Shock Waves (1991) 1: 3-15

Shock Waves

The discovery of the Mach reflection effect and its demonstration in an auditorium

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Received July 10, 1990; accepted August 11, 1990

Abstract. This paper examines the historical background leading to the discovery of the Mach reflection effect and applies original documents from Mach's residue which are kept in the archives of the Ernst-Mach-Institut in Freiburg. Two experimental setups for the generation and demonstration of the Mach reflection effect, incorporating an overhead projector, are described: (a) Mach's historic mechanical shock wave reflection and interaction experiments with soot covered glass plates, performed in 1875. The Mach triple points sharply erase the soot which results in a residual picture of funnel-shaped V-formations. The head-on collision of two shock waves is marked as a narrow line of piled-up soot. (b) CalTech's hydraulic jump reflection experiments in a shallow ripple tank, performed during World War II. Regular reflection and its transition into a Mach reflection wave. Using a slightly inclined tank and providing a "shoreline" in the middle of the tank, Mach stem propagation slows down to zero when hitting the shore line and, therefore, can be observed "live" without the use of a slow motion technique.

Key words: Ernst Mach, History of shock wave research, Mach reflection, Hydraulic analogy

1. Introduction

Ernst Mach (1838-1916), a universalist of science in the classical sense and a leading authority of his time in philosophy, physiology and physics, was also a pioneer in fluid dynamics and ballistics. He was the first and foremost who recognized the characteristics of shock waves and invented various high speed visualization and shock wave diagnostic methods (Blackmore 1972). Figure 1 shows a portrait of the young Ernst Mach.

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For many physicists and engineers the name of Ernst Mach is associated mainly with the Mach number, the ratio of flow speed to acoustic speed which can be less than, equal to or greater than unity according to a steady subsonic, sonic or supersonic flow, respectively. Furthermore, he discovered the occurrence of head or bow waves which surround any body flying with supersonic speed and which e.g. create the well-known sonic boom of supersonically traveling aircraft.

However, the Mach reflection effect is certainly his most important contribution to fluid dynamics. Today more than 115 years since his first conclusions - it now as before remains a phenomenon which is unexpected and fascinating for the newcomer as well as surprising and challenging for the expert because of the continuous discovery of new side effects and the tremendous difficulties in modeling them mathematically. Since 1981 a special international symposium has been dedicated exclusively to this effect [1]. Mach reflection is a very basic effect in shock wave physics and can cover all ranges of shock pressures. Although mostly investigated in gases, it exists also in liquids and even in solids where ultrahigh dynamic pressures can be achieved in very simple arrangements (Fowles and Isbell 1965).

Quite frequently visitors to our institute show a particular interest in the Mach reflection effect, and we can refer to our large collection of photographs on shock propagation and reflection phenomena studied within shock tubes and in the free field, but also in liquids. These pictures, mostly taken in shadowgraph, schlieren or interferometer arrangements with sophisticated and expensive high speed framing cameras, makes one forget that the era of gas dynamics started with very simple, but highly ingenious methods and apparatus. In recent years, already two from our institute (Merzkirch 1970;

[1] The International Mach Reflection Symposia have been held in Victoria, BC, Canada (1981); Sydney, Australia (1982); Freiburg, FRG and Melbourne, Australia (1983); Tokyo and Sendai, Japan (1984); Menlo Park, CA, USA (1985); Beer Sheva, Israel (1986); Albuquerque, USA (1987) and Toronto, Canada (1988)

Fig. 1. A little known portrait of the young Ernst Mach at the age of 23, then appointed "Privatdozent für Physik" at Vienna University

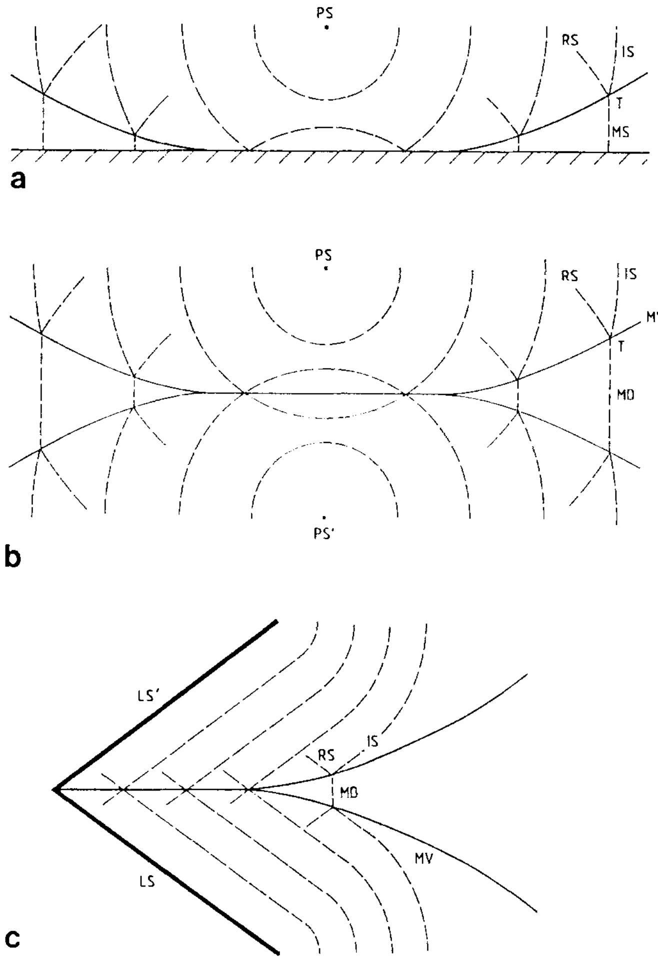

Fig. 2a-c. Different geometrical arrangements to generate Mach reflection: **a** reflection of a shock wave at a rigid boundary, **b** corresponding analogous setup of two identical and simultaneously triggered point shock sources PS and PS' and **c** oblique interaction of shock waves emerging from two simultaneously exploding linear shock sources LS and LS'. Broken lines represent instantaneous shock wave patterns at four successive time instants, solid lines the residual soot picture. IS – incident shock; RS – reflected shock; MS – Mach stem; MD – Mach disk or bridge; T – triple point; MV – Mach-V or Mach funnel

Reichenbach 1983) have reminded us of this glorious beginning. The present paper follows this line by describing two different types of historic Mach reflection experiments modified by us for demonstration purposes. They are both easy and inexpensive to prepare and appropriate to be shown to a large audience with the aid of an overhead projector.

The Mach reflection effect – or in shock physics briefly called Mach effect – is illustrated in Fig. 2a. At sufficiently oblique incidence angles the reflected shock no longer meets the incident wave directly at the boundary, but rather at a point in the fluid. The two waves join to form a third shock wave extending to the wall. This form of reflection is known as irregular or Mach reflection (Courant and Friedrichs 1948). In the beginning of analytical treatment the three-shock configuration was also known as a λ-configuration (Courant and Friedrichs 1948), but later on this designation was rarely used. Today, the third shock is frequently described as the Mach stem and bridge or disc, according to the applied geometrical setup. At the triple point, three regions of different pressures and densities join each other.

Merzkirch (1970) mentioned that the designation 'Mach effect' for an irregular shock reflection originated from Seeger in 1944. However, already in his publication on oblique shock reflection, von Neumann (1943) cited a paper by K. Friedrichs entitled "Remarks on the Mach effect" which was presented in 1943 at the Applied Mathematics Panel of NDRC. Today it is difficult to state with certainty who coined this designation. Obviously, the originator had a profound knowledge on the history of gas dynamics.

2. Shock reflection soot experiments

2.1. Historical remarks on the Mach reflection effect

Ernst Mach discovered the reflection effect in 1875 at the Physical Institute of the German University of Prague. At that time he still had his laboratory at No. 562-1 Obstmarkt 7, a building located in the town center and close to the "Carolineum", the oldest building of the university. The discovery was favored by the coincidence of the following factors.

First – Mach was basically an experimentalist, and his great success in physical interpretation was based on observation. Consequently, he was fascinated by any new visualization technique. There already existed a remarkable body of dust recording methods for acoustical and electrical phenomena. G.C. Lichtenberg (1742-1799) was the first who in 1777 made visible traces of gliding sparks along insulators by dusting them. E.F. Chladni (1756-1827) discovered in 1787 sound figures of oscillating membranes, and A. Kundt (1839-1894) introduced this method in 1866 to mark standing waves in acoustic resonators. Curious as to the acoustic nature of Kundt's dust figures, Mach asked his assistant C. Dvorak to investigate them more closely.

Second – Mach, particularly interested in refraction and reflection of acoustic waves since 1860, had just performed with his assistant A. Fischer physiological-acoustic studies using electric sparks as a source of sound. This combined curiosity in acoustics and electricity continued during his whole life and led to remarkable achievements such as intense electric spark light sources, trigger circuits with tunable microsecond time delays and high-voltage high-current pulse generators.

Third – At the same time, K. Antolik, professor at Kaschau College in Hungary, published a paper entitled "The gliding of electric sparks" (Antolik 1875). At first experimenting also with dust to mark spark traces, he discovered the suitability of soot by chance. Bringing a small soot-coated glass balloon close to the spark of an influence machine, he noticed a well-marked trace of the spark in the soot. He readily developed a new soot recording technique which was later taken over by Mach and observed an important phenomenon: soot covered glass plates, brought close to crooked electrical discharges, showed complicated V–shaped patterns which, however, disappeared when the air between the plates was evacuated. It is important here to point out the fact that, contrary to dust, only the soot method is capable of recording traces of the triple points as will be shown later in Fig. 7. Furthermore, dust is less appropriate in high voltage arrangements, because charges on the plates, induced by electrostatic influences, can also affect dust transportation.

Fourth – It was Dvorak who brought this paper to Mach's attention. Dvorak repeated Antolik's experiments and demonstrated them at the regular institute's seminars. Here Mach, an expert both in acoustics and electricity, speculated for the first time that the origin of these soot figures was merely caused by an interaction of acoustic waves originating from the electric sparks. At that time he was not yet conscious of the phenomenon of shock waves which propagate faster than acoustic waves. Later on, Antolik was also invited into Mach's institute and lectured on his work.

Then, only within a few months after Antolik's publication, Mach together with his student Wosyka eagerly verified his discovery by also applying other experimental methods. Already on April 15, 1875, Mach communicated to the Imperial Academy of Sciences of Vienna in a brief note that he and Wosyka had successfully repeated Antolik's experiments, revealing that Antolik's soot pictures are caused by an interference of acoustic waves (Wiener Akad. Anzeiger 10:83). Their final results, submitted for publication on June 10, 1875, are summarized as follows (Mach and Wosyka 1875):

(a) They repeated the soot experiments, but immersed the glass plates in turpentine. They obtained also the V-formations which, however, were somewhat blurred because the liquid partly dissolved the soot. This proved for the first time the existence of irregular reflection in liquids.
(b) They used a rectangular plate of transparent glue which had just solidified and hit one corner with a hammer over a piece of wood into which a rectangular cutout had been milled and brought into contact with two edges. Applying an optical setup with crossed polarizers he observed the propagation of mechanical waves emitted from both corners and their mutual interaction. Again, they obtained similar results of funnel shaped V-formations as obtained in their previous soot experiments. This method was even recommended for demonstration in an auditorium. As a light source they used a heliostat, a plane mirror driven by a clockwork mechanism which reflected sun light into the darkened lecture hall. Today this method – to optically diagnose stress distributions in solids – is called photoelasticity.
(c) They also successfully used surface waves for modeling purposes which will be described further in section 3.1.

Although the schlieren method, a considerably more powerful recording method, was already discovered in 1865 by A. Toepler, it was infrequently used by Mach in the beginning because of technical problems such as providing an intense flash light source, an electric delay circuit in the microsecond regime, sensitivity of films, etc. However, later on it became his favorite method in photographing supersonic projectiles. It is interesting to note that Mach ingeniously concluded the existence of an irregular reflection behavior merely out of soot pictures which can provide only residual traces of the triple points. On the other hand, shock physicists nowadays are accustomed to think more in terms of instantaneous wave patterns such as provided directly by high speed photo-instrumentation.

We do not know today how Mach himself felt about his discovery. His numerous communication notes with the Vienna Academy as well as his publications reveal that in 1875 he was also working successfully in completely different fields such as optics and physiology. Also his private notebook of this period documents that only a minor part of his entries relates to the reflection effect. Certainly his major research activities were a continuation of previous years, while he was overcome with the discovery of nonlinear reflection phenomena by chance. But after 1875, Mach and his team continued on the soot recording technique and applied it to various problems: mechanical-acoustic effects of electric sparks (Rosicky

1876); measurement of the velocity of shock waves (Mach and Sommer 1877; Mach et al. 1878); study of propagation of two- and three-dimensional shock waves criteria for the occurrence of irregular reflection (Mach 1878), and investigation of the fine structure of the V-formations (Mach et al. 1878; Mach and Simonides 1879). Wosyka was excluded from the team. The reason was described in a short note dated from 1886 and now in the Ernst-Mach-Archives. Mach stated that among other things: "In the year 1876 my assistant J. Wosyka was discharged at once from the institute for book stealing. I have replaced the books. I have fallen, because of this behavior of Wosyka who was indebted to me for many things, into an emotional depression which has made me for years incapable of work". He developed an eye ailment related to high blood pressure which led to migraine headaches (Blackmore 1972).

These publications laid the cornerstones of modern shock wave physics and can be summarized as follows: B. Riemann (1826-1866), professor at Göttingen University, was the first who in 1860 mathematically treated shock waves as waves of finite amplitude and predicted for shock waves a higher propagation velocity than the sound velocity (Riemann 1860). Mach carefully analyzed his first experiments of 1875 and experimentally confirmed Riemann's very important conclusion that shocks are waves of finite amplitude and propagate faster than sound waves. In the case of an oblique intersection of two shocks, a new secondary wave is generated in the plane of symmetry which has a higher strength and which propagates faster than the component of the primary shocks in the direction of the symmetry axis. Also, examining the fine structure of the soot lines under a microscope, he concluded that the secondary wave was of greater strength by the fact that the soot inside the two branches of the V-formation was compressed stronger than outside the 'V'. From simple geometric considerations of the velocity vectors of incident and reflected waves he calculated the starting point of the V-formation in the fluid and found experimental evidence for this. However, the first comprehensive analytical treatment of the Mach effect did not succeed until World War II (von Neumann 1943).

How was then the echo in the physical community of his time on this remarkable achievement? Theoreticians on nonlinear flow did not pay any attention to his results. Notable contemporary theoreticians on hydrodynamics were e.g. Helmholtz (1821-1894), Hugoniot (1851-1887), Kirchhoff (1824-1887), Rayleigh (1842-1919) and Stokes (1819-1903). There was also hardly any interest from experimentalists. This is in contrast to his next great discovery in 1886, the phenomenon of the bow shock which surrounds a supersonic projectile. First strongly doubted by French experts it was quickly accepted worldwide for its direct practical importance in ballistics. Only De Waha in Luxembourg (De Waha 1878) and A. Schuller, professor at the Polytechnicum Budapest, performed similar soot experiments. Schuller had duplicated some of Antolik's and Mach's results, however, using explosives instead of electric sparks. Unfortunately, Schuller did not consider his results to be important enough for publication (Mach and Simonides 1879). Even many years after its first application, Mach himself showed a continuous interest in the soot technique and on Antolik's ongoing research. In a letter of May 14, 1884, Antolik describes to Mach further electric spark experiments.[2]

Then Mach's principle method of diagnostics, the soot technique, fell into oblivion for more than 60 years until it was rediscovered in the USA during World War II. The renewed interest arose also from the revival of interest in the Mach reflection effect for military applications, both in the USA and England, for determining the optimum height of burst of atomic weapons (Reines and von Neumann 1947) and for increasing the efficiency of underwater explosions from multiple charges (Cole 1948), respectively. At that time John von Neumann (1903-1957) developed a theory on oblique reflection of shocks and became highly interested in previous experimental results. He owed his knowledge of Mach's work on the irregular forms of shock collisions to discussions with Col. H. Zornig, U.S. Army Ordnance Department, in 1941. On von Neumann's suggestion, E.B. Wilson Jr., E. Kennedy and J. Frankl started with Mach's soot technique in the same year. However, the next year, Prof. R.A. Wood (1868-1955), professor eremitus of Johns Hopkins University and then already 74 years old, continued the experiments (von Neumann 1943). Wood, a talented experimentalist and interested in the photography of acoustical phenomena since the turn of the century, improved the original method. Much later, Seeger (1970) published some of the soot pictures from Wood's personal files by courtesy of J. Wosyka-Mandansky, probably a descendant of Mach's student J. Wosyka. Therefore, it seems quite possible that Wosyka had met and consulted Wood personally, thus acting as a link between the discovery of the Mach reflection effect and its rediscovery in the USA.

At the Ernst-Mach-Institut, the soot technique was applied previously for various problems on detonations in gases. Inspired by the works of Soloukhin (1966) in the USSR and Strehlow (1968) in the USA, detailed research on the ignition behavior of spherical detonation waves in gas mixtures was carried out at EMI; a review up till 1971 was given by Schultz-Grunow (1971). In these experiments a soot coated lucite cone was used and positioned with its tip into the center of the explosion in such a way that the axis of symmetry of the cone pointed radially outwards. The spatial distribution of reaction centers from deflagration into detonation is marked on the soot as a parallelogram-type cell structure. Schmolinske (1974) extended this method by applying a combined schlieren/soot technique. The suitability of the soot method was also demonstrated for

[2] This letter is in the possession of the EMI Archives together with a large collection of many other letters which Mach had received during his life from such famous scientists as Boltzmann, Einstein, Helmholtz, Hertz, Planck etc. We do not know his replies but some of them can be found as drafts in his notebooks

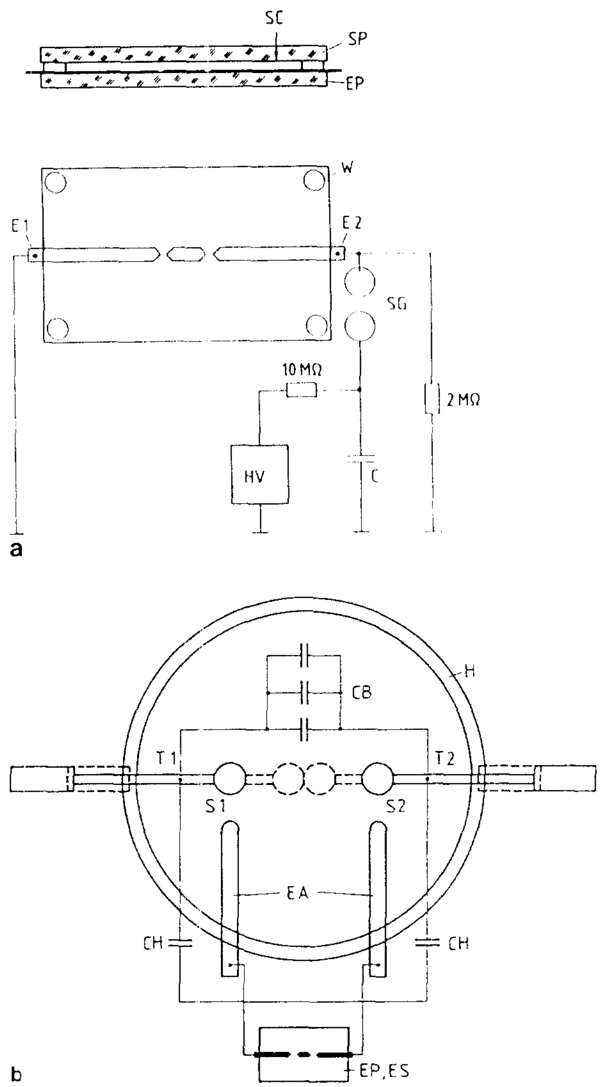

Fig. 3a,b. Schematic of the soot experiments: a modern setup: C - pulse capacitor; HV - high voltage generator; SG - spark gap; EP, SP - electrode/soot covered glass plates; SC - soot coating; E1, E2 - copper foil electrodes; W - washer. b historical setup as used by Antolik and Mach: H - Holtz machine, CB - additional battery of Leiden jars; CH - small Leiden jar; EA - "Einschalt - Apparat", two isolated mounted telescopic brass rods. Switching occurs between the two air spark gaps S1 and S2. The Machine is started with the terminals T1 and T2 shortened (broken lines)

the visualization of Mach-V formations at special exit geometries of shock tubes (Reichenbach 1983).

2.2. Experimental setup

The soot recording technique, although in principle is easy to apply, is not familiar to modern shock physicists. We experienced the practical difficulties when we repeated the historic soot experiments. Many important parameters have not been described in sufficient detail to reproduce the experiments, e.g. the method to provide the soot layer homogeneously, the actual size of the soot figures, etc. Also, since the total capacity of the Leiden jars and their charging voltage – both influencing the shock strength – were not specified, it is difficult to get an idea of the sensitivity of the soot method. Therefore, in the following paragraphs we depict these experimental parameters in more detail.

The shock waves are produced by a pulse discharged from capacitors. To obtain strong shocks from small electrical energies, the shock propagation is confined to a thin two-dimensional channel formed by a set of parallel glass plates facing the electrodes. A schematic illustration is shown in Fig. 3, a modern version in Fig. 3a. A pulse capacitor of approximately 5 nF is charged up to 35-40 kV and discharged via an air spark gap when its self-breakdown voltage is reached. To stabilize its trigger voltage level independently from the applied electrode geometry on the glass plate, a high voltage resistor of 2 MΩ is inserted toward ground. Today, any modern high-voltage power supply and capacitor would be suitable.

Both glass plates are 3 mm thick, have a format 210 mm × 150 mm and are separated by 2 mm thick washers at each corner. The electrodes, consisting of a 0.15 mm thick copper foil, are glued on top of the lower glass plate. For cylindrical shock interactions, Fig. 2b, each spark is generated between the short gap of two pointed electrodes. Linear shock wave fronts as depicted in Fig. 2c can be produced by electric wire explosions. However, a substantial part of the capacitor energy would be needed to evaporate the wire material and only a minor part would be transmitted into the shock wave. In this regard gliding sparks are more effective, but the discharge path of long sparks is random. To force the gliding spark in following a predetermined path, different methods had been used. Antolik applied a gold paint. Wood (see Seeger 1970) drew these lines with a lead pencil. We obtained best results with silver paint, sufficiently thinned with Cyano Solv, both from GC Electronics, Rockford, Ill. and applied to the glass plate with a 0.1 mm wide drawing pen. The lower surface of the upper glass plate is coated with soot. The quality of the soot layer is decisive for the resolution efficiency of the soot pictures. Antolik used a stearin-candle, but it is difficult in this way to achieve a homogeneous layer of soot. Rosicky (1876), one of Mach's assistants, improved the quality of the soot layer by using city gas. We tried various methods, including diesel fuel, petroleum and acetylene gas. We obtained best results by using an acetylene-driven Bunsen burner, operated at 200 kPa and carrying a narrow, 40 mm × 2 mm nozzle adapter. The top of the flame front should just touch the glass plate. For recording strong shock waves, it is advantageous - prior to exposing the glass plate to the flame - to provide a very thin layer of silicone grease on it by using a rubber roller. This will result in a well-adhered soot layer (Giesel 1990).

Mach and Antolik used an array of Leiden jars charged up by a Holtz influence machine, their historic setup is shown in Fig. 3b. The first manually powered high-voltage generator based on friction electricity was invented in 1663 by O. von Guericke (1602-1686). A more

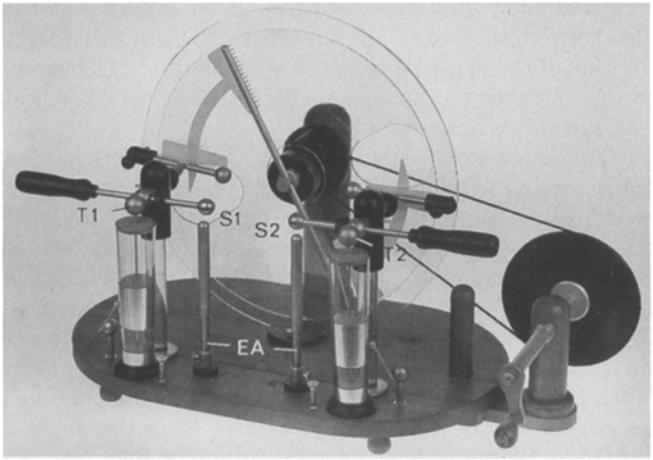

Fig. 4. Reproduction of a Holtz influence machine as originally manufactured in 1872 by E. Borchardt, Berlin. Since the Holtz machine is not self-exciting, it has to be started by providing an external charge to one of the paper segments at the stationary glass disk, such as by touching it with a hard rubber bar which has been rubbed before by a woolen cloth

effective machine based on the principle of electrostatic influence was invented in 1864 by W. Holtz (1836-1913) in Berlin and independently by A. Toepler (1836-1912) in Riga. Considered as the most advanced and powerful high-voltage generator of its time, the Holtz machine became a widespread instrument. Hundreds were produced but very few have survived up to today. Leiden jars were the favored high-voltage capacitors at Mach's time. The Leiden or Kleist jar was invented in 1745 by P. van Musschenbroek in Leiden, Holland and, independently, by E.J. von Kleist in Pomerania, then an eastern province of Germany. In the present example three Leiden jars, each with a capacity of 1.7 nF, have been manufactured from quartz glass beakers 280 mm in height and 140 mm in diameter (commonly used in chemistry laboratories). The influence machine incorporates also an "Einschalt-Apparat" (Holtz 1865), a twin air spark gap device representing the air spark switching device as depicted in Fig. 3a. It completely decouples the discharge load galvanically during the charging cycle, an important advantage for experimenting with high voltage. A view of a Holtz influence machine is shown in Fig. 4. The two small Leiden jars (each with a capacity of 15 pF) connected to the output terminals are from Leybold Didaktik GmbH, Stuttgart.

To demonstrate the Mach reflection experiments in an auditorium, an overhead projector may be used. Fig. 5 shows an overall view of the complete setup. First the lower glass plate is put on the projector to illustrate the applied electrode geometry. Then the electrodes are connected to the discharge circuit, and the opaque, soot covered upper glass plate is laid on top of the lower plate, spaced by four washers. The experiment is started by driving up the Holtz generator. Immediately after discharge the complete soot picture including the electrode geometry is directly visible to the audience.

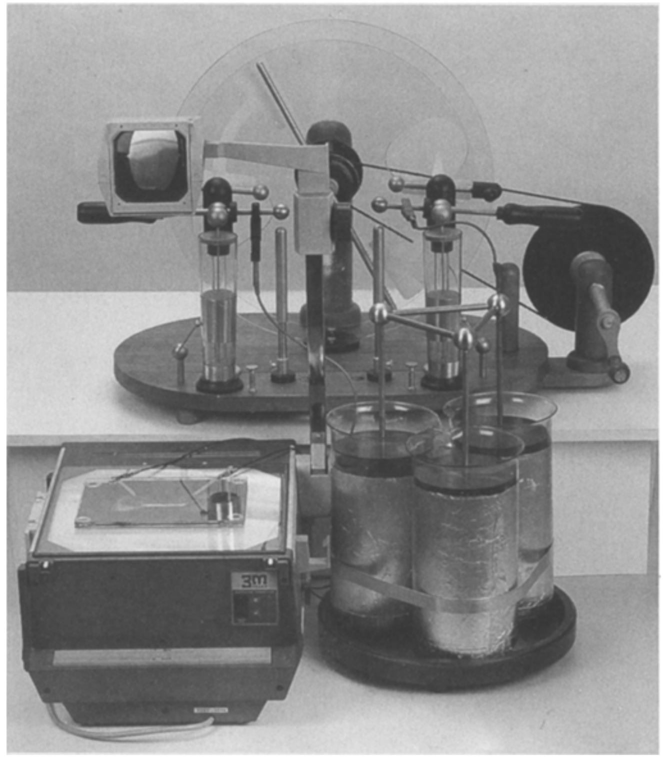

Fig. 5. Total setup with overhead projector for demonstration of soot experiments

2.3. Antolik's gliding spark soot experiments

Antolik (1875) described a large number of soot experiments. In the beginning, he used a setup which consisted only of one glass plate. He glued the electrodes onto the glass plate and covered them with a piece of cardboard, such as a visiting-card. Then he coated the arrangement with soot by holding it over a candle. The cardboard was necessary to isolate the high voltage spark from the semi-conducting soot layer. Later he improved this method by applying a second, upper glass plate which he installed only 2 – 3 mm apart from the lower, clear glass plate carrying the electrodes. Obviously, this "covering method" as he called it had the advantage that cardboard was no longer necessary and that also the close environment of the sparks was nicely marked in the soot.

In the following, we want to confine our attention to a few typical experiments which are summarized in Fig. 6. Generally, gliding sparks along glass surfaces are rarely straight lines, extending from one to the opposite electrode as shown in Fig. 6a. Rather, the spark follows the instantaneous inhomogeneities in local conductivity, thus resulting in a random spark trace. In this case shock waves emerging from such a crooked spark channel interact with each other and can create Mach-Vs as illustrated in Fig. 6b.

Antolik produced crooked spark channels artificially by drawing with gold paint a zigzag line on the glass

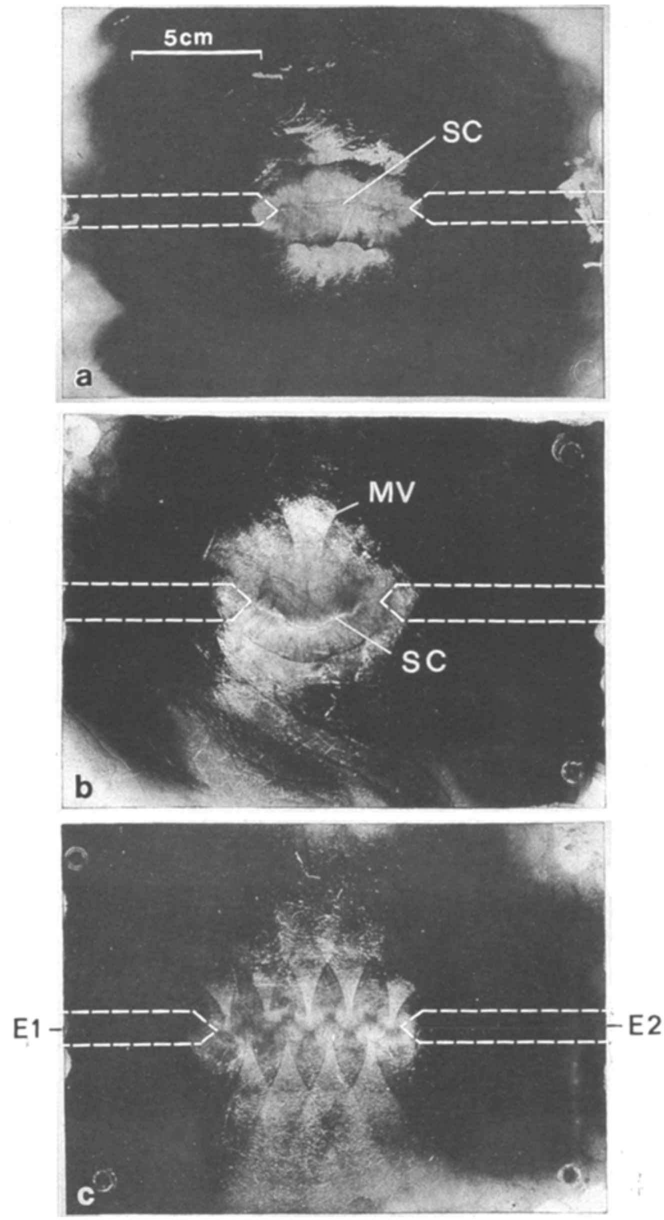

Fig. 6a-c. Duplication of Antolik's soot pictures: **a** gliding spark with a straight and **b** with a discharge channel curved at random. **c** a zigzag-guided gliding spark shows a multitude of Mach-Vs. E1,2 - electrodes; SC - spark channel; MV - Mach-V or Mach funnel

plate, thus forcing the gliding spark to follow this trace. As illustrated in Fig. 6c, every straight section of the zigzag-shaped spark channel becomes then a source of a shock wave which, interacting with its neighbors, form a multitude of funnel-shaped lines. Certainly, this soot picture, which is nice to look at but unexplained by Antoplik, inspired Mach to reduce this complicated phenomenon to the essential mechanism by first experimenting with a single V-arrangement of two straight gliding sparks.

2.4. Mach's shock reflection soot experiments

With the soot technique, Mach studied four basic types of shock interaction geometries: (a) Double discharge experiments, Fig. 2b; (b) Oblique interaction of two plane shock waves, Fig. 2c; (c) Head-on collision of a Mach disk with a plane shock wave; and (d) Interaction of two Mach-V configurations.

Although we still retain in our archives some of Mach's original photographs from his ballistics experiments, none of his soot pictures have passed on to us. It seems that in most cases he generated the shock waves by an electric discharge between two closely spaced glass plates. Although wave propagation is then not strictly two-dimensional because of continuous reflections along the plate surfaces, sharply pronounced shock fronts are generated. This is best illustrated in the schlieren photos Figs. 7c, d for the example of a double discharge. Corresponding pictures in Figs. 7a and b were taken with the same electrical and geometrical parameters, but using different recording methods. These also document that the soot funnels truly represent records of the traces of the triple points. However, the dust method only shows that along the middle line of both discharges the superposition of both shocks is somewhat stronger than a single shock, thus sweeping the cork dust further outwards.

A soot picture of the famous Mach funnel for a V-spark is shown in Fig. 8a. This geometry easily allows one to change the angle of incidence of both shocks which in the close environment of the spark is identical with the angle both spark channels form with each other. Initially the soot funnel starts from a central line between the two sparks, indicating regular reflection. But at the transition angle into Mach reflection it bifurcates into two lines. This is seen more clearly in Figs. 8b, c.

In the configuration of Fig. 8a the Mach disk has not marked itself on the soot. However, using another instantaneous wave, colliding head-on with the Mach disk, soot transportation is stopped along a line where the afterflows of both shocks eliminate each other. This line is well marked as a sharp wall of accumulated soot and illustrated in Figs. 8b and 8c for two different spark arrangements. The straightness of this soot wall in the case of a plane, head-on colliding shock wave, Fig. 8b, proved to Mach that the Mach disk is also a straight line.

It truly surprised Mach that in the near vicinity of the "V"-shaped gliding spark the soot inside the funnel lines was completely erased, thus creating sharply pronounced boundaries. These lines represent the trajectories of the two triple points which separate an outer region composed by the incident and reflected shocks and an inner region created by the Mach disk alone. Today we know that in the inner region closely behind the Mach disk there is also a slipline, extending between the two triple points, which separates the two regions of different density and flow velocity. Principally, at this boundary the velocity vectors have the same direction, but different magnitudes. It is quite possible that this be-

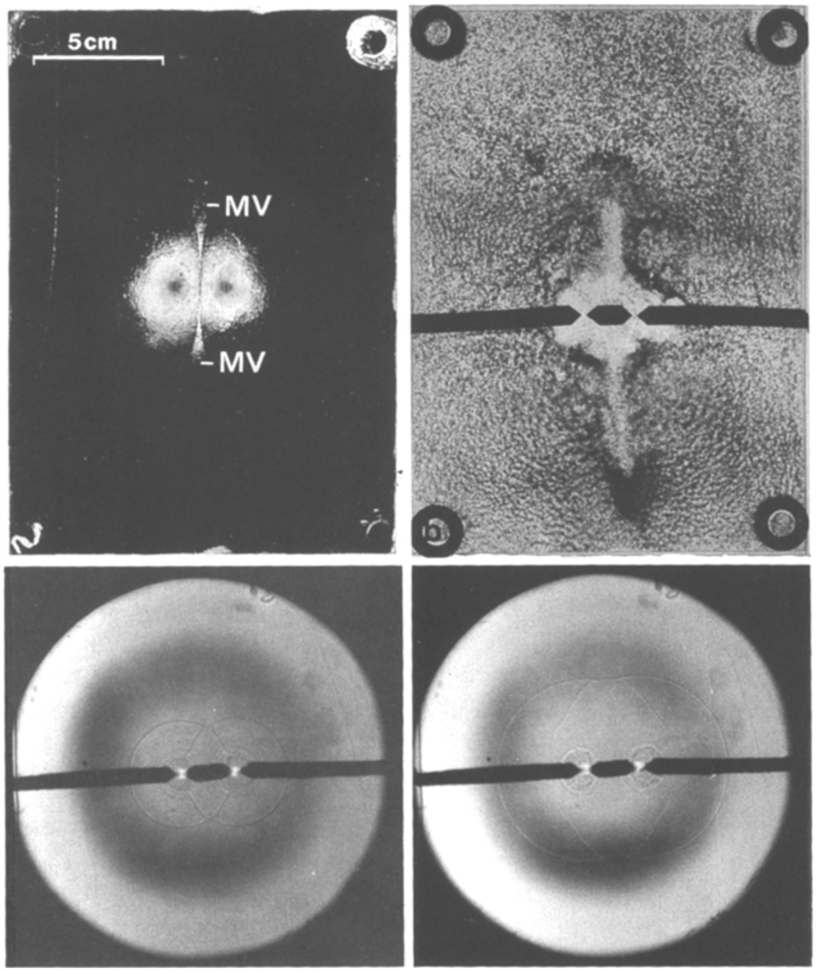

Fig. 7a-d. Comparison of different recording methods of Mach reflection, but applied on the same geometry according to Fig. 2b and Fig. 3: a upper glass plate coated with soot on its lower surface, b lower glass plate covered with cork dust on its upper surface, c,d corresponding high speed schlieren photos of shock wave patterns taken at 30 and 65 μs, respectively, after beginning of discharge

havior leads to micro-vortices which, propagating across the increasing width of the Mach funnel, "mill off" the soot from the glass plate. The afterflow following this slipline sweeps the loose soot off, and finally the Mach funnel becomes then fully visible. However, because of the inertia of the soot particles, this process is slow in comparison to the shock propagation. Current experiments at EMI (Krehl and Heilig 1991) indicated that a clear formation of the "V"-branches does not occur until approximately 100 microseconds after passage of the triple points.

3. Regular and Mach reflection experiments with surface waves

3.1. Historical remarks

Mach (1875) also applied surface waves in liquids to demonstrate irregular reflection. At first he used mercury as a liquid. To store the flow field sweeping over the mercury surface he strewed lycopodium seed on it, a very fine dust. By letting a V-shaped or a zigzag-shaped iron wire fall horizontally onto the mercury surface, he observed wave patterns very similar to the ones obtained in Antolik's soot experiments. Instead of mercury and lycopodium he also used milk and syrup, respectively. These successful experiments which Mach had documented in his notebook as illustrated in Fig. 9 and which he published later (Mach and Wosyka 1875) reveal that he can also be regarded as the spiritual father of modeling the interaction of shock waves by free surface waves. The notebook No. 7 covered the period from February 7, till August 19, 1875. This entry documents his successful interaction experiments with surface waves and their analogy in respect to Antolik's results: "With wires which one let fall on mercury, all Antolik's figures can be reproduced. On syrup and milk)* glue...", compare also (Mach and Wosyka 1875). - Mach used to cross out those pages in his notebooks which he worked out in his later publications.

At this point, some comments regarding Mach's notebooks might be of interest to the reader: The original

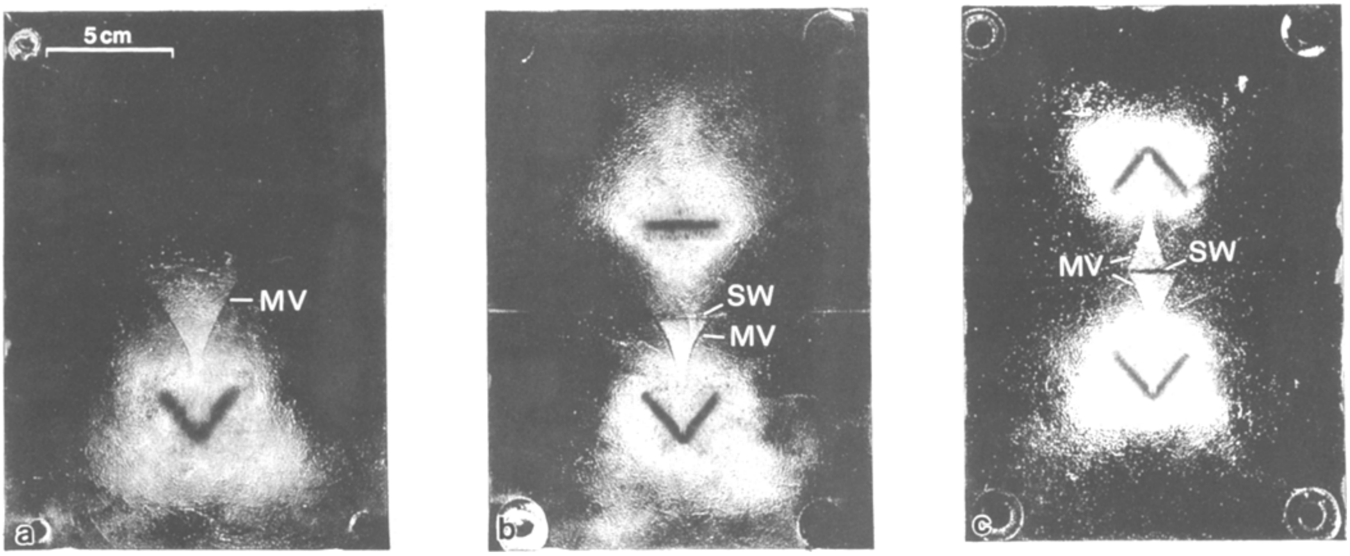

Fig. 8a-c. Further duplication of Mach's soot experiments: **a** Mach funnel MV, generated by interaction of two shock waves according to Fig. 2c, **b** stopping of Mach disk propagation by head-on collision of a plane wave results in a narrow, straight soot wall SW and **c** Mach disk stopping at two oppositely facing V-sparks

notebooks of E. Mach, 65 volumes in all, cover the period 1870-1910: most of them are in the handy format 190 mm × 120 mm and are stored in the archives of the Ernst-Mach-Institut. They do not represent laboratory notebooks in the modern sense with a detailed record of experimental parameters and results, but rather reflect outlines of scientific ideas, sketches of planned experiments and brief résumés of their results as well as personal notes, letter drafts and philosophical thoughts which he sometimes composed into verses. It is striking that none of the entries bear a date, with the exception of the frontispiece of each notebook on which he wrote the starting date of the new notebook. Therefore, it is not possible to state with certainty when he discovered the reflection effect.

The analogy between the two-dimensional supersonic flow of a gas and the shooting of a shallow liquid with a free surface – a so-called hydraulic jump – was first put on a mathematical basis by Jouguet (1920). Experimental testing on this similarity started in 1936 in Pasadena at the Hydrodynamics Laboratory of California Institute of Technology (CalTech). Also Einstein's son, Hans Albert (Einstein 1947) participated in this research. A detailed review covering the time period until 1950 is given by Crossley (1949) and Gilmore et al. (1950). At that time it was hoped to model complicated shock wave structures such as the three-shock Mach interaction on a low-cost water table rather than running the tests in a supersonic wind tunnel which requires expensive high-speed diagnostics. The use of a ripple tank to study the interaction of surface waves dates back to C. Huygens (1629-1695). Its principle suitability to study also the reflection of hydraulic jumps, a non-linear superposition process, could be proved. However, it turned out that the analogy is exact only if the ratio of specific heats of the gas is equal to two. Since for air, the gas of most practical interest, this ratio amounts to 1.4 only, this research method was soon abandoned. Additionally, it was found that just for the most interesting case of strong hydraulic shocks the agreement between theory and experiment is poor, probably caused by the formation of a breaker or roller which results in a curvature of the Mach jump.

Nevertheless, this method, straightforward and inexpensive, proved to be very valuable and illustrative for the basic understanding of irregular reflection phenomena. Therefore, we have resumed it for demonstration purposes and modified it in such a way that – in combination with an overhead projector – shock wave reflection and interaction processes can easily be shown "live" in an auditorium.

It is noteworthy here that Mach-type interactions of hydraulic jumps can also be observed in nature. Probably the earliest document can be found in the book by Cornish (1910), a captain who during his journeys round the world photographed all different kinds of wave phenomena at sea and by sea-shores, in rivers, waterfalls, whirlpools and canals. He observed quite correctly that the curious interaction of the waves of Fig.10 exists only in shallow water and depends on the strength of interacting waves. However, he could not give a satisfying physical explanation of it. Stimulated by his observations, one of us (P.K.) carried out observations at Chesapeake Bay, Virginia. The Mach effect in the case of hydraulic jump interactions can best be observed directly at the shoreline. Since in shallow water the propagation velocity c of

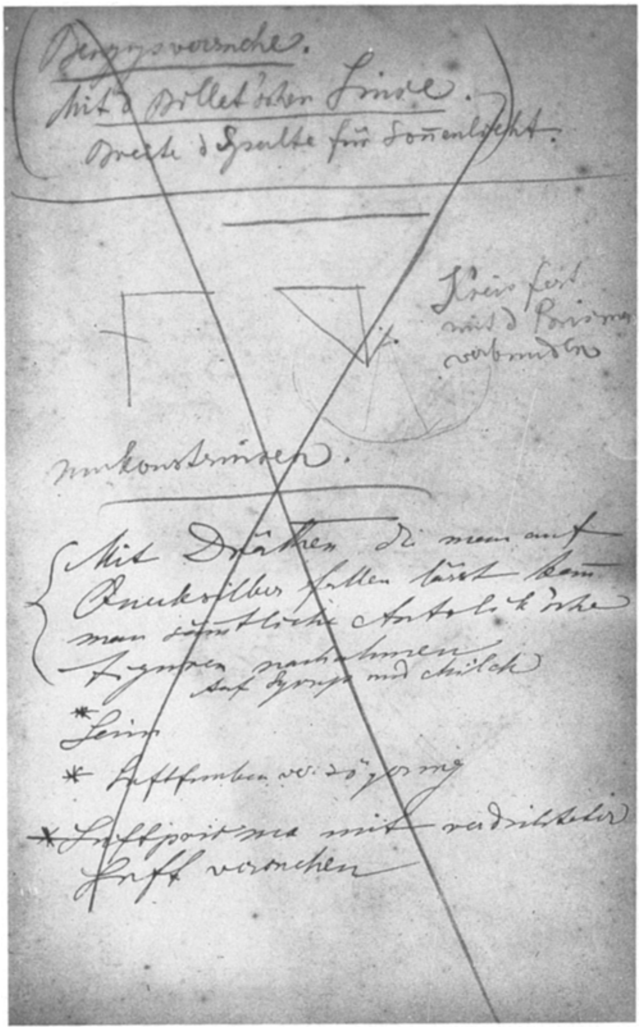

Fig. 9. Copy of a page of Mach's notebook No. 7

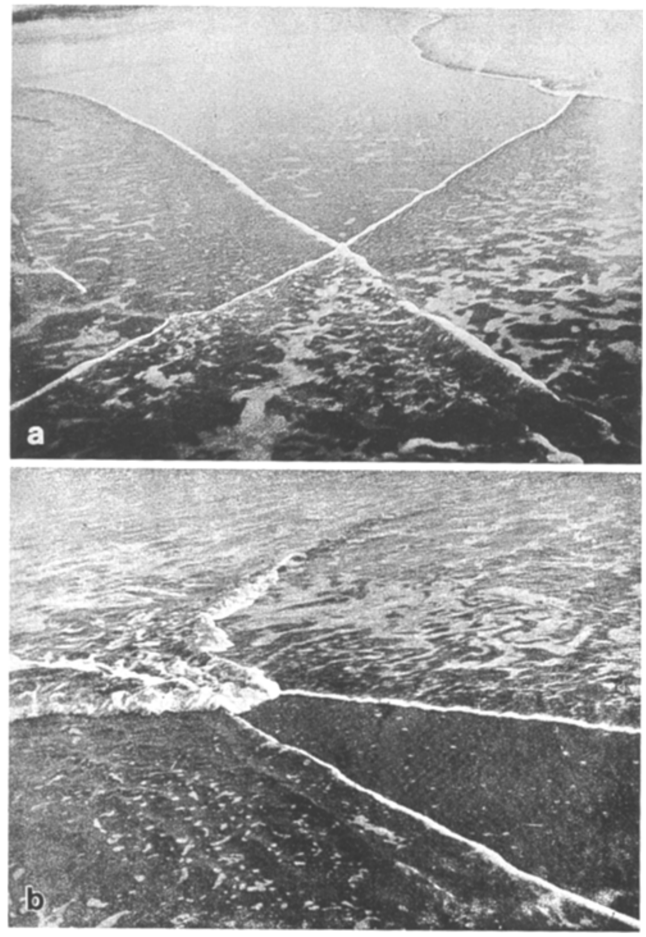

Fig. 10a,b. Meeting and crossing of sea waves in very shallow water (Cornish 1910): **a** regular reflection and **b** Mach interaction phenomenon

an hydraulic jump depends only on gravity g and water depth h, given by the simple equation (Weitzel 1963)

$$c = (g \cdot h)^{1/2}$$

the velocity of any surface wave, such as the Mach stem slows down to zero when running and heading towards the shoreline. At the moment when the velocity reaches zero, the Mach stem becomes distorted and is immediately followed by its disintegration.

It was this observation which initiated the idea that for demonstration purposes a shallow ripple tank with a slight inclination would be most advantageous. This allows one to slow down reflection phenomena just by setting the "shoreline" into the center of the field of view of an overhead projector. Therefore, it enables the audience to catch and resolve quite clearly the geometry of hydraulic jumps during their propagation and reflection, although the screen magnification – depending on the available light intensity output of the applied projector and normally not exceeding 10 – also magnifies the speed of the actual phenomena in the ripple tank.

3.2. Experimental setup

A ripple tank 500 mm × 350 mm × 60 mm of 10 mm thick glass has been used. It was positioned directly on the projection window of the viewgraph projector at a slight inclination and filled with about 0.55 l of water to provide a "shoreline" in the middle of the tank when the liquid is pumped up to a certain height in the reservoir.

The construction of the wave generator was similar to the one used by Crossley (1949) and is shown in Fig. 11. The apparatus extends over the total width of the tank, thus providing wave fronts of the same length. The water is made to rise inside the generator by reducing the pressure by means of a small vacuum pump. An electrically operated valve, Balzers type BPV 43000, is located at the top of the generator. A 2 mm thick soft rubber ribbon is inserted between the bottom of the wave generator and the ripple tank to avoid undesired waves during activation of the electric valve. A uniform, 5 mm wide discharge slot at the bottom of the apparatus provides a uniform wave. To reduce detrimental adhesion effects at low water levels, especially along the

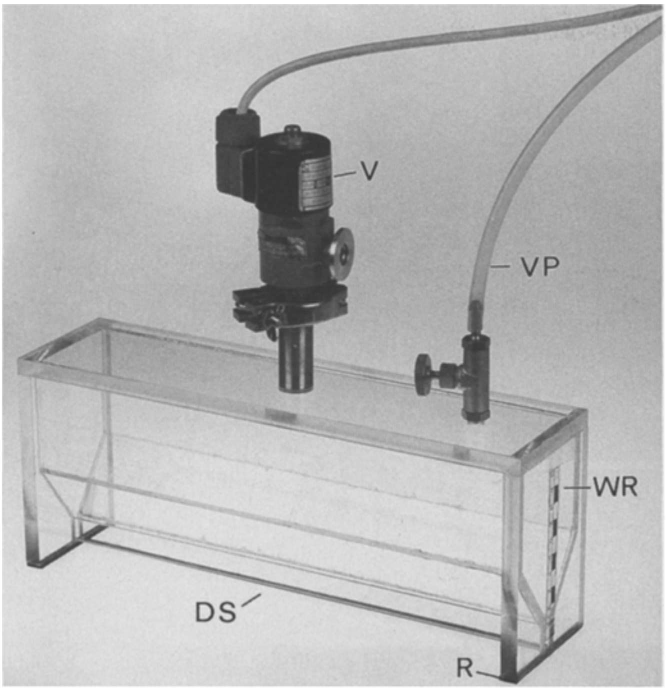

Fig. 11. View of the wave generator: WR – water reservoir; DS – discharge slot; R – rubber; V – air inlet valve; VP – connection to vacuum pump

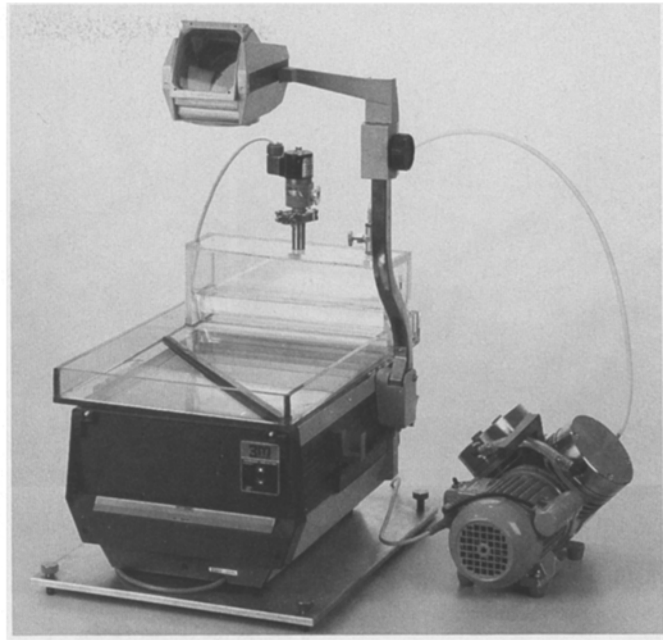

Fig. 13. Total setup with overhead projector to demonstrate reflection of hydraulic jumps

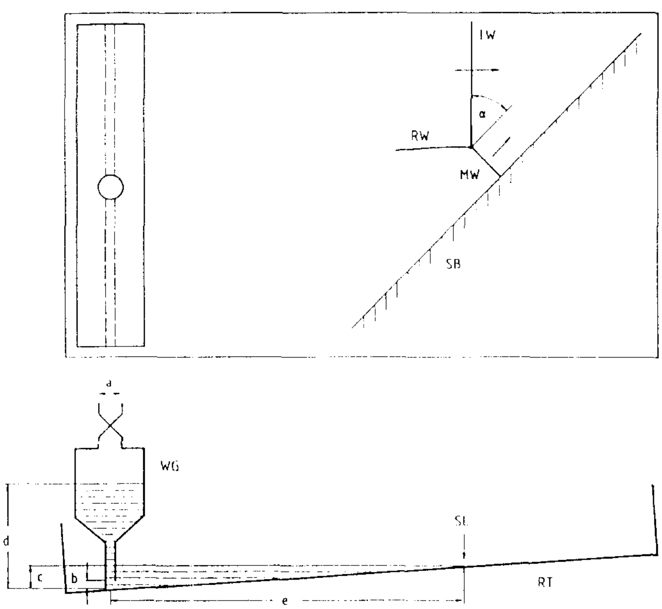

Fig. 12. Schematic of hydraulic jump interaction experiments: WG – wave generator; RT – ripple tank; SB – solid boundary; SL – "shoreline"; IW – incident wave; RW – reflected wave; MW – Mach stem wave; α – angle of the incident wave

"shoreline", a detergent, Agfa Agepon was added to the tap water in a concentration of about 0.2 % by volume.

The strength of the hydraulic jump released by the generator is determined by the diameter a of the air inlet orifice of the valve, the height b of the discharge slot, the water level c in the ripple tank at the position of the discharge slot and the height d to which water is pumped up into the reservoir. These main parameters are illustrated in Fig. 12. For demonstration purposes good results – i.e. slow and clearly resolvable wave propagation – are obtained for a = 10 mm, b = 10 mm, c = 20 mm and d = 45 mm. Before discharge the "shoreline" is set at a distance e = 300 mm apart from the discharge slot, resulting in an inclined water level of 1.9°.

3.3. Regular and Mach reflection experiments at a solid boundary

The total setup for demonstration of hydraulic jump interaction experiments is shown in Fig. 13. Wave reflection at a rigid boundary is studied by using heavy brass cylinders with a rectangular profile and of different lengths. These are placed on the tank bottom, and positioned under different angles in relation to the incident wave front.

Figures 14a, b and Figs. 14c, d show for two successive time instants the propagation of regular and Mach reflection of an hydraulic jump at a solid boundary, respectively. In the case of an angle of incidence of 25°, the reflection remains regular. However, when this angle is increased up to 50°, the regular reflection very soon turns into irregular reflection. The increasing extension of the Mach stem with time is clearly visible. When running up the inclined "beach", the Mach stem is slowed down to a complete standstill. At this moment, depicted in Fig. 14e, the Mach stem front is no longer perpen-

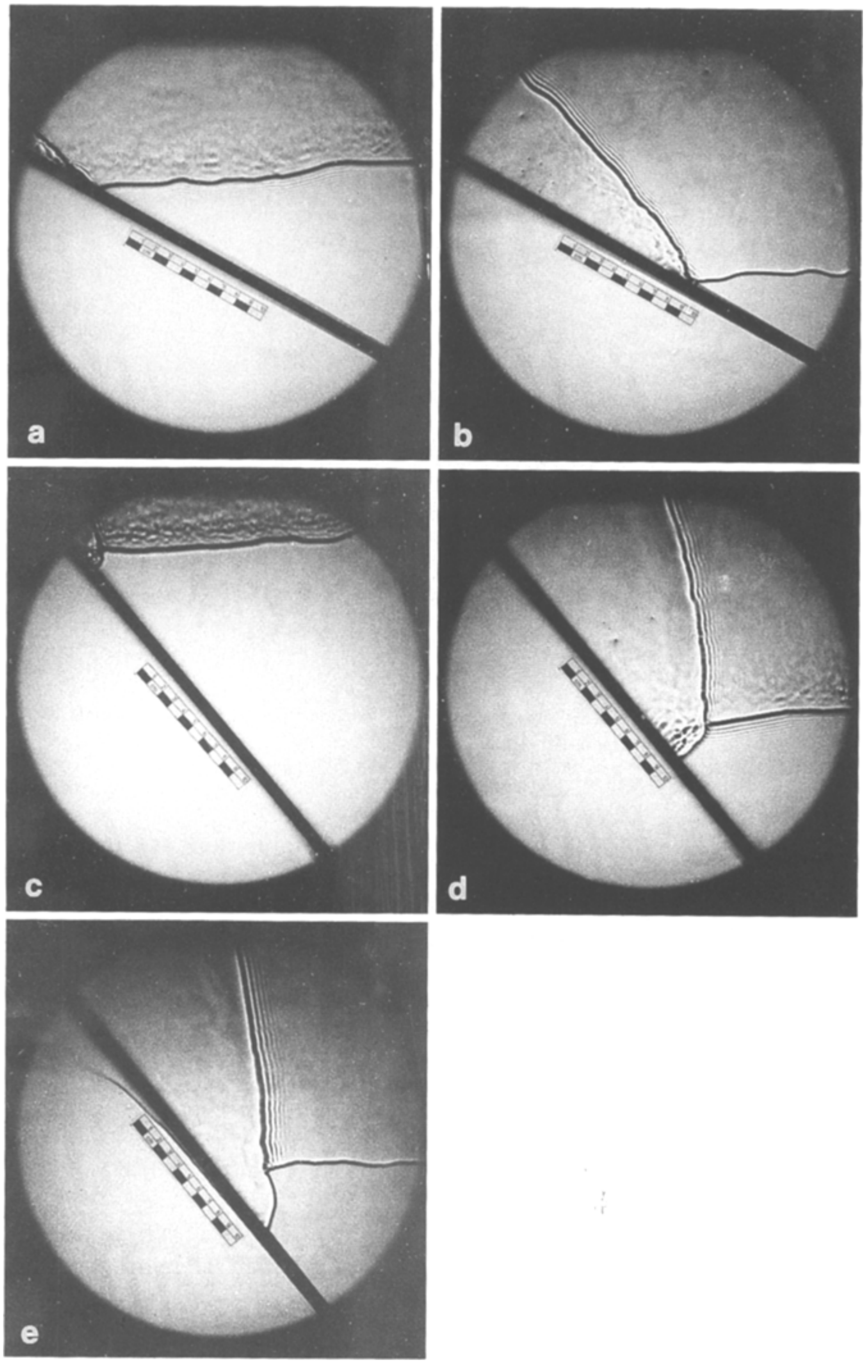

Fig. 14a-e. Reflection of a hydraulic jump at a solid boundary photographed at two successive time instants: **a,b** regular reflection at $\alpha = 25°$ and **c,d** Mach reflection at $\alpha = 50°$. **e** change in geometry of the stagnated Mach stem just before its disintegration

dicular to the boundary, but rather becomes strongly deformed and immediately begins to dissolve.

Previous experiments at EMI on the interaction of hydraulic jumps using a larger ripple tank and a constant water depth have been documented by Krehl et al. (1978) in a 16 mm movie. It shows both in real time and slow motion: (a) propagation of a plane wave; (b) interaction of two plane waves under oblique incidence; (c) oblique reflection of a plane wave at a rigid boundary; and (d) propagation and reflection of a plane wave within straight conical and convex curved nozzles.

Acknowledgements. The permission of the Musée Océanographique de Monaco for inspecting their comprehensive collection of publications on sea wave phenomena as well as the support of the Deutsches Museum München for duplicating their Holtz machine is gratefully acknowledged. The Holtz machine was manufactured at EMI by M. Fischer and W. Schöpflin.

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