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The Relation Between Channel Averages and Energy Averaged Cross Sections

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For nuclear cross sections a very simple relation between channel averages and averages over incident energy is found. The assumptions used are normally satisfied for sets of channels involved in inclusive cross sections in regions without intermediate structure. In the absence of direct reactions the channel averaged cross sections are expressed in terms of channel averaged transmission coefficients; in the presence of such reactions, channel averages of first and second powers of transmission matrix elements occur when channel correlations induced by direct reactions are neglected.

The statistical theory of nuclear reactions connects fluctuating or compound cross sections averaged over the incident energy with the corresponding transmission matrix that reflects the "optical" or "direct" properties of the system. In the absence of precompound structures this theory is well developed and yields simple results for large numbers of open channels, if we disregard certain problems that remain open for very weakly coupled channels. When no direct reactions are present we can use Hauser-Feshbach theory [1] and in general we may use Vager's conjecture [2, 3] to describe the energy average of the fluctuating cross section. More elaborate theories for small channel numbers or weakly coupled channels have been developed partially on numerical basis [4, 5].

On the other hand averages over exit channels have been discussed only scantily in spite of their importance for inclusive cross sections.

The purpose of this note will be to show a very simple relation that exists between the two types of averaging. We shall first summarize briefly the pertinent facts about energy averages. Next we define channel averaged cross sections. The desired relation among the two types of averages is first shown under the restrictions that no direct reactions occur and that the channels averaged over are equivalent in the sense that their transmission coefficients are equal. We finally lift first the latter and then the former restriction.

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In the course of our argument we shall concentrate on a fixed angular momentum and drop all the kinematical factors that are irrelevant in this context. The total cross section

$$\sigma_{ab}=\sigma_{ab}^{\mathrm{fl}}+\sigma_{ba}^{\mathrm{dir}} \tag{1}$$

may be split into direct and fluctuating parts defined by

$$\sigma_{ab}^{\mathrm{dir}}=|1-\bar{S}_{ba}|^2, \quad \sigma_{ba}^{\mathrm{fl}}=|S_{ba}^{\mathrm{fl}}|^2 \tag{2}$$

where in turn the S-matrix is split as

$$S_{ba}=\bar{S}_{ba}+S_{ba}^{\mathrm{fl}}. \tag{3}$$

Here – denotes an energy average and this separation implies

$$\overline{S_{ba}^{\mathrm{fl}}}=0.$$

For the energy averaged cross section we have

$$\overline{\sigma_{ba}}=\sigma_{ba}^{\mathrm{dir}}+\overline{\sigma_{ba}^{\mathrm{fl}}}. \tag{4}$$

The average fluctuating cross section may be expressed as

$$\overline{\sigma_{ba}^{\mathrm{fl}}}=\frac{P_{aa}P_{bb}+P_{ba}^2}{Tr\,P} \tag{5}$$

as conjectured by Vager [2] and proved for large values of $Tr\,P$ by Agassi, Weidenmüller and Mantzouranis [3]. The transmission matrix P is given by

$$P_{ba}=\delta_{ba}-\sum_c \bar{S}_{ac}\,\overline{S_{bc}^*}. \tag{6}$$

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In the absence of direct reactions this simplifies for inelastic channels to the Hauser-Feshbach result (1).

$$\overline{\sigma_{ba}}=\overline{\sigma_{ba}^{\mathrm{fl}}}=\frac{T_b\,T_a}{\sum T_i}(1+\delta_{ab}) \tag{7}$$

where $T_a=P_{aa}$ is the transmission coefficient.

We define a channel average for a subset Δ of the set of all open channels, with the condition that the number of channels N in Δ be large compared to one, but small compared to the total number of open channels. The channel averaged cross section is then given by

$$\langle\sigma_{\Delta a}\rangle=\frac{1}{N}\sum_{i\in\Delta}\sigma_{ia}. \tag{8}$$

To see the basic structure of the problem we first look at the simplest possible example, namely a situation with no direct reactions and equal transmission coefficients for all channels in Δ. We have a set of N functions of a parameter (energy) all with the same mean value and variance, that are uncorrelated among each other.

If we now perform an average at a fixed value of the parameter over the set of functions, the average value is clearly the same as the average of a single function taken with respect to the parameter. Thus we find

$$\langle\sigma_{\Delta a}\rangle=\overline{\sigma_{ia}}=\frac{T_i\,T_a}{\sum\limits_s T_s}. \tag{9}$$

Although the two types of averages agree, it is important to note the basic difference. The fluctuating cross sections are correlated for small differences in incident energy, and the corresponding correlation length is given by Ericson's theory [5] for the compound system. If on the other hand we choose our channel average as a spectral average, the channels remain uncorrelated for discrete spectra of the final states. For overlapping final states we must find a correlation in the spectrum but it is exclusively determined by the overlap of resonances in the final state, and unrelated with the compound system. In such a case the interval must be large enough that the effect of such correlations is cancelled.

Next consider the case of arbitrary transmission coefficients, but no direct reactions. There we partition the interval into k bins Λ_α, $\alpha=1\dots k$. We define subsets of channels Δ_α such that $i\in\Delta_\alpha$ if $T_i\in\Lambda_\alpha$. N_α will be the number of channels in Δ_α and $\langle T_\alpha\rangle=\frac{1}{N_\alpha}\sum_{i\in\Delta_\alpha}T_i$ the average transmission coefficient for the bin Λ_α. If N_α is large and Δ_α small we can use the same argument as above for each bin and we find

$$\langle\sigma_{\Delta_\alpha a}\rangle=\frac{1}{N_\alpha}\sum_{i\in\Delta_\alpha}\sigma_{ia}=\frac{T_\alpha\,T_a}{\sum\limits_s T_s}. \tag{10}$$

Adding all bins we obtain

$$\langle\sigma_{\Delta a}\rangle=\frac{1}{N_\Delta}\sum_\alpha N_\alpha\frac{T_\alpha\,T_a}{\sum\limits_s T_s}$$

$$=\frac{1}{N_\Delta}\frac{T_a}{\sum\limits_s T_s}\sum_\alpha N_\alpha\langle T_\alpha\rangle=\frac{\langle T_\Delta\rangle\,T_a}{\sum\limits_s T_s} \tag{11}$$

if

$$\langle T_\Delta\rangle=\frac{1}{N_\Delta}\sum_{i\in\Delta}T_i \tag{12}$$

is the average transmission coefficient of the channels in Δ. Note that the concept of bins does not appear in the final results. Actually it is not necessary that the relevant conditions be fulfilled for all bins. What we require is that the functions with the largest fluctuations in a given problem appear in sufficient number for these fluctuations to average out. This implies that the "bin" corresponding to the largest transmission coefficients must contain a sufficient number of channels.

In the presence of direct reactions the problem becomes somewhat more involved for two reasons. First the direct part of the cross section has to be included into the average and secondly the direct part may lead to channel cross correlations. The former point is easy to take care of, the latter may destroy the entire argument. If we assume that these correlations are sufficiently small we can write

$$\langle\sigma_{\Delta a}\rangle=\frac{1}{N_\Delta}\sum_{i\in\Delta}\overline{\sigma_{ia}}=\langle\sigma_{\Delta a}^{\mathrm{dir}}\rangle+\frac{1}{N_\Delta}\sum_{i\in\Delta}\overline{\sigma_{ia}^{\mathrm{fl}}}. \tag{13}$$

To proceed further we define averaged quantities

$$\langle P_{\Delta a}^2\rangle=\frac{1}{N_\Delta}\sum_{i\in\Delta}P_{ia}^2,\quad\langle P_{\Delta\Delta}\rangle=\frac{1}{N_\Delta}\sum_{i\in\Delta}P_{ii}. \tag{14}$$

Then using (2) and (5) and (6) we obtain

$$\langle\sigma_{\Delta a}\rangle=|\langle P_{\Delta a}\rangle|+\frac{\langle P_{\Delta\Delta}\rangle\,P_{aa}+\langle P_{\Delta a}^2\rangle}{Tr\,P}. \tag{15}$$

The first term is the averaged direct part and the second is obtained from (5) following similar arguments as those that lead to (11). The result given in the absence of direct reactions retains the form of a Hauser-Feshbach formula for channel averaged quantities. This does no longer hold true in the presence of direct reactions, as (15) does *not* have the form of Vager's conjecture as given in (5). We find the term $\langle P_{\Delta a}^2 \rangle$ rather than a term of the form $\langle P_{\Delta a} \rangle^2$. This difference obviously emphasizes the importance of channels with strong direct coupling to the entrance channels. The result of (15) is to be applied with caution for the following reason: if all couplings involved in the average are very similar, the difference between the two terms above is a higher order effect. If this is not the case our assumption of small channels correlations may well be false, and thus our treatment not valid. Further investigation on this point will be necessary.

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