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Calculation of the Position of a Side-Protruding Electrode Tip in Stereotactic Brain Operations Using a Stereotactic Apparatus with Polar Coordinates

By

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With 3 Figures

Summary

The mathematical derivations of the formulae for the position of the side-protruding electrode are given for a stereotactic apparatus with polar coordinates (Riechert-Mundinger system).

When multiple punctures are performed in a stereotactic brain operation through a single trephine hole, problems of sterility and accuracy arise. To solve these, it has become necessary to calculate target parameters with the aid of a computer. For the past three years we have been successfully employing this calculation, as elaborated for the specially modified Riechert-Mundinger stereotactic apparatus, together with a computer programme that has been tested on over 500 stereotactic brain operations. Given the six coordinates of the target and trephine points, when we have calculated the target parameters (four angles and needle depth), as verified through practical testing, the precise calculation of the path of the side-protruding electrode should be carried out. Due to its passage into the subthalamus, hypothalamus, mesencephalon and brain stem this calculation is very necessary, for here functionally important nuclear and fibre structures lie much closer to one another than, for example, in surgery of the thalamus, capsula interna and pallidum. In such cases, therefore, only when we adapt and adjust the form and the size of the lesion to the structure in question, can we achieve an optimal therapeutic effect and avoid side-effects and complications. This means, however, that the

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position and the track of the electrode can be calculated by a computer programme and that we can also visualize them on individually adaptable brain sections.

For the precise calculation of the electrode track, let us recall the construction of the stereotactic target apparatus (Fig. 1):

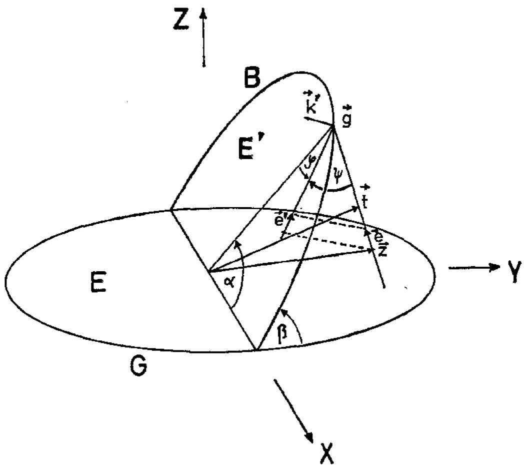

Fig. 1

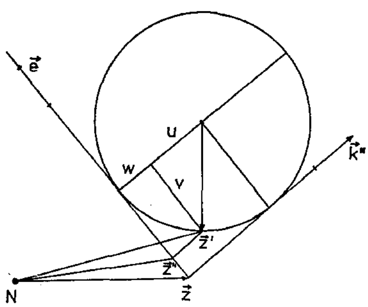

Fig. 2

Calculation

A. As we have shown elsewhere (1), the target parameters α, β, φ, ψ (and the needle depth T which does not interest us here) are calculated from the coordinates $\vec{z}$ (target point) and $\vec{t}$ (trephine point). During the operation the electrode shaft is usually moved backwards or forwards only a few millimeters and revolved about its own axis (angle δ). Thereupon the electrode protrusion is extended by the amount w (Fig. 2), describing a circle with radius $r = 6$ mm, a structurally determined constant. The point with position vector $\vec{z}$ is the target point, $\vec{e}$ is the unit vector in the direction of the electrode, while $\vec{z}$ is the desired point corresponding to the end of the electrode protrusion. According to Fig. 2

$$\vec{z} = \vec{z} + (r - v) \cdot \vec{e} + w \cdot \vec{k}'' \tag{1}$$

v can be calculated from the given amount for w:

$$v = \sqrt{r^2 - u^2} = \sqrt{r^2 - (r - w)^2} = \sqrt{2rw - w^2}$$

which holds when $w < r$. If $w > r$, then $V = \sqrt{2rw + w^2}$. Since the vector $\vec{e}$ has already been determined in the target point calculation $\left(\vec{e} = \frac{t - z}{|t - \mathrm{z}|}\right)$, only the vector $\vec{k}''$ is left to be found. The complicated movement of this vector makes its determination somewhat difficult. The vector $\vec{k}$, located perpendicular to the plane E' of the target arm, can be expressed with the unit vectors $\vec{i}, \vec{j}, \vec{k}$ in the directions x, y and z.

$$\vec{k}' = -\vec{j} \sin \beta + \vec{k} \cos \beta. \tag{2}$$

The vector $\vec{k}''$ is to be found from the rotation of the above vector by the angle ψ. The axis of rotation is the unit vector $\vec{t}'$, which results from the tangent vector

$$\vec{t}'' = \vec{i} \sin \gamma - \vec{j} \cos \alpha - \vec{k} \cos \gamma \sin \beta) \tag{3}$$

rotated by the angle φ at point $\vec{g}$ in the orbit of the target arm.

The angle of rotation γ here is:

$$\begin{aligned} \gamma &= \alpha + \varphi & (\alpha \leq 90 \text{ degrees}) \\ \gamma &= \alpha + \varphi + 180 & (\alpha > 90 \text{ degrees}). \end{aligned} \tag{4}$$

Accordingly, we get $\vec{k}'' = \vec{k}' \cos \psi + (\vec{t}'' \, x \, \vec{k}') \sin \psi$, therefore,

$$\vec{k}'' = \vec{i} \cos \gamma \sin \psi - \vec{j} (\sin \beta \cos \psi + \sin \gamma \cos \beta \sin \psi) + \vec{k} \cdot (\cos \beta \cos \psi - \\ - \sin \beta \sin \gamma \sin \psi) \tag{5}$$

The final position $\vec{k}'''$ for this vector is attained by rotating it by angle δ, letting the axis of rotation be the electrode axis having the unit vector $\vec{e}$:

$$\vec{k}''' = \vec{k}'' \cos \delta - \vec{c} \cdot \sin \delta. \tag{6}$$

Vector $\vec{c}$ is the vector product of $\vec{e} \times \vec{k}''$. With formula (1) the calculation of point $\vec{z}$ has now been completed. We have written the corresponding computer programme in Basic and Fortran. Consequently, the locus of the coagulation can now be ascertained quantitatively and with greatest accuracy.

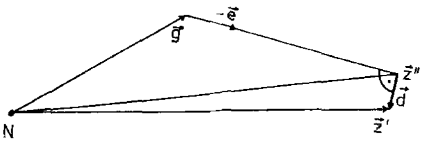

Fig. 3

B. Even more important from a practical, clinical viewpoint, than the determination of the position of the electrode protrusion tip from the angle of revolution and the amount of the protrusion is the reversal of the given problem: what angle δ, what protrusion w, and what electrode depth T, should be used in order to place the side-protruding electrode tip at a desired coagulation position $\vec{z}$ while retaining the electrode tract $\vec{e}$? Again, Fig. 2 aids us in this problem. Given are $\vec{g} = \vec{g}\ (\alpha, \beta)$, $\vec{e} = \vec{e}\ (\vec{t}, \vec{z})$ and the vector of the "electrode protrusion target point" $\vec{z}'$. We are looking for T, w and δ. We first determine the distance of the point $\vec{z}$ from the electrode axis $\vec{e}$: We know that

$$\vec{d} = \vec{z}' - \vec{g} + L\vec{e}; \tag{8}$$

with $\vec{d} \cdot \vec{e} = 0$, $L = (\vec{g} - \vec{z}') \cdot \vec{e} = L_0$, Fig. 3.

Thus we have the electrode depth [2] : $T = A - L_0$, where A is a constant entirely dependent on the measurements of the target apparatus ($A = 312.5$ mm).

L_0 put into (8) gives $\vec{d}_0$ and the desired electrode protrusion

$$W = |\vec{d}_0|$$

Vector $\vec{d}_0/w$ is equal to vector $\vec{k}'''$ in equation (6). We obtain the angle of rotation δ by scalar multiplication of the vector $\vec{k}''$, which depends only on δ, β, φ and ψ, with $\vec{k}''' = \vec{d}_0/w$:

$$\vec{k}''' \cdot \vec{k}'' = \cos \delta.$$

This gives us the angle δ within the limits (o, π). Since $\cos(-\delta) = \cos\delta$, the sign of δ must be determined separately. For this reversal problem as well we have written a computer programme in Basic and Fortran.

Both programmes, together with the standard programme for the target apparatus with polar coordinates [2] enable us to make quantitatively exact statements about the position of the cannula. Further, we may represent it on a display system (reversal problem) and give precise data on the actual coagulation structure when the side-protruding electrode is extended. Both are, for the reasons given at the outset of this exposition, most valuable in improving stereotactic neurosurgery.

References

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