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Formfactors of semileptonic decays $B \to D(D^*)e\nu$ from QCD sum rules

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Abstract. Formfactors of semileptonic decays $B \to D(D^*)e\nu$ are calculated by means of QCD sum rules for three-point Green functions. Partial widths found essentially differ from values obtained in previous papers [1–5].

Introduction

The study of exclusive B-mesons semileptonic decays can provide us with important information on both the Kobayashi-Maskawa matrix elements and the strong interaction dynamics of quarks in hadrons. Many theoretical papers were devoted to this subject which was stimulated by intensive experimental study of the B-mesons exclusive decays [6–8]. But in our opinion the results obtained for the semileptonic transitions of B-mesons into D- and D^*-mesons [1–5] are not grounded enough. In particular, papers [1, 2, 4, 5] are based on some model-dependent assumptions about B- and D-mesons' structure. Voloshin and Shifman [3] did not use the models for B- and D-mesons, but they however made assumptions whose accuracy could not be controlled theoretically in the framework of the approach used. Namely, in the point of maximal momentum transfer squared $q^2_{\max} = (m_B - m_D)^2$ the formfactors f_+, F_0, defined by

$$\langle D(p)|\, \bar{c}\gamma_\mu b\, |B(p')\rangle = f_+(p'+p)_\mu + f_-\, q_\mu,$$

$$\frac{1}{i}\langle D^*(p,\varepsilon)|\, \bar{c}\gamma_\mu\gamma_5 b\, |B(p')\rangle$$

$$= F_0\, \varepsilon^*_\mu(p) + F_+\, (\varepsilon^*(p)\, p')\, (p'+p)_\mu + F_-\, (\varepsilon^*(p)\, p')\, q_\mu,$$

$$\frac{1}{i}\langle D^*(p,\varepsilon)|\, \bar{c}\gamma_\mu b\, |B(p')\rangle = F_V\, i\, \varepsilon_{\mu\nu\alpha\beta}\, \varepsilon^{*\nu}(p)\, (p'+p)^\alpha q^\beta \tag{1}$$

(where $q = p' - p$, $\varepsilon(p)$ is D^*-meson polarization vector) are equal to $f_+ = \sqrt{m_B m_D}/(m_B + m_D)$, $F_0 = 2\sqrt{m_B m_D}$ and the others are equal to zero, which is true only at the limit $m_b, m_c \to \infty$. On the other hand c-quark is not heavy enough in the scale of the typical hadronic masses. For example, the relativistic corrections [9] to the non-relativistic sum rules [10] for the calculation of constant f_D are large. Hence, in [3] the accuracy of this supposition is assumed to be too high. Another source of uncertainty in [3], which is controlled by the parameter $(m_B - m_D)^2/(m_B + m_D)^2 \sim 0.2$, can also give a considerable error.

QCD sum rules method [11] is based directly on the first principles of quantum chromodynamics. Its accuracy can also be obtained within the method. Here we represent the result of calculating the decays $B \to D(D^*)e\nu$ formfactors with the help of QCD sum rules method for the three-point Green functions (see, e.g., [12] and references therein).

From the sum rules for the three-point correlators we calculate the formfactors f_+, $F_{0,+,V}$ which one needs for the evaluation of the total decay widths. In order to obtain their q^2-dependence for $q^2 > 0$ we use the independent sum rules for the derivatives of the formfactors at $q^2 = 0$. This derivatives are small so we can work in the linear approximation in q^2.

Our results for formfactors (1) in the case of D^*-meson are quite different from the estimates of [3] (and other papers [1, 2, 4, 5]), which leads to a lower $B \to D^* e\nu$ decay width. Taking into account the accuracy of the experiment and the matrix element V_{bc} determination uncertainty we conclude that our result does not contradict the experimentally obtained value. As for the parameters characterizing the polarization of vector meson in the $B \to D^* e\nu$ decay, we obtain a good agreement with the recent ARGUS data [8].

In Sect. 1 we discuss the method and evaluate the phenomenological part of the sum rules. The theoretical expressions for the correlators are derived in Sect. 2. Section 3 is devoted to the analysis of the sum rules obtained and at last in Sect. 4 we calculate the widths and other parameters of the decays.

1 The method

For the evaluation of f_+ in the decay $B \to D e \nu$ we considered the correlator

$$\Pi_\mu(p, p', q) = i^2 \int dx\, dy\, e^{ipx-ip'y} \langle T\{\bar{\psi}\, \gamma_5\, c(x), J_\mu^V(0), \bar{b}\, \gamma_5\, \psi(y)\}\rangle_0, \tag{2}$$

and for the formfactors of the decay $B \to D^* e \nu$ we consider the correlators

$$\Gamma_{\mu\nu}^{V,A}(p, p', q) = i^2 \int dx\, dy\, e^{ipx-ip'y} \cdot \langle \mathrm{T}\, \{\bar{\psi}\, \gamma_\nu\, c(x), J_\mu^{V,A}(0), \bar{b}\, \gamma_5\, \psi(y)\}\rangle_0, \tag{3}$$

where $q = p' - p$, $J_\mu^V = \bar{c}\, \gamma_\mu\, b$, $J_\mu^A = \bar{c}\, \gamma_\mu\, \gamma_5\, b$, and $\psi(x)$ is the operator of the light quark (u, d). Here we neglect its mass. The decomposition of correlators (2), (3) into the Lorentz structures takes the form

$$\Pi_\mu = \Pi_+ (p' + p)_\mu + \Pi_-\, q_\mu, \tag{4}$$

$$\Gamma_{\mu\nu}^A = \Gamma_0\, g_{\mu\nu} + \Gamma_1\, p_\mu\, p'_\nu + \Gamma_2\, p'_\mu p'_\nu + \Gamma_3\, p_\mu\, p_\nu + \Gamma_4\, p_\nu\, p'_\mu, \tag{5}$$

$$\Gamma_{\mu\nu}^V = \Gamma_V\, i\, \varepsilon_{\alpha\beta\mu\nu}\, p_\alpha\, p'_\beta. \tag{6}$$

For our purposes we need only the amplitudes Π_+, $\Gamma_{0,+,V}$ (where $\Gamma_+ = \frac{1}{2}(\Gamma_1 + \Gamma_2)$), which correspond to the formfactors f_+, $F_{0,+,V}$. f_- and F_- are inessential due to smallness of electron mass. For each amplitude $\Pi_\pm$, Γ_i we have the following dispersion relation:

$$\Gamma_i(p^2, p'^2, Q^2) = -\frac{1}{(2\pi)^2} \int \frac{\rho_i(s, s', Q^2)\, ds\, ds'}{(s - p^2)(s' - p'^2)} + \text{subtraction terms}, \tag{7}$$

where ρ_i is a corresponding spectral density, $Q^2 = -q^2 > 0$. According to QCD sum rules method the left hand side must be calculated at large Euclidean momenta p^2, p'^2 with the help of Wilson operator product expansion (OPE).

The right hand (phenomenological) side is obtained by saturating (7) with the lowest mesonic resonances. The double Borel transformation [11, 13] over variables p^2, p'^2 will suppress both the higher resonances contribution to the right hand side of (7) and the higher power corrections contribution to the left hand side of the sum rules.

To obtain the phenomenological part we saturate correlator (2) by B- and D-mesons and correlator (3) – by B- and D^*-mesons. Thus we find:

$$\Pi_+^{(ph)}(p^2, p'^2, Q^2) = -\frac{f_D\, f_B\, m_D^2\, m_B^2}{m_c\, m_b} \frac{f_+(Q^2)}{(p^2 - m_D^2)(p'^2 - m_B^2)},$$

$$\Gamma_{0,+}^{(ph)}(p^2, p'^2, Q^2) = -\frac{f_B\, m_{D^*}^2\, m_B^2}{g_{D^*} m_b} \frac{F_{0,+}(Q^2)}{(p^2 - m_{D^*}^2)(p'^2 - m_B^2)},$$

$$\Gamma_V^{(ph)}(p^2, p'^2, Q^2) = -\frac{2 f_B\, m_{D^*}^2\, m_B^2}{g_{D^*}\, m_b} \frac{F_V(Q^2)}{(p^2 - m_{D^*}^2)(p'^2 - m_B^2)}, \tag{8}$$

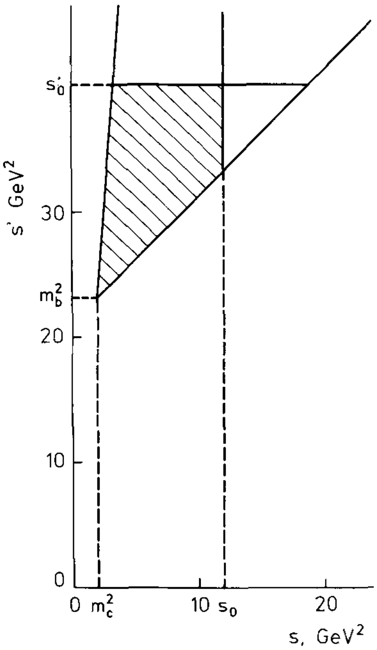

Fig. 1. Integration region in (7) when $Q^2 = 0$. s_0, s_0'-continuum thresholds

where the residues are defined in a standard way: $\langle 0 |\, \bar{\psi}\, \gamma_\mu\, \gamma_5\, c\, | D(p) \rangle = i f_D\, p_\mu$ (and similar for B-meson), $\langle 0 |\, \bar{\psi}\, \gamma_\mu\, c\, | D^*(p, \varepsilon) \rangle = \frac{m_{D^*}^2}{g_{D^*}}\, \varepsilon_\mu(p)$. Quark masses are equal to $m_c = 1.35$ GeV, $m_b = 4.8$ GeV. The residues were found from QCD sum rules: $f_D = 170$ MeV, $f_B = 130$ MeV [9], $g_{D^*} = 9$ [14].

2 Theoretical calculation of the correlators

The theoretical part of the sum rules is calculated by means of OPE at short distances. The coefficient functions of various operators can be evaluated with the help of fixed point gauge technique [15]. We calculate the operator expansion for (2) and (3) up to the operators of dimension six in the lowest order in α_s. We use the analytical calculations system HECAS developed at IHEP [16].

We start from the perturbative contribution (unit operator of the OPE), which is given by bare quark loop. For each amplitude Π_+, Γ_0, Γ_+, Γ_V the corresponding spectral density in (7) can be obtained by substituting the propagators $1/(p^2 - m^2)$ by $-2\pi\, i\, \delta(p^2 - m^2)$ in the initial Feynman integrals for the correlators. The integration region in (7) for perturbative contribution is shown in Fig. 1. As usual we parametrize the contribution of higher states in the phenomenological part of the rules assuming that the corresponding spectral density is equal to the perturbative spectral density starting from $s>s_0$, $s'>s'_0$. If we subtract this contribution from both sides of the sum rules we get a corresponding integration region in (7) (in the theoretical part of the rules) which is shown in Fig. 1. The lines that define this region are given by the formulas

$$s'_{1,2}(s)=\tfrac{1}{2}\left(\frac{s}{m_c^2}(m_c^2+m_b^2+Q^2)+(m_b^2-m_c^2-Q^2)\right)$$
$$\pm\frac{s-m_c^2}{2m_c^2}\sqrt{(m_c^2+m_b^2+Q^2)^2-4m_c^2\,m_b^2}.$$

The parameters s_0 and s'_0 are the so-called continuum thresholds which must be found from the numerical analysis of the sum rules.

The spectral densities we are interested in have the following form:

$$\rho_+(s,s',Q^2)=\frac{3}{2\kappa^{3/2}}\,[(\Delta' s+m_c\,m_b\,\Delta)(s-s'+Q^2)$$
$$+(\Delta s'+m_c\,m_b\,\Delta')(s'-s+Q^2)] \qquad (9)$$

for decay $B\to D e\nu$, and for $B\to D^* e\nu$

$$\rho_0(s,s',Q^2)=\frac{3}{2\kappa^{1/2}}(m_c\,\Delta'+m_b\,\Delta)$$
$$+\frac{3m_b}{\kappa^{3/2}}(s'\Delta^2+s\Delta'^2-\Delta\Delta' u),$$
$$\rho_+(s,s',Q^2)=\frac{3}{2\kappa^{3/2}}\,[m_c(2s'\Delta-u\,\Delta')$$
$$+m_b(2s\Delta'-u\Delta+4\Delta\Delta'+2\Delta^2)]$$
$$+\frac{9m_b}{\kappa^{5/2}}[4s\,s'\Delta\Delta'-u(s\Delta'^2+s'\Delta^2$$
$$+2s\Delta\Delta')+2s^2(\Delta'^2+s'\Delta)],$$
$$\rho_V(s,s',Q^2)$$
$$=\frac{3}{\kappa^{3/2}}[m_c(u\Delta'-2s'\Delta)+m_b\,u\Delta-2s\Delta')],$$

where

$$\kappa=(s+s'+Q)^2-4s\,s',\qquad u=s+s'+Q^2,$$
$$\Delta=s-m_c^2,\qquad \Delta'=s'-m_b^2.$$

Now we proceed to the calculations of dimension 3, 5, and 6-operators contributions. The quark condensate contributions are:

$$\Pi_+^{\langle\bar\psi\psi\rangle}=\frac{1}{2}\langle\bar\psi\,\psi\rangle_0\,\frac{m_c+m_b}{(p^2-m_c^2)(p'^2-m_b^2)} \qquad (10)$$

for $B\to D e\nu$, and

$$\Gamma_0^{\langle\bar\psi\psi\rangle}$$
$$=\tfrac{1}{2}\langle\bar\psi\,\psi\rangle_0\left(\frac{(m_c+m_b)^2+Q^2}{(p^2-m_c^2)(p'^2-m_b^2)}+\frac{1}{p^2-m_c^2}+\frac{1}{p'^2-m_b^2}\right),$$
$$\Gamma_+^{\langle\bar\psi\psi\rangle}=-\tfrac{1}{2}\langle\bar\psi\,\psi\rangle_0\frac{1}{(p^2-m_c^2)(p'^2-m_b^2)},$$
$$\Gamma_V^{\langle\bar\psi\psi\rangle}=\langle\bar\psi\,\psi\rangle_0\,\frac{1}{(p^2-m_c^2)(p'^2-m_b^2)}$$

for the $B\to D^*$ transition. The contributions depending on one of the variables p^2 or p'^2 turn into zero after Borel transformation and so they will be omitted further. We neglect the gluon condensate contribution since it contains a quark loop and must be small in comparison with the contribution from other operators which have no loop suppression.

Now let us calculate the contribution of the quark-gluon condensate $g_s<\bar\psi\,G_{\mu\nu}\,\sigma_{\mu\nu}\,\psi>_0$ $=m_0^2\langle\bar\psi\,\psi\rangle_0$, $m_0^2=0.8\pm0.2\ \mathrm{GeV}^2$ [17]. For the transition $B\to D$ this contribution is equal to

$$\Pi_+^{\langle\bar\psi G\psi\rangle}=-\frac{g_s}{12}\langle\bar\psi\,G_{\mu\nu}\,\sigma_{\mu\nu}\,\psi\rangle_0$$
$$\cdot\left(\frac{2(2m_c+m_b)}{r^2\,r'}+\frac{2(2m_b+m_c)}{r\,r'^2}\right.$$
$$+\frac{3m_c^2(m_c+m_b)}{r^3\,r'}+\frac{3m_b^2(m_c+m_b)}{r\,r'^3}$$
$$\left.+\frac{m_c^2(2m_c+m_b)+m_b^2(2m_b+m_c)}{r^2\,r'^2}+2(m_c+m_b)\,Q^2\right). \qquad (11)$$

where we use the notations $r=p^2-m_c^2$, $r'=p'^2-m_b^2$. The corresponding expressions for $B\to D^*$ transition are given in the Appendix.

As for the four-quark operators contribution, the vacuum dominance hypothesis [11] can be used to express their vacuum averages through the quark condensate. The expression for the amplitude Π_+ is

$$\Pi_+^{\langle\bar\psi\,\psi\rangle^2}=-\frac{\pi}{81}\,\alpha_s\langle\bar\psi\,\psi\rangle_0^2\left\{\frac{12m_c^3(m_c+m_b)}{r^4\,r'}\right.$$
$$+\frac{12m_b^3(m_c+m_b)}{r\,r'^4}+\frac{4m_c}{r^3\,r'^2}[m_c^2(2m_c+m_b)$$
$$+m_b^2(2m_b+m_c)+2Q^2(m_c+m_b)]+\frac{4m_b}{r^2\,r'^3}$$
$$\cdot[m_c^2(2m_c+m_b)+m_b^2(2m_b+m_c)+2Q^2(m_c+m_b)]$$
$$+\frac{8m_c(7m_b-m_c)}{r^3\,r'}+\frac{8m_b(7m_c-m_b)}{r\,r'^3}$$
$$-\frac{12}{r^2\,r'}-\frac{12}{r\,r'^2}-\frac{8}{r^2\,r'^2}[2m_c(2m_b+m_c)$$
$$\left.+2m_b(2m_c+m_b)+Q^2]\right\}. \qquad (12)$$

The contribution of four-quark operators to other amplitudes is shown in the Appendix.

Substituting (8)–(12) into (7) and applying the double Borel transformation ($p^2\to M^2$, $p'^2\to M'^2$) we get the following sum rule for the formfactor f_+:

$$f_+(Q^2)=-\frac{m_c\,m_b}{f_D\,f_B\,m_D^2\,m_B^2}\exp\left(\frac{m_D^2}{M^2}+\frac{m_B^2}{M'^2}\right)$$
$$\cdot\left\{-\frac{1}{(2\pi)^2}\int ds\,ds'\,\rho_+(s,s',Q^2)\,e^{-s/M^2-s'/M'^2}\right.$$
$$\left.+M^2\,M'^2\,\hat{B}\,\Pi_+^{\mathrm{p.c.}}\right\}, \qquad (13)$$

where the last term is the Borel transform of the power corrections contribution. Note that we take into account the renormalization of the quark condensate $\langle \bar{\psi}\psi \rangle_0$ in numerical analysis. The same expressions can be obtained in the case of $B \to D^* e\nu$ decay and we find the sum rules for the formfactors $F_{0,+,V}$.

To estimate the widths of the decays it is necessary to know the formfactors $f_+(q^2)$, $F_{0,+,V}(q^2)$ (let us denote them all by $F_i(Q^2)$ for convenience) in the whole region $0 \leqq q^2 \leqq q^2_{\max}$. In order to find their behaviour as functions of q^2 we use the sum rules for the derivatives $dF_i(Q^2)/dQ^2$ at $Q^2=0$. The last ones are obtained from the sum rules for the formfactors by taking the derivative over Q^2. Note, that when taking the derivatives one should consider the variations of the integration region in perturbative part depending on Q^2. These sum rules must be considered as independent ones.

3 Numerical analysis of the sum rules

In the sum rules for the formfactors F_i in a wide range of the Borel parameters M^2, M'^2 (we vary M^2 and M'^2 independently) and continuum thresholds s_0, s'_0 the quark condensate $\langle \bar{\psi}\psi \rangle_0$ contribution is dominant, while the perturbative contribution is rather small. Nevertheless, the operator expansion is reasonable because the power corrections convergence takes place. The choice of working region for M^2, M'^2 must be made on one hand from the condition of the continuum contribution smallness and on the other hand from the condition of power corrections convergence. Namely, we demand the quark-gluon condensate contribution to be less than 50% of the quark condensate contribution (the four-quark operators contribution is small in all cases). As for the first condition, it allows the parameters M^2, M'^2 to be too large, that is why they can go to the region, where the stability of our sum rules breaks down. The reason for it is that the approximation of the higher states contribution in a standard way becomes untrue at large M^2, M'^2. So we restrict ourselves to the region of M^2, M'^2 where the stability of the sum rules is good. Practically it is possible for all the sum rules for F_i to take $M^2_{\max}=7\ \mathrm{GeV}^2$, $M'^2_{\max}=15\ \mathrm{GeV}^2$.

As for the sum rules for the derivatives dF_i/dQ^2, the quark condensate is absent here (except for dF_0/dQ^2 – these sum rules can be treated in a similar way as the previous ones) so one can introduce a usual condition for the power corrections to be small (less than 50%) in comparison with the whole value of dF_i/dQ^2. After taking the derivative d/dQ^2 we put $Q^2=0$.

The continuum thresholds s_0, s'_0 are chosen from the condition for the sum rules to have the best stability in the allowed M^2, M'^2-region. The dependence on s_0, s'_0 is rather weak and for each case we can take $s_0=12\ \mathrm{GeV}^2$, $s'_0=40\ \mathrm{GeV}^2$.

For example, we show in Fig. 2 and 3 the dependence of the formfactor f_+ and its derivative versus the Borel parameters. The curves for other formfactors and for dF_0/dQ^2 are quite similar to those shown in Fig. 2. The qualitative behaviour of the curves for the derivatives of the formfactors F_+, F_V are similar to those in Fig. 3. In this way we find:

$$
\begin{aligned}
&f_+(0)=1.0\pm0.2,\\
&\frac{df_+(0)}{dQ^2}=-(0.021\pm0.003)\ \mathrm{GeV}^{-2},\\
&F_0(0)=6.2\pm1.4\ \mathrm{GeV},\quad \frac{dF_0(0)}{dQ^2}=-(0.05\pm0.02)\ \mathrm{GeV}^{-1},\\
&F_+(0)=-(0.15\pm0.04)\ \mathrm{GeV}^{-1},\\
&\frac{dF_+(0)}{dQ^2}=0.003\pm0.001\ \mathrm{GeV}^{-3},\\
&F_V(0)=0.19\pm0.045\ \mathrm{GeV}^{-1},\\
&\frac{dF_V(0)}{dQ^2}=-(0.004\pm0.0015)\ \mathrm{GeV}^{-3}.
\end{aligned}
\tag{14}
$$

For comparison see the estimates of [3] which are $f_+(q^2_{\max})\cong 0.88$, $F_0(q^2_{\max})\cong 6.5$ GeV.

One can represent q^2-dependence of the formfactors in the form $F_i(q^2)\cong F_i(0)(1+q^2/q^2_{\mathrm{char},i})$, where $q^2_{\mathrm{char},i}\sim(m_B+m_D)^2$. Note, that $q^2_{\max}/q^2_{\mathrm{char}}\sim 0.2$ so the linear approximation of q^2-dependence for $F_i(q^2)$ is good. If we put $q^2_{\mathrm{char},i}=\kappa_i(m_B+m_D)^2$, results (14) can be represented in the form: $\kappa_+\cong 1.3$ for $B\to D^*$ transition and $\kappa_0\simeq +2.6$, $\kappa_+\simeq 1.3$ for $B\to D^*$ transition. One can see that all κ (excluding κ_0) are close to 1. It confirms the simple pole model for q^2-dependence of the formfactors. The value $\kappa_0=+2.6$ can be obtained in manypole model only.

4 Results and discussion

Now in order to find the total width of the decays considered we must substitute formfactors (14) ob-

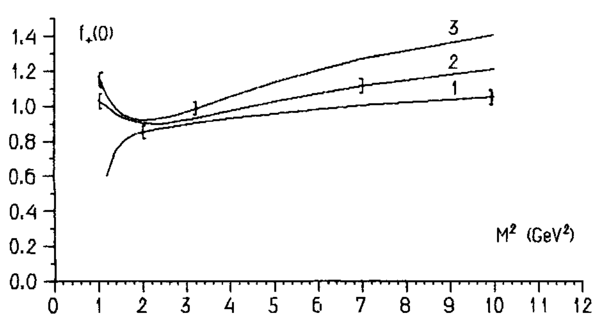

Fig. 2. Borel parameters dependence for formfactor $f_+(0)$. The curves are respected to values $M'^2=5\ \mathrm{GeV}^2$ (1), $M'^2=10\ \mathrm{GeV}^2$ (2), $M'^2=20\ \mathrm{GeV}^2$ (3). The working region of sum rule (13) is marked by brackets

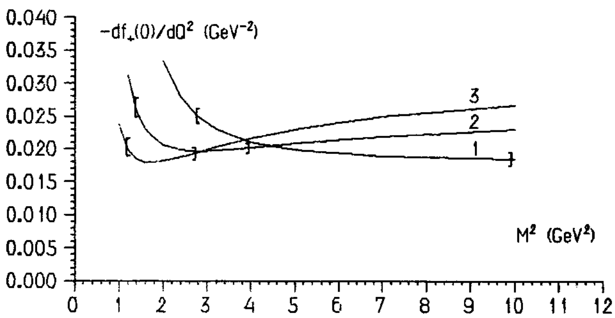

Fig. 3. Borel parameters dependence for derivative df_+/dQ^2. The curves are respected to values $M'^2=5$ GeV^2 (1), $M'^2=10$ GeV^2 (2), $M'^2=20$ GeV^2 (3). The working region of respective sum rule is marked by brackets

tained into the formulas for the total widths which are

$$\Gamma(B\to D)=\frac{G^2\,m_B^5}{768\pi^3}\,|V_{bc}|^2\;0.4 f_+^2(0) \tag{15}$$

for $B\to De\nu$, and

$$\Gamma(B\to D^*)=\frac{G^2 m_B^5}{768\pi^3}\,|V_{bc}|^2\,(1/m_B^2\,F_0^2(0)+0.9\,F_0(0)\,F_+(0) + 0.3\,m_B^2\,F_+^2(0)+0.07\,m_B^2\,F_V^2(0)). \tag{16}$$

for $B\to D^* e\nu$.

The numerical factors in these formulas are the values of some dimensionless integrals over the phase space volume of the decays. For simplicity we omit here the dependence of the formfactors on q^2, which gives only small corrections. Of course, this corrections were taken into account in the final answer. The coefficient at the term $\sim F_0(0)\,F_V(0)$ in (16) is equal to zero.

Substituting (14) into (15) and (16) and using the experimental value for the B-meson lifetime $\tau_B=1.42\cdot 10^{-12}$ s we find for the branching ratios of the decays:

$$\mathrm{Br}(B\to D)=3.7\%\cdot\left|\frac{V_{bc}}{0.045}\right|^2,$$
$$\mathrm{Br}(B\to D^*)=6.4\%\cdot\left|\frac{V_{bc}}{0.045}\right|^2. \tag{17}$$

The experimental value of the second branching ratio is known to be $\mathrm{Br}(B\to D^*)=(7.0\pm 1.2\pm 1.9)\%$ [7]. This value is somewhat higher than our result (17), but if we take into account the uncertainty of the matrix element V_{bc} value and also the uncertainty in (17) connected with the accuracy of our results for the formfactors we see that our theoretical estimate does not contradict the experimental value. Note that the accuracy of our estimate of $\mathrm{Br}(B\to D^*)$ (17) is of order 70% because of the partial cancellation of different terms in (16).

Let us point out a significant difference between our results and the estimates of [3]. We have obtained the probabilities of the transitions $B\to D$ and $B\to D^*$ to be approximately equal to each other, while in [3] their ratio is 1:4. This disagreement is on the one hand due to our value of the formfactor F_0 which is about ~ 1.5 times smaller than the corresponding value from [3], and on the other hand due to the neglection formfactor F_+ in [3]. Really the term, proportional to $F_0\,F_+$ in (16), is rather large and has an opposite sign as compared with the term $\sim F_0^2$, which reduces significantly the width of the decay $B\to D^* e\nu$.

If one assumes the parton formula for the total semileptonic width of B-meson decays

$$\Gamma_{sl}=\frac{G^2\,m_b^5}{192\pi^3}\,(|V_{bc}|^2\;0.5+|V_{bu}|^2), \tag{18}$$

(we neglect $b\to u$ transition) to be true with the accuracy $\sim 10\%$, and uses the experimental results $\mathrm{Br}(B\to Xe\nu)=10\%$, it is possible to exclude V_{bc} and find $\mathrm{Br}(B\to D)=(3.7\pm 1.5)\%$, $\mathrm{Br}(B\to D^*)=(6.4\pm 3.5)\%$. We see that our value for the branching ratios $\mathrm{Br}(B\to D^* e\nu)$ is in agreement with the experimental data. Two transitions $B\to D$ and $B\to D^*$ saturate the total width of the semileptonic transitions of B-meson. Note that it will be possible to compare our predictions with experiment soon because ARGUS collaboration is now working on the determination of the decay $B\to De\nu$ width.

For the decay $B\to D^* e\nu$ it is also possible to determine the polarization of the D^*-meson. We have obtained the dependence of the asymmetry parameter, characterizing the polarization of D^*-meson, $\alpha=2\Gamma^{(L)}/\Gamma^{(T)}-1$ ($\Gamma^{(L)}$ and $\Gamma^{(T)}$ are total widths of the decays to the states with longitudinal and transverse polarizations of D^*-meson) on the cutoff in the energy of electron E_e (the events with electron energy less than E_e are not taken into account because of the large experimental uncertainties in this kinematical region). The curve presenting the dependence $\alpha(E_e)$ is shown in Fig. 4. For $E_e=1$ GeV $\alpha=0.1$, and for $E_e=1.2$ GeV we have $\alpha=0.0$, which corresponds to $\Gamma^{(L)}/\Gamma^{(T)}=0.55$ and 0.50, respectively. These values are in a good agreement with the experimental data [8], although the accuracy of our prediction for the ratios $\Gamma^{(L)}/\Gamma^{(T)}$ is not better than 100% (the accuracy of the experimental result [8] is rather low too).

The distributions in the electron energy for each of the channels considered are shown in Fig. 5. Our curve for $B\to D^*$ transition is going considerably lower than in other models [1–5] for this decay.

In conclusion let us stress once more that our study of the decays $B\to De\nu$, $B\to D^* e\nu$ in the present paper is based on the standard assumptions of QCD sum rules method and do not use any model-depen-

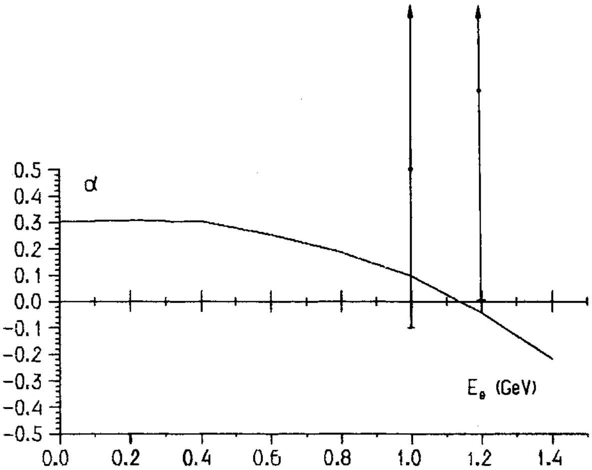

Fig. 4. The asymmetry parameter α dependence on electron energy cut-off E_e in decay $B \to D^* e \nu$. Experimental values are taken from [8]

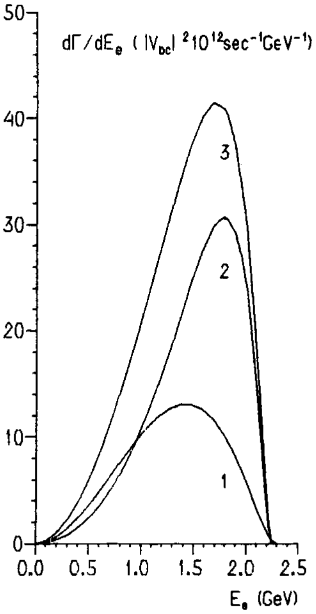

Fig. 5. The electron spectra for decays $B \to D e \nu$ (1), $B \to D^* e \nu$ (2) and for their sum (3)

dent assumptions. But this approach does not allow to achieve the accuracy higher than 20–30% for our formfactors and that leads to the accuracy of order 50% for the decay widths.

The results of our paper for $B \to D e \nu$ decay were independently obtained by Bayer and Grozin [18] with the same method. Their results are in agreement with ours.

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Appendix

Here we represent the expressions for quark-gluon and four-quark condensate contributions to the amplitudes Γ_i defined by (5) and (6) in the case of $B \to D^* e \nu$ decay.

$$\Gamma_0^{\langle\bar\psi G\psi\rangle} = -\frac{g_s}{12}\langle\bar\psi\, G_{\mu\nu}\,\sigma_{\mu\nu}\,\psi\rangle_0 \cdot\left(\frac{3m_c^2}{r^3\, r'}(m_c^2+m_b^2+2m_c\, m_b+Q^2) + \frac{3m_b^2}{r\, r'^3}(m_c^2+m_b^2+2m_c\, m_b+Q^2) + \frac{1}{r^2\, r'^2}[3m_c\, m_b(m_c^2+m_b^2+Q^2) + 2((m_c^2+m_b^2+Q^2)^2-m_c^2\, m_b^2)] + \frac{1}{r^2\, r'}[3m_c(m_c+m_b)+2(m_b^2+Q^2)] + \frac{1}{r\, r'^2}[3m_b(3m_c+m_b)+4(m_c^2+Q^2)] - \frac{2}{r\, r'}\right),$$

$$\Gamma_+^{\langle\bar\psi G\psi\rangle} = \frac{g_s}{12}\langle\bar\psi\, G_{\mu\nu}\,\sigma_{\mu\nu}\,\psi\rangle_0 \cdot\left(\frac{3m_c^2}{r^3\, r'} + \frac{3m_b^2}{r\, r'^3} - \frac{2}{r\, r'^2} + \frac{1}{r^2\, r'^2}(2m_c^2+2m_b^2-m_c\, m_b+2Q^2)\right),$$

$$\Gamma_V^{\langle\bar\psi G\psi\rangle} = -\tfrac{1}{6}\, g_s\langle\bar\psi\, G_{\mu\nu}\,\sigma_{\mu\nu}\,\psi\rangle_0 \cdot\left(\frac{3m_c^2}{r^3\, r'} + \frac{3m_b^2}{r\, r'^3} + \frac{2}{r\, r'^2} + \frac{1}{r^2\, r'^2}(2m_c^2+2m_b^2-m_c\, m_b+2Q^2)\right),$$

$$\Gamma_0^{\langle\bar\psi\psi\rangle^2} = -\frac{4\pi}{81}\,\alpha_s\langle\bar\psi\,\psi\rangle_0^2 \cdot\left(\frac{4(2m_c-m_b)}{r^2\, r'} + \frac{2(7m_b-8m_c)}{r\, r'^2} + \frac{3m_c^3}{r^4\, r'}(m_c^2+m_b^2+2m_c\, m_b+Q^2) + \frac{3m_b^3}{r\, r'^4}(m_c^2+m_b^2+2m_c\, m_b+Q^2) + \frac{1}{r^3\, r'^2}[3m_c^2\, m_b(m_c^2+m_b^2+Q^2) + 2m_c((m_c^2+m_b^2+Q^2)^2-m_c^2\, m_b^2)] + \frac{1}{r^2\, r'^3}[3m_c\, m_b^2(m_c^2+m_b^2+Q^2) + 2m_b((m_c^2+m_b^2+Q^2)^2-m_c^2\, m_b^2)]\right.$$

$$+\frac{m_c}{r^3\,r'}(17m_c^2+16m_b^2+19m_c\,m_b+16Q^2)$$

$$+\frac{m_b}{r\,r'^3}(16m_c^2+15m_b^2-3m_c\,m_b+16Q^2)$$

$$-\frac{1}{r^2\,r'^2}[2m_c(m_c^2+Q^2)-6m_b(m_b^2+Q^2)$$

$$\left.+m_c\,m_b(11m_c+m_b)]\right),$$

$$\Gamma_+^{\langle\bar\psi\psi\rangle^2}=\frac{4\pi}{81}\alpha_s\langle\bar\psi\,\psi\rangle_0^2\left(\frac{3m_c^3}{r^4\,r'}+\frac{3m_b^3}{r\,r'^4}\right.$$

$$+\frac{m_c}{r^3\,r'^2}(2m_c^2+2m_b^2-m_c\,m_b+2Q^2)$$

$$+\frac{m_b}{r^2\,r'^3}(2m_c^2+2m_b^2-m_c\,m_b+2Q^2)$$

$$\left.+\frac{14m_c}{r^3\,r'}+\frac{10m_b}{r\,r'^3}+\frac{2(2m_b-m_c)}{r^2\,r'^2}\right).$$

$$\Gamma_V^{\langle\bar\psi\psi\rangle^2}=-\frac{2\pi}{81}\alpha_s\langle\bar\psi\,\psi\rangle_0^2\left(\frac{3m_c^3}{r^4\,r'}+\frac{3m_b^3}{r\,r'^4}\right.$$

$$+\frac{m_c}{r^3\,r'^2}(2m_c^2+2m_b^2-m_c\,m_b+2Q^2)$$

$$+\frac{m_b}{r^2\,r'^3}(2m_c^2+2m_b^2-m_c\,m_b+2Q^2)$$

$$\left.+\frac{14m_c}{r^3\,r'}+\frac{14m_b}{r\,r'^3}+\frac{2(2m_b-m_c)}{r^2\,r'^2}\right).$$

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