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Investigation of the Stability of Superheavy Nuclei around $Z=114$ and $Z=164$*

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The potential-energy-surface has been calculated using a method developed in a previous paper. Two regions of relative stability against spontaneous fission around $Z=114$ and $Z=164$ are discussed in greater detail. The stability of the quasistable elements against fission is discussed and the stability against alpha- and beta-decay is estimated by using the mass formula of Myers-Swiatecki. It is found that quasistable elements should exist in the regions around $Z=114$ and $Z=164$.

I. Introduction

The various extrapolations of the shell model predict magic nuclear configurations far beyond the stable elements. These predictions are nearly independent of the specific ansatz for the shell model, though the sequence of the levels between magic numbers may be quite different. A potential of the Wood-Saxon type[1] results in the magic numbers 114, 164 for protons and 184 for neutrons while the oscillator shell model[2,3] predicts 114, 124, 164 for protons and 184, 196, 236, 318 for neutrons. The investigation of the potential-energy-surface (PES) in an earlier paper[3] shows that there are some nuclei around $Z=114$ and N between 184 and 196 (this is referred later on as region I) and around $Z=164$ and $N=318$ (this is referred later on as region II) which are stable against spontaneous fission.

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It is our aim to investigate in this paper these islands of stability in greater detail. Especially the size of these quasistable areas will be searched for specifically. For this purpose the deformations of the groundstates, the heights and locations of the fission barriers are calculated. Also the life-times against spontaneous fission, alpha- and beta-stability will be estimated.

II. The Computation of the Potential-Energy-Surface

The collective potential energy surface (PES) plays a central role in the discussion of the stability of nuclei against spontaneous fission. Since fission is one of the most important decay channels for superheavy elements we shall sketch the method of computation of the PES briefly. More details can be found in Ref.[3] and [4].

We start with the Hamiltonian for the three-dimensional anisotropic oscillator shell model of the form[5, 6]

$$H = T + \frac{m}{2}(\omega_x^2 x^2 + \omega_y^2 y^2 + \omega_z^2 z^2) + C\vec{l}\cdot\vec{s} + D(\vec{l}^2 - \langle\vec{l}^2\rangle_N). \qquad (1)$$

Transforming, as usual, the nuclear surface to the intrinsic system one obtains through a selfconsistency argument the connection between the oscillator frequencies and the deformation parameters (a_0, a_2) which specify the axes of the ellipsoidal nuclear shape. In detail the axes of the ellipsoid are given by

$$\begin{aligned} a &= R_0\left(1 - \frac{1}{4}\sqrt{\frac{5}{\pi}}\,a_0 + \frac{1}{2}\sqrt{\frac{15}{2\pi}}\,a_2\right) \\ b &= R_0\left(1 - \frac{1}{4}\sqrt{\frac{5}{\pi}}\,a_0 - \frac{1}{2}\sqrt{\frac{15}{2\pi}}\,a_2\right); \qquad a\cdot b\cdot c = \mathring{R}_0^3 = \text{const} \qquad (2) \\ c &= R_0\left(1 + \frac{1}{2}\sqrt{\frac{5}{\pi}}\,a_0\right) \end{aligned}$$

with $\mathring{R}_0 = r_0 A^{\frac{1}{3}}$ fm and $r_0 = 1.2$ fm. The parameters κ and μ occuring in (1) are extrapolated with a formula given by Seeger[7] into the region

4 Mosel, U., Greiner, W.: Z. Physik **217**, 256 (1968).

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7 Seeger, P. A., Perisho, R. C.: LA 3751 Los Alamos 1967.

of the superheavy nuclei

$$\kappa = -\frac{1}{2}\frac{C}{\hbar\overset{\circ}{\omega}_0}$$

$$\mu = \frac{2D}{C}$$

$$\kappa = \kappa_0\left(\tfrac{1}{2}[N+1][N+2]\right)^{-\frac{1}{3}}$$

$$\mu = \mu_0$$

	Z	N
κ_0	0.180	0.210
μ_0	0.620	0.308

(3)

The short range part of nuclear forces is important for the stability, since it prefers spherical nuclear shapes. It is taken into account by the residual pairing force. This has been treated in a BCS-calculation by considering 24 levels symmetrically to the Fermi surface, each filled with two particles according to the Ω-degeneracy. The strength of the pairing force was adjusted to the even-odd-massdifferences of the actinides [8]. This yields

$$G*A = 32\ \text{MeV (protons)}, \qquad G*A = 29\ \text{MeV (neutrons)} \\ \text{for } \hbar\overset{\circ}{\omega}_0 = 41A^{-\frac{1}{3}}\ \text{MeV}. \tag{4}$$

We have also to take into account the Coulomb-energy $E_c(a_0, a_2)$ for which we make the ansatz of a homogeneously charged ellipsoid [3]. The PES is then obtained in the form

$$E(a_0, a_2) = \sum_{Z,N}\left\{\sum_i \varepsilon_i(a_0, a_2)\, V_i^2 - \frac{\Delta^2}{G}\right\} + E_c(a_0, a_2) \tag{5}$$

where $\varepsilon_i(a_0, a_2)$ are the eigenvalues of (1), and V_i^2 and Δ are the occupation probability of the level $|i\rangle$ and the gap as occuring in the BCS-formalism. A few words seem to be adequate concerning the special method of computing the PES (5) by summing essentially the single particle energies. This corresponds to the point of view where the collective field is considered as being generated by a quadrupole-quadrupole force [9]. At first it seems that a kind of Hartree-Fock treatment should be more appropriate [10]. It is true, however, that recent results on Hartree-Fock calculations indicate that, for example, the binding energies cannot be obtained by this treatment alone and that, moreover, one has to include at least second order perturbation terms beyond the Hartree-Fock-calculation. This simply expresses the fact, that a Hartree-Fock calculation is not a good approximation in nuclear

8 Grumann, J.: Diplomarbeit Frankfurt a. M., Jan. 1969.
9 Moszkowski, S. A.: Phys. Letters **6**, 237 (1963).
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physics. On the other hand the potential energy surfaces calculated for many vibrational and rotational nuclei on the basis of (5) do agree surprisingly well with various experimental facts such as deformations, β- and γ-vibrational energies, predictions of deformed and spherical nuclei, etc.[4]. This indicates, at least phenomenologically, that the computational method (5) is adequate for the calculation of the PES.

III. The Properties of Superheavy Nuclei. The Quasistable Islands

As shown in Ref.[4] the PES (5) can be computed with minimal expense of computer time by using the symmetries of the intrinsic $\{a_0, a_2\}$ representation of the nuclear surface. It is known that the a_2-coordinate is mainly describing the collective dynamics of the asymmetry vibrations (γ-vibrations) while the deformation of the ground states, the location and the height of the fission barriers are described nearly completely by the a_0-coordinate. Therefore, we restrict ourselves in most cases to the calculation of the potential curves, i.e. the collective potential energy along the a_0-axis (i.e. $a_2 \equiv 0$). These potential-energy-curves (PEC) are calculated for both islands (region I and II) of superheavy nuclei. It must be emphasized that the approximation of the two-dimensional (and in general even more-dimensional) potential barrier by a one dimensional potential curve is a very crude approximation which has to be revised in the future. Nevertheless it is this basic approximation which allows relatively easy estimates of fission half-lives.

In the discussion of the PEC – some of them are shown for the $Z=114$ and $Z=164$ isotopes in Figs. 1 and 2 – one has to remember, that in the ansatz (1) only ellipsoidal surfaces are taken into account. This is a good approximation only for small deformations while for larger deformations more complicated surfaces must be used. On the other hand, the maxima of the fission barriers of superheavy nuclei appear at relatively small deformations, i.e. $|a_0| \sim 0.2$ to 0.3 (see Figs. 1, 2 and also 5, 6). Thus it is possible to describe the fission process up to and somewhat beyond the barrier by quadrupole deformations. This is the second important approximation of our calculation. This approximation is justified since the inclusion of higher multipoles in the surface influences the PES only beyond the saddle points of the fission barriers[5]; the location and height of the barriers are not altered at all.

It turns out that the typical properties and behaviour of the known deformed elements, namely prolate deformations in the ground state and a prolate fission barrier are not completely retained for superheavy nuclei. They are mostly spherical or possess a small oblate deformation. Furthermore in many cases the height of their potential barrier is several MeV lower for oblate deformation than for prolate, i.e. many super-

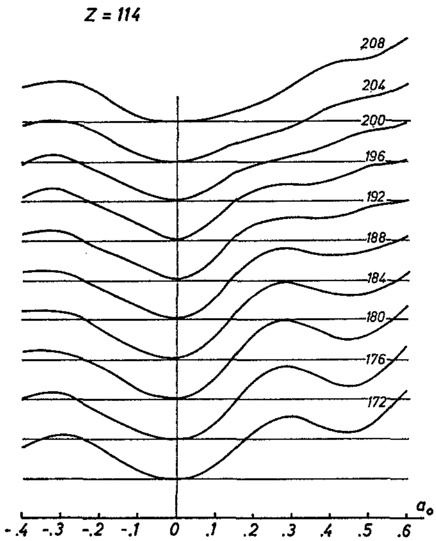

Fig. 1

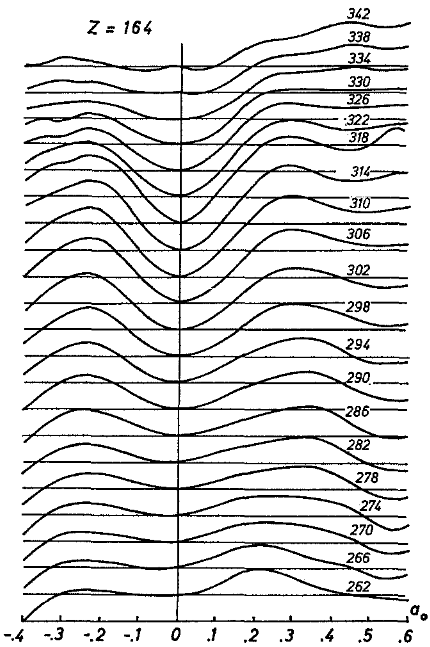

Fig. 2

Figs. 1 and 2. The PES along the a_0-axis for $Z=114$ and $Z=164$ isotopes. The separation between successive horizontal lines is 5 MeV

heavy nuclei seem to fission through oblate barriers (see Fig. 1, 2 and also 5, 6).

This effect is already contained within the liquid drop model, if one looks at the deformation dependent surface- and Coulomb-energy of the mass formulas. For the known stable elements the deformation energy of the surface term which strongly prefers the prolate deformation dominates. However, with increasing number of protons the influence of the Coulomb-energy which favours the oblate form increases and in region II, i.e. for $Z=164$, both energies compensate nearly completely. This implies that mainly shell effects and to some extent the Coulomb-energy will determine the shape of superheavy nuclei. This effect can be seen within the two quasistable islands (Figs. 3 and 4) when along the expected valley of beta-stability the deformations of the nuclei with incompletely filled shells are dominantly prolate, while the high Z-numbers of an isotonic family always favour the oblate shape.

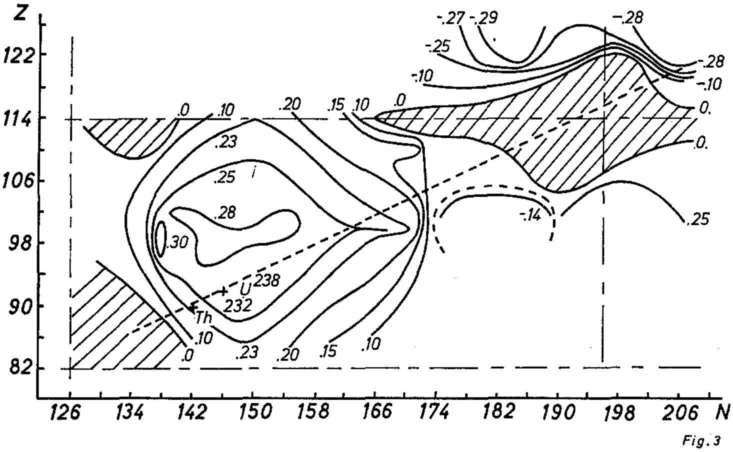

Fig. 3. Deformations of the ground states of the actinides and the following superheavy nuclei (region I). The hatched regions indicate spherical shapes. The long diagonal dashed line shows the beta-stable valley according to the mass formula[11]. The other dashed line separates prolate deformations from oblate ones

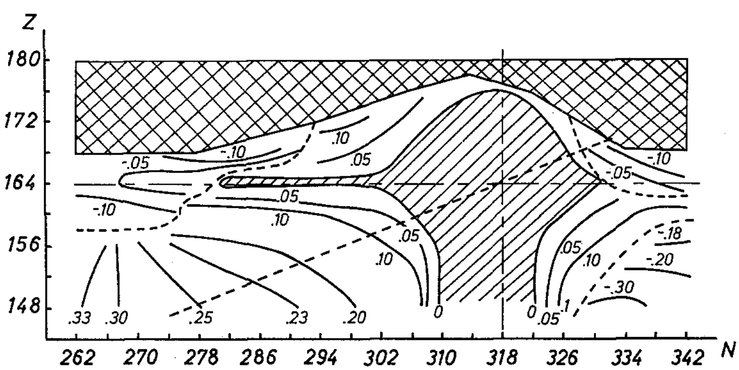

Fig. 4. Deformations of the ground state in region II. The hatched region markes the spherical shapes. The long diagonal dashed line shows the beta-stable valley according to the mass formula[11]. The other dashed lines separate prolate deformations from oblate ones. The proton rich superheavy elements ($Z \approx 176$) posses no energy minimum (crossed hatched region)

Most impressive is the lowering of the oblate fission barriers as compared to the prolate ones. This can be seen directly from Figs. 1 and 2 and also from Figs. 5 and 6, where the location of the maxima of the lowest fission barriers are shown. It is noticed that most elements

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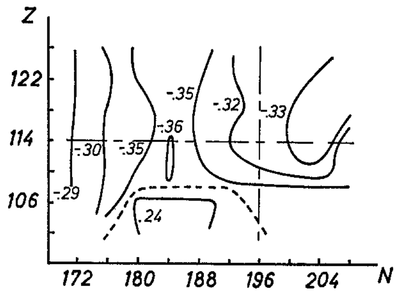

Fig. 5

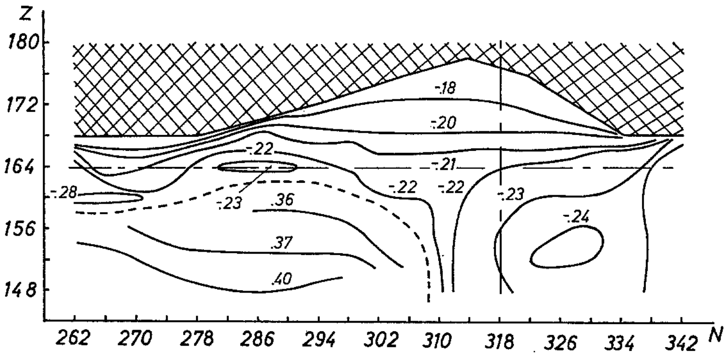

Fig. 6

Figs. 5 and 6. The a_0-values of the maxima of the fission barriers in regions I and II. The dashed line separates prolate barriers from oblate ones. Nuclei with $Z \approx 168$ (Fig. 6) possess no barrier; they fission immediately

of the two islands of stability possess oblate fission barriers. Hence the question arises how fission through the oblate barriers has to be interpreted. From a physical point of view it seems intuitively clear that nuclei which fission over the oblate shape should – with high probability – favour a break-up in more than two parts. Indeed this is also clear from pure energetical considerations which show that the energy gain of these superheavy elements is in multiple fission much greater than in a binary process. Especially ternary and quadruple fission processes should therefore be characteristic for superheavy elements. In fact as an extreme case of an oblate nucleus one may consider a disk, the node lines of which may be considered as the breaking lines for multiple fission. Alternatively the inclusion of higher multipoles in the expansion of the nuclear surface and their φ-dependence (φ is the polar angle about the symmetry axis) may yield a vivid picture of oblate fission.

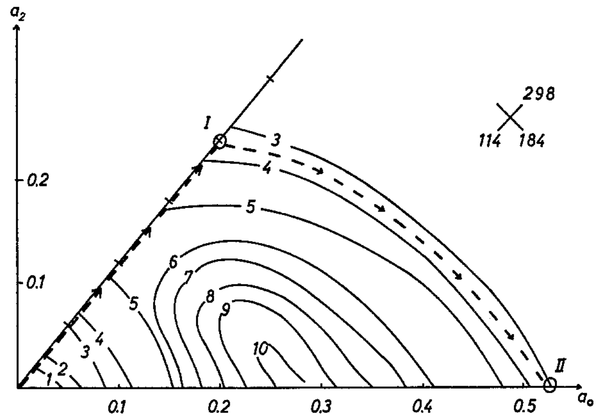

Fig. 7. The two-dimensional potential-energy-surface for the nucleus $_{114}X_{184}^{298}$ is shown. The numbers at the curves give the energies in MeV. The dashed line shows the fission path with the points I and II discussed in the text

There is also the question whether a nucleus, if it has passed the oblate barrier, does come back to the prolate fission path. This is made more transparent in Fig. 7, where a typical potential surface for the superheavy nucleus $_{114}X_{184}^{298}$ is shown. Also the possible fission path is indicated by the dashed curve. The question then precisely is wether fission takes place in the points I or II indicated in Fig. 7. This problem cannot be solved within the framework of the present calculation, since the Nilsson-type calculation gives rise to the increase of the potential to infinity for large deformations (a_0). The surface can be trusted from these calculation only up to $a_0 \sim 0.4$. The surface in Fig. 7 shows, however, that the oblate deformation beyond the barrier $a_0 \approx -0.3$ is large enough for fission to take place before eventually the oblate deformation transforms into prolate deformation.

We thus suppose that oblate fission takes place and that it may be considered as the doorway process for multiple fission. This possibly could serve as an experimental indication for superheavy nuclei.

In the areas of stable nuclei the double magic configurations cause a great stiffness (large stiffness parameters C_{20}) of the potential for vibrations of the surface near the ground state. For example among the nuclei of the first island (region I) the element $_{114}X_{196}^{310}$ has the largest C_{20}-value with 720 MeV, and in region II the element $_{164}X_{318}^{482}$ has the enormous C_{20}-value of 1450 MeV. This has to be compared with the stiffnes of the nearly magic nucleus Pb^{206}, which is $C_{20} \approx 1500$ MeV. Up to the transition from spherical to weakly deformed nuclei, along

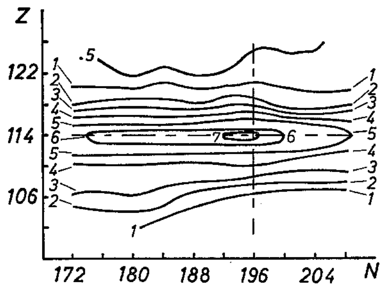

Fig. 8

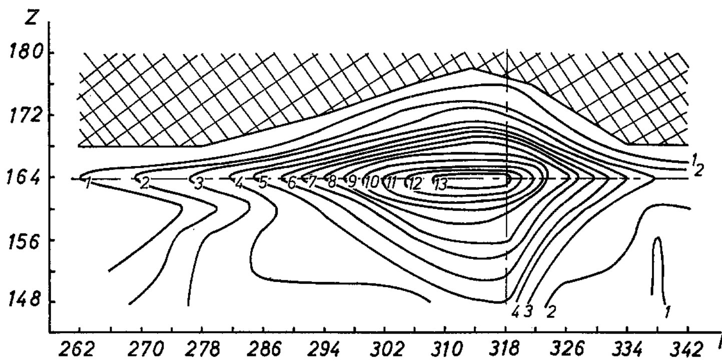

Fig. 9

Figs. 8 and 9. The heights of the fission barriers in regions I and II. The separation between the contour lines is 1 MeV.

the magic proton numbers down to neutron-deficient isotopes, the stiffness decreases to 200 or 300 MeV. This again has to be compared with the stiffness of nuclei in the transition region as, for example, Sm^{150} which has a C_{20}-value of $C_{20}=43$ MeV or Os^{192} which has a C_{20}-value of about 32 MeV [4]. The numerous spherical nuclei along $Z=114$ and $Z=164$ of Figs. 3 and 4 show the influence of the Coulomb-energy which will become even more evident if one considers the barrier heights (Figs. 8 and 9). They have a maximum of 6.7 MeV for ${}_{114}X^{310}_{196}$ which decreases about 0.6 MeV per proton, but only about 0.1 MeV per neutron. A similar behaviour show the elements of region II around ${}_{164}X^{482}_{318}$. There the heights are nearly twice as large as in region I, which indicates the great stability of this island against spontaneous fission. In this region, with its enormous fission barriers, one has to consider, however, that the uncertainty in the extrapolation of the shell model parameters is much greater than for region I (for this point see

Ref.[3]). Even if we take half of the calculated heights only, a great area remains around ${}_{164}X^{482}_{318}$ in which we can expect many stable nuclei.

In order to obtain an estimate for the half-life-times against spontaneous fission, we approximate the barriers by parabolas. In this way we always underestimate the barrier and thus the life-times because of the long tails of the actual barrier*. One obtains in WKB-approximation for the penetration probability[3]

$$P = e^{-K}$$

where

$$K = \frac{2\pi(E_s - E_f)}{\hbar\omega_f} \quad \text{and} \quad \omega_f = \sqrt{\frac{C_f}{B}}. \tag{6}$$

Here E_s is the height of the fission barrier and ω_f is the fission frequency which depends on the curvature C_f of the barrier at the maximum. For E_f we take the zero-point energy of a five-dimensional harmonic oscillator, i.e. $E_f = 5/2\,\hbar\omega_2$, because we are interested only in spontaneous fission from the groundstate which is spherical for nearly all stable superheavy elements. In the literature (see e.g. Ref.[2]) one usually uses only $1/2\,\hbar\omega_2$ with the tacit assumption that the fission mode, e.g. a_0, is completely decoupled from the other degrees of freedom and that one therefore has to subtract the zero-point energy of all the bounded oscillations at the saddle-point which is equivalent to the use of the value $E_f = 1/2\,\hbar\omega_2$ from the beginning. In other words, it is claimed by these authors that E_f should be only $1/2\,\hbar\omega_2$ instead of $5/2\,\hbar\omega_2$ since the fission problem is reduced to one dimension only and since the residual four degrees of freedom should not be affected during the fission process. This argument seems to be incorrect because we have shown in a separate calculation within this model (1) and (5) that the a_2-degree of freedom is very strongly coupled with the fission-stretching mode a_0 and that the energy for a_2-vibrations, E_{a_2}, at the saddle point is less than 50% of $\hbar\omega_2$ but very likely much more. A spherical nucleus has in its groundstate the five degrees of freedom $\alpha_{2\mu}$. Equivalent to these five coordinates are for a deformed state the three Euler angles α, β, γ and the two vibrational amplitudes a_0 and a_2. Since the rotations which are connected with the Euler angles have no zero-point-energy the zero-point-energy for a spherical nucleus is $5/2\,\hbar\omega_2$ while it amounts for a deformed state to $1/2 \cdot E_{a_0} + E_{a_2}$[12]. Here a_0 is just the fission-mode and therefore the *total* zero-point-energy at the saddle point reduces to E_{a_2} which is less than $1/2\,\hbar\omega_2$ (ground state) (see above).

* We are, anyway, only interested in the lower limits for the life-times.

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Therefore, the critical difference between the zero-point-energy in the ground state and that of the bounded a_2-oscillations at the saddle point is

$$E_0(\text{ground state}) - E_{a_2}(\text{saddle point}) \gtrsim 4/2\,\hbar\,\omega_2\,.$$

Because of this all life-time calculations should be done with $E_f = E_{\text{zero}} = 5/2\,\hbar\omega_2$ *instead of* $E_{\text{zero}} = 1/2\,\hbar\omega_2$ *the use of which overestimates the life-times quite appreciably**.

The logarithm of the half-live $T_{1/2}$ for spontaneous fission is given by

$$^{10}\log T_{1/2}\,[\text{years}] = -28.04 - {}^{10}\log E_{\text{vib}} + 0.434\,K\,. \tag{7}$$

It is seen from (6) that the massparameter B enters in (7) sensitively. Instead of calculating this parameter microscopically we use an empirical formula, which has been obtained from the $B(E2)$ values and tested for the rare earth and actinide nuclei[4,13]:

$$B\,\hbar^{-2} = 2.5 \times 10^{-3} A^{\frac{7}{3}} [\text{MeV}^{-1}]\,. \tag{8}$$

The correct B-value is certainly deformation dependent. However, it is well known that B increases strongly with deformation. We therefore always take that constant B-value which is appropriate for the description of the nuclear ground state[4]. In this way we again underestimate the penetration probability of the fission barrier, i.e. the fission half-lives (7). To obtain a feeling for the influence of the uncertainties of this inertial parameter we also calculated all the lifetimes with half of the B-values given in (8). These lower B-parameters diminish the logarithm of the life-times up to 50% (see Fig. 11a and b).

With the B-parameters (8) we get for the $Z = 114$-isotopes with neutron numbers between $N = 180$ and 190 life-times which are greater than one year (Fig. 10). Also the elements with $Z = 110$ have similar life-times. This can be explained by their relatively great stiffness parameters C_{20} and large curvatures of the barrier C_f, which are less sensitive to the Coulomb-energy than the barrier heights. In fact, a few elements with fission half-lives up to 10^9 years exist in the region I. Nevertheless Fig. 10 clearly indicates that the first island of stability is rather small. The fission life-times of the elements of region II are shown in the Fig. 11. In order to estimate the uncertainties introduced by the mass parameter, life-times are computed as well with the smaller mass-parameter (Fig. 11a) as with the larger one (Fig. 11b). In the first case

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* We thank J. R. Nix, Los Alamos, for enlightening discussions on this point.
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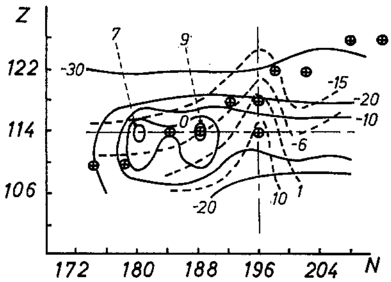

Fig. 10

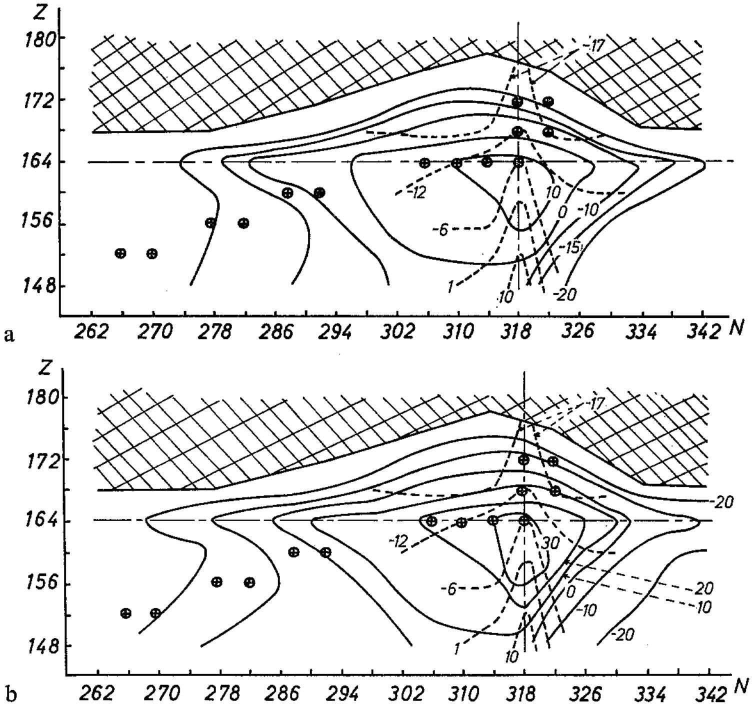

Fig. 11 a and b

Figs. 10 and 11. The half-life-times in $^{10}\log$ [years] for the nuclei in regions I and II. The full lines show the life-times against spontaneous fission, the dashed lines against alpha-decay. In Fig. 11a we used the lower mass parameter and in Fig. 11b the inertial parameter (8). The circled crosses indicate the region, where the nuclei are stable against beta-minus-decay (using the mass formula of Myers and Swiatecki[16])

the life-times are again strongly underestimated. In reality the life-times of superheavy elements against fission will occur somewhere between the predictions of Fig. 11a and b.

The area containing nuclei with life-times longer than one year is much greater in region II than in region I. Though there is an uncertainty in the parameters (see above) we see that around ${}_{164}X^{382}_{318}$ some nuclei exist which have extremely long life-times against spontaneous fission. To obtain an estimate for the stability against alpha-decay one has to know the shape of the barriers for the emission of alpha-particles. Since up to now there exists only insufficient information about these alpha-barriers, we also take the WKB-approximation in solving the one dimensional problem of the penetration of alpha-particles through a pure Coulomb wall. In this way one obtains the following dependence for the half-live $t_{1/2}$ on the Q-values for alpha-decay[14]

$$ {}^{10}\log t_{1/2} = C_1 (Z\,Q^{-\frac{1}{2}} - Z^{\frac{2}{3}}) - C_2 \tag{9} $$

where Z is the charge-number of the daughter-element. The coefficients $C_1 = 1.61$ and $C_2 = 28.9$ are determined by an empirical fit of the alpha-decay-data for nuclei with $N \geqq 126$[15].

For spherical nuclei the Q-values for the alpha-particles have been calculated by using the extrapolated massformula of Myers and Swiatecki[16]. With this formula we investigated also the energetical possibility of β^--decay*.

The life-times against alpha-decay (Figs. 10 and 11) diminish considerably the number of stable nuclei. If we also take into account beta-stability, there remain the $Z = 114$-isotopes ($N = 188$ to 200), $Z = 112$-isotopes ($N = 184$ to 196) and the $Z = 110$-isotopes ($N = 178$ to 190) which all possess a stability against spontaneous fission, alpha- and beta-decay of more than one minute. ${}_{114}X^{310}_{196}$ should exist a few months.

In region II we obtain around the magic neutron number 318 life-times against alpha-decay from a minute up to a great number of years (see Fig. 11). However, after considering also beta-stability, there remain only some nuclei around ${}_{164}X^{482}_{318}$ with life-times of a minute up to some hours. This depends on the fact, that with the used extrapolation the double magic nuclei are located near the border of the beta-stable-valley. One has to bear in mind, however, that in this region far off the known nuclei, the beta-stable-valley changes significantly by using the different mass formulas (see Fig. 2 and Ref.[3]). Therefore these alpha-decay rates and also the results on beta-stability may serve only as a qualitative and perhaps semi-quantitative guide. Small changes in the shell structure and in particular in the binding energies (mass formula) may change the results considerably.

* We thank T. Morović who provided the computer program.

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Table. *In this table are given the quantities of the PES and the corresponding life-times for the Z=114 and Z=164 isotopes. C_{20} gives the stiffness of the nuclei against vibrations in the ground state, C_f is the curvature of the barriers, E_{vib} is the a_0-vibrational energy with the inertial parameter B, K is the quantity defined in Eq. (6) and $T_{1/2}$ is the half life-time against spontaneous fission. Finally E_β and Q_α give the decay-energies for β and α decay respectively and $t_{\frac{1}{2}}$ the half-life against alpha decay*

Z	N	C_{20} (MeV)	C_f (MeV)	E_s (MeV)	$B\hbar^{-2}$ (MeV^{-1})	E_{vib} (MeV)	K	$\log T_{\frac{1}{2}}$ (years)	E_β (MeV)	Q_α (MeV)	$\log t_{\frac{1}{2}}$ (years)
114	176	120	−699	6.0	1.392	0.29	47	−7	−2.8	8.8	−12
114	180	334	−225	6.2	1.437	0.48	80	7	−1.8	8.1	−10
114	184	503	−310	6.3	1.483	0.58	67	1	−0.8	7.4	−8
114	188	472	−183	6.1	1.530	0.56	86	10	0.1	6.7	−6
114	192	551	−496	6.5	1.577	0.59	56	−3	1.1	6.0	−3
114	196	724	−489	6.7	1.626	0.67	57	−3	0.3	3.6	13
114	200	347	−537	6.0	1.675	0.45	53	−4	4.2	7.1	−7
114	204	187	−483	5.5	1.726	0.33	56	−3	5.0	6.3	−4
164	262	522	−833	0.9	3.414	0.39	0	−28	−7.3	19.7	−21
164	266	263	−442	1.3	3.489	0.27	11	−23	−6.3	16.6	−17
164	270	337	−681	2.1	3.565	0.31	20	−19	−5.9	19.0	−20
164	274	309	−744	2.8	3.642	0.29	29	−15	−5.2	18.3	−19
164	278	161	−676	3.3	3.720	0.21	41	−9	−4.5	17.6	−19
164	282	138	−664	4.2	3.799	0.19	55	−3	−3.9	17.3	−18
164	286	207	−834	5.3	3.879	0.23	64	1	−3.2	16.8	−18
164	290	302	−692	6.6	3.960	0.28	88	11	−2.6	16.3	−17
164	294	432	−1025	8.1	4.042	0.33	90	12	−2.0	15.8	−16
164	298	424	−1320	9.5	4.125	0.32	97	14	−1.3	15.2	−15
164	302	534	−1518	11.0	4.209	0.36	106	18	−0.7	14.7	−14
164	306	876	−1589	12.5	4.294	0.45	117	23	−0.1	14.1	−13
164	310	887	−1589	13.4	4.379	0.45	123	28	0.5	13.6	−12
164	314	769	−1521	13.5	4.466	0.42	134	31	1.1	13.1	−11
164	318	1447	−1302	13.6	4.554	0.56	143	34	−0.3	10.6	−6
164	322	654	−1108	10.8	4.643	0.38	127	28	3.3	14.2	−14
164	326	411	−925	8.3	4.732	0.29	108	19	3.9	13.7	−13
164	330	275	−760	6.0	4.823	0.24	86	10	4.4	13.1	−11
164	334	226	−690	4.0	4.914	0.21	59	−2	5.0	13.0	−11
164	338	116	−602	2.9	5.007	0.15	46	−7	5.7	13.3	−12
164	342	439	−946	2.9	5.101	0.29	32	−14	6.3	12.6	−10

Summary

We have computed the collective potential energy surface in the region of superheavy nuclei within an extrapolated shell model. The deformations of the ground states, the locations and heights of the fission barriers, the stability against alpha- and beta-decay have been discussed. It is shown that around the double magic nuclei ${}_{114}X^{310}_{196}$ and ${}_{164}X^{482}_{318}$ some chance for stable superheavy elements exists. One important result found is that the heights of the oblate barriers are lower

than the prolate potential barriers. Therefore the possibility of oblate fission has been proposed and discussed. It is supposed that oblate fission is to be interpreted as the first step for multiple fission and thus multiple fission processes will become important for superheavy nuclei. We calculated by the WKB-approximation the half-life-times against spontaneous fission by underestimating the fission barriers by a parabola and ignoring the deformation dependence of the inertial parameters. Around the above mentioned double magic nuclei we obtained two great areas of stable elements. The extension of these islands is drastically diminished by the alpha- and beta-decay channels, which are estimated with the massformula of Myers and Swiatecki. Some nuclei remain around ${}_{114}X_{196}^{310}$ with a half-life-time up to some months. In spite of the uncertainties of the extrapolations around ${}_{164}X_{318}^{482}$ we obtain half life-times up to hours. Slight changes in the extrapolations of the shell model and of the massformula may change these results even more favourably.

It arises the question, which reactions are favourable to reach these islands of stability. There are naturally several possibilities and we show here only some typical examples (see also[17]). Reactions with heavy ions, reaching directly the quasistable islands around the double magic nuclei, as for example,

$${}_{96}\mathrm{Cm}_{152}^{248}+{}_{18}\mathrm{Ar}_{22}^{40}\rightarrow{}_{114}X_{174}^{288}$$

lead mostly to the neutron-deficient elements of the island. Moreover, the superheavy nuclei are formed in such direct reactions with an excitation energy of the order of 30 – 50 MeV. Hence there exists a very large probability that immediate fission will take place and thus the chances for forming the superheavy nucleus in its ground state are small. There exists, however, the possibility for the nucleus to get rid of its excitation energy by emission of nucleons or alpha-particles.

For example one obtains in the reaction

$${}_{98}\mathrm{Cf}_{154}^{252}+{}_{20}\mathrm{Ca}_{28}^{48}\rightarrow{}_{114}X_{176}^{290}+2\,{}_{2}\mathrm{He}_{2}^{4}+{}_{0}n^{1}$$

an element which lies at the border of the island of stability (see Fig. 10). The evaporating particles are helpful in decreasing the energy of the compound system and thus make the formation of the superheavy nucleus ${}_{114}X_{176}^{290}$ more likely.

For reaching the island around $Z=164$ one has to use fusion of very heavy ions like

$${}_{92}\mathrm{U}_{146}^{238}+{}_{92}\mathrm{U}_{146}^{238}\quad\text{or}\quad{}_{98}\mathrm{Cf}_{154}^{252}+{}_{92}\mathrm{U}_{146}^{238}.$$

17 Seaborg, G. T.: Amer. Chem. Soc., Mendeleev Centennial Symposium, Minneapolis, Minnesota, April 1969. — Seaborg, G. T.: Ann. Rev. Nucl. Sci. **18**, 53 (1968).

This reaction leads to superheavy elements around $Z=164$ only after the multiple emission of protons. Moreover, by inspection of Fig. 11b one realizes that even reactions with the heaviest ions lead only to the neutron deficient side of the second island of superheavy elements. Hence it is necessary to look for other indications of the elements of the second island: If these elements exist at all it seems possible that they either occur – in a dynamical equilibrium – in supernovae or in neutron stars. Hence, perhaps, there might be some chance of finding indications of such elements in the very heavy components of cosmic rays (either directly or via the distribution of their fission fragments) or, since they are possibly highly ionized, via their typical X-ray spectra[18]. Furthermore, the island around $Z=164$ may have quite important consequences for the proposed mechanism of the fusion of heavy nuclei with following fission into the $Z=114$ region. This decay-channel may be influenced quite drastically by the structure of the intermediate compound nuclei. The last two reactions discussed above may serve also as an entrance channel for such a fusion-fission process.

One further point should be said on the advantage of producing odd nuclei in the superheavy regions. This possibility has not been discussed in this paper. It is however expected that the life-time of these nuclei is enhanced by about 10^3 years compared with that of the neighbouring even-even nuclei. This comes from the fact that the fission barriers of odd nuclei are usually higher because of the conservation of the K-quantum number during the fission process. This conservation-law leads to the fact that the change of the quantum state of the odd nucleon at a level-crossing in a deformed shell model (in order to retain always the lowest possible energy) is now forbidden by the K-conservation. This possibility should be discussed more thoroughly in further papers on this subject*.

Note Added in Proof. There seems to be now for the first time an experimental indication that the γ-vibration in the neighbourhood of the saddle point is lowered in energy by about a factor of 3 compared to its energy at the groundstate-deformation (Britt, H. C., paper SM-122/102 at the Second Symposium on the Physics and Chemistry of Fission, Vienna 1969, and Britt, H. C., Los Alamos, private communication). This supports our discussion of the treatment of the zero-point-energy in the fission process on pg. 380 and 381.

* Very recently this odd-even effect has been discussed semiquantitatively by H. Meldner and G. Hermann (submitted to Z. Naturforschung).

18 Pieper, W., Greiner, W.: Z. Physik **218**, 327 (1969).

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