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Developing a Postprocessor for Three Types of Five-Axis Machine Tools

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This paper presents a postprocessor capable of converting cutter location (CL) data to machine control data for three typical five-axis machine tools to establish an interface between computer-aided manufacturing (CAM) systems and numerically controlled (NC) machines. The analytical equations for NC data are obtained using the homogeneous coordinate transformation matrix and inverse kinematics. In addition, the developed postprocessor method is implemented through a trial-cut on a five-axis machine and verified on the coordinate measurement machine. Experimental results confirmed the effectiveness of the proposed postprocessor method which can be used to integrate the various five-axis machine tools employed in manufacturing systems.

Keywords: Coordinate transformation matrix; Cutter location data; Five-axis machines; NC programming; Postprocessor

1. Introduction

Freeform surfaces (or sculptured surfaces) have found extensive industrial applications, such as in automobile bodies, ship hulls and aerospace parts. The mould or stamping die used to create such a part is usually machined using numerically controlled (NC) machine tools. The conventional way of machining freeform surfaces is by using a three-axis machine tool with a ball end-mill. Three-axis machine tools cannot change the tool orientation, so a five-axis machine tool is introduced to set the cutting tool in an appropriate orientation for efficient machining. With the rapidly advancing computer technology, commercial CAD/CAM systems can design freeform surfaces and generate either the three-axis or five-axis tool path. The cutter location (CL) data, composed of cutter tip position and the tool orientation, can then be directly obtained from a CAD model of a product design created in the CAD/CAM systems. However, difficulty frequently arises in communication between the CAM systems and the NC machine tools, especially when various machine tools are employed. The interface that links the CAM systems and NC machines is called the postprocessor and it converts CL data to machine code. Essentially, different combinations of machine tool and control unit require different postprocessors. Consequently, a manufacturing system with a variety of machine tools requires several postprocessors.

Various studies have addressed the issue of developing postprocessors for machine tools. Bedi and Vickers [1] developed a postprocessor program for FANUC 6MB machine tool. Balaji [2] presented the development and implementation of a postprocessor by converting APT source codes to a machine code format. Lin and Chu [3] derived the NC data for machine tools to manufacture cams with flat-face followers using a modified Denavit–Hartenberg (D-H) notation. However, the above works are only related to three-axis machining. In addition, since the tool axis orientation is fixed for a three-axis machine tool, the transformation from CL data to NC data is straightforward and no additional coordinate transformation technique is necessary. To fulfil the industrial demand for geometric variety and high precision, the use of multi-axis machining has increased, especially for machining sculptured surfaces. Suh and Lee [4] developed a four-axis CAM system including CL data generation and postprocessing. Vickers et al. [5] used a G-surf method to define and machine compound curvature surfaces on three-, four- and five-axis machine tool. Takeuchi and Watanabe [6] proposed a five-axis control collision-free tool path and postprocessor method for two types of machine configuration. Sakamoto and Inasaki [7] classified the configuration of five-axis machine tools into three types. However, the analytical NC data expression is not available in the above works.

Lin and Tsai [8] recently used the D-H notation to generate an NC data equation for machining a spatial cam on a four-axis machine tool. Furthermore, Warkentin et al. [9] presented a technique for machining spherical surfaces. In their work, the desired NC data is derived for only one configuration of five-axis machine. Meanwhile, Rao et al. [10] developed the principal-axis method to machine complex surfaces on two configurations of five-axis machines. Nevertheless, only rotational movements were determined. Translational movements for the machine tool were not investigated. Since there may be a considerable number of combinations of five-axis

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machine tool configuration, the postprocessor will inevitably be developed individually [6]. However, according to the classification proposed by Sakamoto and Inasaki [7], the structure of the five-axis machine tool can be divided into three basic types. None of the above studies has derived the complete analytical equations for NC data which contain three linear motions and two rotary motions.

This work aims to develop a postprocessor for three kinds of five-axis machine tools, based on the homogeneous coordinate transformation matrix. The analytical equations for NC data can be obtained by equating the form-shaping function matrix and the known CL data and solving the simultaneous algebraic equations. Moreover, the machine tool setting procedure and the characteristics for different configurations are discussed. To verify the validity and effectiveness of the developed postprocessor, a designed Bezier surface is machined with model material on a typical five-axis machining centre and then measured on the coordinate measurement machine (CMM).

2. Kinematics Model

Machine tools are articulated open chains of serially connected links with joints. The joints may either be revolute or prismatic. Actuation of a revolute joint rotates the link about the joint axis, while actuation of a prismatic joint translates the link along the joint axis. To adequately control the position and orientation of the cutting tool and the machine tool, a kinematics model establishing the mathematical description of the geometry and motion of a machine tool is required. Denavit and Hartenberg [11] first introduced the spatial transformation between two successive link coordinate systems using a 4×4 homogeneous coordinate transformation matrix, which was later adopted by Paul [12]. It is a conventional modelling technique used for mechanisms, robotics, error analysis and computer visions. In this paper, four fundamental transformation matrices using Paul's notation are introduced. They can be expressed as follows:

$$\mathrm{Trans}(a,b,c) = \begin{bmatrix} 1 & 0 & 0 & a \\ 0 & 1 & 0 & b \\ 0 & 0 & 1 & c \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

$$\mathrm{Rot}(X,\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \mathrm{C}\theta & -\mathrm{S}\theta & 0 \\ 0 & \mathrm{S}\theta & \mathrm{C}\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

$$\mathrm{Rot}(Y,\theta) = \begin{bmatrix} \mathrm{C}\theta & 0 & \mathrm{S}\theta & 0 \\ 0 & 1 & 0 & 0 \\ -\mathrm{S}\theta & 0 & \mathrm{C}\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{3}$$

$$\mathrm{Rot}(Z,\theta) = \begin{bmatrix} \mathrm{C}\theta & -\mathrm{S}\theta & 0 & 0 \\ \mathrm{S}\theta & \mathrm{C}\theta & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4}$$

$\mathrm{Trans}(a,b,c)$ implies a translation given by the vector $a\mathbf{i} + b\mathbf{j} + c\mathbf{k}$, and $\mathrm{Rot}(X,\theta)$, $\mathrm{Rot}(Y,\theta)$, $\mathrm{Rot}(Z,\theta)$ imply rotations of θ about the X, Y and Z coordinate axis, respectively; and "C" and "S" refer to cosine and sine functions, respectively. The spatial transformation from one coordinate system to another coordinate system can therefore be decomposed by combining the fundamental transformation matrices.

3. Definition of CL Data for Five-Axis Machining

The cutter location data for five-axis milling consists of the position and orientation of the cutter with respect to the workpiece coordinate system, as shown in Fig. 1. In this paper, the point vector is written as $[Q_x\, Q_y\, Q_z\; 1]^{\mathrm{T}}$, and vectors of the form $[K_x\, K_y\, K_z\; 0]^{\mathrm{T}}$ are used to represent directions for homogeneous coordinate notation; the superscript "T" denotes the transposed matrix. It is worth mentioning that the significant cutter position is defined as the cutter centre tip and not the cutter contact point. For a given parametric design surface to be machined using five-axis milling with a generalised cutting tool defined according to DIN 66215 where any point of the cutting tool can be defined as the contact point, the appropriate CL data can be determined [13] by differential geometry and the homogeneous coordinate transformation matrix.

4. Postprocessor for Five-Axis Machining

The CL file, once obtained, should be transformed into five reference inputs (i.e. three linear motions plus two rotational motions) using the inverse kinematics transformation for controllers of the five-axis machine. This translation is known as postprocessing and the translating software is called the

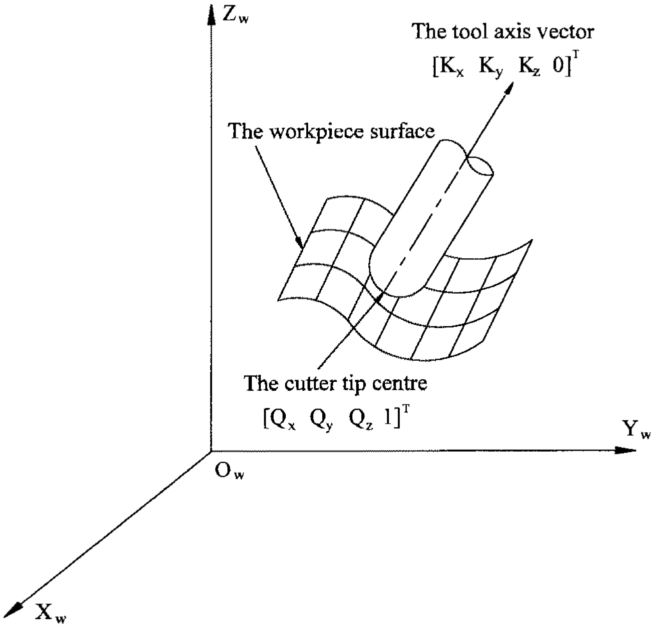
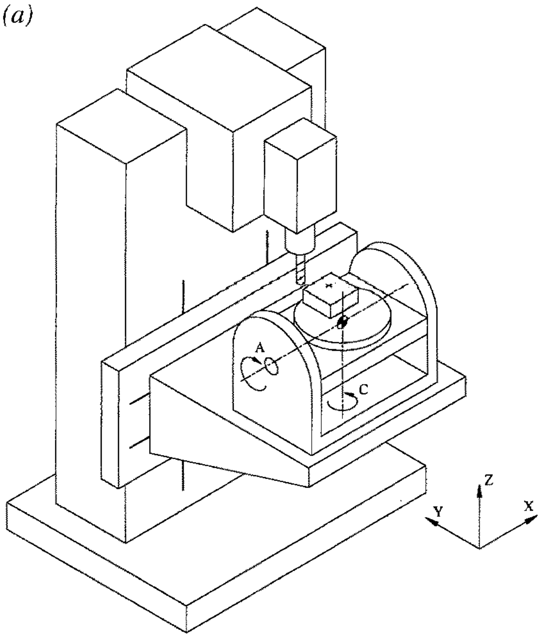
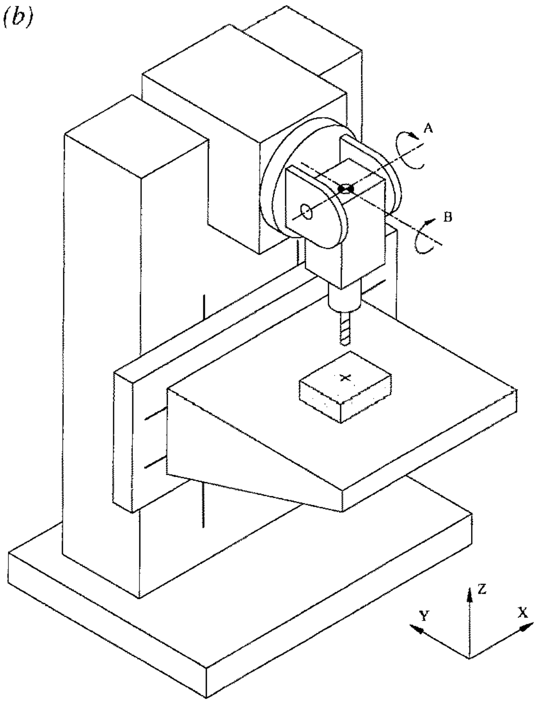


Fig. 1. Geometric definition of CL data.

(a)



(b)



(c)

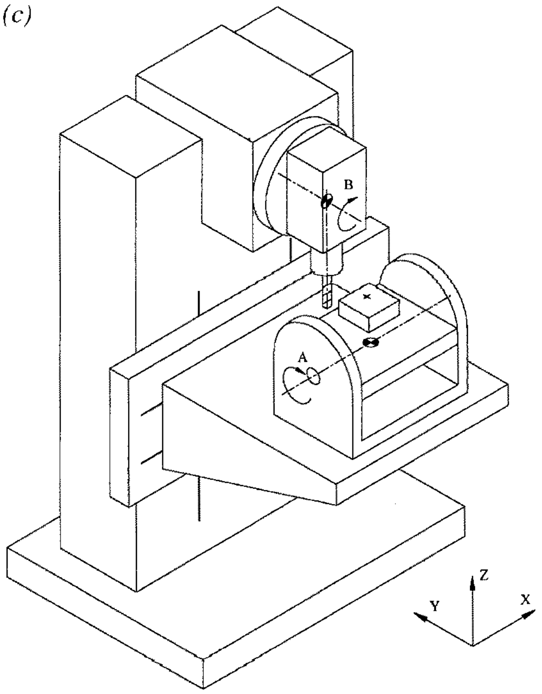


Fig. 2. Configuration for three kinds of five-axis machining centres. (*a*) Table-tilting type. (*b*) Spindle-tilting type. (*c*) Table/spindle-tilting type.

postprocessor. The inverse kinematics transformation depends on the geometric structure of the five-axis machine tool used. Theoretically, there are numerous combinations to yield the five-axis machine tool configuration. However, in practice, the configuration can be classified into three basic types according to the distribution of the two rotational movement units [6,7]:

1. Table-tilting with two rotations on the table.
2. Spindle-tilting with two rotations on the spindle.
3. Table/spindle-tilting with one rotation each on the table and spindle.

The postprocessor for five-axis machining presented in this paper is applied to the above three types. Moreover, the kinematic structures in Figs 2(*a*) to 2(*c*), are selected to demonstrate the derivation procedure. In addition, the two rotary axes are assumed to be continuously intersected with each other for table-tilting type and spindle-tilting type of configurations to simplify the formulations.

Generally speaking, to machine a workpiece on a CNC machine according to a part program, a coordinate system and an axis designation system must first be established. In addition to three linear perpendicular axes (X, Y, Z), CNC machine tools may involve rotary motion around the rotational axes, which are designated by the characters A, B, and C to indicate the rotary motions around the X, Y, and Z axes, respectively. Therefore, the five-axis machine tool configuration in Figs 2(*a*) to 2(*c*) can be characterised by (X, Y, Z, A, C) and (X, Y, Z, A, B). All the movements should be referred to the program coordinate system, e.g. the G54 ~ G59 code is used to define the program coordinate system in the FANUC controller. In most cases, the program coordinate system is coincident with the workpiece coordinate system and is thus applied in this paper.

4.1 Table-Tilting Type

Regarding the table-tilting type, Fig. 3 depicts relevant coordinate sytems. Coordinate systems $O_wX_wY_wZ_w$ and $O_tX_tY_tZ_t$ are attached to the workpiece and the cutting tool, repsectively.

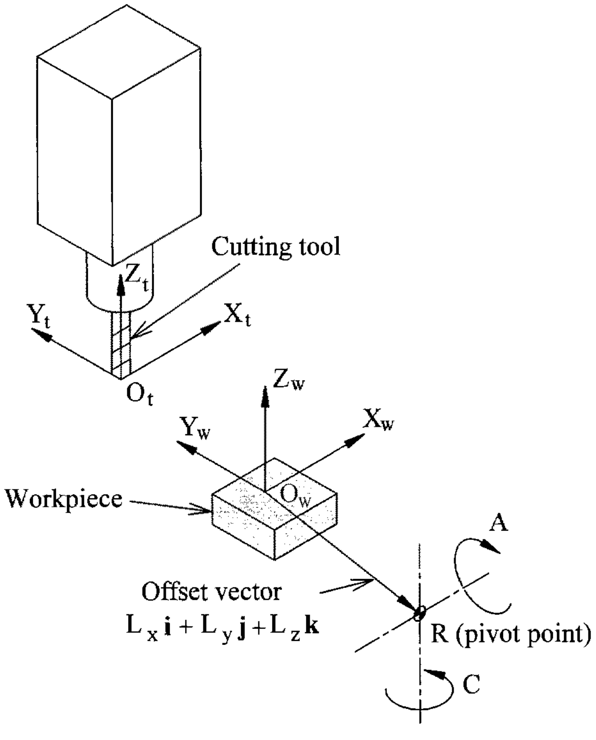


Fig. 3. Coordinate systems of table-tilting type configuration.

The pivot point R is the intersection of the two rotary axes. The offset vector $L_x\mathbf{i} + L_y\mathbf{j} + L_z\mathbf{k}$ as determined from origin O_w to point R is required for the coordinate transformation. Since the structural elements of the machine tool consist of a rotary table, linear table, machine bed, spindle and the cutting tool, the generating motion of the machine tool, which determines the designed characteristics for machine tool and is referred to as the form-shaping function [14], can be characterised sequentially starting from the workpiece and ending at the cutting tool. For the machine tool configuration shown in Fig. 2(*a*), the consecutive structural elements can be described according to Fig. 4.

Consequently, the relative orientation and position of the cutting tool with respect to the workpiece coordinate system can be determined by multiplying the corresponding fundamental transformation matrices in series, and should be equal to the known CL data, $[K_x\ K_y\ K_z\ 0]^T$ and $[Q_x\ Q_y\ Q_z\ 1]^T$, respectively. The mathematical expression is described as follows:

$$[K_x\ K_y\ K_z\ 0]^T = \mathbf{Trans}(L_x, L_y, L_z)\ \mathbf{Rot}(Z,-\phi_C)\ \mathbf{Rot}(X,-\phi_A)\ \mathbf{Trans}(P_x, P_y, P_z)[0\quad 0\quad 1\quad 0]^T \tag{5}$$

$$[Q_x\ Q_y\ Q_z\ 1]^T = \mathbf{Trans}(L_x, L_y, L_z)\ \mathbf{Rot}(Z,-\phi_C)\ \mathbf{Rot}(X,-\phi_A)\ \mathbf{Trans}(P_x, P_y, P_z)[0\quad 0\quad 0\quad 1]^T \tag{6}$$

where ϕ_A and ϕ_C are the rotation angles about the X, and Z axes, respectively, and positive rotation is in the direction to advance a right-hand screw in the $+X$ and $+Z$ axis directions. P_x, P_y, P_z are the relative translation distances of the X, Y, and Z tables, respectively.

Multiplying equations (5) and (6), yields:

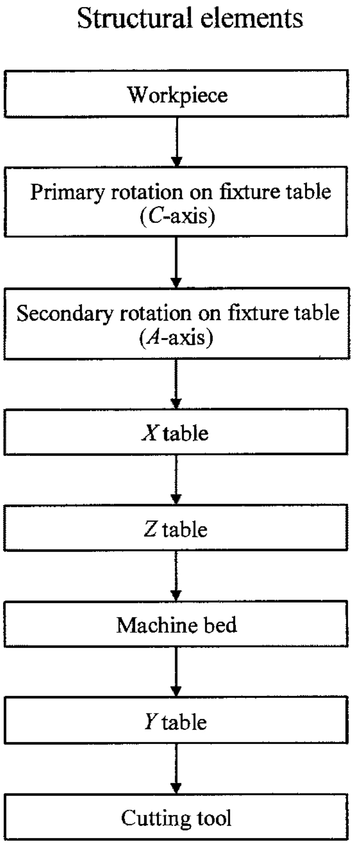


Fig. 4. Relationship of structural elements of table-tilting type configuration.

$$\begin{bmatrix} K_x \\ K_y \\ K_z \\ 0 \end{bmatrix} = \begin{bmatrix} S\phi_A\ S\phi_C \\ S\phi_A\ C\phi_C \\ C\phi_A \\ 0 \end{bmatrix} \tag{7}$$

$$\begin{bmatrix} Q_x \\ Q_y \\ Q_z \\ 1 \end{bmatrix} = \begin{bmatrix} L_x+P_xC\phi_C+P_yC\phi_AS\phi_C+P_zS\phi_AS\phi_C \\ L_y-P_xS\phi_C+P_yC\phi_AC\phi_C+P_zS\phi_AC\phi_C \\ L_z-P_yS\phi_A+P_zC\phi_A \\ 1 \end{bmatrix} \tag{8}$$

From the above equations, the rotation angles (ϕ_A, ϕ_C) and the relative translation distances (P_x, P_y, P_z) can be solved. On the other hand, the X, Y, Z values of the NC data in programming are obtained using equation (6) under the condition $\phi_A = \phi_C = 0$, and $[Q_x\ Q_y\ Q_z\ 1]^T = [X\ Y\ Z\ 1]^T$ since the program coordinate system is coincident with the workpiece coordinate system. This leads to:

$$[X\quad Y\quad Z\quad 1]^T = [L_x+P_x\quad L_y+P_y\quad L_z+P_z\quad 1]^T \tag{9}$$

Thus, the desired equations for NC data of this configuration can be expressed as follows:

$$A = \phi_A = \arccos(K_z) \quad (0 \le \phi_A \le \pi) \tag{10}$$

$$C = \phi_C = \text{arctan2}(K_x, K_y) \quad (-\pi \le \phi_C \le \pi) \tag{11}$$

$$X = L_x+P_x = (Q_x-L_x)C\phi_C-(Q_y-L_y)S\phi_C+L_x \tag{12}$$

$$Y = L_y+P_y = (Q_x-L_x)C\phi_A S\phi_C+(Q_y-L_y)$$
$$C\phi_A C\phi_C-(Q_z-L_z)S\phi_A+L_y \quad (13)$$
$$Z = L_z+P_z = (Q_x-L_x)S\phi_A S\phi_C+(Q_y-L_y)$$
$$S\phi_A C\phi_C+(Q_z-L_z)C\phi_A+L_z \quad (14)$$

where arctan2(y, x) is the function that returns angles in the range $-\pi \leq \theta \leq \pi$ by examining the signs of both y and x [12].

4.2 Spindle-Tilting Type

For the spindle-tilting type configuration, two rotational axes (A and B axes) are applied to the spindle (Fig. 2b) so that the pivot point R (Fig. 5) is selected to be the intersection of these two axes. Furthermore, since the spindle will rotate during machining, the effective tool length, L_t, as determined from the pivot point R to the cutter tip centre O_t, is required for NC data derivation. The same coordinate transformation procedure, similar to the table-tilting type configuration, leads to the following equations:

$$[K_x \quad K_y \quad K_z]^T = \mathbf{Trans}(P_x, P_y, P_z)\ \mathbf{Rot}(Y,\phi_B)$$
$$\mathbf{Rot}(X,\phi_A)\ [0 \quad 0 \quad 1 \quad 0]^T \quad (15)$$
$$[Q_x \quad Q_y \quad Q_z \quad 1]^T = \mathbf{Trans}(P_x, P_y, P_z)\ \mathbf{Rot}(Y,\phi_B)$$
$$\mathbf{Rot}(X,\phi_A)\ [0 \quad 0 \quad -L_t \quad 1]^T \quad (16)$$
$$[X \quad Y \quad Z \quad 1]^T = [P_x\ P_y\ P_z-L_t\ 1]^T \quad (17)$$

Therefore, the analytical equations for NC data can be obtained by solving equations (15)–(17):

$$A = \phi_A = \arcsin(-K_y) \quad (-\pi/2 \leq \phi_A \leq \pi/2) \quad (18)$$
$$B = \phi_B = \text{arctan2}(K_x,K_z) \quad (-\pi \leq \phi_B \leq \pi) \quad (19)$$
$$X = P_x = Q_x + L_t C\phi_A S\phi_B \quad (20)$$

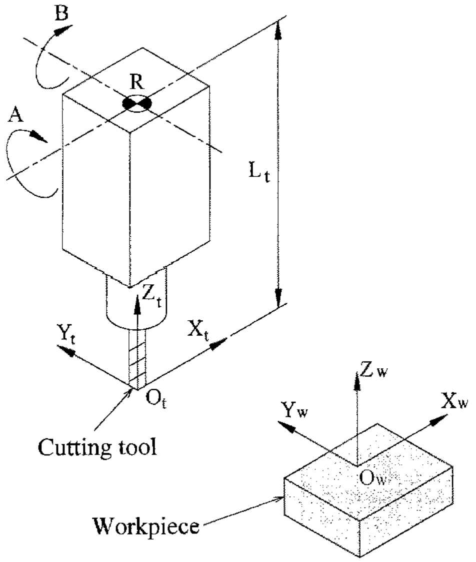


Fig. 5. Coordinate systems of spindle-tilting type configuration.

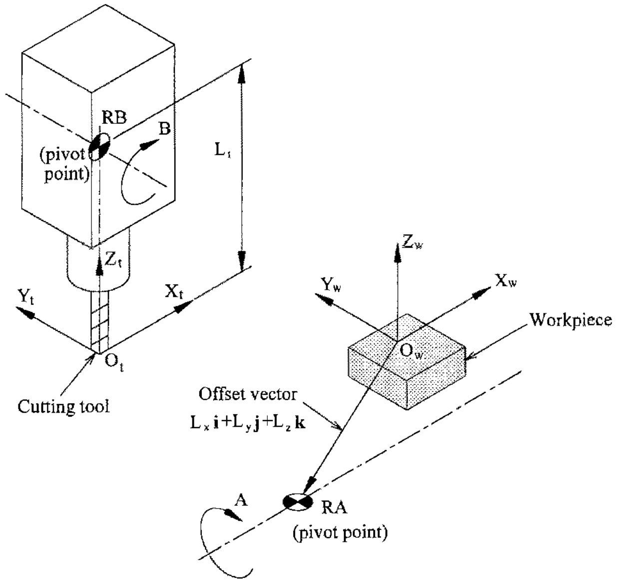


Fig. 6. Coordinate systems of table/spindle-tilting type configuration.

$$Y = P_y = Q_y - L_t S\phi_A \quad (21)$$
$$Z = P_z - L_t = Q_z + L_t C\phi_A C\phi_B - L_t \quad (22)$$

4.3 Table/Spindle-Tilting Type

In the case of this configuration, there is one rotation axis on the rotary table and spindle, and the pivot points are located on the A and B axes, respectively. As shown in Fig. 6, the pivot point RA is located on the A axis arbitrarily, and the pivot point RB is chosen to be the intersection of the spindle tilting axis (B axis) and the cutting tool's axis. The offset vector $L_x\mathbf{i} + L_y\mathbf{j} + L_z\mathbf{k}$ is calculated from the origin O_w to the point RA and the effective tool length, L_t, is the distance between the pivot point RB and the cutter tip centre O_t. As before, the following equations can be obtained using the coordinate transformation matrices:

$$[K_x\ K_y\ K_z\ 0]^T = \mathbf{Trans}(L_x, L_y, L_z)\ \mathbf{Rot}(X,-\phi_A)$$
$$\mathbf{Trans}(P_x, P_y, P_z)\ \mathbf{Rot}(Y,\phi_B)\ [0 \quad 0 \quad 1 \quad 0]^T \quad (23)$$
$$[Q_x\ Q_y\ Q_z\ 1]^T = \mathbf{Trans}(L_x, L_y, L_z)\ \mathbf{Rot}(X,-\phi_A)$$
$$\mathbf{Trans}(P_x, P_y, P_z)\ \mathbf{Rot}(Y,\phi_B)\ [0 \quad 0 \quad -L_t \quad 0]^T \quad (24)$$
$$[X \quad Y \quad Z \quad 1]^T = [L_x+P_x \quad L_y+P_y \quad L_z+P_z-L_t \quad 1]^T \quad (25)$$

Once again, by solving equations (23)–(25), the analytical equations for NC data of this machine configuration can be expressed as:

$$B = \phi_B = \arcsin(K_x) \quad (-\pi/2 \leq \phi_B \leq \pi/2) \quad (26)$$
$$A = \phi_A = \text{arctan2}(K_y, K_z) \quad (-\pi \leq \phi_A \leq \pi) \quad (27)$$
$$X = L_x+P_x = Q_x+L_t S\phi_B \quad (28)$$
$$Y = L_y+P_y = (Q_y-L_y)C\phi_A-(Q_z-L_z)S\phi_A+L_y \quad (29)$$

$$Z = L_z + P_z - L_t = (Q_y - L_y)\mathrm{S}\phi_A + (Q_z - L_z)\mathrm{C}\phi_A + L_t(\mathrm{C}\phi_B - 1) + L_z \tag{30}$$

5. Discussion

From the derivation described in the preceding sections, some findings can be stated as follows:

1. If there is one rotational movement on the rotary table (e.g. table-tilting type and table/spindle-tilting type), the offset vector which correlates the workpiece origin with the pivot point must be determined by the touch sensor tool after the workpiece has been clamped onto the fixture table.
2. When the rotational movement is applied to the spindle (e.g. spindle-tilting type and table/spindle-tilting type), the effective tool length which is the distance between the pivot point and the cutting tool tip centre and can be considered as the total swing radius for the tool tip, should be measured. The tool presetter unit is used to measure the distance, which is called the set length, from the gauge plane to the tip of the tool, as shown in Fig. 7. The gauge plane is at a specific diameter of the taper shank to ensure that all tools fit into the same position in the spindle nose. Then, the effective tool length can be calculated by adding the set length and the distance, which is a constant value given by the machine tool manufacture, from the spindle nose to the pivot point.
3. The pivot point is defined as the intersection of two rotational axes. However, for the table/spindle-tilting configuration, the rotational axes do not intersect in space. As mentioned previously, the pivot point for the rotary table can be arbitrarily selected on the rotational axis. This phenomenon can be explained by observing the NC data expressions of equations (28)–(30), since these equations are independent of the L_x value.

6. Implementation and Verification

6.1 Experimental Implementation

To verify the feasibility of the proposed postprocessor method, a trial-cut experiment was conducted on a table/spindle-tilting five-axis machining centre at National Cheng Kung University. The NC data involve terms of both the offset vector and the effective tool length for this configuration. A Bezier surface with a 4 × 4 control point matrix given by:

(−20, −40, 0)	(0, −40, 10)	(20, −40, 10)	(40, −40, 0)
(−20, −20, 0)	(0, −20, 10)	(20, −20, 10)	(40, −20, 0)
(−20, 0, 0)	(0, 0, 10)	(20, 0, 10)	(40, 0, 0)
(−20, 20, 0)	(0, 20, 10)	(20, 20, 10)	(40, 20, 0)

is machined. In this paper, the tool path generation is based on the isoparametric method with a ball-end cutter whose cutter orientation is assumed to be normal to the contact point of the surface. Mathematically, a Bezier surface can be expressed as $\mathbf{P}(u,v)$, where u and v are independent parameters [15]. The isoparametric step size definition is the input of incremental change in each parameter. Once the u and v parameters have been specified, the point can be defined on the surface and used as the cutter location point $[Q_x\ Q_y\ Q_z\ 1]^{\mathrm{T}}$. Meanwhile, the cutting tool orientation can be calculated using differential geometry [15]:

$$[K_x\ \ K_y\ \ K_z\ \ 0]^{\mathrm{T}} = \frac{\dfrac{\partial \mathbf{P}}{\partial u} \times \dfrac{\partial \mathbf{P}}{\partial v}}{\left|\dfrac{\partial \mathbf{P}}{\partial u} \times \dfrac{\partial \mathbf{P}}{\partial v}\right|} \tag{31}$$

Therefore, the complete CL data for an isoparametric tool path can be determined. A C program was used to generate the CL data and convert the CL data to machine control data (NC code) using the proposed postprocessor method. The trial-cut has been conducted on the basis of the following experimental conditions:

1. The diameter of the ball-end milling cutter is 10 mm.
2. The spindle speed is 500 r.p.m. and the feedrate is 200 mm/min.
3. The step-over for the tool path is 0.5 mm.
4. The offset vector $L_x = 0$, $L_y = -10.0$ mm and $L_z = -25.0$ mm.
5. Effective tool length L_t is 409.571 mm.
6. The workpiece material is acrylic resin.

Fig. 8 depicts the actual cutting on the five-axis machining centre.

6.2 CMM Verification

The finished part (Fig. 9) is measured on the Mitutoyo (model BHN710) CMM consisting of a bridge type main body and a personal computer. In real measurement, sixteen sets of typical measurement data (see Fig. 10) were taken by the CMM using a 2 mm diameter Renishaw PH-9 touch-trigger probe. The touch-trigger probe can be driven toward the part along the normal direction. The outside guide point and inside guide point (Fig. 11) were generated according to the surface normal vector obtained from equation (31). The probing paths can be expressed in terms of the specific CNC code and sent to the CNC controller. While processing the measuring operation, the probe initially moves to the specified outside guide point quickly. Next, the probe moves to the inside guide point slowly until it touches the surface point. A measured point with the positional coordinate of the probe ball centre is collected and saved in an ASCII format file. The accumulated data are compensated by offsetting the probe centre coordinate with the radius of the probe ball along the inner normal direction of the surface. A comparison of the measured sample points with the design surface is shown in Fig. 12. This figure reveals that the maximum deviation of the machined surface, compared to the design surface, is 0.02 mm. These results demonstrate that the proposed postprocessor method is highly effective and reliable.

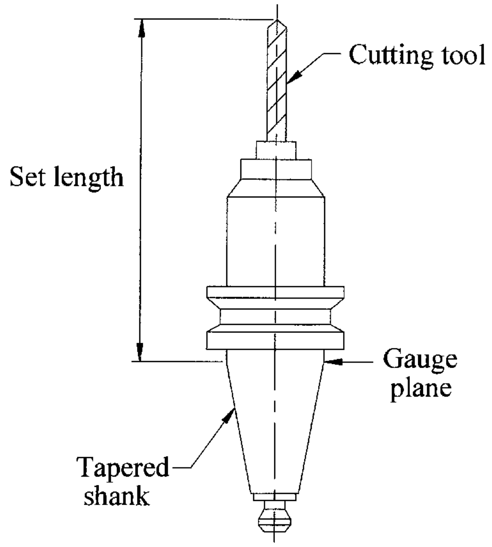


Fig. 7. Set length of the cutting tool.

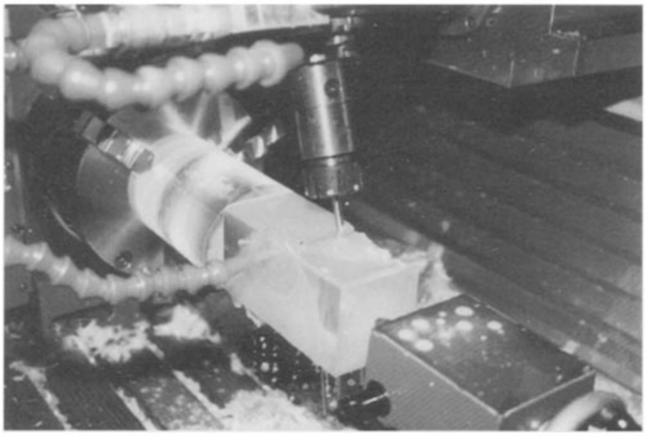


Fig. 8. Machining by the table/spindle-tilting type of machining centre.

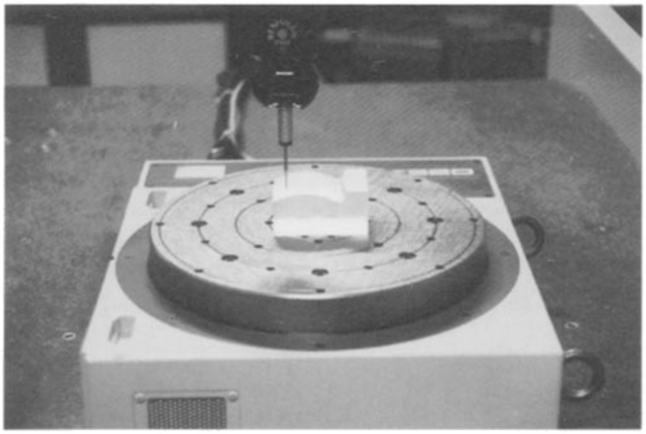


Fig. 9. Measurement of the machined workpiece on CMM.

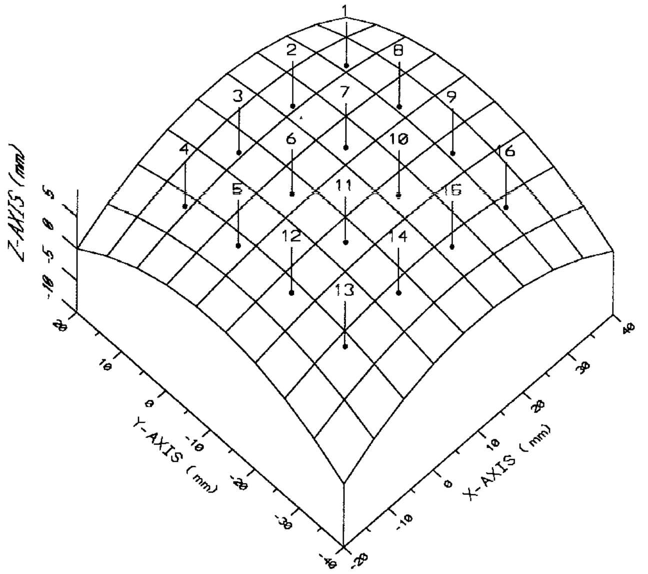


Fig. 10. Checking points for the machined surface.

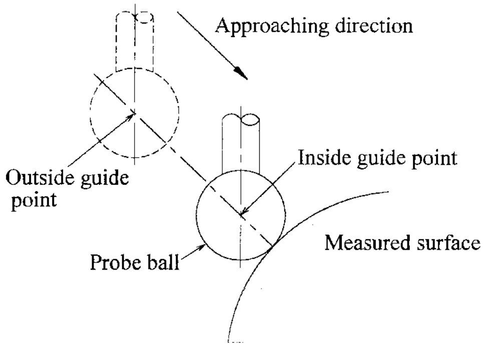


Fig. 11. Outside and inside guide point for the probe.

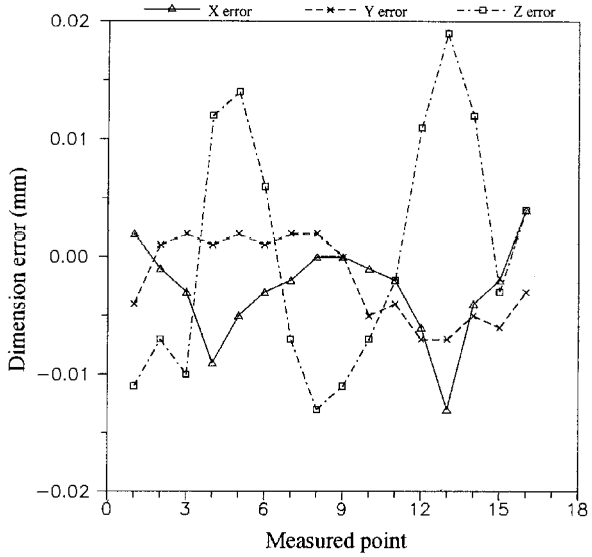


Fig. 12. Dimension errors of the machined surface from CMM measurement.

7. Conclusions

This paper presents an analytical methodology to develop a postprocessor for three typical five-axis machine tools. The form-shaping function is derived according to the homogeneous coordinate transformation matrix. The complete analytical equations for NC data are obtained by equating the form-shaping function and the CL data. Implementation with a trial-cut on a five-axis machining centre and verification on the CMM confirms that the proposed postprocessor method is reliable. Moreover, to integrate the variety of configurations of five-axis machine tools, development of a generalised postprocessor method is currently in progress.

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Notation

A, B, C	machine rotary axes
K_x, K_y, K_z	components of cutting tool's axis orientation
L_x, L_y, L_z	components of offset vector from origin O_w to pivot point
L_t	effective tool length
$O_wX_wY_wZ_w$	coordinate system of workpiece
$O_tX_tY_tZ_t$	coordinate system of cutting tool
P_x, P_y, P_z	relative translation distances of the X, Y and Z tables
$\mathbf{P}$	parametric equation of the surface
Q_x, Q_y, Q_z	coodinate of cutter tip centre
Rot	rotation matrix
Trans	translation matrix
u, v	parameters of the parametric equation
X, Y, Z	machine linear axes
ϕ_A, ϕ_B, ϕ_C	rotation angles about X-, Y- and Z- axes