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Band Mixing in ^{156}Gd and ^{158}Gd

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A detailed investigation of the energies and intensities of the γ-rays that depopulate the low spin levels of the β- and γ-vibrational bands of ^{156}Gd and the γ-vibrational band of ^{158}Gd has been conducted. Both singles and γ-γ coincidence measurements were made on sources of 15-d ^{156}Eu and 46-min ^{158}Eu by use of large volume, high resolution Ge(Li) detectors.

In addition to the γ-band at 1154.09 keV, two $K^{\pi}=0^{+}$ bands were observed in ^{156}Gd with band heads at 1049.45 and 1168.11 keV, respectively. The 2^{+} and 3^{+} members of the γ-vibrational band in ^{158}Gd were observed at 1187.12 and 1265.43 keV, respectively, as well as a new $K^{\pi}=0^{+}$ band at 1195.98 keV. A first order perturbational treatment of the branching ratios was applied to both nuclei. In addition, the mixing between the ground state, the β-, and the γ-vibrational bands of ^{156}Gd is considered from two approaches, but neither satisfactorily explains all the experimental $B(E2)$ ratios.

1. Introduction

In even-even deformed nuclei, it is well-known that the ground state rotational and β- and γ-vibrational levels couple with each other through the rotation-vibration interaction. For inter-band transitions this coupling alters the ratios of reduced transition probabilities from the simple adiabatic predictions of Alaga *et al.*[1], based on the collective model of Bohr and Mottelson[2,3]. The extent of the coupling between the β- and γ-vibrational bands and the ground state rotational band has been analyzed in terms of Z_{β} and Z_{γ}, respectively, the spin-independent band mixing parameters[3,4].

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In the first approximation, a single value of Z_β is expected to explain all of the observed branching ratios for transitions between members of the β-vibrational and ground-state bands, and likewise a single value of Z_γ should account for transition intensities between the γ-vibrational levels and the ground state band. However, it has been necessary to make further developments of the theory in which the effects of interactions between the β- and γ-vibrational bands are considered[4–7]. Investigations[6–8] have shown that in ^{152}Sm and ^{154}Gd a consistent Z_γ parameter is obtained only after the effects of mixing between the β- and γ-vibrational bands are included. Unfortunately, this new consideration of β-γ mixing still did not yield an internally consistent Z_β mixing parameter for these nuclei[6–8]. It should be pointed out that a detailed investigation[9] of highly deformed ^{166}Er has shown that single-parameter mixing theory provides a consistent value of Z_γ for the 2^+ through 8^+ members of the γ-vibrational band in this nucleus.

It seemed important to extend the earlier studies of Riedinger *et al.*[6,7] on the band mixing aspects of ^{152}Sm and ^{154}Gd (which are just at the onset of permanent deformation) to the more strongly deformed nuclei ^{156}Gd and ^{158}Gd. We have made detailed studies[10] of the level properties of these two nuclei as populated in the decay of 15-d ^{156}Eu and 46-min ^{158}Eu, respectively, but here will report only on branching intensities from excited $K^\pi = 0^+$ and $K^\pi = 2^+$ bands.

2. Source Preparation

For these experiments, the 15-d ^{156}Eu activity was produced through the double neutron capture process by irradiation of 10 mg samples of Sm_2O_3 in the Oak Ridge Research Reactor. The samarium oxide, 99.54% enriched in ^{154}Sm, was obtained from the Oak Ridge Isotopes Division. Following irradiation, an ion exchange chemistry was performed to remove any impurities. One half-life after irradiation, about 6% of the source activity resulted from 1.8-y ^{155}Eu and no observable amounts of any other activity were present.

The (d, α) reaction was utilized to produce 46-min ^{158}Eu. Targets of Gd_2O_3 obtained from the Oak Ridge Isotopes Division were bom-

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barded in a 13.5 MeV beam of deuterons at the Oak Ridge Isochronous Cyclotron. The gadolinium oxide was 95.95% enriched in ^{160}Gd and contained 2.16% of ^{158}Gd, 0.67% of ^{157}Gd, 0.72% of ^{156}Gd and 0.41% of ^{155}Gd plus very small quantities of other gadolinium and rare-earth isotopes. After bombarding 50 mg samples for 90 minutes, a precipitation chemistry was performed on the target material. The reduction of Eu^{+3} to Eu^{+2} was used to effect a separation of the europium from other rare-earth ions. The entire procedure required about 45 minutes. About 10 minutes after counting began, ^{157}Eu comprised about 1% of the source activity.

3. Experiments

The singles γ-ray energies and intensities were measured with a system which included a large volume (40 cm^3), high resolution Ge(Li) detector (FWHM = 2.1 keV and peak-to-Compton background ratio = 30/1 for 1333 keV), an Ortec 450 amplifier having pole-zero cancellation and baseline restoration, and a Nuclear Data 2200 analyzer. The coincidence experiments were performed with the detector described for the singles measurements and another of similar volume which had a resolution of 3.5 keV at 1333 keV.

For the coincidence experiments we used constant fraction, leading edge timing, a time-to-amplitude converter, pile-up rejection, and baseline restoration. The pulses were analyzed with a Nuclear Data 3300 system which included a buffer memory and a magnetic tape transport for related-address storage. Approximately six magnetic tapes of related-address data were accumulated for each experiment. A computer code called MPASØRT2 was developed to sort the data into a maximum of 25 coincidence spectra in a single pass through the magnetic tapes. Spectra coincident with 54 γ-rays in ^{156}Gd and 50 γ-rays in ^{158}Gd were extracted from the total coincidence data. Although we only report on those measurements pertaining to the excited $K=0$ and $K=2$ bands in the present paper, a detailed account of all the measurements will be presented later.

In both the ^{156}Eu and ^{158}Eu singles spectra, there exist several complex γ-ray peaks that make it difficult to extract accurate intensities for the transitions from the $K^\pi=2^+$ and $K^\pi=0^+$ states of interest. However, by carefully selecting the coincidence gating energies we were able to obtain these transition intensities. The coincidence data were analyzed quantitatively by using several well-known cascades in each experiment to normalize the data for other cascades. Wherever possible we use our singles intensities since they were determined with greater accuracy, but in all cases the coincidence measurements are in good agreement with them.

4. Results of ^{156}Eu Experiments

In Fig. 1 we show the ^{156}Eu γ-ray spectra coincident with the following gating energies: (a) 1140.51, (b) 811.77, and (c) 1011.87 keV. From these three spectra it is possible to obtain the desired γ-ray branching

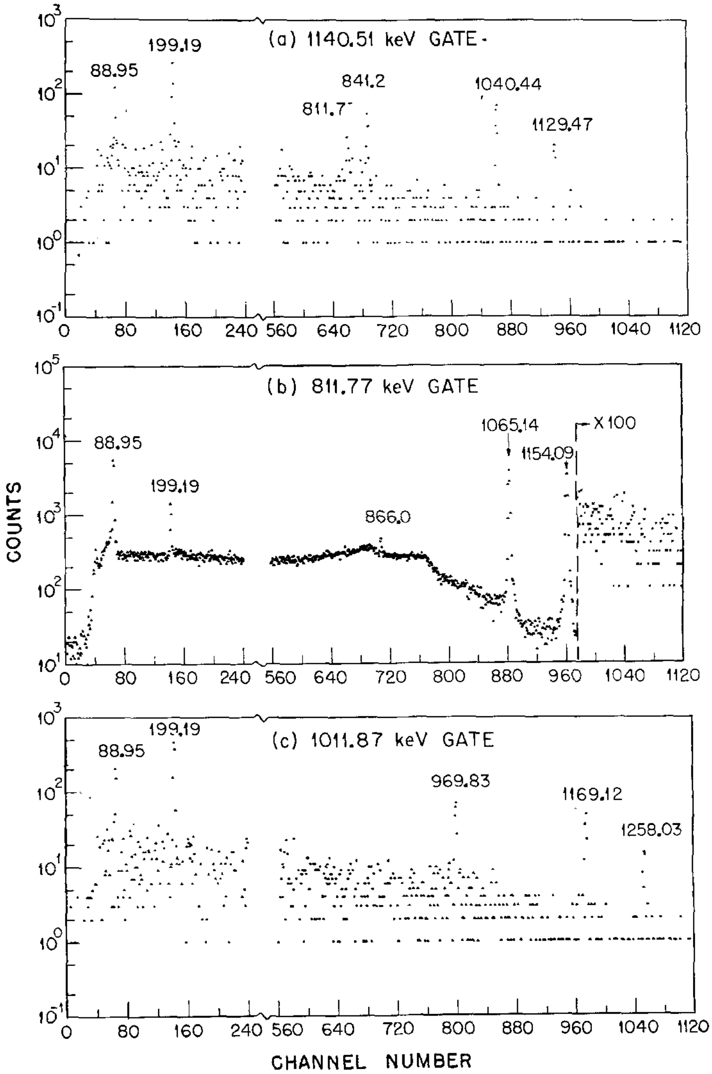

Fig. 1. The γ-ray spectrum of ^{156}Eu in coincidence with the (a) 1140.51, (b) 811.77, and (c) 1011.87 keV transitions. These spectra show the γ-rays which depopulate the 1129.41 (02$^+$), 1154.09 (22$^+$), and 1258.03 (02$^+$) keV levels, respectively. The information in parentheses denotes KI^π

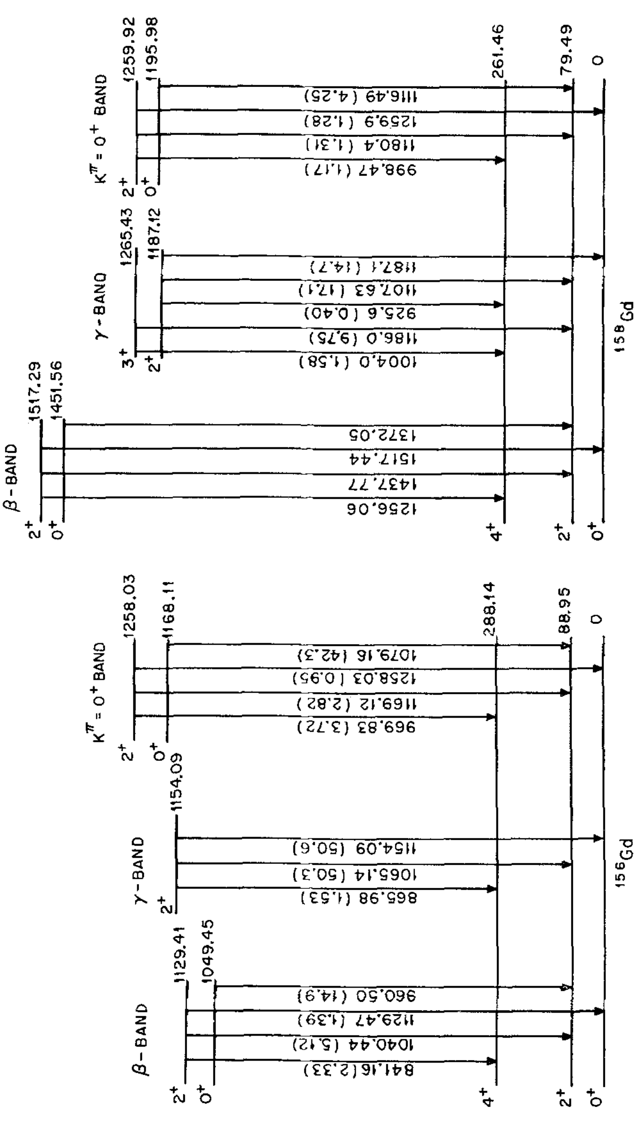

Fig. 2. Partial level schemes showing the ground, $K^\pi = 0^+$, and $K^\pi = 2^+$ bands in ^{156}Gd and ^{158}Gd. The data on the β-vibrational band of ^{158}Gd are from Ref. 17

ratios from the 2^+ members of the so-called β-band, the γ-vibrational band, and another $K^\pi = 0^+$ band, respectively. Extraction of these branching ratios is a simple matter since in each case the gating transition cascades directly into the level of interest. In addition, we have observed the γ-ray transition from the ground state of both of the excited $K^\pi = 0^+$ bands. These data are summarized in the partial level scheme of ^{156}Gd shown in Fig. 2. Note, we have labeled the $K^\pi = 0^+$ band at 1049.45 keV the β-vibrational band since it is the one populated more strongly in Coulomb excitation studies[11]. Higher spin members for any of these three bands were not observed from the decay of ^{156}Eu whose ground state spin is 0^+.

Before considering the problem of band mixing for these states, it is important to establish in as many cases as possible that the transitions from them are pure $E2$ in nature. In the case of the 1040.44 ($02^+ \rightarrow 02^+$) and 1065.14 ($22^+ \rightarrow 02^+$) keV transitions, the multipolarities have been determined[12] experimentally by directional correlation measurements (note that the designation used for a transition is $K_i I_i^\pi \rightarrow K_f I_f^\pi$). These two transitions were found[12] to be 97% and 99.7% $E2$, respectively. In addition, we normalized our γ-ray data to the internal conversion electron data of Ewan *et al.*[13] from the decay of ^{156}Eu in order to obtain K-conversion coefficients for comparison with theory[14]. A similar analysis was done with the aid of data[15,16] from the decay of ^{156}Tb. The results support the $E2$ character of transitions from these states. Since neither electron data nor angular correlation data are available for ^{158}Gd, we have assumed the γ- to ground-band transitions are predominantly $E2$ as this is the trend exhibited by other nuclei in the transition region.

Summarized in Table 1 are the data on the $K^\pi = 0^+$ and 2^+ bands observed in ^{156}Gd. Column 3 shows experimental ratios of $B(E2)$ values whereas column 4 gives the simple adiabatic predictions for the symmetric-rotor model. In all cases we have assumed that the γ-ray intensities are pure $E2$ radiation, except for the 1040.44 keV transition where corrections were made for a 3% M1 admixture.

The error limits assigned to the branching ratios in Tables 1 and 2 are conservative estimates that include both the contributions from detector-efficiency calibrations and peak area determinations as well as a contribution to account for any indeterminable behavior of the entire experimental system. It is felt that the improved resolution of our detection system, the careful cross checks provided by the large number of coincidence measurements, and the definitive information gained by gating on transitions that feed directly some of the vibrational states considered lend to these branching ratio values a degree of reliability not previously realized.

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Table 1. $B(E2)$ ratios and band mixing parameters for transitions between the $K^\pi=0^+$ and $K^\pi=2^+$ bands and the ground state band of ^{156}Gd

$\frac{I_i^\pi \to I_f^\pi}{I_i^\pi \to I_f'^\pi}$	$\frac{E(I_i \to I_f)}{E(I_i \to I_f')}$	$\frac{B(E2, K_iI_i \to K_fI_f)}{B(E2, K_iI_i \to K_fI_f')}$ Experimental value	Adiabatic theory[a]	Z_β or $Z_\gamma \times 10^3$ Present work	McMillan *et al.*[b]	Greenwood and Reich[c]	With β-γ mixing
β-vibrational band							
$\frac{2^+ \to 0^+}{2^+ \to 2^+}$	$\frac{1129.47}{1040.44}$	0.185 ± 0.019	0.70	83 ± 4	87 ± 6	74 ± 4	64 ± 13
$\frac{2^+ \to 4^+}{2^+ \to 2^+}$	$\frac{841.16}{1040.44}$	1.36 ± 0.10	1.80	-9 ± 2	-17 ± 4	-15 ± 2	-5 ± 4
$\frac{2^+ \to 0^+}{2^+ \to 4^+}$	$\frac{1129.47}{841.16}$	0.137 ± 0.016	0.39	28 ± 3	25 ± 5	18 ± 2	24 ± 5
$K^\pi=0^+$ band							
$\frac{2^+ \to 0^+}{2^+ \to 2^+}$	$\frac{1258.03}{1169.12}$	0.223 ± 0.025	0.70	72 ± 5	68 ± 30	77 ± 5	54 ± 13
$\frac{2^+ \to 4^+}{2^+ \to 2^+}$	$\frac{969.83}{1169.12}$	3.35 ± 0.27	1.80	26 ± 4	24 ± 12	18 ± 4	31 ± 7
$\frac{2^+ \to 0^+}{2^+ \to 4^+}$	$\frac{1258.03}{969.83}$	0.070 ± 0.007	0.39	48 ± 3	44^{+10}_{-8}	47 ± 4	39 ± 4
γ-vibrational band							
$\frac{2^+ \to 0^+}{2^+ \to 2^+}$	$\frac{1154.09}{1065.14}$	0.67 ± 0.03	0.70	7 ± 6	13 ± 10		9 ± 8
$\frac{2^+ \to 4^+}{2^+ \to 2^+}$	$\frac{865.98}{1065.14}$	0.086 ± 0.017	0.05	48 ± 22	47 ± 13		39 ± 23
$\frac{2^+ \to 0^+}{2^+ \to 4^+}$	$\frac{1154.09}{865.98}$	7.9 ± 1.6	13.9	32 ± 12	34 ± 8		28 ± 13
$\frac{3^+ \to 2^+}{3^+ \to 4^+}$	$\frac{1159.14}{959.89}$	1.52 ± 0.08[b]	2.50		39 ± 4		

[a] This is simply the ratio of squares of Clebsch-Gordan coefficients.
[b] From Ref. [16].
[c] From Ref. [17].

Applying the first order perturbational corrections (i.e., corrections only due to mixing between the vibrational and ground-state bands) in the same manner as described in our previous work[6,7] on ^{152}Sm and ^{154}Gd leads to the spin-independent band mixing parameters Z_β and Z_γ shown in column 5. In column 6 these mixing parameters are compared with those obtained by McMillan *et al.*[16] from a study of the ^{156}Gd levels populated in the decay of ^{156}Tb; in column 7 a comparison is made with the values from the recent neutron capture work of Greenwood and Reich[17]. The Z_β mixing parameters obtained by these various techniques are in rather good agreement, but show no indication of a consistent value. A first-order perturbational treatment leaves a similar inconsistency in the Z_γ mixing parameters, as was found in the work[6-8] on ^{152}Sm and ^{154}Gd and in the more recent studies[18] on ^{160}Dy.

We now proceed to include in the perturbational treatment the second order effects of mixing between the β- and γ-vibrational bands. Note that the factor $\sqrt{2}$ was omitted from some of the expressions in our original consideration[7] of these second order effects. To account for any mixing between the β- and γ-vibrational bands, it is necessary to have branching ratios from the 3^+ member of the γ-band, a fact due to the condition that there can be no direct mixing between this state of odd spin and the β-band. Since the 3^+ state is not populated in the decay of ^{156}Eu, we have used $Z_\gamma = 0.039 \pm 0.004$ from the ^{156}Tb decay data of McMillan *et al.*[16]. This value was used to determine from the branching ratios in column 3 of Table 1 three values for $Z_{\beta\gamma}$, the β-γ spin-independent mixing parameter for the γ-band. From these three values a weighted average of $Z_{\beta\gamma} = 0.005 \pm 0.002$ was computed. Then the result $B(E2, 00^+ \rightarrow 22^+)/B(E2, 00^+ \rightarrow 02) = 10 \pm 2$ from Coulomb excitation data[11] and the weighted average value of $Z_{\beta\gamma}$ was used to determine the quantity $\zeta_{\beta\gamma} = 0.006 \pm 0.004$, the β-γ mixing parameter for the β-band. Finally, this amount of β-γ mixing was used to redetermine the Z_γ and Z_β mixing parameters as shown in column 8 of Table 1. It is evident that the β-vibrational band mixing parameters still are not brought into alignment but that within just over one standard deviation in the error limits, these mixing parameters for the other $K^\pi = 0^+$ band are consistent. The values for Z_γ shown in column 8 appear to be consistent as a result of including β-γ mixing.

5. Results of the ^{158}Eu Experiment

The transitions which depopulate the 1187.12 and 1265.43 keV members of the γ-vibrational band in ^{158}Gd and the 1259.92 keV level ($KI^\pi = 02^+$) are seen in the spectra of Fig. 3. In each case the gating transition directly populates the level of interest. The three respective gates were set at 606.39, 528.05, and 763.94 keV. Our data on these two

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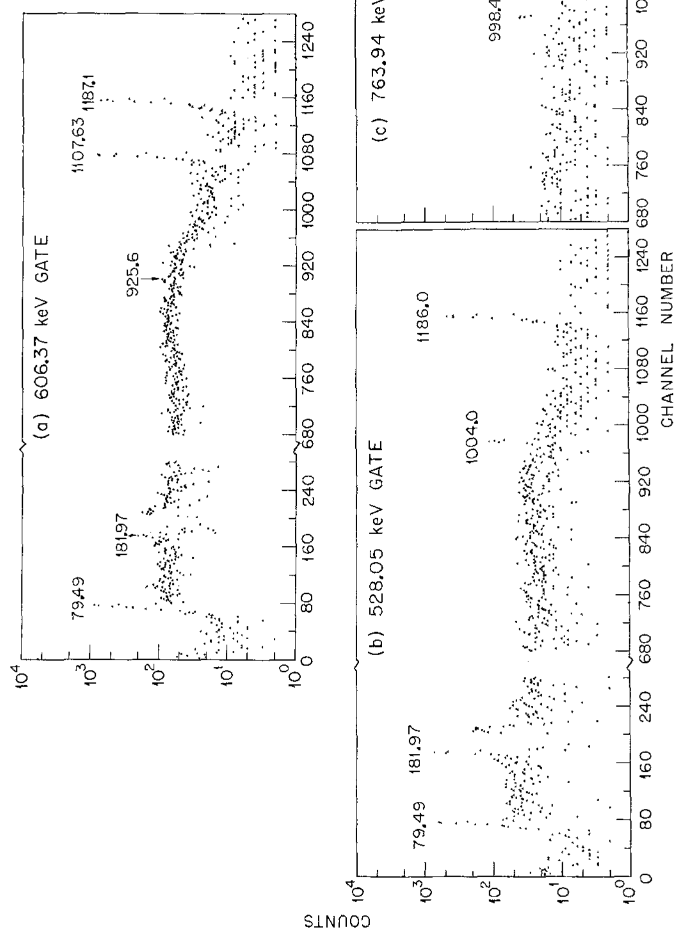

Fig. 3. The γ-ray spectrum of ^{158}Eu in coincidence with the (a) 606.39, (b) 528.05, and (c) 763.94 keV transitions. These spectra show the γ-rays which depopulate the 1 187.12 (22^+), 1 265.43 (23^+), and 1 259.92 (02^+) keV levels, respectively. The information in parentheses denotes KI^π

bands are summarized in Fig. 2. Also shown in Fig. 2 are the 0^+ and 2^+ members of the so-called β-band reported in (d, d') reaction work[19] and (n, γ) studies[20]. There is evidence in our data for the latter two states,

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Table 2. Ratios of reduced transition probabilities and values of Z_γ and $Z_{\beta\gamma}$ for transitions from the γ-vibrational band of ^{158}Gd to the ground state rotational band

$\frac{I_i^\pi \to I_f^\pi}{I_i^\pi \to I_f'^\pi}$	$\frac{E(I_i \to I_f)}{E(I_i \to I_f')}$	$\frac{B(E2, K_i I_i \to K_f I_f)}{B(E2, K_i I_i \to K_f I_f')}$		$Z_\gamma \times 10^3$		$Z_{\beta\gamma} \times 10^3$
		Experimental value	Adiabatic theory [a]	Present work	Paperiello *et al.* [b]	
$\frac{2^+ \to 0^+}{2^+ \to 2^+}$	$\frac{1187.1}{1107.63}$	0.608 ± 0.030	0.70	24 ± 8	41 ± 15	-23 ± 8
$\frac{2^+ \to 4^+}{2^+ \to 2^+}$	$\frac{925.6}{1107.63}$	0.057 ± 0.012	0.05	9 ± 17	9 ± 16	8 ± 9
$\frac{2^+ \to 0^+}{2^+ \to 4^+}$	$\frac{1187.1}{925.6}$	10.6 ± 2.5	13.9	14 ± 13	17 ± 12	22 ± 16
$\frac{3^+ \to 2^+}{3^+ \to 4^+}$	$\frac{1004.2}{1186.0}$	0.37 ± 0.04	0.40	-7 ± 7		

[a] From the predictions of the adiabatic symmetric rotor model.
[b] Values calculated from the data of Paperiello *et al.*[21]

but it is not sufficiently strong to draw any conclusions on the decay properties of these levels. The energies listed for the members of the β-band are taken from the work of Greenwood and Reich[20].

A perturbational treatment similar to that used above was applied to the 2^+ and 3^+ members of the γ-vibrational band of ^{158}Gd. In column 5 of Table 2 are listed our Z_γ values for ^{158}Gd and in column 6 are shown the Z_γ values calculated from the ^{158}Tb decay data of Paperiello *et al.*[21]. Although conservative error limits have been set on our γ-ray intensities, it is evident that just as was the case for ^{156}Gd, there is very poor agreement among the values of the Z_γ parameter. (Here the transitions are assumed to be pure $E2$ radiation.) Contrary to the case of ^{156}Gd, including β-γ mixing effects does not improve the agreement with theory. This can be seen from the values of $Z_{\beta\gamma}$ shown in column 7.

6. Discussion

Recently, Rud *et al.*[22] have attempted to account for the $B(E2)$ ratios from levels of the β-band of ^{152}Sm, ^{154}Gd, and ^{156}Gd by calculating the amount of β-γ mixing necessary to bring these ratios into agreement with adiabatic symmetric rotor theory[1]. They also include an additional diagonal matrix element in the standard perturbational treat-

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ment used by Riedinger *et al.*[7], which leads to another term proportional to Z_γ. Including this additional matrix element and utilizing the format of our previous work[7] gives the perturbed $B(E2)$ values for the β-band as

$$B(E2, 0\,I_i \to 0\,I_f) = B_0(E2, 0\,I_i \to 0\,I_f)\,[1 + Z_\beta F_\beta(I_i, I_f) + \zeta_{\beta\gamma} F'_{\beta\gamma}(I_i, I_f) - Z_\gamma \zeta_{\beta\gamma} F_{\gamma\beta}(I_i, I_f)]^2, \quad (1)$$

where $B_0(E2)$ is the unmixed reduced transition probability, Z_β and $\zeta_{\beta\gamma}$ are the spin-independent mixing parameters, and the F's are functions of the initial and final pins. Except for the last term which results from the additional matrix element, Eq. (1) is identical to Eq. (10) of Ref. [7] and to that used for the treatment of the $K^\pi = 0^+$ bands of ^{156}Gd shown in Table 1. By using the value of Z_γ from the 3^+ level of the γ-band from ^{156}Tb decay[16], unique values of Z_β and $\zeta_{\beta\gamma}$ can be determined from the branching ratios of the β-band. For the ^{156}Eu decay, this method yields a value of $\zeta_{\beta\gamma}$ which is four times larger than that determined in our treatment described above and produces a satisfactory fit to the branching ratios from the 2^+ member of the β-band. However, we feel that the effect of this approach on the γ-band of ^{156}Gd cannot be disregarded as was done by Rud *et al.*[22] who contend that the β-γ mixing parameter determined in Eq. (1) is not the same as that obtained through a treatment of the γ-vibrational band. They attribute this to the fact that higher lying $K^\pi = 0^+$ bands may admix into the γ-band while higher $K^\pi = 2^+$ bands may be admixed to a different extent into the β-band. In general, this argument appears valid but it seems that since the 2^+ members of the β- and γ-bands of ^{156}Gd are separated by only 25 keV, a higher $K^\pi = 0^+$ band at 1168.11 keV should interact with both the β- and γ-bands (Note that out results indicate no additional $K^\pi = 2^+$ bands within over 1 MeV above the γ-band). Therefore we have retained our assumption that in ^{156}Gd there is a close connection between the β-γ mixing parameter for the γ-band and that for the β-band and have applied a treatment similar to that in Eq. (1) to the γ-vibrational band to obtain the following expression for the perturbed $B(E2)$ values

$$B(E2, 2\,I_i \to 0\,I_f) = B_0(E2, 2\,I_i \to 0\,I_f)\,[1 + Z_\gamma F_\gamma(I_i, I_f) + Z_{\beta\gamma} F_{\beta\gamma}(I_i, I_f) + Z_\beta Z_{\beta\gamma} F'(I_i, I_f)]^2, \quad (2)$$

where $Z_{\beta\gamma}$ is a spin-independent mixing parameter. The relationship between $Z_{\beta\gamma}$ and $\zeta_{\beta\gamma}$ is

$$Z_{\beta\gamma} = 3\,\frac{M_\beta^2}{M_\gamma^2}\,\zeta_{\beta\gamma}, \quad (3)$$

where M_β and M_γ are the intrinsic matrix elements between the β- and ground and γ- and ground bands, respectively. Using $\zeta_{\beta\gamma}$ obtained in Eq. (1) and values of M_β and M_γ determined from Ref. [11], we calculated

$Z_{\beta\gamma}$ in Eq. (3). With this value and $Z_{\beta\gamma}$ and Z_β from Eq. (1), Z_γ can be calculated in Eq. (2). The Z_γ values obtained in this manner for the transitions from the 2^+ level of the γ-band of ^{156}Gd differ from one another by a factor of 4. An alternative method is to use the value $Z_\beta=0.013$ determined from Eq. (1) and $Z_\gamma=0.039\pm0.004$ from ^{156}Tb decay[16] to calculate $Z_{\beta\gamma}$ in Eq. (2). This avoids the direct transformation from $\zeta_{\beta\gamma}$ to $Z_{\beta\gamma}$, but the three $Z_{\beta\gamma}$ values for the branching ratios from the 2^+ member of the γ-band still differ from one another by a factor of 4.

The inclusion of the additional diagonal matrix element in the approach of Rud *et al.*[22] produces only a second order effect which is probably of minor importance comparable to the other second order matrix elements not included in Eq. (1). It appears that the main reason they are able to explain the branching ratios from the β-band of ^{156}Gd is that they calculate the amount of β-γ mixing necessary to produce agreement for the branching ratios of the β-band, while neglecting the effect on the γ-band. Therefore, it seems reasonable to conclude that the perturbational treatments considered to this point are unable to yield a satisfactory picture for the γ-ray branching ratios between members of the β-vibrational and ground state bands and in the case of ^{158}Gd, between members of the γ- and ground-bands also.

Perhaps an appropriate approach to the solution of this problem for these two nuclei is a perturbational calculation involving in each case mixing between both $K=0$ bands, the γ-vibrational band and the ground state band. The recently suggested procedure of Stokstad *et al.*[23] also deserves consideration. In the latter approach, which involves three bands and matrix diagonalization, energy level information is introduced into the rotation-vibration interaction by means of the variable moment of inertia model[24].

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