Shock Waves (1992) 2: 211-223

Shock Waves

The phenomena of shock wave reflection – a review of unsolved problems and future research needs

G. Ben-Dor * and K. Takayama

Shock Wave Research Center, Institute of Fluid Science, Tohoku University, Sendai, Japan

Received February 2, 1992; Accepted June 1,1992

Abstract. Although the phenomenon of shock wave reflection was discovered more than a hundred years ago, active research related to this phenomenon still goes on in many countries in the world (e.g., Australia, Canada, China, Germany, Israel, Japan, Poland, Russia and United States of America). As a matter of fact the research activity increased so drastically in the past decade and a half that a special scientific meeting dedicated to better understanding the reflection phenomena of shock waves, namely "The International Mach Reflection Symposium" was initiated in 1981 and was held since then in the major research centers actively involved in the research of shock wave reflections. In the present paper the status of the research of the phenomenon of shock wave reflection will be discussed in general, and unresolved problems and future research needs will be pointed out.

Key words: Mach reflection

1. Introduction and historical background

Probably the first scientist to notice and record the phenomenon of shock wave reflection was the distinguished philosopher Ernst Mach who reported his discovery as early as 1878. In his ingenious experimental study which was recently repeated and demonstrated by Krehl and van der Geest (1991), he recorded two different shock wave reflection configurations. The first, a two shock wave configuration, is known as regular reflection, and the second, a three shock wave configuration, was later named after him, and is known today as Mach reflection.

Intensive research of the shock wave reflection phenomenon was re-initiated in the early 1940's by von Neumann (1943a and 1943b). Since then it has been realized that the Mach reflection wave configuration can be further divided into more specific wave structures.

A general illustration of various shock wave reflections is given in Fig. 1. In general, the reflection of shock waves can be divided into regular reflection (RR) or irregular reflections (IR). The RR wave configuration consists of two shock waves: the incident shock wave – i, and the reflected shock wave – r. These two shock waves intersect at the reflection point, which is located on the reflecting surface. All the other wave configurations which are obtained when an incident shock wave reflects over an oblique surface are termed irregular reflection – IR. The IR can be divided, in general, into two categories: von Neumann reflection – vNR and Mach reflection – MR. The MR wave configuration consists of three shock waves, namely: the incident shock wave – i, the reflected shock wave – r, the Mach stem – m, and one slipstream – s. These four discontinuities intersect at a single point called the triple point, which is located above the reflection surface. The reflection point is at the foot of the Mach stem where it touches the reflecting surface. Colella and Henderson (1990) recently hypothesized that there are cases in which the reflected shock wave – r degenerates to a compression wave near the triple point. In such cases the reflection is not an MR. They termed it von Neumann reflection – vNR.

Following the re-initiation of the investigation of the shock wave reflection phenomenon in the early 1940's, Courant and Friedrichs (1948) indicated that, theoretically, three different types of MR are possible, depending on the direction of propagation of the triple point. If the triple point moves away from the reflecting surface, then the MR is called direct, DiMR; if it moves parallel to the reflecting surface, then it is called stationary, StMR; and if it moves toward the reflecting surface, then it is called inverse, InMR. (Courant and Friedrichs originally termed it inverted Mach reflection). The existence of these three types of MR was later validated experimentally by Ben-Dor and Takayama (1986/7). Since the InMR is an MR in which the triple point moves towards the reflecting surface, it terminates as soon as its triple point interacts with the reflecting surface. The termination of the InMR leads to the

Correspondence to: G. Ben-Dor

* Visiting Professor from Pearlstone Center for Aeronautical Engineering Studies, Department of Mechanical Engineering, Ben-Gurion University of the Negev, Beer Sheva, Israel

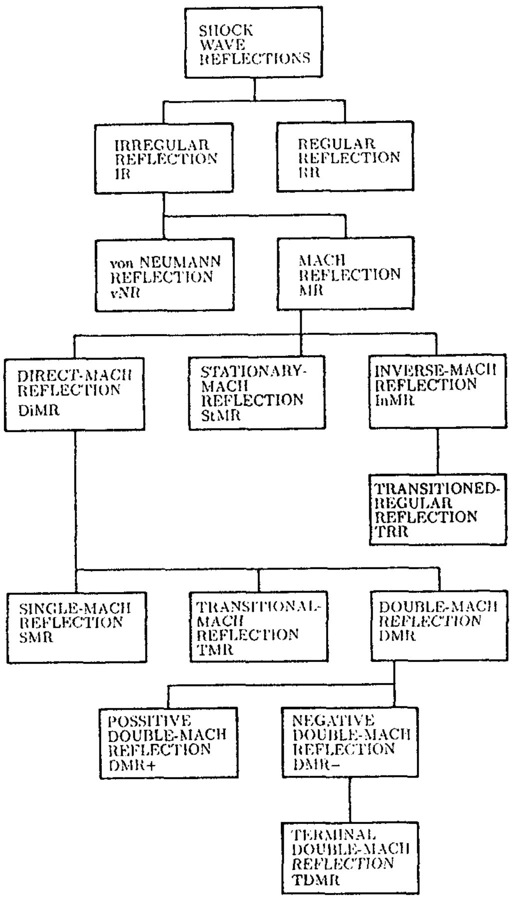

Fig. 1. The various shock wave reflections

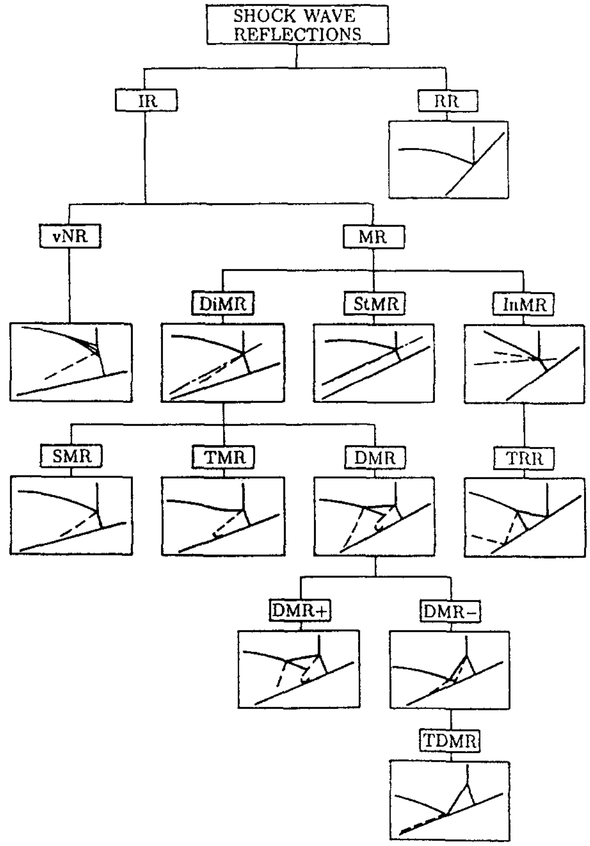

Fig. 2. Schematic illustration of the various shock wave reflection configurations

formation of a new wave configuration, first mentioned by Ben-Dor and Takayama (1986/7). The wave configuration of this reflection consists basically of an RR followed by an MR. Since it is formed following the termination of an InMR, and since it has the basic structure of an RR, it is called transitioned regular reflection – TRR.

While experimentally investigating the reflection phenomenon, Smith (1945) noted that in some cases he observed a kink in the reflected shock wave of the MR. However, only after White (1951) discovered a new type of reflection which he called double Mach reflection, DMR, was the wave configuration observed by Smith (1945), i.e., an MR with a kink in the reflected shock wave, recognized as yet another type of reflection. Throughout the past 50 years it has been referred to as a complex Mach reflection, as opposed to the simple Mach reflection with a reflected shock wave without a kink. However, since the so-called simple Mach reflection is not simple at all, it was later renamed and is known today as single Mach reflection – SMR. Similarly, since the so-called complex Mach reflection is less complex than some of the other reflection configurations and since, as will be shown subsequently, its wave configuration can be viewed as an intermediate wave configuration between the SMR and the DMR, it is called transitional Mach reflection – TMR, as originally suggested by Professor I. I. Glass.

The reflection structure discovered by White (1951) was termed double Mach reflection – DMR, because its structure consists of two triple points. Ben-Dor (1981) showed that the trajectory angle of the second triple point, χ', could be either larger or smaller than the trajectory angle of the first triple point, χ, depending on the initial conditions. Lee and Glass (1984) termed the DMR when $\chi' > \chi$ as DMR+ and the DMR when $\chi' < \chi$ as DMR−. Lee and Glass (1984) also argued that there are conditions for which the second triple point could be located on the reflecting surface, i.e., $\chi'=0$. Such a reflection configuration was termed by them as a terminal double Mach reflection – TDMR.

In summary there are ten different wave configurations which are associated with the reflection of a shock wave over an oblique surface, namely: RR, vNR, StMR, InMR, TRR, SMR, TMR, DMR+, DMR− and TDMR. Schematic drawings of these ten reflection configurations are shown in Fig. 2 which for the reader's convenience is arranged in a way similar to that of Fig. 1. For more details the interested reader is referred to the book entitled *Shock Wave Reflection Phenomena* which was recently published by Ben-Dor (1991).

Although, as stated in the introduction, the research activity related to the shock wave reflection phenomenon continues for more than one hundred years there are still many doubts regarding past findings, numerous open and unsolved questions, and in our opinion vast ground for future research. This will be presented in detail in the following sections. Owing to the fact that the reflection phenomenon, as reported in various publications in the past decades, depends on whether the flow field under consideration, is steady, pseudo-steady or unsteady, in the following, the discussion will also be divided into steady, pseudo-steady, and unsteady reflections. Naturally, the most treated one, namely, the reflection in pseudo-steady flows, will be presented first.

2. The transition criteria between the various reflection configurations

2.1. Pseudo-steady flows

Since the discovery of the reflection phenomenon, major attention was put on determining the transition criteria between the various reflection configurations. All the proposed criteria were based on either the two- or the three-shock theories which were originally formulated by von Neumann (1943a and 1943b). In his formulation, von Neumann assumed that:

1. the flow field is steady,
2. the fluid is ideal, i.e., inviscid ($\mu = 0$) and thermally non-conductive ($k = 0$),
3. the gas obeys the equation of state of a perfect gas, i.e., $P = \rho RT$,
4. the discontinuities at the vicinities of the reflection point of a regular reflection and the triple point of a Mach reflection are straight, and as a consequence, the flow states bounded by the discontinuities are uniform, and
5. the contact discontinuity of the MR is infinitely thin, i.e., it is a slipstream.

Based on these simplifying assumptions (for details see Ben-Dor 1991) the two-shock theory consists of a set of 9 equations and 13 parameters, namely P_0, P_1, P_2, T_0, T_1, T_2, U_0, U_1, U_2, ϕ_1, ϕ_2, θ_1, and θ_2 and the three-shock theory consists of a set of 14 equations with 18 parameters, namely P_0, P_1, P_2, P_3, T_0, T_1, T_2, T_3, U_0, U_1, U_2, U_3, ϕ_1, ϕ_2, ϕ_3, θ_1, θ_2, and θ_3. The parameters P_i, T_i, and U_i are the pressure, the temperature, and the flow velocity in state (i), respectively, ϕ_i is the angle of incidence of the flow on the shock wave across which it enters to state (i), and θ_i is the flow deflection angle while passing through the shock wave and entering state (i). In both theories $i = 0$ is the flow state ahead of the incident shock wave, $i = 1$ is the flow state between the incident and the reflected shock waves, and $i = 2$ is the flow state behind the reflected shock wave. The additional flow state, in the case of a Mach reflection, behind the Mach stem is $i = 3$.

The above brief presentation of the two- and three-shock theories clearly implies that in order to solve the appropriate equations four of the parameters must be known. The 4 parameters which are usually chosen are P_0, T_0, U_0 and ϕ_1.

The presently accepted set of transition criteria, which are all based on either the two- or the three-shock theories, is shown schematically in Fig. 3, which for the reader's convenience is again arranged similarly to Figs. 1 and 2.

The parameter determining whether the reflection is regular (RR) or irregular (IR) is the flow Mach number behind the reflected shock wave in the vicinity of the reflection point and with respect to it, namely M_2^R. As long as $M_2^R > 1$ the reflection is an RR. Thus, the RR⇌IR transition criterion, which is also known as the "sonic" criterion is:

$$M_2^R = 1. \tag{1}$$

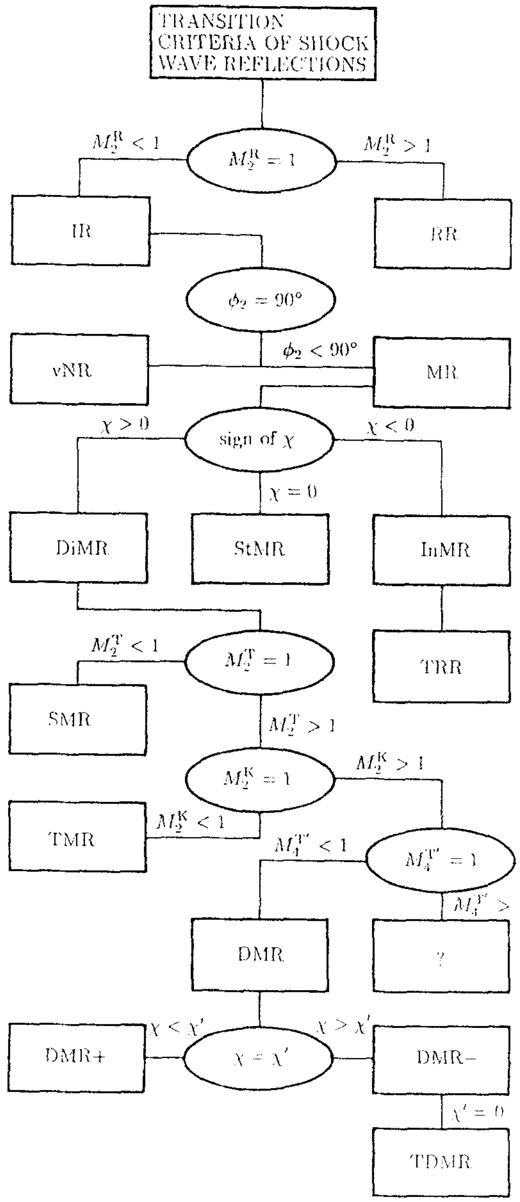

Fig. 3. Schematic summary of the transition criteria between the various types of shock wave reflection configurations

It should be noted here that other RR⇌IR transition criteria, (i.e., the "detachment" and the "mechanical eqiuilibrium" criteria) have been suggested. However, as discussed in Ben-Dor (1991), there is little doubt that the most appropriate RR⇌IR transition criterion in pseudo-steady flows is the "sonic" criterion given by (1). This criterion is also obtained by applying Hornung et al's (1979) "length scale" concept for the RR⇌IR transition.

If the reflection is an IR then it can be either a Mach reflection (MR) or a von Neumann reflection (vNR) depending on the angle of incidence between the reflected shock wave and the flow that passes through it, ϕ_2, in the vicinity of the triple point and in a frame of reference attached to it. The reflection is an MR as long as $\phi_2 < 90°$. Consequently, the MR⇌vNR transition occurs when

$$\phi_2 = 90°. \tag{2a}$$

Note that when ϕ_2=90°, the flow passing through the reflected shock wave is not deflected. However since the flow behind the reflected shock wave must be parallel to the slipstream, it is obvious that the above condition for the MR⇌vNR transition can also be written as

$$\omega_{rs} = 90°, \tag{2b}$$

where ω_{rs} is the angle between the reflected wave and the slipstream in the vicinity of the triple point.

Since a stationary Mach reflection (StMR) and an inverse Mach reflection (InMR) cannot occur in pseudo-steady flows, the MR in pseudo-steady flows is always a direct Mach reflection (DiMR). Once the condition for the existence of a DiMR is met, the value of the flow Mach number in state (2), behind the reflected shock wave of a DiMR in the vicinity of the triple point, T, and with respect to it, i.e., M_2^T, becomes the significant parameter in determining the particular type of the DiMR. As long as $M_2^T < 1$, the reflection is a single Mach reflection (SMR), characterized by a curved reflected shock wave along its entire length. The fact that the reflected shock wave is curved along its entire length implies that a physical length scale is communicated through state (2) up to the triple point (from which the reflected shock wave emanates). This communication path is possible only as long as $M_2^T < 1$. When the flow in state (2) in the vicinity of the triple point becomes supersonic, i.e., $M_2^T > 1$, the communication path is blocked by a supersonic flow zone, and the reflected shock wave develops a straight portion, terminated by a kink in it, which most likely indicates the point along the reflected shock wave which has been reached by the corner-generated signals. A direct Mach reflection with a kink in its reflected shock wave is a transitional Mach reflection (TMR). Thus, the SMR⇌TMR transition criterion is,

$$M_2^T = 1. \tag{3}$$

Once a kink is formed in the reflected shock wave, the value of the flow Mach number, in state (2), behind the reflected shock wave of a DiMR with respect to the kink, K, i.e., M_2^K, becomes the significant parameter in determining whether the reflection remains a transitional Mach reflection (TMR) or changes to a double Mach reflection (DMR). As long as $M_2^K < 1$, the reflection is a TMR, characterized by a centered compression wave at the kink. When the flow in state (2) becomes supersonic with respect to the kink, K, $M_2^K > 1$, the compression waves converge into a shock wave, and the kink changes into a triple point, the second triple point, T′. Thus, the TMR⇌DMR transition occurs when

$$M_2^K = M_2^{T'} = 1. \tag{4}$$

It should be noted that since a TMR is formed and a straight portion is developed in the reflected shock wave, the flow in state (2) is most likely uniform between the triple point, T, and the kink, K.

The DMR can be either a positive double Mach reflection (DMR+) or a negative double Mach reflection (DMR−) depending on whether the trajectory angle of the first triple point, χ, is larger or smaller than that of the second triple point, χ'. Thus the DMR+⇌DMR− transition occurs at:

$$\chi = \chi'. \tag{5}$$

Once the condition for the onset of a DMR− is met, the reflection could be a terminal double Mach reflection (TDMR). This occurs if

$$\chi' = 0. \tag{6}$$

Shirouzu and Glass (1986) proposed an additional necessary condition for the SMR⇌TMR transition. Using basic gasdynamic arguments they concluded that

$$\omega_{ir} \geq 90° \tag{7}$$

is a necessary condition for the SMR⇌TMR transition. Here ω_{ir} is the angle between the incident and the reflected shock waves in the vicinity of the triple point.

By applying the two- and three-shock theories while imposing the above listed transition criteria the corresponding transition lines can be obtained. The RR⇌IR transition line is calculated by applying the two-shock theory at the reflection point and requiring that (1) is satisfied. The vNR⇌MR and the SMR⇌TMR transition lines are calculated by applying the three-shock theory at the first triple point and requiring that (2a) and (3) be satisfied, respectively. The TMR⇌DMR, the DMR+⇌DMR−, and the DMR−⇌TDMR transition lines are calculated by applying the three-shock theory at the first and the second triple points and requiring that (4), (5) and (6) are, respectively, satisfied.

Note that in order to apply the three-shock theory at the second triple point, the relative motion of the second triple point with respect to the first triple point must be known.

Based on the assumption that the horizontal velocity of the second triple point is identical to that induced by the incident shock wave, Law and Glass (1971) showed that the velocity of the kink (of a TMR) or the second triple point (of a DMR) with respect to the first triple point is

$$V_K^T = \frac{\rho_0}{\rho_1} V_s \mathrm{cosec}(\phi_1 + \phi_2 - \theta_1), \tag{8}$$

where ρ is the density and V_s is the incident shock wave velocity. This relation was found by Bazhenova, Fokeev and Gvozdeva (1976) to be very good in the range $\theta_w < 40°$ and poor to fairly good elsewhere. Based on their findings it is evident that there is a need for a better model relating the motion of the second triple point to that of the first one. Furthermore, as long as the Law and Glass (1971) assumption is used, it is clear that an inherent error is introduced into calculations and results which are based on it.

By applying the two- and three-shock theories for the above listed transition criteria the reflection domains in the (M_0, ϕ_1) plane can be obtained. Here M_0 is the flow Mach number ahead of the incident shock wave, i.e., $M_0 = u_0/a_0$, where a_0 is the local speed of sound ahead of the incident shock wave.

As shown by many investigators and summarized by Ben-Dor (1991) the comparison of experimental results with the above listed transition criteria in the (M_0, ϕ_1) plane revealed that the transition lines could be considered as sufficiently good from an engineering point of view only, i.e., the reflection configurations, as well as their associated properties, can be predicted a priori quite well as long as the initial conditions (M_0 and ϕ_1) are not too close to the transition lines. There is li ttle doubt that from a scientific point of view the transition lines as calculated using the above transition criteria are still far from being accurate.

In addition to the lack of perfect agreement between theory and experiments, scientists have been reporting since the late 1940's (see Birkhoff 1950), that RR and MR wave configurations were observed in parameter domain where the two- and three-shock theories have no real solutions. Unfortunately, when treated separately each of these two paradoxes was referred to as the von Neumann paradox. For clarity purposes we will refer to the persistence of RR beyond its theoretical limit as determined by the two-shock theory as the 1st von Neumann paradox, and to the persistence of MR beyond its theoretical limit as determined by the three-shock theory as the 2nd von Neumann paradox.

The question whether or not the existence of these two paradoxes, as well as the above mentioned lack of perfect agreement between the reported experimental results and the transition lines arising from the above presented transition criteria, could be attributed to the fact that the transition lines were calculated using oversimplified two- and three-shock theories, is yet to be answered.

In order to further emphasize this point it should be noted that while the existing RR$\rightleftharpoons$IR experiments do not agree with the transition line as calculated from the two-shock theory and (1), they agree excellently with the experimental " sonic" transition line, for further details see Lock and Dewey (1989).

In our opinion some clarification to this question could be obtained through CFD by solving the full set of the governing equations including viscous, thermal conduction, and real gas effects. Unfortunately, such numerical computing capability does not exist yet. However, in view of the extremely fast developments in CFD capabilities it is our belief that such code will eventually evolve in the future.

It should be noted here that viscous, thermal conduction, and real gas effects are non-self-similar effects as they depend on the Reynolds, Prandtl, and Mach numbers of the flow. Hence the use of the classical two- and three-shock theories, which were de veloped under a steady flow assumption, is yet to be justified in pseudo-steady reflections.

In addition, it should be noted that the additional necessary condition for the SMR$\rightleftharpoons$TMR transition, given by (7), was checked experimentally by Shirouzu and Glass (1976) (i.e., for a variety of gases air, N_2, O_2, Ar and CO_2) and found to be correct beyond any doubt. The fact that the calculated transition line arising from it fails to clearly distinguish between the SMR- and the TMR-experiments clearly indicates that the three-shock theory upon which it is based is oversimplified.

In order to obtain the reflection in the (M_s, θ_w) plane, which is more useful since in pseudo-steady reflections the incident shock wave Mach number, M_s, and the reflecting wedge angle, θ_w, are the known paramet ers, the following transformation should be made

$$M_s = M_0 \sin\phi_1 \tag{9}$$

$$\theta_w = 90^\circ - (\phi_1 + \chi) \tag{10}$$

where χ, the first triple point trajectory angle, is

$\chi = 0$ in the RR domain

$\chi \neq 0$ in the IR domain.

However, since χ is an unknown parameter, one faces a situation in which the three-shock theory in terms of the known parameters in pseudo-steady flows, consists of a set of 14 equations with 19 parameters (χ in addition to the 18 parameters listed earlier) of which only four are known as initial conditions, namely: M_s, θ_w, P_0 and T_0.

Consequently, in order to get useful solutions using the pseudo-steady three shock theory an additional equation is required. Using the following simplifying assumption:

1. the Mach stem is straight,
2. the Mach stem is perpendicular to the reflecting wedge surface, and
3. the triple point originates from the leading edge of the reflecting wedge

Law and Glass (1971) suggested the following relation

$$\chi = 90^\circ - \phi_1 \tag{11}$$

as the additional equation required to complete the pseudo-steady three-shock theory.

Unfortunately, comparisons of the experimentally measured triple point trajectory angles with those predicted theoretically clearly indicated that the model suggested by Law and Glass for predicting χ was not sufficiently good. Hence a better model f or predicting the first triple point trajectory angle is required.

Owing to the fact that the analytical model for predicting the first triple point trajectory angle, χ, is not sufficiently good, it is obvious, that an inherent error is introduced when the transition lines are transformed from the (M_0, ϕ_1) plane to the (M_s, θ_w) plane. However, when comparing the transition lines in the (M_0, ϕ_1) and the (M_s, θ_w) planes with experimental results, surprisingly in many cases the agreement with the transition lines in the (M_s, θ_w) plane is better than that with the transition lines in the (M_0, ϕ_1) plane, in spite the inherent error which is introduced in obtaining the transition lines in the (M_s, θ_w) plane. This peculiar fact clearly indicates that calculations based on von-Neumann's classical two- and three-shock theories are not sufficiently good.

As mentioned earlier, in spite of the clear disagreement between the experimental results and the appropriate transition lines, the transition criteria, as summarized in Fig. 3, cannot be discarded on these grounds since the transition lines were calculated using oversimplified two- and three-shock theories, i.e., theories which neglect viscous, thermal conduction, and non-equilibrium real gas effects.

The RR$\rightleftharpoons$IR transition criterion, given by (1), implies that the flow behind the reflected shock wave of an RR is always supersonic. This in turn implies that the reflected shock wave, in the vicinity of the reflection point, must be straight, in contradiction to reported RR-photographs which clearly shock curved reflected shock waves near the reflection points. Hence is the sonic criterion indeed the RR⇌IR transition criterion? If yes, then how can the existence of a curved reflected shock wave be explained?

2.2. Steady flows

Unlike pseudo-steady flows, in the case of steady flows only two reflection configurations are obtainable, namely the RR and the SMR.

The RR⇌SMR transition criterion in steady flows is different from that in pseudo-steady flows. Based on the "length scale" concept of Hornung et al. (1979), the RR⇌SMR transition occurs at the point where the deflect ion angle satisfies the following expression :

$$\theta_1 - \theta_2 = \theta_3 = 0. \qquad (12)$$

Note that the condition given by (12) is identical to that of the "mechanical equilibrium" criterion for the RR⇌IR transition.

Unfortunately, the transition line arising from the condition given by (12) has not been compared with experiments in a wide range of flow parameters and a variety of gases. Hence there is a definite need to conduct a comprehensive experimental investigation in steady flows in order to have enough evidence to be in a position to either accept or reject this transition criterion. In addition, it should be noted here that unlike pseudo-steady flows, the parameters in the three-shock theory which are usually chosen as known parameters, namely P_0, T_0, U_0, and ϕ_1 are indeed known in steady flows, as they are the initial conditions. Hence in steady flows there is no need for an additional equation ,e.g. (11), like in pseudo-steady flows, to complement the three-shock theory.

2.3. Unsteady flows

Unfortunately, the transition criteria in unsteady flows are not established at all. The most promising concept so far is the "length scale" concept of Hornung et al. (1979) which led to the correct IR⇌RR transition criteria both in steady and pseudo-steady flows, and was shown by Ben-Dor and Takayama (1985, 1986/7), Ben-Dor, Takayama and Dewey (1987), Takayama and Ben-Dor (1989) and Ben-Dor and Rayevski (1993); to be applicable to some unsteady reflection phenomena.

Due to the complexity of the governing equations of unsteady shock reflections, simple transition criteria, such as those presented earlier for steady and pseudo-steady flows, cannot be established. Consequently the transition lines between the various reflection configurations should be established using numerical computations. The transition lines should be established at least for the following three cases which have been experimentally investigated quite extensively in the past decade and the details of which are given in Ben-Dor (1991): reflection of a planar shock wave off a concave cylinder, reflection of a planar shock wave off a convex cylinder, and reflection of a spherical shock wave off a plane surface.

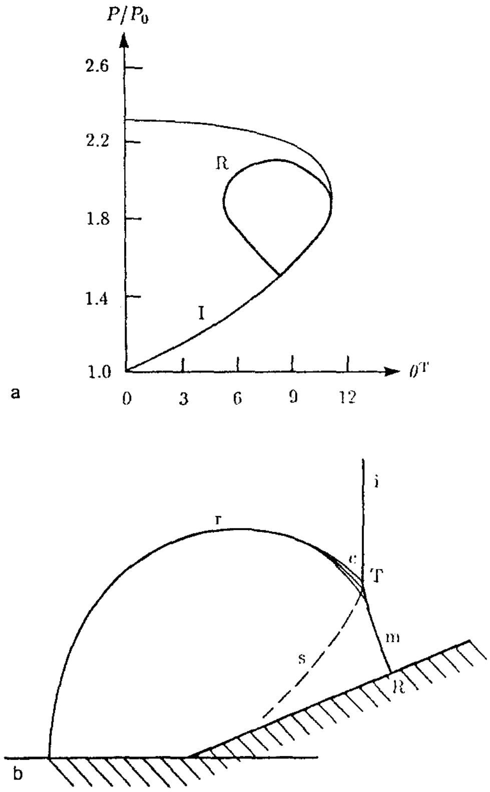

Fig. 4. a a shock polar combination for which a Mach reflection is impossible, **b** schematic illustration of a von Neumann reflection

In the former two cases the dependence of the transition lines on both the radius of curvature of the cylindrical surfaces and their initial angles should also be investigated. Similarly, the dependence on the height from the plane surface where the spherical shock wave is generated should be investigated in the latter case.

Once the transition lines are numerically established they should be compared with experiments. These comparisons could indicate how good the numerical codes are in simulating relatively simple unsteady reflection processes. As a further step, the numerical codes could be used to numerically predict the transition lines over more complicated specific geometries over which the reflection process is unsteady.

3. The wave configuration of the various reflections

3.1. Pseudo-steady flows

For quite a few decades the scientific community has been trying to explain the so-called "von Neumann paradox" which was first mentioned by Birkhoff (1950). The 2nd von Neumann paradox referred to the fact that Mach reflection (MR) wave configurations were observed experimentally for incident shock wave Mach numbers, M_s, and reflecting wedge angle, θ_w, for which von Neumann's classical three-shock theory has no correct solutions (see Fig. 4a). Recently, based on numerical simulations, Colella and Henderson (1990) hypothesized that the experimentally observed wave configurations were not Mach reflections but belong to another type of reflection which they termed von Neumann reflection – vNR. Hence, since the three-shock theory was never meant to be applied to the wave configuration of a vNR they concluded that the 2nd von Neumann paradox was not a paradox at all.

Colella and Henderson (1990) claimed that their numerical study clearly indicated that the reflected disturbance at the triple point was not a shock wave but a "smoothly disturbed self-similar band of compression waves of finite thickness," which according to Colella and Henderson (1990) "is too small to be resolved experimentally." They further explained that "as the compression waves retreat from the triple point they converge and steepen into a shock wave," as shown in Fig. 4b. "The distance over which this happens," according to Colella and Henderson (1990) is again "too small to be resolved experimentally." However, since the reflection as they claim is self-similar, the wave configuration of a vNR must grow linearly with time, and eventually the structure of the reflected disturbance, in the vicinity of the triple point, should reach a resolvable size. Especially in light of the modern flow visualization techniques which have extremely high resolution capabilities (see Takayama 1992). Since the structure described by Colella and Henderson (1990) has never been observed, one is left with the following question: does the von Neumann reflection as hypothesized by Colella and Henderson actually exist? If no, then what is the reflection that is obtained when the classical three-shock theory has no correct solutions? If yes, what is the exact structure of the reflected disturbance in the vicinity of the triple point and how can it be observed experimentally?

Except for the vNR, the other wave configurations shown in Fig. 2, have all been recorded experimentally. However, the actual existence of one of them, namely the terminal double Mach reflection (TDMR) was doubted by many investigators, who argued that the second triple point cannot touch the reflecting surface, since in reality a boundary layer, inside which the flow is subsonic, develops along the reflecting surface. Consequently, according to these investigators, the second triple point trajectory angle is always greater than zero, i.e., $\chi' > 0$. This argument was opposed by the claim that the boundary layer could be separated by the vortex that is induced by the curled contact discontinuity. Unfortunately, the unclear photographs of TDMR configurations which have been reported so far only add to the uncertainty as to whether a TDMR is indeed possible. The above mentioned fact, in addition to the fact that the flow is self similar and hence growing linearly with time, imply that if the so called TDMR is an underdeveloped negative DMR and if it is allowed to develop by generating it over sufficiently long reflecting surface, then it will eventually resemble a negative double Mach reflection wave configuration (DMR−). Consequently the following questions are yet to be answered: does a terminal double Mach reflection actually exist? If yes, how can the second triple point lie on the reflecting surface and what exactly is its structure? If no, then is the reflection reported so far as TDMR actually an undeveloped DMR−?

Although, as mentioned earlier, beside the vNR and the TDMR there are no doubts about the validity of the other wave configurations shown in Fig. 2, there are still many unresolved questions, as outlined subsequently, regarding the nature of the various d iscontinuities consisting the wave configurations.

It has been reported by some investigators (e.g., Ben-Dor 1978; Dewey and McMillin 1985a and 1985b) that the Mach stem of a Mach reflection is curved, and that its curvature could be either concave or convex. However, no criterion by which one can decide a priori what curvature will the Mach stem have has been forwarded yet. Hence, the criterion determining whether the Mach stem's curvature is concave or convex is yet to be determined. This question raises another one, namely: is there some, yet unknown, significance associated with this difference in curvature of the Mach stem?

The nature of the contact discontinuity of a Mach reflection wave configuration is also not clear yet. In early days, when von-Neumann (1943a, 1943b) developed his three-shock theory he regarded the contact discontinuity as an infinitely thin discontinuity, i.e., a slipstream, across which the pressure and the flow directions are identical. Dewey and McMillin (1985a), using an ingenious experimental technique, by which the velocity vectors were measured, reported that the flows on both sides of the contact discontinuity are not parallel. These findings, together with the undoubtful fact that the classical three-shock theory consistently fails in predicting the angles between the various discontinuities at the first triple point of a Mach reflection, as reported by Sternberg (1959) more than 30 years ago and reconfirmed recently by Ben-Dor (1987, 1990) raises the following questions: what is the exact nature of the contact discontinuity of a Mach reflection? Is it a slipstream as modelled by von-Neumann (1943a and 1943b) when he forwarded the classical three-shock theory? Or is it an angular diverging zone as hypothesized by Skews (1972) and later modelled by Ben-Dor (1990)? Or is it shear layer between two parallel jets, along which two boundary layer develop, as modelled by Ben-Dor (1987)? Or perhaps it is an angular mixing zone?

Regardless of its nature, there are cases in which the contact discontinuity merges smoothly into the boundary layer along the wedge as shown in Fig. 5a while there are cases in which it curves forward as shown in Fig. 5b. Consequently, what is the criterion determining whether the contact discontinuity is curled or not?

It has been suggested by some investigators (for details see Ben-Dor 1991) that the curling of the contact discontinuity is caused by its interaction with the compression wave which develops at the kink of the transitional Mach reflection (TMR) or the second Mach stem of the double Mach reflection (DMR). Hence, do single Mach reflections always have contact discontinuities which smoothly merge into the boundary layer developing along the reflecting surface? Or alternatively, do transitional and double Mach reflections always have curled contact discontinuities? If yes, does the appearance of a curled discontinuity indicate the onset of a transitional Mach reflection even though a clear kink in the reflected shock wave is not visible?

The above mentioned interaction of the second Mach stem of a double Mach reflection with the contact discontinuity emanating from the first triple point is yet another mystery from a gas dynamics point of view. Consider Fig. 6a

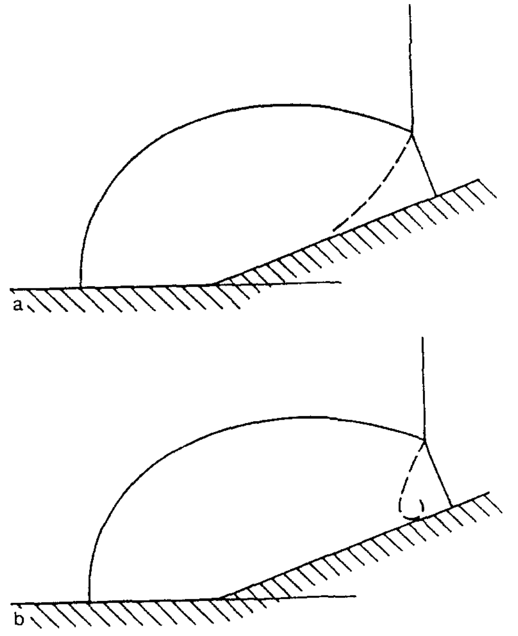

Fig. 5. Schematic illustration of two MR wave configurations. **a** with a contact discontinuity which merges smoothly into the wedge, **b** with a curled contact discontinuity

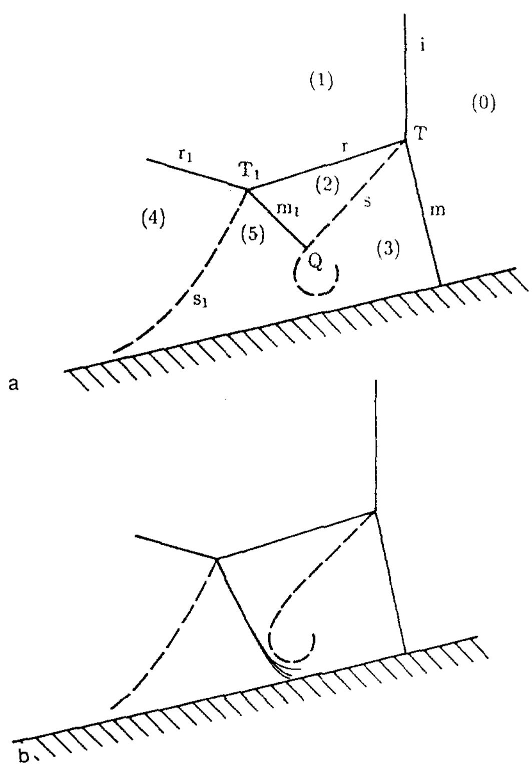

Fig. 6. Schematic illustration of two wave configurations of a double Mach reflection. **a** m_1 terminating on s, **b** m_1 breaks down to a compression wave that goes around s

where the wave configuration of a DMR is schematically drawn. Note how the second Mach stem, m_1, emanating from the second triple point, T_1, terminates at the point where it reaches the contact discontinuity, s. Since the acoustic

Fig. 7. An enlarged photograph of a DMR illustrating the interaction of the second Mach stem with the first contact discontinuity (courtesy of Prof. I. I. Glass)

impedances on both sides of the contact discontinuity are different, one should expect the shock wave m_1 to reflect from the contact discontinuity either as a shock or as a rarefaction wave. However, this is obviously not the case. Hence, what is the gas dynamic explanation for the above described interaction? Is the interaction always tailored so that the wave which reflects from the contact discontinuity is always an invisible Mach wave? In addition, the wave configuration shown in Fig. 5a implies that at point Q where the shock, m_1, touches the contact discontinuity, s, there is a sudden jump in the pressure, from P_2 to P_5 on the upper side of the contact discontinuity. Consequently, how is this sudden change in the pressure on one side of the contact discontinuity balanced on its other side? Or alternatively, where is the transmitted shock wave which is required to balance this kind of sudden pressure jump along interfaces?

Some photographs (e.g., Fig. 7) suggest that there are cases in which the second Mach stem, m_1, goes around the first contact discontinuity, s, and breaks down to a compression wave. This interaction is shown schematically in Fig. 6b. If this indeed is the case, then what is the nature of this interaction?

The above discussed peculiar shock wave-contact discontinuity interactions clearly suggest that the nature of the contact discontinuity of a Mach reflection is far from being understood and undoubtedly needs further investigation.

The discussion in the previous section regarding the transition criteria between the various shock wave reflection configurations raises some questions regarding the possibility of obtaining more complicated Mach reflection configurations.

The onset of a kink in the reflected shock wave of a MR was attributed to the fact that the flow, in state (2),

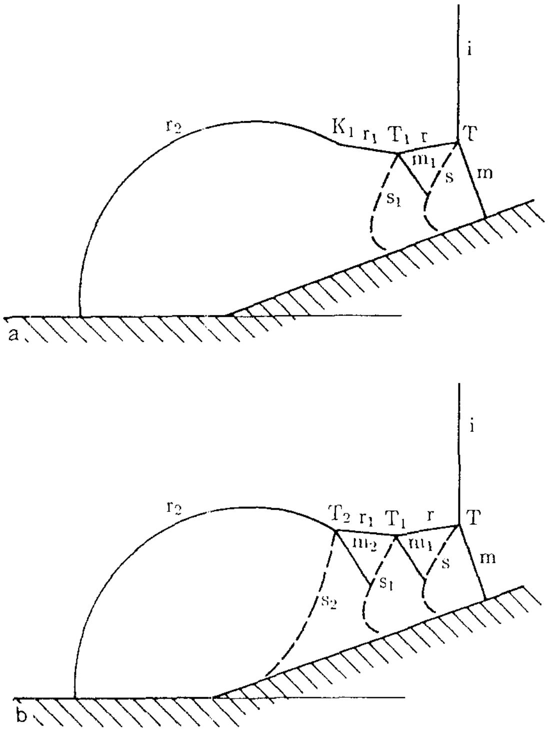

Fig. 8. Schematic drawings of hypothesized shock wave reflection configurations, which perhaps could be obtained at extremely high incident shock wave Mach numbers: **a** a double Mach reflection with a kink in the second reflected shock wave, **b** a triple Mach reflection

behind the reflected shock wave became supersonic with respect to the first triple point. If this is true then one may ask (consider Fig. 6a) whether or not a kink develops in the second reflected shock wave, r_1, of a DMR when the flow behind the second reflected shock wave, in state (4), becomes supersonic with respect to the second triple point (i.e., $M_4^{T_1} > 1$), or, whether or not the double Mach reflection terminates when $M_4^{T_1} = 1$ to form a new type of Mach reflection? The wave configuration of this new Mach reflection which was first hypothesized by Ben-Dor and Glass (1979) is shown in Fig. 8a.

The onset of the second triple point was attributed to the fact that the flow, in state (2), behind the reflected shock wave, became supersonic with respect to the kink. Hence if the wave configuration shown in Fig. 8a indeed materializes, does the kink, k_1, shown in Fig. 8a change to a triple point if the flow behind r_1 becomes supersonic with respect to k_1 to form a triple Mach reflection as shown in Fig. 8b?

Finally, if indeed the wave configurations shown in Figs. 8a and 8b do materialize, provided the conditions for their formation are met, the question whether this sequence goes on forever is unavoidable.

It should be mentioned here that in order to increase the flow Mach number, in state (4), behind the second triple point, the incident shock wave Mach number should be increased. This in turn will cause the internal degrees of freedom of the gas under consideration to excite. The excitation of the internal degrees of freedom, which as shown by Ben-Dor (1991) significantly affects the flow around the second triple point, could make it impossible to reach the above mentioned conditions for obtaining the above hypothesized transition criteria and reflection configurations. Hence can the above hypothesized transition criteria be actually met?

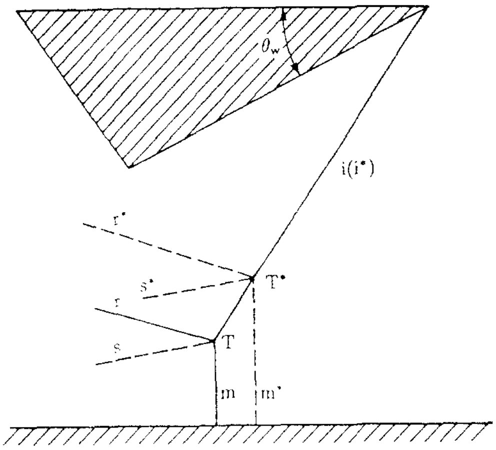

Fig. 9. Schematic illustration of two theoretically possible MR wave configurations, in a steady flow, for identical initial conditions

As a final remark, we note that in addition to the above examples of unresolved questions and not completely understood phenomena regarding the transition criteria between the various reflections and their wave configurations (Ben-Dor, Takayama and Needham 1987), several questions have been raised recently regarding the nature of the triple point as well as its formation whether it is a hot spot?

Recent experimental studies by Reichenbach (1985) and Schmidt (1989) raised some doubt about the commonly used assumption that the first triple point originates at the leading edge of the reflecting wedge. Hence where does the first triple point originate from and is its trajectory straight?

Another, yet hardly investigated, phenomenon related to the triple point is its reflection mechanism from solid surfaces when it collides with them.

As a final remark it should be noted that all the above questions which have been asked about the first triple point are of course relevant to the second triple point of a DMR as well.

3.2. *Steady flow*

Probably the only unclear question regarding the wave configurations of the possible shock wave reflections in steady flows is: how can one determine the length of the Mach stem of a steady Mach reflection?

Consider Fig. 9, where the solid lines describe the four discontinuities of a steady MR-configuration with triple point, T, shock waves i, r, and m, and slipstream, s. If one selects any point along the incident shock wave, i., and draws three lines from it parallel respectively to the reflected shock wave, r, to the Mach stem, m, and to the slipstream, s, then one has a new triple point T*, with its four discontinuities. The two triple points, T and T*, as well as all the other triple points which could be obtained by choosing a different location for T* along the incident shock wave, completely satisfy the conservation equations given by von Neumann's classical three-shock theory. However, if an ex-

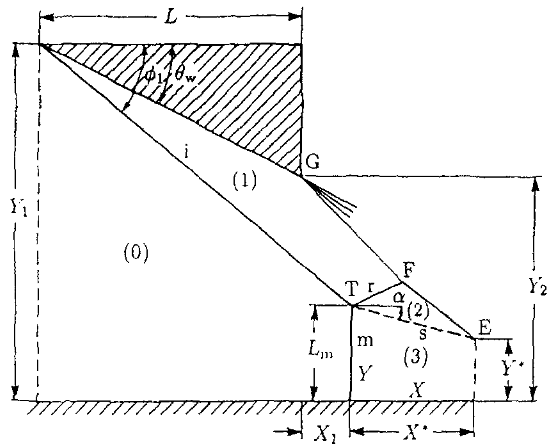

Fig. 10. Schematic illustration of Azevedo's (1989) model

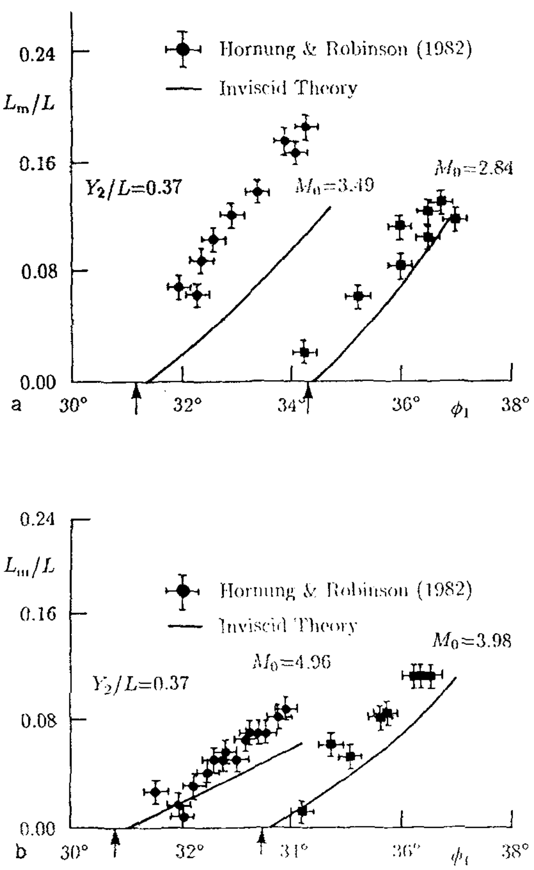

Fig. 11. Predicted values of the Mach stem height as obtained by Azevedo (1989) and comparison with the experimental results of Hornung and Robinson (1982). **a** M_0=2.84 and 3.49, **b** M_0=3.98 and 4.96

periment with the same initial conditions (i.e., M_0 and θ_w) is repeated, then only one reflection configuration out of the infinity of possible ones is always obtained. Thus, the classical three-shock theory is incapable of predicting the actual size of the MR since it is, inherently, independent of any physical length scale.

Azevedo (1989) suggested an ingenious physical model for predicting the height of the Mach stem. Consider Fig. 10

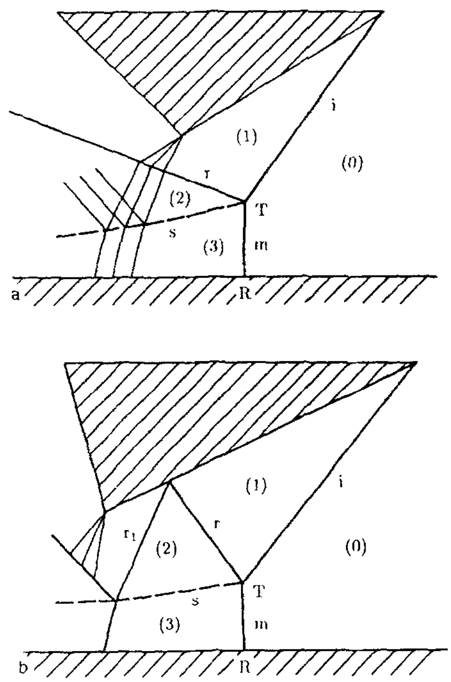

Fig. 12. Schematic illustration of two possibilities of generating steady Mach reflections. **a** short reflecting wedge, **b** long reflecting wedge

where a schematic drawing of a reflecting wedge, which generates an MR, is shown. Azevedo (1989) assumed that: the Mach stem, the slipstream, and the bottom surface form a one-dimensional converging nozzle, the throat of this converging nozzle is at the point where the leading characteristic of the expansion wave, generated by the shoulder of the reflecting wedge, intersects the slipstream (point E in Fig. 10), the flow in region (3) is isentropic and reaches sonic conditions at the throat, and the gas is an ideal fluid, i.e., $\mu = 0$ and $k = 0$. By applying the conservation laws of mass and linear momentum to the control volume shown in Fig. 10, Azevedo (1989) developed a simplified model capable of predicting the Mach stem height.

The predicted values of the height of the Mach stem as obtained using Azevedo's model are shown in Figs. 11a and 11b, together with the experimental results of Hornung and Robinson (1982). The predicted results show a trend similar to that for the Mach stem height dependence on the angle of incidence in the experimental results. In addition, the RR⇌MR transition angles, as predicted by Azevedo's analytical model, agree excellently with those predicted by the length scale criterion (compare value obtained for ϕ_1, at L_m=0 and the corresponding arrowheads along the ϕ_1 axis which indicate the measured transition angles). Although the agreement between the predicted values for the Mach stem height and those obtained experimentally is far from being satisfactory, one must admit that, in general, the analytical predictions are surprisingly good in view of the oversimplifying assumptions upon which the analytical model is based. Furthermore, it should be noted that no other model capable of predicting the height of the Mach stem exists!

As a final remark, it should be mentioned that Azevedo's model was developed for the case when the reflecting wedge, by which the incident shock wave is generated, is short and

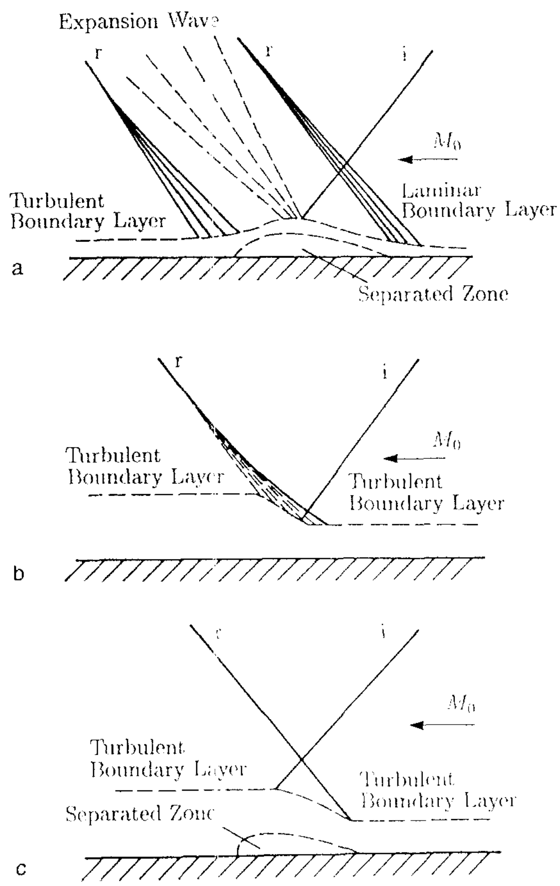

Fig. 13. Schematic illustration of the way by which the boundary layer over the bottom surface affects the structure of the reflected shock wave of a steady RR near the reflection point

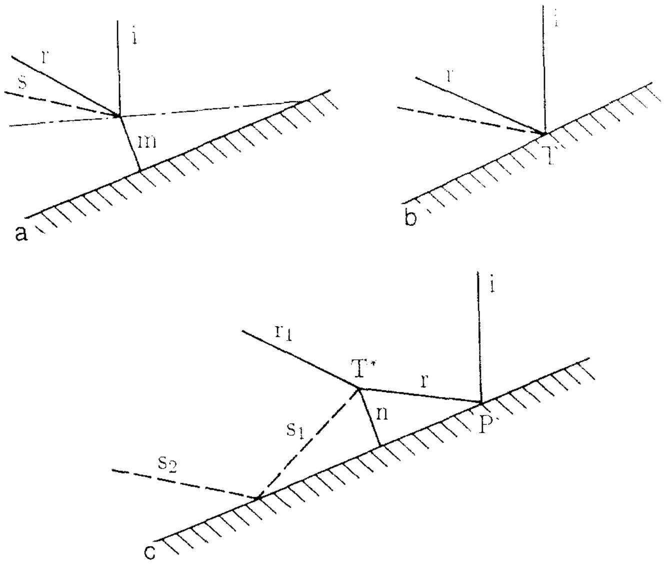

Fig. 14. Schematic illustration of the wave configuration during the InMR⇌TRR transition. **a** an InMR prior to its interaction with the reflecting surface, **b** an InMR at the moment its triple collides with the reflecting surface, **c** a TRR which is formed after the InMR terminates

hence the reflected shock wave, r, interacts with the expansion wave, prior to any other interactions (see Fig. 12a). Therefore, Azevedo's model cannot be used for the case of a long reflecting wedge, such as the one shown in Fig. 12b, where the reflected shock wave is reflected from the surface of the reflecting wedge prior to its interaction with the expansion wave. (Note that the interaction of r_1 with s, as shown in Fig. 12b, which results in a reflected and a transmitted shock wave is the one which for some, yet unknown, reason does not occur in the supposedly equivalent case shown in Fig. 6a where m_1 terminats at s without any reflection or transmitting any waves). The foregoing discussion clearly indicates that a physical length associated with the reflecting wedge, which is communicated by the expansion wave to the triple point, determines the actual height of the Mach stem. Whether this occurs in the way suggested by Azevedo (1989) or in another way is yet to be clarified.

Another issue regarding the wave structure in steady flow reflections which requires further investigation is the interaction of the incident shock wave with the bottom wall boundary layer and the formation of the reflected shock wave of a regular reflect ion. Consider Fig. 13 where the incident shock wave, i., is seen to interact with the boundary layer that develops along the bottom wall. The formation of the reflected shock wave, r, and its structure near the reflection point is seen to strongly depend on whether the boundary layer is laminar or turbulent. The details of this complicated interaction are yet to be investigated and clarified.

3.3. Unsteady flows

Unsteady flows give rise, in addition to the wave configurations that appear in pseudo-steady flows, to some additional shock wave reflection configurations, namely; stationary Mach reflection (StMR), inverse Mach reflection (InMR), and transitioned regular reflection (TRR). Since the wave configurations are continuously changing with time, questions like those asked earlier in the case of steady and pseudo-steady reflections cannot be asked in the case of unsteady reflections. However, in view of the earlier remarks regarding the unclear nature of the contact discontinuity of a triple point of a Mach reflection, the following interesting phenomena are pointed out.

Consider Fig. 14 where the InMR⇌TRR is shown schematically. Figure 14a shows an InMR prior to the interaction of its triple point with the reflecting surface; an InMR at the moment when its triple point collides with the reflection surf ace is shown in Fig. 14b; and a TRR which is formed when the InMR terminates is shown in Fig. 14c. Note how the contact discontinuity, s_1, of the new triple point, T*, reflects from the reflecting wedge. This reflection, which could be termed as a regular reflection of a contact discontinuity is yet to be investigated. (Some details regarding the wave configuration of a TRR can be found in Ben-Dor and Elperin 1991).

Some other interesting phenomena, related to the behavior of the contact discontinuity, are obtained when a planar shock wave reflects over a concave double wedge. It was shown by Ben-Dor, Dewey and Takayama (1987) that for concave double wedges having an gles θ_w^1 and θ_w^2 that satisfy the following condition

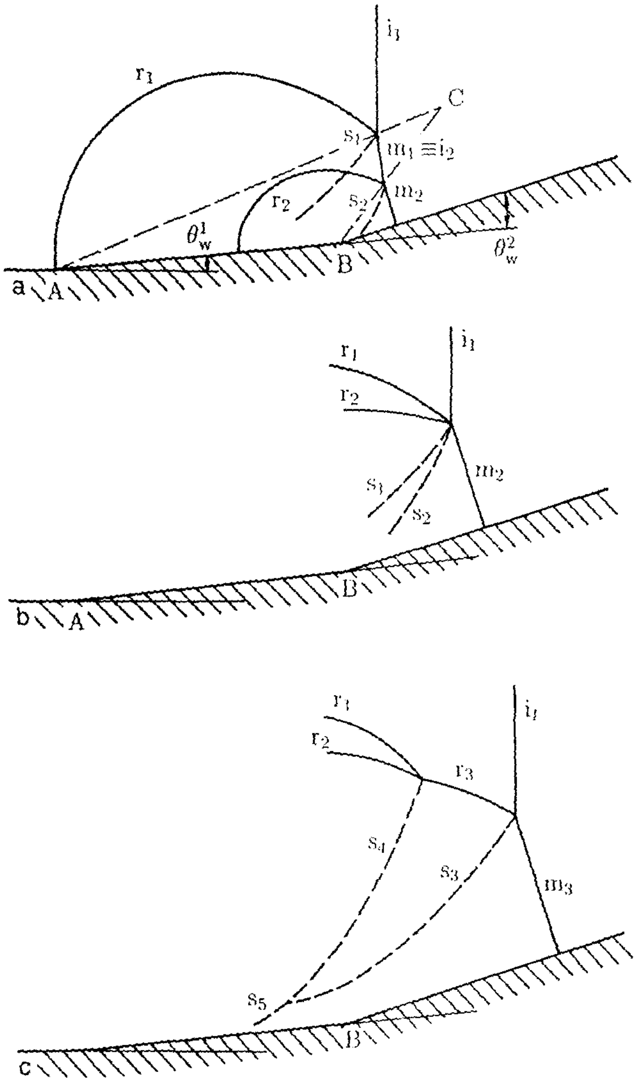

Fig. 15. The interaction process of an incident planar shock wave with a concave double wedge. **a** prior to the interaction of the two triple points, **b** at the moment the two triple points interact, **c** the resulted configuration after the interaction is completed

$$\theta_w^1 + \theta_w^2 < \theta_w^{tr}$$

where θ_w^{tr} is the pseudo-steady RR⇌IR transition wedge angle for the incident shock wave under consideration, the reflection process is as follows: first the incident shock wave reflects as a MR over the first reflecting surface; then the Mach stem of this MR reflects as a secondary MR over the second reflecting surface as shown in Fig. 15a; at a later time the two triple points of these two MR's coalesce at point C (see Fig. 15a) to result in the configuration shown in Fig. 15b where six discontinuities, four shock waves and two contact discontinuities, originate from a single point; finally, the wave configuration shown in Fig. 15c is obtained.

The termination of the first contact discontinuity, s_1, at the point where it meets with the second reflected shock wave, r_2, as shown in Fig. 15a, the six discontinuities confluence, shown in Fig. 15b, and the interaction of the two contact discontinuities, s_3 and s_4, as shown in Fig. 15c, to form a new contact discontinuity, s_5, are all clear demonstration of interesting and quite puzzling interactions which are yet to be investigated and understood, e.g., how can the flow bounded by the slipstreams s_1 and s_2 be parallel simultaneously to these two slipstreams?

4. Conclusions

It was demonstrated in the foregoing discussion that although the shock wave reflection phenomena have been investigated quite intensively in the past five decades, there are still many open questions and unresolved problems regarding the reflection phenomena in steady, pseudo-steady and unsteady flows. Additional unsolved problems and suggested research topics can be found in Ben-Dor (1991). It is hoped that the present paper will encourage the gasdynamic community to continue its effort for better understanding of the shock wave reflection phenomena.

References

Azevedo DJ (1989) Analytic prediction of shock patterns in a high-speed, wedge-bounded duct. Ph.D.Thesis, Dept Mech and Aero Eng, State Univ, N.Y., Buffalo, N.Y., USA

Bazhenova TV, Fokeev VP, Gvozdeva LG (1976) Regions of various forms of Mach reflection and its transition to regular reflection. Acta Astro 3:131-140

Ben-Dor G (1978) Regions and transitions of nonstationary oblique shock wave diffractions in prefect and imperfect gases. UTIAS Rept 232

Ben-Dor G (1981) Relation between first and second triple point trajectory angles in double Mach reflection. AIAA J 19:531-533

Ben-Dor G (1987) A reconsideration of the three shock theory of a pseudo steady Mach reflection. J Fluid Mech 181:467-484

Ben-Dor G (1990) Structure of the contact discontinuity of nonstationary Mach reflections. AIAA J 28:1314-1316

Ben-Dor G (1991) Shock wave reflection phenomena. Springer-Verlag, New York, N.Y., USA

Ben-Dor G, Dewey JM, Takayama K (1987) The reflection of a plane shock wave over a double wedge. J Fluid Mech 176:483-520

Ben-Dor G, Elperin T (1991) Analysis of the wave configuration resulting from the termination of an inverse Mach reflection. Shock Waves 1:237-241

Ben-Dor G, Glass II (1979) Domains and boundaries of nonstationary oblique shock wave reflections. 1 Diatomic gas. J Fluid Mech 92:459-496

Ben-Dor G, Rayevski D, Shock wave interaction with a high density layer. submitted for publication

Ben-Dor G, Takayama K (1985) Analytical prediction of the transition from Mach to regular reflection over cylindrical concave wedges. J Fluid Mech 158:365-380

Ben-Dor G, Takayama K (1986/7) The dynamics of the transition from Mach to regular reflection over concave cylinders. Israel J Tech 23:71-74

Ben-Dor G, Takayama K, Dewey JM (1987) Further analytical considerations of the reflection of weak shock waves over concave wedges. Fluid Dynamics Research 20:77-85

Ben-Dor G, Takayama K, Needham CE (1987) The thermal nature of the triple point of a Mach reflection. Physics Fluids 30:1287-1293

Birkhoff G (1950) Hydrodynamics, a study in Logic, fact and similitude. Princeton University Press, N.J., USA

Colella P, Henderson LF (1990) The von Neumann paradox for the diffraction of weak shock waves. J Fluid Mech 213:71-94

Dewey JM, McMillan DJ (1985a) Observation and analysis of the Mach reflection of weak uniform planar shock waves. Pt. I. Observation. J Fluid Mech 152:49-66

Dewey JM, McMillan DJ (1985b) Observation and analysis of the Mach reflection of weak uniform planar shock waves. Pt. II. Analysis. J Fluid Mech 152:67-81

Hornung HG, Oertel H Jr, Sandeman RJ (1979) Transition to Mach reflection of shock waves in steady and pseudo-steady flow with and without relaxation. J Fluid Mech 90:541-560

Hornung HG, Robinson ML (1982) Transition from regular to Mach reflection of shock waves. Part 2. The steady-flow criterion. J Fluid Mech 123:155-164

Krehl P, van der Geest (1991) The discovery of the Mach reflection effect and its demonstration in an auditorium. Shock Waves 1:3-15

Law CK, Glass II (1971) Diffraction of strong shock waves by a sharp compressive corner. CASI Trans 4:2-12

Lee J-H, Glass II (1984) Pseudo-stationary oblique-shock wave reflections in frozen and equilibrium air. Prog Aerospace Sci 21:33-80

Lock GD, Dewey JM (1989) An experimental investigation of the sonic criterion for transition from regular to Mach reflection of weak shock waves. Exp in Fluids 7:289-292

Mach E (1878) Uber den Verlauf von Funkenwellen in der Ebene und im Raume. Sitzungsbr Akad Wiss Wien 78:819-838

Neumann J von (1943a) Oblique reflection of shocks. Explos Res Rep 12, Navy Dept Bureau of ordinance, Washington, DC, USA

Neumann J von (1943b) Reflection, intersection and reflection of shock waves. NAVORD Rep 203-45, Navy Dept, Bureau of Ordinance, Washington, DC, USA

Reichenbach H (1985) Roughness and heated layer effects on shock-wave propagation and reflection – experimental results. Ernst Mach Int, Rep E24/85, Freiburg, West Germany

Schmidt B (1989) Structure of incipient triple point at the transition from regular reflection to Mach reflection. In: Muntz EP, Weaver DP, Campbell DH (eds) Rarefied Gas Dynamics: Theoretical and Computational Techniques, Progress in Astronautic and Aeronautics 118:597-607

Shirouzu M, Glass II (1986) Evaluation of assumptions and criteria in pseudo-stationary oblique shock wave reflections. Proc Roy Soc London, Ser A 406:75-92

Skews BW (1972) A departmental note from McMaster University by private communication

Smith LG (1945) Photographic investigation of the reflection of plane shocks in air. OSRD Rep 6271, Off Sci Res Dev, Washington, D.C., USA, or NDRC Rep A-350

Sternberg J (1959) Triple shock wave interactions. Phys Fluids 2:179-206

Takayama K (1992) Optical flow visualization of shock wave phenomena. Springer Verlag, New York, N.Y., USA

Takayama K, Ben-Dor G (1989) A reconsideration of transition criterion from Mach to regular reflection over cylindrical concave surfaces. KSME J 3:6-9

White DR (1951) An experimental survey of the Mach reflection of shock waves. Princeton Univ, Dept Phys Tech Rep II-10, Princeton, N.J., USA

This article was processed using Springer-Verlag TEX Shock Waves macro package 1.0 and the AMS fonts, developed by the American Mathematical Society.