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A Loop Theorem for Duality Spaces and Fibred Ribbon Knots

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1. Introduction

In this paper we prove a version of the loop theorem for surfaces in the boundary of a 3-dimensional duality space, i.e. a space which resembles a 3-manifold only in that it satisfies the appropriate form of Poincaré-Lefschetz duality over some field of untwisted coefficients. Our motivation comes from the fact that such spaces occur as the infinite cyclic coverings of certain 4-manifolds which arise in the study of knot concordance, and as the main application of our theorem we show that if a fibred knot in the 3-sphere is a ribbon knot, then its monodromy extends over a handlebody.

We approach the loop theorem via the study of planar coverings of a surface, as in the original paper of Papakyriakopoulos [11] and the subsequent work of Maskit [9]. §2 contains a simple geometric treatment of these matters. The main result of §3 is that a duality space actually satisfies duality with (twisted) coefficient module the quotient of the fundamental group ring by any power of the augmentation ideal. In §4, the results of §§2 and 3, together with an algebraic lemma on the intersection of the powers of the augmentation ideal of a group ring, are used to prove the loop theorem for 3-dimensional duality spaces. (For the reader's convenience a proof of the algebraic lemma is included as an appendix.) §5 contains the application to fibred ribbon knots mentioned above. In §6 the result of §5 is used to obtain a limited amount of information on some questions about knots in the boundaries of contractible 4-manifolds. In §7 we apply our methods to another aspect of knot concordance, and show that for any concordance with a rationally anisotropic fibred knot (see [7]) at one end, the inclusion of the complement of the knot into the complement of the concordance induces an injection of fundamental groups. For torus knots, this question was raised by Scharlemann [14].

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2. Planar Coverings

By a *surface* we shall mean a connected 2-manifold (not necessarily orientable, compact, or without boundary). A surface F is *planar* if it embeds in S^2; equivalently, the intersection form $H_1(F;\mathbf{Z}_2)\times H_1(F;\mathbf{Z}_2)\to\mathbf{Z}_2$ is zero. The free homotopy class of a loop u in F determines a conjugacy class in $\pi_1(F)$; we denote by $[u]$ any representative of this conjugacy class.

The following theorem establishes a close connection between planar coverings of a surface F and simple loops in F. (Compare Papakyriakopoulos [11, 12], and Maskit [9].) The proof we give is geometric. This approach is by now widely known; see for example Scharlemann [15].

Theorem 2.1. *Let $p\colon \tilde{F}\to F$ be a regular covering of a surface F, with $\tilde{F}$ planar, and let N be a normal subgroup of $\pi_1(F)$ such that $p_*\pi_1(\tilde{F})-N\neq\emptyset$. Then there exists a simple orientation-preserving loop u in F and an integer n such that $[u]^n\in p_*\pi_1(\tilde{F})-N$.*

Proof. We may assume without loss of generality that F is compact. (If not, choose a loop w in F such that $[w]\in p_*\pi_1(\tilde{F})-N$, and replace F by a compact subsurface F_0 containing w, $\tilde{F}$ by a component of $p^{-1}(F_0)$, and N by $i_*^{-1}(N)$, where $i\colon F_0\to F$ is inclusion.) Now choose a Riemannian metric on F with respect to which ∂F is totally geodesic. Then every non-trivial free homotopy class in F has at least one shortest (piecewise-smooth) representative loop, and any such is a closed geodesic. Also, for any l, there are only finitely many free homotopy classes with representatives of length $\leqq l$. This is because, in the universal covering of F, only finitely many fundamental regions are accessible from a given one by paths of length $\leqq l$.

Let v be a loop in F such that $|v|$ (the length of v) is minimal with respect to the property $[v]\in p_*\pi_1(\tilde{F})-N$.

Let $\tilde{v}$ in $\tilde{F}$ be any lift of v. We claim that $\tilde{v}$ is simple. For if not, then we could express $\tilde{v}$ as the composition $\tilde{v}_1*\tilde{v}_2$ of two non-trivial loops $\tilde{v}_1,\tilde{v}_2$ in $\tilde{F}$. Writing $v_i=p(\tilde{v}_i)$, $i=1,2$, this would give $v=v_1*v_2$, with $[v_i]\in p_*\pi_1(\tilde{F})$ and $|v_i|<|v|$, $i=1,2$. But $[v]=[v_1][v_2]\notin N$ implies $[v_1]$ or $[v_2]\notin N$, contradicting the definition of v.

Now let $\tilde{v}'$ be another lift of v. Since v is a geodesic, the intersections of $\tilde{v}$ with $\tilde{v}'$ are transverse. We claim that $\tilde{v}\cap\tilde{v}'=\emptyset$. For if not, then since $\tilde{F}$ is planar, $\tilde{v}\cap\tilde{v}'$ contains at least two points, say P and Q. Then $\tilde{v}=\alpha\cup\beta$, $\tilde{v}'=\gamma\cup\delta$, say, where α, β, γ, δ are arcs joining P and Q. Interchanging $\tilde{v}$, $\tilde{v}'$ if necessary, (and lifting the metric on F to $\tilde{F}$), we may suppose $|\alpha|\leqq|\beta|$, $|\gamma|$, $|\delta|$. Let $v_1=p(\alpha\cup\gamma)$, $v_2=p(\alpha\cup\delta)$. Then $[v_i]\in p_*\pi_1(\tilde{F})$, $i=1,2$, and (orienting v_1 and v_2 appropriately) $[v]=[v_1][v_2]\notin N$, giving $[v_1]$ or $[v_2]\notin N$. Also, each v_i satisfies $|v_i|\leqq|v|$, and has corners (at $p(P)$ and $p(Q)$), so by a small homotopy yields a loop with length $<|v|$. As before, this contradicts the definition of v.

Since the lifts of v in $\tilde{F}$ are disjoint simple loops, it follows that $[v]=[u]^n$ for some simple loop u in F and some integer n.

Finally, u may be chosen to be orientation-preserving (see [9, Lemma 1]). For if it is not, then n must be even, since $\tilde{F}$ is orientable, whilst $[u]^2$ can be represented by a simple (orientation-preserving) loop, since a regular neighbourhood of u in F is a Möbius band. □

Maskit's planarity theorem [9, Theorem 3] characterizes the regular planar coverings of a compact surface F as precisely those corresponding to the normal closure ($\langle\ldots\rangle$) in $\pi_1(F)$ of a set of powers of disjoint simple orientation-preserving loops in F. This follows easily from Theorem 2.1. In fact, for completeness, we observe that, suitably stated, this characterization is valid for arbitrary F.

Corollary 2.2. *Let $p: \tilde{F} \to F$ be a regular connected covering of a surface F. Then $\tilde{F}$ is planar if and only if there exist disjoint annuli $A_1, A_2, \ldots$ in $\operatorname{int} F$ and integers $n_1, n_2, \ldots$ such that $p_* \pi_1(\tilde{F}) = \langle [A_1]^{n_1}, [A_2]^{n_2}, \ldots \rangle$.*

Proof. The "if" part is straightforward; see [9, Theorem 1].

Conversely, suppose that $\tilde{F}$ is planar, and let $\mathfrak{A} = \{A_1, A_2, \ldots\}$ be a maximal collection of disjoint, essential, non-parallel annuli in $\operatorname{int} F$ such that $[A_i]^{n_i} \in p_* \pi_1(\tilde{F})$ for some non-zero integer n_i, $i = 1, 2, \ldots$. We assume that each n_i is chosen to be the least such positive integer. Note that $\mathfrak{A}$ is countable, since F is separable.

For notational convenience, given a subspace Y of a path-connected space X, we whall write $\langle Y \rangle$ for the smallest normal subgroup of $\pi_1(X)$ containing the conjugacy classes represented by loops in Y.

Let $\tilde{A} = p^{-1}(\bigcup_i A_i)$, a disjoint union of annuli in $\tilde{F}$. Observe that $\langle [A_1]^{n_1}, [A_2]^{n_2}, \ldots \rangle = p_* \langle \tilde{A} \rangle$. We claim that this subgroup is in fact the whole of $p_* \pi_1(\tilde{F})$. For suppose not. Then $\langle \tilde{A} \rangle \neq \pi_1(\tilde{F})$. Now since $\tilde{F}$ is planar, each component of $\tilde{A}$ separates $\tilde{F}$. Hence $\langle \tilde{A} \rangle \neq \pi_1(\tilde{F})$ implies that for some component $\tilde{F}_0$ of $\overline{\tilde{F} - \tilde{A}}$, we have $\pi_1(\tilde{F}_0) \neq \langle \partial \tilde{F}_0 \cap \partial \tilde{A} \rangle$. Note that $F_0 = p(\tilde{F}_0)$ is a component of $\overline{F - \bigcup_i A_i}$, $p_0 = p | \tilde{F}_0$: $\tilde{F}_0 \to F_0$ is a regular planar covering, and $N_0 = p_{0*} \langle \partial \tilde{F}_0 \cap \partial \tilde{A} \rangle$ is the normal closure in $\pi_1(F_0)$ of $\{[C_j]^{n_{i_j}}\}$, where $\{C_j\}$ is the set of components of $\partial F_0 \cap \bigcup_i \partial A_i$, with $C_j \subset \partial A_{i_j}$, say. By hypothesis, $p_{0*} \pi_1(\tilde{F}_0) - N_0 \neq \emptyset$, so by Theorem 2.1 there exists an annulus $A \subset \operatorname{int} F_0$ and an integer n such that $[A]^n \in p_{0*} \pi_1(\tilde{F}_0) - N_0$. By the maximality of $\mathfrak{A}$, A is parallel to A_k for some $A_k \in \mathfrak{A}$ which meets F_0. But then $[A]^n \notin N_0$ implies that the positive integer g.c.d. (n, n_k) is strictly less than n_k, contradicting the choice of n_k. □

3. Duality Triads

Let $\mathbf{F}$ be a field, and let $(X, \partial X)$ be a pair of topological spaces with the homotopy type of a pair of CW complexes, such that $H_q(X; \mathbf{F})$ and $H_q(X, \partial X; \mathbf{F})$ are finite-dimensional vector spaces over $\mathbf{F}$ for all q. These conditions imply that $H^q(X; \mathbf{F})$, $H^q(X, \partial X; \mathbf{F})$, $H_q(\partial X; \mathbf{F})$ and $H^q(\partial X; \mathbf{F})$ are all finite-dimensional, and also that each component of X has a universal covering. We call $(X, \partial X)$ an **F**-*duality pair of dimension n* if there is a *fundamental class* $\xi \in H_n(X, \partial X; \mathbf{F})$ such that the bilinear map

$$H^r(X; \mathbf{F}) \times H^{n-r}(X, \partial X; \mathbf{F}) \to \mathbf{F}$$

sending the pair (α, β) to the Kronecker product $\langle \alpha \cup \beta, \xi \rangle$ is non-singular for all r.

Similarly, a triad of spaces $(X, \partial_+ X, \partial_- X)$ is an **F**-*duality triad of dimension* n if, writing $\partial X = \partial_+ X \cup \partial_- X$ and $Y = \partial_+ X \cap \partial_- X$, $(X, \partial X)$, $(\partial_+ X, Y)$, and $(\partial_- X, Y)$ are **F**-duality pairs of dimensions n, $n-1$, and $n-1$ respectively, with fundamental classes $\xi, \partial_+ \xi, \partial_- \xi$, say, where $\xi \mapsto (\partial_+ \xi, -\partial_- \xi)$ under the map

$$H_n(X, \partial X; \mathbf{F}) \xrightarrow{\partial} H_{n-1}(\partial X, Y; \mathbf{F}) \cong H_{n-1}(\partial_+ X, Y; \mathbf{F}) \oplus H_{n-1}(\partial_- X, Y; \mathbf{F}).$$

It is also convenient to regard an **F**-duality pair $(X, \partial X)$ as an **F**-duality triad with $\partial_- X = \emptyset$. This allows us to work exclusively with triads.

If $(X, \partial_+ X, \partial_- X)$ is a compact manifold triad, (i.e. $(X, \partial X)$, $(\partial_+ X, Y)$, and $(\partial_- X, Y)$ are compact manifolds with boundary), with an **F**-orientation, then $(X, \partial_+ X, \partial_- X)$ is an **F**-duality triad. Our motivation, however, is provided by the following less obvious examples. Let W be the exterior of a concordance in $S^n \times I$ between two knots of S^{n-2} in $S^n \times 0$, $S^n \times 1$. Then ∂W can be naturally expressed as $\partial_+ W \cup \partial_- W$, where $\partial_+ W$, $\partial_- W$ are homeomorphic to the exteriors of the corresponding knots, and $\partial_+ W \cap \partial_- W \cong S^{n-2} \times S^1$. Let $(\tilde{W}, \partial_+ \tilde{W}, \partial_- \tilde{W})$ be the (unique) infinite cyclic covering of $(W, \partial_+ W, \partial_- W)$. Then by a theorem of Milnor [10], $(\tilde{W}, \partial_+ \tilde{W}, \partial_- \tilde{W})$ is an **F**-duality triad of dimension n, for all **F**.

It will be convenient to work with cap products rather than cup products. In this context we have the following lemma (compare Wall [20, p. 225]).

Lemma 3.1. *Let* $(X, \partial_+ X, \partial_- X)$ *be an* **F**-*duality triad with fundamental class* $\xi \in H_n(X, \partial X; \mathbf{F})$. *Then there is a skew-commutative diagram*

$$\begin{array}{ccccccc}
\cdots \to H^{n-r-1}(X, \partial_- X; \mathbf{F}) & \xrightarrow{j^*} & H^{n-r-1}(\partial_+ X, Y; \mathbf{F}) & \xrightarrow{\delta} & H^{n-r}(X, \partial X; \mathbf{F}) \to \cdots \\
\downarrow{\scriptstyle \cap \xi} & & \downarrow{\scriptstyle \cap \partial_+ \xi} & & \downarrow{\scriptstyle \cap \xi} \\
\cdots \to H_{r+1}(X, \partial_+ X; \mathbf{F}) & \xrightarrow{\partial} & H_r(\partial_+ X; \mathbf{F}) & \xrightarrow{i_*} & H_r(X; \mathbf{F}) \to \cdots
\end{array}$$

in which the rows are exact and the vertical maps are isomorphisms.

Proof. The bottom row is the exact sequence of the pair $(X, \partial_+ X)$, and the top row is derived from the exact sequence of the triple $(X, \partial X, \partial_- X)$ by means of the excision isomorphism $H^*(\partial X, \partial_- X; \mathbf{F}) \cong H^*(\partial_+ X, Y; \mathbf{F})$. Skew-commutativity is a consequence of the formula

$$\partial(\gamma \cap \zeta) = (\delta \gamma) \cap \zeta \pm \gamma \cap (\partial \zeta).$$

That the right-hand and middle vertical maps are isomorphisms follows from the hypothesis that $(X, \partial X)$ and $(\partial_+ X, Y)$ are **F**-duality pairs, together with the relation

$$\langle \alpha \cup \beta, \zeta \rangle = \langle \alpha, \beta \cap \zeta \rangle.$$

The fact that the left-hand vertical map is an isomorphism now follows from the 5-lemma. □

We wish to study the homology of certain coverings of **F**-duality triads. For this we use homology and cohomology with twisted coefficients; our notation (summarized below) is similar to that of Wall [20, p. 223].

Let X be a space equipped with a regular covering $\tilde{X}$ having translation group G. In general we do not insist that $\tilde{X}$ be the universal covering of X (or even that X or $\tilde{X}$ be connected) unless we specify that $G=\pi_1(X)$. The singular chain groups $C_q(\tilde{X};\mathbf{F})$ are naturally regarded as (left) modules over the group ring $\mathbf{F}G$. The involution "bar" on $\mathbf{F}G$, defined by

$$\overline{\Sigma\lambda_g g}=\Sigma\lambda_g g^{-1} \qquad (g\in G, \lambda_g\in\mathbf{F}),$$

enables us to regard any left $\mathbf{F}G$-module A as a right $\mathbf{F}G$-module $\bar{A}$ by defining ar to be $\bar{r}a$ $(a\in A, r\in\mathbf{F}G)$.

Let $C_*(X;A)$, $C^*(X;A)$ be the chain complexes $\bar{A}\otimes_{\mathbf{F}G}C_*(\tilde{X};\mathbf{F})$, $\mathrm{Hom}_{\mathbf{F}G}(C_*(\tilde{X};\mathbf{F}),A)$, and let $H_*(X;A)$, $H^*(X;A)$ denote the homology of these complexes. Observe that if G acts trivially on A then $H_*(X;A)$ and $H^*(X;A)$ coincide with the ordinary homology and cohomology groups of X with (untwisted) coefficients in A.

For any subspace Y of X let $\tilde{Y}$ be the covering of Y induced by $\tilde{X}$, and let $H_*(X,Y;A)$, $H^*(X,Y;A)$ be the groups obtained in the same way from the relative singular chain complex $C_*(X,Y;\mathbf{F})$.

The vector subspace of $\mathbf{F}G$ generated by $\{g-1: g\in G\}$ is a 2-sided ideal I (the *augmentation ideal*) such that $\mathbf{F}G/I\cong\mathbf{F}$. We shall frequently use $\mathbf{F}G/I^k$ as coefficient module; observe that I^k is the vector subspace spanned by all products $(g_1-1)(g_2-1)\dots(g_k-1)$ with $g_i\in G$.

Lemma 3.2. *If X is connected and $G=\pi_1(X)$ then $H_0(X;A)\cong A/IA$ for any left $\mathbf{F}G$-module A, and $H_1(X;\mathbf{F}G/I^k)\cong I^k/I^{k+1}$ for any k.*

Proof. Let $\tilde{X}$ be the universal covering of X. Tensoring the exact sequence

$$C_1(\tilde{X};\mathbf{F})\to C_0(\tilde{X};\mathbf{F})\to\mathbf{F}\to 0$$

with $\bar{A}$ gives $H_0(X;A)\cong\bar{A}\otimes_{\mathbf{F}G}\mathbf{F}\cong A/IA$.

It follows that the natural map

$$H_0(X;\mathbf{F}G)\to H_0(X;\mathbf{F}G/I^k)$$

is an isomorphism. From the definition,

$$H_1(X;\mathbf{F}G)\cong H_1(\tilde{X};\mathbf{F})=0.$$

The exact sequence

$$0\to I^k\to\mathbf{F}G\to\mathbf{F}G/I^k\to 0$$

gives rise to an exact homology sequence showing that $\partial: H_1(X;\mathbf{F}G/I^k)\to H_0(X;I^k)$ is an isomorphism. Since $H_0(X;I^k)\cong I^k/I^{k+1}$, the lemma is proved. □

Since the natural map $I^{k+1}/I^{k+2}\to I^k/I^{k+1}$ is zero, the following corollary is immediate.

Corollary 3.3. *If* $G=\pi_1(X)$ *then the natural map* $H_1(X;\mathbf{F}G/I^{k+1}) \to H_1(X;\mathbf{F}G/I^k)$ *is zero for all* k.

Lemma 3.4. *If* $G=\pi_1(X)$ *and* $H_1(X;\mathbf{F})$ *is finite-dimensional then* I^k/I^{k+1} *is finite-dimensional for all* k.

Proof. By Lemma 3.2, $I/I^2 \cong H_1(X;\mathbf{F})$ is finite-dimensional. The formula

$$(r_1, r_2, \ldots, r_k) \mapsto r_1 r_2 \ldots r_k$$

defines a k-linear map $(I/I^2)^k \to I^k/I^{k+1}$ whose image generates I^k/I^{k+1}. It follows that I^k/I^{k+1} is finite-dimensional. □

Remark. Suppose that $H_q(X;\mathbf{F})$ is finite-dimensional, for some q. Then $H_q(X;A)$ is finite-dimensional whenever A is a finite-dimensional vector space over $\mathbf{F}$ with trivial G-action. In particular, taking $G=\pi_1(X)$ and assuming that $H_1(X;\mathbf{F})$ is finite-dimensional, it follows from Lemma 3.4 that $H_q(X;I^k/I^{k+1})$ is finite-dimensional for all k. The exact sequence

$$0 \to I^k/I^{k+1} \to \mathbf{F}G/I^{k+1} \to \mathbf{F}G/I^k \to 0$$

gives rise to an exact homology sequence

$$H_q(X;I^k/I^{k+1}) \to H_q(X;\mathbf{F}G/I^{k+1}) \to H_q(X;\mathbf{F}G/I^k).$$

Hence, by induction on k, $H_q(X;\mathbf{F}G/I^k)$ is finite-dimensional for all k.

To study duality we need product operations in twisted homology and cohomology. Let $\tilde{X}$ be a regular covering of X with translation group G, and let A, B be left $\mathbf{F}G$-modules. Define a *Kronecker product* pairing

$$C^q(X;A) \times C_q(X;B) \to \bar{B} \otimes_{\mathbf{F}G} A,$$
$$\text{by} \quad \langle c, b \otimes \sigma \rangle = b \otimes c(\sigma),$$

where $c \in C^q(X;A)$, $b \in \bar{B}$ and σ is a singular q-simplex in $\tilde{X}$. It is easy to check that

$$\langle \delta c, b \otimes \sigma \rangle = \langle c, \partial(b \otimes \sigma) \rangle,$$

so a Kronecker product

$$H^q(X;A) \times H_q(X;B) \to \bar{B} \otimes_{\mathbf{F}G} A$$

is defined.

It is useful to note that if $A=\mathbf{F}G/J$, where J is an ideal such that $\bar{J}=J$, then $\bar{A} \otimes_{\mathbf{F}G} A \cong A$.

Define the *cap product* pairing

$$C^q(X;A) \times C_n(X;\mathbf{F}) \to C_{n-q}(X;A)$$

by $c \cap \sigma = c(\sigma_f) \otimes \sigma_b$, where $c \in C^q(X;A)$, σ is a singular n-simplex in $\tilde{X}$, and σ_f, σ_b are the 'front' q-face and 'back' $(n-q)$-face of σ. To see that $c \cap \sigma$ is well-defined, note that if $g \in G$ then

$$\begin{aligned} c \cap (g\sigma) &= c(g\,\sigma_f) \otimes g\sigma_b = g\,c(\sigma_f) \otimes g\sigma_b \\ &= c(\sigma_f)\,g^{-1} \otimes g\sigma_b = c(\sigma_f) \otimes \sigma_b. \end{aligned}$$

It may be verified that

$$\partial(c \cap \sigma) = (\delta c) \cap \sigma + (-1)^q c \cap (\partial \sigma),$$

so a cap product pairing

$$H^q(X; A) \times H_n(X; \mathbf{F}) \to H_{n-q}(X; A)$$

is defined.

For any subspaces $Y_2 \subset Y_1 \subset X$ we obtain a Kronecker product

$$H^q(X, Y_1; A) \times H_q(X, Y_2; B) \to \bar{B} \otimes_{\mathbf{F}G} A.$$

For any pair of subcomplexes $\{Y_1, Y_2\}$ of X, we obtain a cap product pairing

$$H^q(X, Y_1; A) \times H_n(X, Y_1 \cup Y_2; \mathbf{F}) \to H_{n-q}(X, Y_2; A).$$

Lemma 3.5. *Let $(X, \partial_+ X, \partial_- X)$ be an $\mathbf{F}$-duality triad with fundamental class $\xi \in H_n(X, \partial X; \mathbf{F})$, and let $G = \pi_1(X)$. Then for any k there is a skew-commutative diagram*

$$\begin{array}{ccccccc}
\cdots \to H^{n-r-1}(X, \partial_- X; \mathbf{F}G/I^k) & \xrightarrow{j^*} & H^{n-r-1}(\partial_+ X, Y; \mathbf{F}G/I^k) & \xrightarrow{\delta} & H^{n-r}(X, \partial X; \mathbf{F}G/I^k) \to \cdots \\
\downarrow \cap \xi & & \downarrow \cap \partial_+ \xi & & \downarrow \cap \xi \\
\cdots \to H_{r+1}(X, \partial_+ X; \mathbf{F}G/I^k) & \xrightarrow{\partial} & H_r(\partial_+ X; \mathbf{F}G/I^k) & \xrightarrow{i_*} & H_r(X; \mathbf{F}G/I^k) \to \cdots
\end{array}$$

in which the rows are exact and the vertical maps are isomorphisms.

Proof. Exactness and skew-commutativity follow by arguments analogous to those in the proof of Lemma 3.1.

Since $(X, \partial X)$ is an $\mathbf{F}$-duality pair,

$$\cap \xi : H^{n-r}(X, \partial X; \mathbf{F}) \to H_r(X; \mathbf{F})$$

is an isomorphism, by Lemma 3.1. Therefore

$$\cap \xi : H^{n-r}(X, \partial X; A) \to H_r(X; A)$$

is an isomorphism whenever A is a finite-dimensional vector space over $\mathbf{F}$ with trivial G-action. In particular (by Lemma 3.4)

$$\cap \xi : H^{n-r}(X, \partial X; I^k/I^{k+1}) \to H_r(X; I^k/I^{k+1})$$

is an isomorphism for all k.

Now assume inductively that for all r

$$\cap \xi : H^{n-r}(X, \partial X; \mathbf{F}G/I^k) \to H_r(X; \mathbf{F}G/I^k)$$

is an isomorphism (the induction starts at $k=1$). The exact sequence

$$0 \to I^k/I^{k+1} \to \mathbf{F}G/I^{k+1} \to \mathbf{F}G/I^k \to 0$$

gives rise to a commutative diagram with exact rows

$$\begin{array}{ccccccc}
\cdots\to H^{n-r}(X,\partial X;I^k/I^{k+1}) & \longrightarrow & H^{n-r}(X,\partial X;\mathbf{F}G/I^{k+1}) & \longrightarrow & H^{n-r}(X,\partial X;\mathbf{F}G/I^k)\to\cdots \\
\downarrow{\cap\xi} & & \downarrow{\cap\xi} & & \downarrow{\cap\xi} \\
\cdots\to H_r(X;I^k/I^{k+1}) & \longrightarrow & H_r(X;\mathbf{F}G/I^{k+1}) & \longrightarrow & H_r(X;\mathbf{F}G/I^k)\to\cdots
\end{array}$$

By the 5-lemma,

$$\cap\,\xi\colon H^{n-r}(X,\partial X;\mathbf{F}G/I^{k+1})\to H_r(X;\mathbf{F}G/I^{k+1})$$

is an isomorphism for all r, completing the induction.

Similarly,

$$\cap\,\xi\colon H^{n-r}(X,\partial_- X;\mathbf{F}G/I^k)\to H_r(X,\partial_+ X;\mathbf{F}G/I^k)$$

is an isomorphism for all r and k, and a final application of the 5-lemma completes the proof. □

4. A Loop Theorem for Duality Triads

For any group G define the kth *dimension subgroup of* G *over* $\mathbf{F}$ by $G_k=\{g\in G\colon g-1\in I^k\}$, and let $G_\omega=\bigcap_{k=1}^{\infty} G_k$. The following lemma is easily established; a proof is included in the appendix.

Lemma 4.1. *G_k and G_ω are characteristic subgroup of G. G/G_ω is torsion-free if* $\operatorname{char}\mathbf{F}-0$ *and has only p^r-torsion if* $\operatorname{char}\mathbf{F}=p\neq 0$.

Let $I^\omega=\bigcap_{k=1}^{\infty} I^k$. We shall need the following lemma.

Lemma 4.2. *I^ω is the kernel of the natural map* $\mathbf{F}G\to\mathbf{F}[G/G_\omega]$.

This is equivalent to Remark 2.27 on page 91 of Passi's book [13]. However, we give a direct self-contained proof in the appendix.

Lemma 4.3. *If $(X,\partial_+X,\partial_-X)$ is a 3-dimensional* $\mathbf{F}$*-duality triad and $G=\pi_1(X)$, then the Kronecker product pairing*

$$H^1(\partial_+X,Y;\mathbf{F}[G/G_\omega])\times H_1(\partial_+X;\mathbf{F}[G/G_\omega])\to\mathbf{F}[G/G_\omega]$$

is zero.

Proof. Let $\alpha\in H^1(\partial_+X,Y;\mathbf{F}[G/G_\omega])$ and $\beta\in H_1(\partial_+X;\mathbf{F}[G/G_\omega])$ have images $\alpha_k\in H^1(\partial_+X,Y;\mathbf{F}G/I^k)$ and $\beta_k\in H_1(\partial_+X;\mathbf{F}G/I^k)$. The Kronecker product $\langle\alpha,\beta\rangle\in\mathbf{F}[G/G_\omega]$ has image $\langle\alpha_k,\beta_k\rangle$ in $\mathbf{F}G/I^k$. By Lemma 4.2 it will suffice to prove that $\langle\alpha_k,\beta_k\rangle=0$ for all k.

Let $p\colon\mathbf{F}G/I^{k+1}\to\mathbf{F}G/I^k$ be projection, and $i\colon\partial_+X\to X$ inclusion. By Corollary 3.3,

$$p_*\colon H_1(X;\mathbf{F}G/I^{k+1})\to H_1(X;\mathbf{F}G/I^k)$$

is zero, so $i_*\beta_k = i_* p_* \beta_{k+1} = p_* i_* \beta_{k+1} = 0$. Therefore $\beta_k = \partial b$ for some $b \in H_2(X, \partial_+ X; \mathbf{F}G/I^k)$. By Lemma 3.5,

$$(\delta\alpha_k) \cap \xi = i_*(\alpha_k \cap \partial_+ \xi) = p_* i_*(\alpha_{k+1} \cap \partial_+ \xi) = 0.$$

Also by Lemma 3.5, $\cap\, \xi$ is an isomorphism, so $\delta\alpha_k = 0$. It follows that

$$\langle \alpha_k, \beta_k \rangle = \langle \alpha_k, \partial b \rangle = \langle \delta\alpha_k, b \rangle = 0. \quad \square$$

Corollary 4.4. *If $(\partial_+ X, Y)$ is a surface $(F, \partial F)$, and $\tilde{X}$ is the covering of X with $\pi_1(\tilde{X}) = G_\omega$, then each component of the induced covering $\tilde{F}$ of F is planar.*

Proof. Let $\eta \in H_2(F, \partial F; \mathbf{F})$ be a fundamental class for F. Since F is a genuine manifold,

$$\cap\, \eta \colon H^1(F, \partial F; \mathbf{F}[G/G_\omega]) \to H_1(F; \mathbf{F}[G/G_\omega]) \cong H_1(\tilde{F}; \mathbf{F})$$

is an isomorphism. Let c, c' be closed curves in $\tilde{F}$ and let $\gamma \in H^1(F, \partial F; \mathbf{F}[G/G_\omega])$ be such that $[c] = \gamma \cap \eta$. Then (up to multiplication by a non-zero element of $\mathbf{F}$ depending on η)

$$\langle \gamma, [c'] \rangle = \sum_{g \in G/G_\omega} (gc, c')\, g,$$

where (gc, c') is the intersection number of gc and c'. By Lemma 4.3, $\langle \gamma, [c'] \rangle = 0$ for all c, c'. Hence $(c, c') = 0$ for all c, c', proving that each component of $\tilde{F}$ is planar. $\square$

The following theorem is analogous to Stallings' version [16] of the loop theorem for 3-manifolds.

Theorem 4.5. *Let $(X, \partial_+ X, \partial_- X)$ be a 3-dimensional $\mathbf{F}$-duality triad such that $(\partial_+ X, Y)$ is a surface $(F, \partial F)$. Let $K = \ker(\pi_1(F) \to \pi_1(X)/\pi_1(X)_\omega)$, and let N be a normal subgroup of $\pi_1(F)$ such that $K - N \neq \emptyset$. Then there exists a simple orientation-preserving loop u in F and an integer n such that $[u]^n \in K - N$. Furthermore, n may be taken to be 1 if* $\operatorname{char} \mathbf{F} = 0$, *and of the form p^r if* $\operatorname{char} \mathbf{F} = p \neq 0$.

Proof. Let $p \colon \tilde{F} \to F$ be the regular covering of F with $p_* \pi_1(\tilde{F}) = K$. By Corollary 4.4, $\tilde{F}$ is planar. The result now follows from Theorem 2.1 and Lemma 4.1. $\square$

Remark. We do not know whether or not it is always possible to take $n = 1$.

In the same way, Corollary 2.2 yields the following.

Theorem 4.6. *Given the data of Theorem 4.5, there exist disjoint simple orientation-preserving loops $u_1, \ldots, u_m$ in F and integers $n_1, \ldots, n_m$ such that $K = \langle [u_1]^{n_1}, \ldots, [u_m]^{n_m} \rangle$. Furthermore, n_i may be taken to be 1 if* $\operatorname{char} \mathbf{F} = 0$, *and of the form p^{r_i} if* $\operatorname{char} \mathbf{F} = p \neq 0$, $i = 1, \ldots, m$.

5. Fibred Ribbon Knots

Recall that a knot K in S^3 is *slice* if it bounds a smooth disc D in B^4, and *ribbon* if it bounds an immersed disc in S^3 with only "ribbon" singularities; see

[4, Problem 25]. It is well-known that any ribbon knot is slice, and in fact bounds a smooth disc D in B^4 such that the map $\pi_1(S^3-K)\to\pi_1(B^3-D)$ induced by inclusion is onto. It is unknown whether or not all slice knots are ribbon.

More generally, let K be a knot in a homology 3-sphere M. If $(M,K)=\partial(V,D)$ for some smooth disc in a homology 4-ball V, we say that K is *slice* in V. If such a pair (V,D) exists with $\pi_1(M-K)\to\pi_1(V-D)$ onto, we say that K is *homotopically ribbon* (in V). Note that this implies that $\pi_1(M)\to\pi_1(V)$ is onto. In particular, if M is S^3 then V is a homotopy 4-ball (and hence at least homeomorphic to B^4, by [5]).

Recall that if K is a *fibred* knot, then there is a compact orientable surface F with $\partial F\cong S^1$, and a diffeomorphism $f\colon F\to F$ with $f|\partial F$ equal to the identity (the *monodromy* of K), such that $F\times I/f(=F\times I/((x,1)\sim(fx,0)$ for all $x\in F))$ is diffeomorphic to $\overline{M-N(K)}$, in such a way that under the natural identification of $\partial F\times I/(f|\partial F)$ with $\partial F\times S^1$, a loop $x_0\times S^1$, $x_0\in\partial F$, corresponds to a meridian of K, i.e. the boundary of a meridian disc of $N(K)$. Let $\hat{F}$ be the closed surface $F\cup_\partial B^2$, and define $\hat{f}\colon\hat{F}\to\hat{F}$ by $\hat{f}=f\cup\mathrm{id}$. We shall call $\hat{f}$ the *closed monodromy* of K; it is uniquely determined by K, up to conjugacy and isotopy.

Finally, by a *handlebody* we shall mean an orientable 3-manifold which has a handle decomposition consisting of one 0-handle and some finite number of 1-handles.

Theorem 5.1. *A fibred knot in a homology 3-sphere is homotopically ribbon if and only if its closed monodromy extends over a handlebody.*

In particular, if a fibred knot in S^3 is a ribbon knot, then its closed monodromy extends over a handlebody; this can be used to obtain much explicit information about which fibred knots are ribbon. We intend to discuss this elsewhere. We also remark that a (practical) algorithm has recently been found to decide whether or not a given diffeomorphism of a closed orientable surface extends over a handlebody (or indeed any 3-manifold) [3].

Generalizing the terminology of [2] to allow surfaces with boundary, define a *compression body* T to be a 3-manifold with a handle decomposition of the form $F\times I\cup$2-handles$\cup$3-handles, where F is a compact orientable surface, and where there is a 3-handle for each 2-sphere component of $\partial(F\times I\cup$2-handles$)-F\times 0$. Note that $\partial T=\partial_e T\cup\partial F\times I\cup\partial_i T$, say, where $\partial_e T=F=F\times 0$, and $\partial F\times I\cap\partial_i T=\partial F\times 1=\partial(\partial_i T)$. Also, since $T\cong\partial_i T\times I\cup$0-handles$\cup$1-handles, T is irreducible and $\partial_i T$ is incompressible in T.

Any set of disjoint simple loops in the interior of a compact orientable surface F determines a unique compression body, namely that whose 2-handles are attached along regular neighbourhoods of the given loops in $F\times 1$.

The following lemma will be needed in § 7, whilst the special case contained in Corollary 5.3 will be used in the proof of Theorem 5.1.

Lemma 5.2. *Let $f\colon F\to F$ be a diffeomorphism of a compact orientable surface. Let $u_1,\ldots,u_m$ be a set of disjoint simple loops in* int F, *and let T be the corresponding compression body. Then f extends to a diffeomorphism of T if and only if $f_*(N)=N$, where $N=\langle[u_1],\ldots,[u_m]\rangle\subset\pi_1(F)$.*

Proof. Since $N=\ker(\pi_1(F)\to\pi_1(T))$, the necessity of the condition $f_*(N)=N$ is immediate.

Conversely, suppose $f_*(N)=N$. Then $f_*([u_i])\in N$, and hence, by Dehn's lemma, $f(u_i)=\partial D_i$ for some properly embedded disc $D_i\subset T$, $i=1,\dots,m$. By a standard innermost circle argument, we may suppose that $D_1,\dots,D_m$ are pairwise disjoint. Consider a regular neighbourhood R of $F\cup\bigcup_{i=1}^{m} D_i$ in T. Since T is irreducible, any 2-sphere component of ∂R bounds a 3-ball in T, enabling us to enlarge R to the image of an embedding $g\colon T\to T$ such that $g|F=f$. Since $\partial_i T$ is incompressible in T, and $g_*\colon\pi_1(T)\to\pi_1(T)$ is an isomorphism, $g(\partial_i T)$ is incompressible in T. Therefore each component of $g(\partial_i T)$ is parallel to a component of $\partial_i T$. (The argument, which is almost identical to that given in [2, Lemma 2.3] for the case in which F is closed, is as follows. Regard T as $\partial_i T\times I\cup$0-handles$\cup$1-handles. Then, using a standard innermost circle argument, and the irreducibility of T, we may isotope $g(\partial_i T)$ (rel ∂) off the co-cores of the 1-handles, so that we may assume $g(\partial_i T)\subset\partial_i T\times I$. Since $\partial g(\partial_i T)=g(\partial F\times 1)$ is parallel in $\partial F\times I$ to $\partial F\times 1=\partial(\partial_i T)$, the result is now a consequence of [18, Proposition 3.1].) It follows easily that $g(\partial_i T)$ is parallel to $\partial_i T$, and hence that there exists a diffeomorphism $g'\colon T\to T$ such that $g'|F=f$, as required. □

Corollary 5.3. *A diffeomorphism $f\colon F\to F$ of a closed orientable surface of genus n extends over a handlebody if and only if there exists a set of disjoint simple loops $u_1,\dots,u_m$ in F containing n homologically independent members, such that $f_*(N)=N$, where $N=\langle[u_1],\dots,[u_m]\rangle\subset\pi_1(F)$.*

Proof. The compression body corresponding to such a set of loops is a handlebody with boundary F. □

Proof of Theorem 5.1. First, let K be a fibred knot in a homology 3-sphere M, with monodromy $f\colon F\to F$, and suppose that there exists a handlebody T with $\partial T=\hat F$ and a diffeomorphism $g\colon T\to T$ such that $g|\partial T=\hat f$. Let $W=T\times I/g$, the bundle ever S^1 with fibre T and monodromy g. Note that $\partial W\cong\hat F\times I/\hat f$, which is diffeomorphic to the manifold obtained by doing 0-framed surgery on M along K. Let $V=M\times I\cup H^2\cup_{\partial W}W$, where H^2 is a 2-handle attached along $K\times 1\subset M\times 1$, with the 0-framing. One calculates easily that V is a homology 4-ball. Also, $K\subset M=M\times 0$ bounds the disc $D=K\times I\cup$(core of H^2) in V. Since $\pi_1(F)\to\pi_1(T)$ is onto, the same is true of

$$\pi_1(M-K)\cong\pi_1(F\times I/f)\to\pi_1(T\times I/g)\cong\pi_1(V-D).$$

Thus K is homotopically ribbon.

Conversely, suppose that $(M,K)=\partial(V,D)$ for some smooth disc D is a homology 4-ball V, with $\pi_1(M-K)\to\pi_1(V-D)$ onto, where K is fibred with monodromy $f\colon F\to F$. Let $W=\overline{V-N(D)}$; then $\partial W\cong\hat F\times I/\hat f$. Let $\tilde W$ be the infinite cyclic covering of W; $(\tilde W,\partial\tilde W)$ is a 3-dimensional **Q**-duality pair by [10]. Note that $(\tilde W,\partial\tilde W)$ is homotopy equivalent to $(\tilde W,\hat F)$, where $\hat F=\hat F\times 0\subset\hat F\times R^1\cong\partial\tilde W$. Therefore, by Theorem 4.6, there exist disjoint simple loops $u_1,\dots,u_m$ in $\hat F$ such that $N=\ker(\pi_1(\hat F)\to\pi_1(\tilde W)/\pi_1(\tilde W)_\omega)=\langle[u_1],\dots,[u_m]\rangle$. Now $\pi_1(\partial W)\to\pi_1(W)$ is onto, since $\pi_1(M-K)\to\pi_1(V-D)$ is, by hypothesis. Since

$\pi_1(\hat{F})$, $\pi_1(\tilde{W})$ are identified, via the covering projection $\tilde{W} \to W$, with the commutator subgroups of $\pi_1(\partial W)$, $\pi_1(W)$ respectively, it follows that $\pi_1(\hat{F}) \to \pi_1(\tilde{W})$ is onto. Therefore $\pi_1(\tilde{W})/\pi_1(\tilde{W})_\omega \cong \pi_1(\hat{F})/N$. Hence $H_1(\tilde{W}; \mathbf{Q}) \cong H_1(\hat{F}; \mathbf{Q})/U$, where U is the subspace spanned by the classes of $u_1, \ldots, u_m$; consequently U is the kernel of $H_1(\hat{F}; \mathbf{Q}) \to H_1(\tilde{W}; \mathbf{Q})$. But since $(\tilde{W}, \hat{F})$ is a $\mathbf{Q}$-duality pair, this kernel has dimension $n = \text{genus } \hat{F}$, and hence $\{u_1, \ldots, u_m\}$ contains n homologically independent loops.

Let t: $\tilde{W} \to \tilde{W}$ generate the group of covering translations. Then $t | \partial \tilde{W} \cong \hat{F} \times R^1$ is given by $(x, s) \mapsto (\hat{f}x, s+1)$. Since $\pi_1(\tilde{W})_\omega$ is a characteristic subgroup of $\pi_1(\tilde{W})$, $t_*(\pi_1(\tilde{W})_\omega) = \pi_1(\tilde{W})_\omega$, and hence $\hat{f}_*(N) = N$. (Regarding basepoints, recall that $f | \partial F$ is the identity, so we may take some point $x_0 \in \partial F$ as base-point for $\hat{F}$, and noting that $t(x_0 \times R^1) = x_0 \times R^1$, take $x_0 \times R^1 \subset \hat{F} \times R^1 \cong \partial \tilde{W}$ as base-"point" for $\tilde{W}$.) The fact that $\hat{f}$ extends over a handlebody now follows from Corollary 5.3. □

Remark. If K is a fibred knot in a homology 3-sphere M, and $(M, K) = \partial(V, D)$ with $\pi_1(M-K) \to \pi_1(V-D)$ onto, then the commutator subgroup of $\pi_1(V-D)$ is finitely generated. By analogy with Stallings' fibration theorem for 3-manifolds, one might ask whether $\overline{V - N(D)}$ necessarily fibres over S^1. The proof of Theorem 5.1 yields a partial answer.

Corollary 5.4. *If a fibred knot K in a homology 3-sphere M is homotopically ribbon in V, then $(M, K) = \partial(V', D)$ where $\overline{V' - N(D)}$ fibres over S^1 with a handlebody as fibre.*

(V' is not known to be homeomorphic to V.)

6. Knots in Contractible 4-manifold Boundaries

The main motivation of this section is the following question, raised (in a special case) by Zeeman [21].

Question 6.1. *Does every knot in the boundary of a compact contractible 4-manifold V bound a PL disc in V?*

Note that the disc is not assumed to be locally flat.

If V is B^4, then such a disc may be obtained simply by coning. In [21, Conjecture (5)], however, Zeeman conjectures that the answer is "no" in general.

Being unable to settle this question, we consider it in conjunction with the following analogue of the slice implies ribbon question for knots in S^3.

Question 6.2. *If a knot in the boundary of a compact contractible 4-manifold V is slice in V, is it homotopically ribbon in V?*

The curious piece of information which we offer with regard to Questions 6.1 and 6.2 is that they cannot both have affirmative answers. In fact, we have

Theorem 6.3. *There exists a knot K in the boundary of a compact contractible 4-manifold such that*

(1) *K is homotopically ribbon in a homology 4-ball,*

(2) *K is not homotopically ribbon in any contractible 4-manifold,*

(3) *K bounds a PL disc in a contractible 4-manifold if and only if K is slice in a contractible 4-manifold.*

It follows immediately from (2) and (3) that such a K is an example of a knot for which the answer to at least one of Questions 6.1 and 6.2 is negative.

Before describing how such examples may be constructed, we observe that (3) follows from (1) and the following lemma.

Lemma 6.4. *Let W be a homology 4-ball such that $\pi_1(\partial W)\to\pi_1(W)$ is onto, and let K be a k not in ∂W which is slice in W. Then K bounds a PL disc in a contractible 4-manifold if and only if K is slice in a contractible 4-manifold.*

Proof. Let V be a contractible 4-manifold and E a PL disc in V such that $\partial(V,E)\cong(\partial W,K)$. We may assume that E is locally flat except at a single point $x\in\operatorname{int}E$, and that x has a neighbourhood B in V such that $(B,B\cap E)\cong(B^4$, cone on $J)$, for some knot J in S^3. By hypothesis, there is a smooth disc D in W with $\partial D=K$. Let $N=V\cup_\partial W$. Since V, W are homology 4-balls, N is a homology 4-sphere. Moreover, since $\pi_1(V)=1$ and $\pi_1(\partial W)\to\pi_1(W)$ is onto, $\pi_1(N)=1$ by van Kampen's theorem. Now $S=E\cup_K D$ is a 2-sphere in N, locally flat except at x, and if $(N_0,S_0)=\overline{(N,S)-(B,B\cap E)}$, then N_0 is a homotopy 4-ball, and S_0 is a locally flat disc in N_0 with $\partial S_0=J$. Finally, let

$$(V',E')=\overline{(V,E)-(B,B\cap E)}\cup_\partial(N_0,S_0).$$

Then V' is contractible, E' is a locally flat disc in V', and $\partial(V',E')=\partial(V,E)$. □

Remarks. (1) Clearly the assumption that $i_*\colon\pi_1(\partial W)\to\pi_1(W)$ be onto could be weakened to the condition that $\pi_1(W)$ be the normal closure of $i_*\pi_1(\partial W)$. (2) Since N_0 is homeomorphic to B^4 by [5], V' is homeomorphic to V.

Our construction of a knot K as described in Theorem 6.3 will use the following lemma.

Lemma 6.5. *Let $f\colon F\to F$ be a diffeomorphism of a compact orientable surface, and let $\chi(t)$ be the characteristic polynomial of $f_*\colon H_1(F;\mathbf{Q})\to H_1(F;\mathbf{Q})$. Suppose*

(a) *$\chi(t)$ is irreducible over $\mathbf{Q}[t]$,*

(b) *$\chi(t)$ does not divide t^n-1, for any $n\geqq 1$,*

(c) *$\chi(t)$ is not a polynomial in t^n, for any $n>1$.*

Then

(1) *f^n is not isotopic to the identity, for any $n\geqq 1$,*

(2) *F has at most one boundary component, and any essential 1-submanifold of F which is invariant under a diffeomorphism isotopic to f consists of parallel copies of ∂F.*

Proof. (1) Since $\chi(t)$ is irreducible, it is the minimal polynomial of f_*. If f^n were isotopic to the identity, we would have $f_*^n=1$, implying that $\chi(t)$ divides t^n-1.

(2) Let C be a non-empty essential 1-submanifold of F such that (after an isotopy of f) $f(C)=C$. Suppose there exists a component C_1 of C such that $[C_1]\neq 0$ in $H_1(F;\mathbf{Q})$. Then, for some $n\geqq 1$, $f_*^n[C_1]=[C_1]$, and hence f_* has an n-th root of unity as an eigenvalue. This implies (again since $\chi(t)$ is irreducible) that $\chi(t)$ divides t^n-1. In particular, since ∂F is invariant under f, we conclude that F has at most one boundary component.

So suppose that each component of C is null-homologous in F, and therefore bounds a subsurface of F. Among the components of F cut along C, choose one, F_0, say, with a single boundary component. Let $F_r=f^r(F_0)$, and let n be the least positive integer such that $F_n=F_0$. Let $G=F-\bigcup_{r=0}^{n-1} F_r$. We then have a decomposition $H_1(F;\mathbf{Q})\cong H_1(F_0;\mathbf{Q})\oplus\ldots\oplus H_1(F_{n-1};\mathbf{Q})\oplus V$, where $V=\operatorname{im}(H_1(G;\mathbf{Q})\to H_1(F;\mathbf{Q}))$, such that the first n summands are cyclically permuted by f_* and V is f_*-invariant. It follows that $\chi(t)=\varphi(t^n)\psi(t)$, where $\varphi(t)$, $\psi(t)$ are the characteristic polynomials of $(f^n|F_0)_*$ and $f_*|V$ respectively. Since $H_1(F_0;\mathbf{Q})\neq 0$, and $\chi(t)$ is irreducible, we must have $\chi(t)=\varphi(t^n)$, which implies (by (c)) that $n=1$. We must also have $V=0$, and therefore (since C is essential), $\partial F\neq\emptyset$, G is a collar of ∂F, and C consists of parallel copies of ∂F, as claimed. □

Let K_0 be a knot in a homology 3-sphere M_0 which bounds a contractible 4-manifold V_0, and let $(M,K)=(M_0,K_0)\#-(M_0,K_0)$. Then M bounds the contractible 4-manifold $V_0\#_\partial -V_0$. Also, K is homotopically ribbon, in a homology 4-ball. To see this, simply choose a trivial ball-pair $(B,B\cap K_0)\subset(M_0,K_0)$, and consider $(W,D)=\overline{(M_0,K_0)-(B,B\cap K_0)}\times I$. (An alternative description of (W,D) is that it is obtained by spinning (M_0,K_0) through an angle π.)

Now suppose that K_0 is fibred, with monodromy f_0: $F_0\to F_0$, say, and suppose also that the characteristic polynomial $\chi(t)$ of f_{0*}: $H_1(F_0;\mathbf{Q})\to H_1(F_0;\mathbf{Q})$ (in other words, the Alexander polynomial of K_0) satisfies the hypotheses of Lemma 6.5. To establish assertion (2) of Theorem 6.3, we shall show that, under these conditions, if K is homotopically ribbon in a homology 4-ball V, then there is surjection $\pi_1(V)\to\pi_1(M_0)$. It follows that V cannot be contractible if $\pi_1(M_0)\neq 1$.

Let $\hat{f}$: $\hat{F}\to\hat{F}$ be the closed monodromy of K. Note that we may regard $\hat{F}$ as $\partial(F_0\times I)$, and take $\hat{f}$ to be $(f_0\times\mathrm{id})|\partial(F_0\times I)$. Suppose that K is homotopically ribbon, in some homology 4-ball. Then, by Theorem 5.1, there exists a handlebody T with $\partial T=\hat{F}$ and a diffeomorphism g: $T\to T$ such that $g|\hat{F}=\hat{f}$.

Lemma 6.6. *With the above data, there exists a homeomorphism h: $T\to F_0\times I$ such that $h|F_0\times 0\cup\partial F_0\times I$ is the identity.*

Proof. The set $\beta=\{F_0\times 0, F_0\times 1, \partial F_0\times I\}$ of submanifolds of $\partial T\cong\partial(F_0\times I)$ is a (complete) boundary pattern for T in the sense of Johannson [8] (see also [19]). Moreover, for $i=0$ or 1, the kernel of the map $H_1(F_0\times i;\mathbf{Q})\to H_1(T;\mathbf{Q})$

induced by inclusion is an f_{0*}-invariant subspace of dimension at most the genus of F_0. Since the characteristic polynomial of f_{0*} is irreducible, it follows that this kernel is zero, so that $H_1(F_0 \times i; \mathbf{Q}) \to H_1(T; \mathbf{Q})$ is an isomorphism, $i = 0, 1$. Therefore, $\pi_1(F_0 \times i) \to \pi_1(T)$ is injective, $i = 0, 1$, by [17, Theorem 7.3]. Thus the boundary pattern β is useful [8, §2], [19, p. 27], and hence there exists a characteristic submanifold X for (T, β) [8, Chap. III], [19, p. 28]. Note that, here, each component of X must be an I-bundle.

Since g is an automorphism of (T, β), it can be isotoped, admissibly with respect to β, so that $g(X) = X$ and $(g | \overline{T - X})^n$ is isotopic to the identity for some $n \geqq 1$ [8, Corollary 10.9 and Proposition 27.1]. After this isotopy, f_0 leaves the essential 1-submanifold $\partial(X \cap F_0 \times 0)$ of $F_0 \times 0$ invariant, and so, by Lemma 6.5, each component of $\partial(X \cap F_0 \times 0)$ must be parallel to $\partial F_0 \times 0$. This gives two possibilities for X: (a collar of $\partial F_0 \times 0$ in $F_0 \times 0) \times I$, or T. In the former case, f_0^n would be isotopic to the identity, contradicting Lemma 6.5. Therefore $X = T$, so that T is an I-bundle over $F_0 \times 0$. This is equivalent to the statement of the lemma. □

Now suppose that K is homotopically ribbon in a homology 4-ball V, so that we have $(M, K) = \partial(V, D)$, with $\pi_1(M - K) \to \pi_1(V - D)$ onto. Let $W = \overline{V - N(D)}$, and let $\tilde{W}$ be the infinite cyclic covering of W, with $t \colon \tilde{W} \to \tilde{W}$ a generator of the group of covering translations. Let $g \colon T \to T$ be the diffeomorphism of the handlebody T with $\partial T = \hat{F}$ and $g | \partial T = \hat{f}$ guaranteed by Theorem 5.1. The proof of that theorem shows that there is a surjection $\tilde{\varphi}$: $\pi_1(\tilde{W}) \to \pi_1(\tilde{W}) / \pi_1(\tilde{W})_\omega \cong \pi_1(T)$ such that $\tilde{\varphi} t_* = g_* \tilde{\varphi}$. (Take, as base-point for $\hat{F}$, the point $(x_0, \frac{1}{2}) \in \partial F_0 \times I$, say, for some $x_0 \in \partial F_0$, and take $(x_0, \frac{1}{2}) \times R^1 \subset \hat{F} \times R^1 \cong \partial \tilde{W}$ as base-"point" for $\tilde{W}$.) Therefore $\tilde{\varphi}$ induces a surjection $\varphi \colon \pi_1(W) \to \pi_1(T \times I / g)$.

By Lemma 6.6, $T \times I / g \cong (F_0 \times I) \times I / h g h^{-1}$, which in turn is homotopy equivalent to $F_0 \times I / f_0$ (recall that $h | F_0 \times 0 = \mathrm{id}$, and $g | F_0 \times 0 = f_0$). Hence we obtain a surjection $\varphi' \colon \pi_1(W) \to \pi_1(F_0 \times I / f_0)$. Also, observe that if $\mu = [(x_0, \frac{1}{2}) \times S^1] \in \pi_1(W)$, a meridian of D, then $\varphi'(\mu) = [x_0 \times S^1] \in \pi_1(F_0 \times I / f_0)$, a meridian of K_0. Therefore φ' induces a surjection $\pi_1(W) / \langle \mu \rangle \cong \pi_1(V) \to \pi_1(F_0 \times I / f_0) / \langle \varphi'(\mu) \rangle \cong \pi_1(M_0)$.

To complete the proof of Theorem 6.3, we give the following specific example of a pair (M_0, K_0) satisfying the necessary hypotheses. Consider Dehn surgery of type $\{1, \frac{1}{2}\}$ on the positive Whitehead link in S^3. Using the fact that each component is unknotted, one sees easily that the resulting 3-manifold M_0 can be obtained either by Dehn surgery of type 1 on the stevedore's knot, or by Dehn surgery of type $\frac{1}{2}$ on the figure eight knot. Since the stevedore's knot is slice, the first description shows that M_0 bounds a contractible 4-manifold (see [6, Corollary 3.1.1]), whilst the second description shows that M_0 contains a fibred knot K_0 whose complement is homeomorphic to that of the figure eight knot, and which therefore has Alexander polynomial $1 - 3t + t^2$. Finally, M_0 is not simply-connected, being in fact the Brieskorn homology sphere (2, 3, 13) (see [1, Fig. 23]). Hence (M_0, K_0) may be used as described above to construct a knot K satisfying the conclusions of Theorem 6.3.

7. Injectivity of Anisotropic Fibred Knots

In [7] is defined the notion of a knot (in S^3, or a homology 3-sphere) being *rationally anisotropic*. For a fibred knot, with monodromy $f\colon F\to F$, this is equivalent to the condition that $H_1(F;\mathbf{Q})$ have no non-zero f_*-invariant subspace on which the intersection form of F is identically zero. Fibred knots in S^3 which are rationally anisotropic include the torus knots [14, Proposition 2.3], [7, Corollary 4.8], and, more generally, all connected sums of coherently oriented torus knots [7, Proposition 4.9].

In [14, Question 3.1], Scharlemann asks: given a concordance C in $S^3\times I$ with $C\cap S^3\times 0=K$ a torus knot, is the map $\pi_1(S^3-K)\to\pi_1(S^3\times I-C)$ necessarily injective? He shows that this is indeed the case under the additional hypothesis that $S^3\times I-C$ is fibred, and the results of [7] show that it is also true if C is a ribbon concordance (in either direction). An affirmative answer to Scharlemann's question is contained in the following theorem.

Theorem 7.1. *Let K be a rationally anisotropic fibred knot in a homology 3-sphere M, and let $C\subset Q$ be a concordance in a homology $S^3\times I$ such that (Q,C) has (M,K) at one end. Then the map $\pi_1(M-K)\to\pi_1(Q-C)$ induced by inclusion is injective.*

The proof of Theorem 7.1 will be based on Theorem 5.1 and an additional lemma concerning certain duality triads, which we now discuss.

Let $\mathscr{T}=(X,\partial_+X,\partial_-X)$ be a 3-dimensional $\mathbf{F}$-duality triad with X connected and $Y=\emptyset$. Thus $\partial X=\partial_+X\sqcup\partial_-X$, and, in particular, $H_1(\partial X;\mathbf{F})\cong H_1(\partial_+X;\mathbf{F})\oplus H_1(\partial_-X;\mathbf{F})$. Let $i\colon\partial X\to X$ be inclusion, $L=\ker(i_*\colon H_1(\partial X;\mathbf{F})\to H_1(X;\mathbf{F}))$, and $L_\pm=L\cap H_1(\partial_\pm X;\mathbf{F})$. We are interested in knowing when L splits as $L_+\oplus L_-$. Let ξ be a fundamental class for $\mathscr{T}$, with corresponding fundamental classes $\partial_\pm\xi$ for $\partial_\pm X$. Let $G=\pi_1(X)$, and let $\varepsilon\colon\mathbf{F}G/I^2\to\mathbf{F}$ be the map induced by augmentation. Define $\sigma(\mathscr{T})\in\operatorname{coker}(\varepsilon_*\colon H_2(X;\mathbf{F}G/I^2)\to H_2(X;\mathbf{F}))$ to be the image of the class $i_*\partial_+\xi$ $(=i_*\partial_-\xi)\in H_2(X;\mathbf{F})$.

Lemma 7.2. *Let $\mathscr{T}$ be a 3-dimensional $\mathbf{F}$-duality triad as above, such that $\sigma(\mathscr{T})=0$. Then $L=L_+\oplus L_-$.*

Proof. Suppose $\alpha=\alpha_++\alpha_-\in L$, where $\alpha_\pm\in H_1(\partial_\pm X;\mathbf{F})$. We must show that $\alpha_\pm\in L$. Since $i_*\alpha=0$, $\alpha=\partial\beta$ for some $\beta\in H_2(X,\partial X;\mathbf{F})$. By hypothesis, $\sigma(\mathscr{T})=0$; let $\omega\in H_2(X;\mathbf{F}G/I^2)$ be such that $\varepsilon_*\omega=i_*\partial_+\xi$. Let $\bar\omega\in H^1(X,\partial X;\mathbf{F}G/I^2)$ be dual to ω, i.e. $\bar\omega\cap\xi=\omega$ (see Lemma 3.5). Note that the cap product $\bar\omega\cap\beta\in H_1(X;\mathbf{F}G/I^2)$ satisfies $\varepsilon_*(\bar\omega\cap\beta)=0$ by Corollary 3.3.

Let $\overline{\partial_+\xi}\in H^0(\partial X;\mathbf{F})$ be dual to $\partial_+\xi$, i.e. $\overline{\partial_+\xi}\cap\partial\xi=\partial_+\xi$. Then $\overline{\partial_+\xi}\cap\alpha=\overline{\partial_+\xi}\cap\alpha_+=\alpha_+$. Also, since $(\varepsilon_*\bar\omega)\cap\xi=\varepsilon_*\omega=i_*\partial_+\xi$, we have $\varepsilon_*\bar\omega=\delta\overline{\partial_+\xi}$. Hence $i_*\alpha_+=i_*(\overline{\partial_+\xi}\cap\alpha)=i_*(\overline{\partial_+\xi}\cap\partial\beta)=(\delta\overline{\partial_+\xi})\cap\beta=(\varepsilon_*\bar\omega)\cap\beta=\varepsilon_*(\bar\omega\cap\beta)=0$. Thus $\alpha_+\in L$, as required. □

Proof of Theorem 7.1. Let $W=\overline{Q-N(C)}$, and let $\tilde W$ be its infinite cyclic covering. Then $\partial\tilde W\cong F\times R^1\cup_{\partial F\times R^1}\partial_-\tilde W$, where F is the fibre of K, and $\partial_-\tilde W$ is the infinite cyclic covering of the exterior ∂_-W of the knot at the other end of (Q,C). By [10], $(\tilde W,F,\partial_-\tilde W)$ is a $\mathbf{Q}$-duality triad, where $F=F\times 0\subset\partial\tilde W$. A generator of the group of covering translations defines an automorphism t of $(\tilde{W}, F, \partial_- \tilde{W})$ such that $t|F=f$, the monodromy of K.

Let $N=\ker(\pi_1(F)\to\pi_1(\tilde{W})/\pi_1(\tilde{W})_\omega)$. By Theorem 4.6, there exist disjoint, essential, simple loops $u_1, \ldots, u_m$ in $\operatorname{int} F$ such that $N=\langle [u_1], \ldots, [u_m]\rangle$. Let T be the compression body obtained by attaching 2-handles to $F\times I$ along regular neighbourhoods of $u_i\times 1$, $i=1,\ldots,m$, and let $X=\tilde{W}\cup_F T$. Since $f_*(N)=N$, f extends to a diffeomorphism g: $T\to T$, by Lemma 5.2. Let $\bar{t}$: $X\to X$ be defined by $\bar{t}=t\cup g$.

Since K is rationally anisotropic, and $(\tilde{W}, F, \partial_- \tilde{W})$ is a $\mathbf{Q}$-duality triad, a standard argument shows that the map $H_1(F;\mathbf{Q})\to H_1(\tilde{W};\mathbf{Q})$ is injective (see for example [14, Theorem 2,4] or [7, Lemma 3.3]). Therefore each u_i separates F. Let F_0 be the closure of the component of $F-\bigcup_{i=1}^{m} u_i$ which contains ∂F, and let F_0' be the corresponding component of $\partial_i T$, containing $\partial F\times 1$. Now consider the $\mathbf{Q}$-duality triad $\mathcal{T}=(X, \partial_+ X, \partial_- X)$, where $\partial_+ X=\partial_i T-F_0'$, and $\partial_- X=F_0'\cup\partial F\times I\cup\partial_- \tilde{W}$. Note that $\partial_+ X\cap\partial_- X=\emptyset$. We claim that $\sigma(\mathcal{T})=0$.

To see this, let $G=\pi_1(\tilde{W})$, and $H=\pi_1(X)\cong G/\langle i_*[u_1], \ldots, i_*[u_m]\rangle$, where i: $F\to\tilde{W}$ is inclusion. Now $i_*[u_i]\in G_2$, (in fact, $i_*[u_i]\in G_\omega$), $i=1,\ldots,m$, and therefore the quotient map $G\to H$ induces an isomorphism $\mathbf{Q}G/I^2\cong\mathbf{Q}H/I_H^2$, where I_H denotes the augmentation ideal of $\mathbf{Q}H$. Write $\operatorname{cok}_{\tilde{W}}=\operatorname{coker}(\varepsilon_*: H_2(\tilde{W};\mathbf{Q}G/I^2)\to H_2(\tilde{W};\mathbf{Q}))$, and $\operatorname{cok}_X=\operatorname{coker}(\varepsilon_*: H_2(X;\mathbf{Q}G/I^2)\to H_2(X;\mathbf{Q}))$. Let α_i be the class represented by u_i in $H_1(F;\mathbf{Q})$. Since $i_*[u_i]\in G_2$, $\alpha_i=\varepsilon_*\hat{\alpha}_i$ for some $\hat{\alpha}_i\in H_1(F;\mathbf{Q}G/I^2)$. Since $i_*[u_i]\in G_3$, $i_*\hat{\alpha}_i=0\in H_1(\tilde{W};\mathbf{Q}G/I^2)\cong I^2/I^3$ (see Lemma 3.2). Therefore $\hat{\alpha}_i=\partial\beta_i$ for some $\beta_i\in H_2(\tilde{W}, F;\mathbf{Q}G/I^2)$. An easy Mayer-Vietoris argument now shows that inclusion $\tilde{W}\to X$ induces an isomorphism $\operatorname{cok}_{\tilde{W}}\cong\operatorname{cok}_X$.

Since $H_2(W;\mathbf{Q})=0$, t_*-1: $H_2(\tilde{W};\mathbf{Q})\to H_2(\tilde{W};\mathbf{Q})$ is an epimorphism (see [10]), and hence t_*-1: $\operatorname{cok}_{\tilde{W}}\to\operatorname{cok}_{\tilde{W}}$ is an epimorphism. This implies that it is in fact an isomorphism, since $H_2(\tilde{W};\mathbf{Q})$, and hence $\operatorname{cok}_{\tilde{W}}$, is a finitely-generated module over the principal ideal domain $\mathbf{Q}[t,t^{-1}]$. Therefore $\bar{t}_*-1$: $\operatorname{cok}_X\to\operatorname{cok}_X$ is also an isomorphism. But $\bar{t}(\partial_\pm X)=\partial_\pm X$, showing that $(\bar{t}_*-1)\,\sigma(\mathcal{T})=0$. Hence $\sigma(\mathcal{T})=0$, as claimed.

It now follows from Lemma 7.2 that $L=L_+\oplus L_-$, where $L=\ker(H_1(\partial X;\mathbf{Q})\to H_1(X;\mathbf{Q}))$ and $L_\pm=L\cap H_1(\partial_\pm X;\mathbf{Q})$. Since $\dim L=\frac{1}{2}\dim H_1(\partial X;\mathbf{Q})$ and $\dim L_\pm\leq\frac{1}{2}\dim H_1(\partial_+ X;\mathbf{Q})$, $L=L_+\oplus L_-$ implies $\dim L_\pm=\frac{1}{2}\dim H_1(\partial_\pm X;\mathbf{Q})$. Since each u_i separates F, inclusion induces an isomorphism $H_1(W;\mathbf{Q})\cong H_1(X;\mathbf{Q})$, and we also have an isomorphism $H_1(\partial_+ X;\mathbf{Q})\cong H_1(\overline{F-F_0};\mathbf{Q})$, under which L_+ corresponds to $\ker(H_1(\overline{F-F_0};\mathbf{Q})\to H_1(\tilde{W}\,\mathbf{Q}))$. The latter is therefore a non-zero f_*-invariant selfannihilating subspace of $H_1(F;\mathbf{Q})$, contradicting the assumption that K is rationally anisotropic. The collection $\{u_i$: $i=1,\ldots,m\}$ must therefore be empty, implying that $\pi_1(F)\to\pi_1(\tilde{W})$ is injective. Hence $\pi_1(M-K)\to\pi_1(Q-C)$ is injective, as stated. □

Appendix

Recall that, for any group G with group ring $\mathbf{F}G$ and augmentation ideal I, $G_k=\{g\in G\colon g-1\in I^k\}$ and $G_\omega=\bigcap_{k=1}^{\infty} G_k$.

Proof of Lemma 4.1. If $g, g' \in G_k$, then $g^{-1}g' - 1 = g^{-1}(g'-1) - g^{-1}(g-1) \in I^k$, showing that G_k is a subgroup of G. It follows that G_ω is a subgroup of G; clearly G_k and G_ω are invariant under automorphisms of G. The map $g \mapsto g-1$ induces an injective homomorphism from G_k/G_{k+1} to I^k/I^{k+1}, implying that G_k/G_{k+1} is torsion-free if char $\mathbf{F}=0$ and p-torsion if char $\mathbf{F}=p \neq 0$. It follows that all torsion elements of G/G_ω have order p^r, where $p=$ char $\mathbf{F}$. □

The rest of this appendix is devoted to a proof of Lemma 4.2, asserting that $\mathbf{I}^\omega = \bigcap_{k=1}^{\infty} \mathbf{I}^k$ is the kernel of the natural map $\mathbf{F}G \to \mathbf{F}[G/G_\omega]$. Clearly I^ω contains this kernel. To prove the opposite inclusion, we shall use the following inductive definition of a *polynomial map* from a group G to a vector space V over $\mathbf{F}$. A map $\varphi: G \to V$ is *polynomial of degree* $\leqq k$ if, for each $g \in G$, the map $x \mapsto \varphi(gx) - \varphi(x)$ is polynomial of degree $\leqq k-1$. The zero map is polynomial of degree -1. The unique $\mathbf{F}$-linear extension of φ over $\mathbf{F}G$ will also be denoted by $\varphi: \mathbf{F}G \to V$. Lemma 1 below shows that this definition agrees with that of Passi [13, Chap. V].

Lemma 1. *A map $\varphi: G \to V$ is polynomial of degree $\leqq k-1$ if and only if $\varphi | I^k = 0$.*

Proof. Induction on k, starting with $k=1$.

Given φ and $g \in G$, define $\psi_g: G \to V$ by $\psi_g(x) = \varphi((g-1)x) = \varphi(gx) - \varphi(x)$. Since $\{(g-1)x: g \in G, x \in I^k\}$ spans I^{k+1}, $\varphi | I^{k+1} = 0$ if and only if, for each $g \in G$, $\psi_g | I^k = 0$, that is (by inductive hypothesis) ψ_g is polynomial of degree $\leqq k-1$. □

Lemma 2. *If $G_\omega = \{1\}$ and $g_1, \dots, g_n$ are distinct elements of G, then there is a polynomial map φ: $G \to \mathbf{F}$ such that $\varphi(g_1) = 1$ and $\varphi(g_2) = \dots = \varphi(g_n) = 0$.*

Proof. For k sufficiently large, $g_1, \dots, g_n$ have distinct images in G/G_k. Let $\pi_k: G \to \mathbf{F}G/I^k$ be the natural map, and let $x_i = \pi_k(g_i)$; then $x_1, \dots, x_n$ are distinct. For $i = 2, \dots, n$ there is a linear map α_i: $\mathbf{F}G/I^k \to \mathbf{F}$ such that $\alpha_i(x_i) \neq \alpha_i(x_1)$. Define β_i: $\mathbf{F}G/I^k \to \mathbf{F}$ by

$$\beta_i(x) = \frac{\alpha_i(x) - \alpha_i(x_i)}{\alpha_i(x_1) - \alpha_i(x_i)};$$

then $\beta_i(x_1) = 1$, $\beta_i(x_i) = 0$. Clearly $\varphi_i = \beta_i \pi_k$: $G \to \mathbf{F}$ is polynomial, of degree $\leqq k-1$. It follows from the inductive definition that the map φ: $G \to \mathbf{F}$ defined by $\varphi(x) = \varphi_2(x) \dots \varphi_n(x)$ is polynomial of degree $\leqq (n-1)(k-1)$. By construction, $\varphi(g_1) = 1$, $\varphi(g_2) = \dots = \varphi(g_n) = 0$. □

Completion of proof of Lemma 4.2. First consider the special case in which $G_\omega = \{1\}$; we must prove that $I^\omega = 0$. Any element of I^ω can be written as $\sum_{i=1}^{n} \lambda_i g_i$ with $g_1, \dots, g_n$ distinct elements of G. By Lemma 1, $\sum_{i=1}^{n} \lambda_i \varphi(g_i) = 0$ for all polynomial maps φ: $G \to \mathbf{F}$. Using the map φ of Lemma 2 gives $\lambda_1 = 0$, and similarly $\lambda_2 = \dots = \lambda_n = 0$.

The general case follows by applying the special case to the group G/G_ω, noting that $(G/G_\omega)_\omega = \{1\}$. □

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