be shown that there is a holomorphic map $\varphi: X \to C^n$ with $\varphi^{-1}(0)=A$; this result cannot be improved in general.

Lemma 2. *If A is an analytic set on a Stein space of dimension n, there is a holomorphic map $\psi: X \to C^{2n+1}$ such that $\psi(A)=0$ and $\psi|X-A$ is injective.*

Proof. Let E be the space of holomorphic functions on X which vanish on A equipped with the topology of compact convergence. E is a Fréchet space. Further, for any pair of points $a, b \notin A$, $a \neq b$, there is $g \in E$ with $g(a) \neq g(b)$ (one can, in fact, prescribe the values of g on a and b).

For $a, b \notin A$, $a \neq b$, let $E(a, b)$ denote the set of $g \in E$ with $g(a) \neq g(b)$. It is immediate that $E(a, b)$ is open and dense in E. Since the intersection of countably many open dense sets in a Fréchet space is non-empty, we deduce:

(*) Given any countable set of points (a_ν, b_ν), $a_\nu, b_\nu \notin A$, $a_\nu \neq b_\nu$, there is $g \in E$ with $g(a_\nu) \neq g(b_\nu)$ for all ν.

Let now $Y=(X-A)\times(X-A)-\Delta$ $\{\Delta$ being the diagonal in the product $(X-A)\times(X-A)\}$ and let $\psi_1\in E$ not be identically zero on any irreducible component of X not contained in A. Let $Y_1=\{(x,y)\in Y\,|\,\psi_1(x)=\psi_1(y)\}$. Clearly Y_1 is an analytic set of dimension $\leqq 2n-1$. Choose a point (a_ν, b_ν) on each irreducible component $Y_1^{(\nu)}$ of Y_1; then $a_\nu, b_\nu \notin A$, $a_\nu \neq b_\nu$, so that, by (*), there is $\psi_2\in E$ with $\psi_2(a_\nu)\neq\psi_2(b_\nu)$. Let

$$Y_2=\{(x,y)\in Y\,|\,\psi_1(x)-\psi_1(y)=\psi_2(x)-\psi_2(y)=0\}.$$

Then, since $\psi_2(x)-\psi_2(y)$ does not vanish identically on any irreducible component of Y_1, we have $\dim Y_2\leqq \dim Y_1-1\leqq 2n-2$. Continuing in this way, we find $\psi_1,\dots,\psi_{2n+1}\in E$ with

$$\{(x,y)\in Y\,|\,\psi_j(x)-\psi_j(y)=0,\ j=1,\dots,2n+1\}=\emptyset.$$

Clearly, if $\psi=(\psi_1,\dots,\psi_{2n+1})$, we have $\psi(A)=0$, $\psi\,|\,X-A$ is injective.

Lemma 3. *Let X, A be as in Lemmas 1, 2. If the dimension of the Zariski tangent space to X is bounded on $X-A$, then there is a holomorphic map $\chi: X\to C^p$ (some p) such that $\chi(A)=0$ and χ is a local imbedding at any point of $X-A$.*

The proof is similar to that of Lemma 2.

Lemma 4. *Let X be a Stein space of dimension n. Then there are $2n+1$ open sets $\mathfrak{U}_1,\dots,\mathfrak{U}_{2n+1}$ with the following properties.*

(a) Each $\mathfrak{U}_j$ is the disjoint union of relatively compact open sets $U_{j,\nu}$, $\nu=1,2,\dots$, which form a locally finite family.

(b) $\bigcup_j \mathfrak{U}_j=X$.

(c) For any coherent sheaf $\mathscr{I}$ of ideals on X, the following approximation theorem holds for each j:

Let K_ν be a compact subset of $U_{j,\nu}$, let $\varepsilon_\nu>0$ and let $s_\nu\in\Gamma(U_{j,\nu},\mathscr{I})$. Then, there exists a section $s\in\Gamma(X,\mathscr{I})$ such that

$$|s(x)-s_\nu(x)|<\varepsilon_\nu \quad \textit{for} \quad x\in K_\nu, \qquad \nu=1,2,\dots.$$

The proof of this lemma follows from Theorem 1 and the proof of Theorem 2 in [*4*] if we note the following two facts.

I. If X is Stein and Y an open subset of X which is Runge in X [i.e. holomorph-convex relative to X] then for any coherent sheaf of ideals $\mathscr{I}$ on X, the restriction map $\Gamma(X,\mathscr{I})\to\Gamma(Y,\mathscr{I})$ has a dense image.

II. $\Gamma(X,\mathscr{I})$ is complete with respect to the topology of compact convergence.

It is now easy to prove Theorem 1.

Because of Lemmas 1, 2, 3, it is sufficient to prove that there is a holomorphic map $h: X\to C^q$ with $h(A)=0$ such that $h\,|\,X-U$ is proper.

Choose open sets $\mathfrak{U}_1, \ldots, \mathfrak{U}_{2n+1}$ as in Lemma 4. Then, since $U_{j,\nu}$ is relatively compact and $\bigcup U_{j,\nu}=X$, there exist compact sets $K_{j,\nu}\subset U_{j,\nu}$ for which $\bigcup K_{j\nu}=X$. Let $L_{j,\nu}=K_{j,\nu}-U$, and let $\varphi=(\varphi_1, \ldots, \varphi_{n+1}):X\to C^{n+1}$ be such that $\varphi^{-1}(0)=A$. Let $\mathscr{I}$ be the sheaf of germs of holomorphic functions vanishing on A. Since $\varphi(L_{j,\nu})$ does not contain $0\in C^{n+1}$, there is a constant $c_\nu>0$ such that

$$c_\nu \max_l |\varphi_l(x)|>\nu+1 \quad \text{for} \quad x\in L_{j,\nu}, \qquad j=1,\ldots,2n+1, \nu\geqq 1.$$

Now, by property (c) in Lemma 4, there exist holomorphic functions $h_{j,l}\in\Gamma(X, \mathscr{I})$, $1\leqq j\leqq 2n+1$, $1\leqq l\leqq n+1$, such that for fixed j and l, we have

$$|h_{j,l}(x)-c_\nu \varphi_l(x)|<1 \quad \text{for} \quad x\in L_{j,\nu}, \nu\geqq 1.$$

Then, each $h_{j,l}$ vanishes on A and

$$\max_l |h_{j,l}(x)|>\nu \quad \text{for} \quad x\in L_{j,\nu}.$$

We see, since $\bigcup L_{j,\nu}=X-U$, that, for any constant $t>0$, the set

$$\{x\in X-U \mid \max_{j,l} |h_{j,l}(x)|\leqq t\}$$

is contained in

$$\bigcup_j \bigcup_{\nu\leqq t} L_{j,\nu}$$

and so is compact. Thus the restriction of the mapping $h:X\to C^q$, $q=(n+1)(2n+1)$, defined by the $h_{j,l}$ to $X-U$ is proper.

This proves Theorem 1.

As a corollary, we obtain

Theorem 2. *Let A be an analytic set in a Stein space X (of finite dimension). Then A has a fundamental system of neighborhoods which are Runge in X.*

Proof. Let U be any neighborhood of A, and let $f:X\to C^N$ be a holomorphic map with the properties stated in Theorem 1. Then $f(X-U)$ is a closed set in C^N not containing 0. Let Z be a polycylinder in C^N with $0\in Z$, $Z\cap f(X-U)=\emptyset$. Then $f^{-1}(Z)=V\subset U$ and $A\subset V$. Since Z is Runge in C^N, V is Runge in X. {If Y, Y' are Stein spaces, $Z'\subset Y'$ a Runge domain and $f: Y\to Y'$ is *any* holomorphic map, then $f^{-1}(Z')$ is Runge in Y; see [4].}

§ 3. In this section we give another proof of Theorem 2. We shall use the following theorem, due essentially to OKA [5] in C^n.[1]

[1] Dr. O. FORSTER has pointed out that one may use instead elementary properties of REINHARDT domains.

Theorem of Approximation. *If Ω is a Stein open set in $\boldsymbol{C}^n$ and p is a plurisubharmonic function in Ω, then the set*

$$\{x \in \Omega \mid p(x) < 0\}$$

is Runge in Ω; in particular, it is again a Stein open set.

Let now X, A be as in Theorem 2 and let $\varphi: X \to \boldsymbol{C}^p$ be a holomorphic map such that $\varphi^{-1}(0) = A$ and let $\psi: X \to \boldsymbol{C}^r$ be a proper, injective holomorphic map (which exists by [*4*]). Let $f: X \to \boldsymbol{C}^N$, $N = p + r$ be the map given by $f(x) = (\varphi(x), \psi(x))$ and let H be the subspace of $\boldsymbol{C}^N = \boldsymbol{C}^p \times \boldsymbol{C}^r$ given by $z = (z_1, \ldots, z_p) = 0$. {We denote a point of $\boldsymbol{C}^N$ by (z, w), where $z = (z_1, \ldots, z_p) \in \boldsymbol{C}^p$, $w = (w_1, \ldots, w_r) \in \boldsymbol{C}^r$.} Clearly $f^{-1}(H) = A$. Further, since f is a homeomorphism of X onto its image, for any open set $U \supset A$, there is an open set Ω in $\boldsymbol{C}^N$, $H \subset \Omega$, such that $f^{-1}(\Omega) = U$. Hence it is sufficient to prove that there is an open set W, $H \subset W \subset \Omega$, which is Runge in $\boldsymbol{C}^N$.

Clearly, there is a positive continuous function $\eta(w) > 0$ on $\boldsymbol{C}^r$ such that the set

$$\{(z, w) \in \boldsymbol{C}^N \mid \max_i |z_i| < \eta(w)\} \subset \Omega .$$

We prove below (Lemma 5) that there is a plurisubharmonic function $p(w)$ on $\boldsymbol{C}^r$ such that

$$p(w) > \log \frac{1}{\eta(w)} .$$

Thus there is a plurisubharmonic function p on $\boldsymbol{C}^r$ such that

$$W = \{(z, w) \in \boldsymbol{C}^N \mid \max |z_i| < e^{-p(w)}\} \subset \Omega .$$

If we set $q(z, w) = \max \log |z_i| + p(w)$, q is plurisubharmonic on $\boldsymbol{C}^N$ and $W = \{(z, w) \in \boldsymbol{C}^N \mid q(z, w) < 0\}$, so that W is Runge in $\boldsymbol{C}^N$ by Oka's approximation theorem stated above. Thus, we have only to prove

Lemma 5. *Let α be any continuous function on $\boldsymbol{C}^r$. Then there is a plurisubharmonic function p on $\boldsymbol{C}^r$ with $p(w) > \alpha(w)$ for all w.*

Proof. We may suppose that $\alpha > 0$. Let β be a continuous function on $\boldsymbol{R}^+$ such that $\beta(|w|^2) > \alpha(w)$. We have only to find a positive increasing convex function γ on $\boldsymbol{R}^+$ such that $\gamma(t) \geqq \beta(t)$ for $t \geqq 0$, for then we may take $p(w) = \gamma(|w|^2)$. Let $v \geqq 0$ be a continuous function for which

$$c + \int_0^t v(s)\, ds > \beta(t) \quad \text{for} \quad t \geqq 0 \quad (c \text{ a constant}).$$

We have only to put

$$\gamma(t) = c + \int_0^t u(s)\, ds ,$$

where

$$u(t) = \sup_{s \leq t} v(s).$$

§ 4. In this section, we apply Theorem 2 to prove the following result.

Theorem 3. *Let X be a Stein space of dimension n. Then, if $H_k(X, \mathbf{Z})$ denotes the k-th singular homology group of X with integer coefficients, we have*

$$H_k(X, \mathbf{Z}) = 0 \quad \text{for} \quad k > n,$$

$$H_n(X, \mathbf{Z}) \quad \text{is torsion free}.$$

Proof. We proceed by induction on the dimension of X. Suppose that for any abelian coefficent group G we have $H_k(A, G) = 0$ for $k > \dim A$ for any Stein space A of dimension $< n$. Let h be a holomorphic function on X which is zero on the singular set of X but does not vanish identically on any irreducible component of X, and let $A = \{x \in X \mid h(x) = 0\}$. Then A is a Stein space of dimension $< n$ and $X - A$ is a Stein manifold of dimension n. Let Y be any neighborhood of A in X which is Runge in X. Then $Y - A$ is Runge in $X - A$, so that, by the proof of Theorem 1 in [*2*], for any abelian group G, we have $H_k(X - A \bmod Y - A, G) = 0$ for $k > n$. Hence, by excision,

$$H_k(X \bmod Y, G) = H_k(X - A \bmod Y - A, G) = 0 \quad \text{for} \quad k > n.$$

By Theorem 2, Runge neighborhoods form a fundamental system of neighborhoods of A. Further, the singular and the Čech homologies of the pairs (X, A), (X, Y) coincide. Hence, by the continuity of Čech homology, we deduce that

$$H_k(X \bmod A, G) = \varprojlim H_k(X \bmod Y, G) = 0 \quad \text{for} \quad k > n.$$

Now, by induction, $H_k(A, G) = 0$ for $k \geq n$. The exact homology sequence of the pair (X, A) shows that $H_k(X, G) = 0$ for $k > n$. Since this is true for an arbitrary abelian group G, the universal coefficient theorem gives us the required result.

Corollary. *If X is a Stein space of dimension n, the singular cohomology groups $H^k(X, \mathbf{Z}) = 0$ for $k > n+1$. Also $H^{n+1}(X, \mathbf{Q}) = 0$.*

This follows from Theorem 3 and the universal coefficient theorem. Of course, $\mathbf{Q}$ can be replaced by any divisible group.

It is not known if $H^{n+1}(X, \mathbf{Z}) = 0$. This would involve proving that $H_n(X, \mathbf{Z})$ is free, which is the case if the singularities of X are isolated [*2*] or if $H_n(X, \mathbf{Z})$ is finitely generated. The question is open in general.

Theorem 2 and the above corollary have also been obtained by KAUP.

§ 5. This section deals with the construction of Runge domains D in C^n with prescribed $H_k(D, Z)$ when k is small compared to n.

Lemma 6. *Let U be an open set in R^n and consider R^n as a subset of C^n. Then there is a Runge domain $D \subset C^n$ with $U \subset D$ such that U is a deformation retract of D.*[2]

Proof. We denote the coordinates in C^n by $z=(z_1, \ldots, z_n)$, $z_j = x_j + i y_j$ and $R^n = \{z \in C^n \mid y_j = 0, j = 1, \ldots, n\}$. If U is an open set in R^n, we assert that there is a C^∞ function g on R^n such that

$$U = \{x \in R^n \mid g(x) > 0\}, \quad g(x) = 0 \quad \text{for} \quad x \in R^n - U,$$

and such that all derivatives of g are bounded on R^n. In fact, let $\{K_\nu\}$, $\nu = 1, 2, \ldots$ be a sequence of compact subsets of U with $K_\nu \subset \mathring{K}_{\nu+1}$, $\bigcup K_\nu = X$, and let $K_0 = \emptyset$. Let φ_ν be a C^∞ function with compact support in U, $\varphi_\nu(x) > 0$ for $x \in K_\nu - K_{\nu-1}$ $(\nu \geqq 1)$. It is easily seen that we may take

$$g(x) = \sum_{\nu=1}^{\infty} \delta_\nu \varphi_\nu(x)$$

for a suitable sequence $\delta_\nu > 0$.

Let

$$p(z) = y_1^2 + \cdots + y_n^2 - \varepsilon g(x), \quad \varepsilon > 0.$$

We see at once that the Levi form of p is given by

$$L(p) \equiv \sum_{\mu, \nu=1}^{n} \frac{\partial^2 p}{\partial z_\mu \partial \bar{z}_\nu} \alpha_\mu \bar{\alpha}_\nu = \frac{1}{4} \sum_{\mu, \nu=1}^{n} (2\delta_{\mu\nu} - \varepsilon g_{\mu\nu}) \alpha_\mu \bar{\alpha}_\nu$$

where $\delta_{\mu\nu}$ is the Kronecker delta and

$$g_{\mu\nu} = \frac{\partial^2 g}{\partial x_\mu \partial x_\nu}.$$

Since $g_{\mu\nu}$ is bounded on R^n, if ε is sufficiently small, $L(p)$ is positive definite, so that p is plurisubharmonic on the whole of C^n.

Let

$$D = \{z \in C^n \mid p(z) < 0\}.$$

By Oka's approximation theorem stated in § 3, D is Runge in C^n. Now, if $x \in R^n - U$, $p(z) = y_1^2 + \cdots + y_n^2 \geqq 0$, so that $D \subset U \times R^n$. Hence

$$D = \{z = (x, y) \mid x \in U, y_1^2 + \cdots + y_n^2 < \varepsilon g(x)\},$$

so that U is clearly a deformation retract of D.

[2] By a refinement of the argument used here, one can show that any C^∞-manifold of real dimension n with a countable base is contained in a Stein manifold of complex dimension n as a deformation retract.

Lemma 7. *Given a finitely generated abelian group G, there is a connected finite 2-dimensional complex K which can be imbedded in $\mathbf{R}^4$ for which*

$$H_1(K, \mathbf{Z}) \approx G.$$

Proof. We assert that it is sufficient to construct K when G is cyclic. In fact, if G is the direct sum of cyclic groups G_i and $K_i \subset \mathbf{R}^4$ corresponds to G_i, one has only to consider a large line segment and attach the K_i to points of this segment so as to be mutually disjoint.

If G is infinite cyclic, we set $K = S^1$. If $G = \mathbf{Z}/m\mathbf{Z}$, we take for K the 2-disc attached to S^1 by a map of degree m of its boundary onto S^1. To see that K can be imbedded in $\mathbf{R}^4$ we proceed as follows. Consider the map f of the disc $|z| \leqq 1$ into $\mathbf{C}^2$ given by

$$f(z) = \left(z^m, (1 - |z|)\, z\right).$$

Clearly, the restriction of f to $|z| = 1$ maps with degree m onto the circle, and f is injective on $|z| < 1$. This gives an imbedding of K into $\mathbf{C}^2 = \mathbf{R}^4$.

Theorem 4. *Let G be any finitely generated abelian group, let k be an integer $\geqq 1$ and let n be an integer $\geqq k+3$. Then there is a connected Runge domain D in $\mathbf{C}^n$ for which $H_k(D, \mathbf{Z}) \approx G$.*

Proof. We assert that if $n \geqq k+3$, there is a connected finite complex L imbedded in $\mathbf{R}^n$ for which $H_k(L, \mathbf{Z}) \approx G$. In fact, by Lemma 7, there is a finite complex (connected) $K \subset \mathbf{R}^4$ with $H_1(K, \mathbf{Z}) \approx G$. The $(k-1)$-fold suspension $S^{k-1}(K)$ is then a connected finite complex imbedded in $\mathbf{R}^{k+3}$ for which $H_k(S^{k-1}(K), \mathbf{Z}) \approx G$. We have only to take for L the product of $S^{k-1}(K)$ by the $(n-k-3)$-disc.

Now, there is a neighborhood U of L in $\mathbf{R}^n$ of which L is a deformation retract [any locally finite complex in Euclidean space has this property]. By Lemma 6, there is a Runge domain D in $\mathbf{C}^n$ with the homotopy type of U. Clearly D is connected and $H_k(D, \mathbf{Z}) \approx G$.

By using the fact that for an arbitrary countable abelian group G, there is a locally finite complex X of dimension $k+1$ with $H_k(X, \mathbf{Z}) \approx G$, we prove, in the same way as above, the following:

Theorem 5. *If G is any countable abelian group, and k is an integer $\geqq 1$, there is a Runge domain $D \subset \mathbf{C}^n$ with $H_k(D, \mathbf{Z}) \approx G$ if $n \geqq 2k+3$.*

Theorem 4 shows that for a Runge domain D in $\mathbf{C}^n$, $H_k(D, \mathbf{Z})$ can have torsion if $k \leqq n-3$. It is known [2] that if $k \geqq n$, these groups are zero and $H_{n-1}(D, \mathbf{Z})$ is torsion free. It is trivial that any countable free group can occur. All the domains constructed by our method have a torsion free H_{n-2} as one sees by using the Alexander duality theorem.

References

[*1*] ANDREOTTI, A., and T. FRANKEL: The Lefschetz theorem on hyperplane sections. Annals of Math. **69**, 713—717 (1959).

[*2*] —, and R. NARASIMHAN: A topological property of Runge pairs. Annals of Math. **76**, 499—509 (1962).

[*3*] GRAUERT, H.: Charakterisierung der Holomorphiegebiete durch die vollständige Kählersche Metrik. Math. Annalen, **131**, 38—75 (1956).

[*4*] NARASIMHAN, R.: Imbedding of holomorphically complete complex spaces. American Journ. of Math. **82**, 917—934 (1960).

[*5*] OKA, K.: Sur les fonctions analytiques de plusieurs variables, IX. Domaines finis sans point critique intérieur. Japanese Journ. of Math. **27**, 97—155 (1953).

[*6*] RAMSPOTT, K.-J.: Existenz von Holomorphiegebieten zu vorgegebener erster Bettischer Gruppe. Math. Annalen, **138**, 342—355 (1959).

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