INTERNAL ROTATION IN THE HYDRODYNAMICS OF WEAKLY CONDUCTING DIELECTRIC SUSPENSIONS

A. O. Tsebers

UDC 533:538

Hydrodynamic phenomena in weakly conducting single-phase media due to interphase electric stresses are reviewed in [1]. In the present paper, a model is constructed of a dielectric suspension with body couples due to the field acting on free charges distributed on the surface of the particles of the suspension. Averaging of the microscopic fields yields macroscopic equations for the field and the polarization of the dielectric suspension with allowance for the finite relaxation time of the distribution of the free charge on the phase interface. The developed model is used to consider the occurrence of spontaneous rotation of a dielectric cylinder in a weakly conducting suspension in the presence of an electric field; compared with the case of single-phase media [2], this is characterized by a significant reduction in the threshold intensity of the electric field with increasing concentration of the particles [3]. In the present model of a dielectric suspension, the destabilization of the cylinder is due to the occurrence of rotations of the particles of the suspension due to the interaction between the polarization and the motion of the medium. The relaxation equation for the polarization for the given model is analogous to the corresponding equation for media which can be magnetized [4-6].

1. The equations describing a many-phase medium on a macroscopic scale are obtained by averaging the microscopic fields [7]. Under the assumption that surface and volume averaging of microscopic fields are equivalent [7], the macroscopic field equations in a weakly conducting suspension are obtained by averaging the microscopic fields over areas with a characteristic dimension appreciably greater than the mean distance between the particles of the suspension. For this, one considers the problem of the perturbation of a homogeneous field by an isolated spherical particle of radius R rotating with angular velocity $\mathbf{\Omega}=(\Omega, 0, 0)$ in a liquid whose permittivity and conductivity are, respectively, ε_1 and γ_1. The permittivity and conductivity of the solid phase are, respectively ε_2 and γ_2.

The Laplace equations for the potential ψ of the electric field in the solid and liquid phases are solved in a spherical coordinate system whose polar axes is along the z direction. The system of boundary conditions for the problem is obtained from the condition of continuity of the potential, the relation for the jump of the normal component of the electric displacement $\mathbf{D}$ on the surface of the particle, and the condition that the perturbations of the field vanish far from the particle:

$$\psi_1=\psi_2,\ D_{n1}-D_{n2}=4\pi\sigma,\quad \mathbf{E}|_{r\to\infty}\to\mathbf{E}_0=(0, E_{0y}, E_{0z}) \tag{1.1}$$

Here, the indices 1 and 2 refer, respectively, to the liquid and solid phases.

The law of conservation of the charge $\partial\rho/\partial t=-\operatorname{div}(\rho\mathbf{v}+\mathbf{j})$ under the assumption that free charge is localized on the boundary between the phases gives, after integration along the direction normal to the interface, a relaxation equation for the surface density of the free charge in a coordinate system fixed relative to the particle [1]:

$$\frac{\partial\sigma}{\partial t}=-\frac{1}{R\sin\vartheta}\frac{\partial}{\partial\vartheta}\left(\sin\vartheta\frac{\partial(\sigma v_\vartheta)}{\partial\vartheta}\right)-\frac{1}{R\sin\vartheta}\frac{\partial(\sigma v_\varphi)}{\partial\varphi}-j_n^1+j_n^2 \tag{1.2}$$

Here, $\mathbf{j}$ is the conduction current determined in the solid and liquid phases by Ohms law $\mathbf{j}_{1,2}=\gamma_{1,2}\mathbf{E}_{1,3}$, $\sigma\mathbf{v}$ is the convective current due to the rotational motion of the particles

Riga. Translated from Izvestiya Akademii Nauk SSSR, Mekhanika Zhidkosti i Gaza, No. 2, pp. 86-93, March-April, 1980. Original article submitted December 8, 1978.

$v=[\Omega\times r]$, and ϑ and φ are the angles of the spherical coordinate system.

The solution of the Laplace equations with allowance for (1.1) and (1.2) gives

$$\psi_1=-\mathbf{E}_0\mathbf{r}+\mathbf{A}\mathbf{r}/r^3,\quad \psi_2=\mathbf{B}\mathbf{r},\quad \mathbf{A}=(0,A_y,A_z),\quad \mathbf{B}=(0,B_y,B_z),\quad \mathbf{A}=\mathbf{A}_0+\mathbf{A}_1$$
$$\overset{\circ}{\varepsilon}_{1,2}=\frac{\varepsilon_{1,2}}{2\varepsilon_1+\varepsilon_2},\quad \overset{\circ}{\gamma}_{1,2}=\frac{\gamma_{1,2}}{2\gamma_1+\gamma_2},\quad \mathbf{A}_0=(\varepsilon_2{}^\circ-\varepsilon_1{}^\circ)\mathbf{E}_0R^3 \tag{1.3}$$

The surface density of the charge is

$$\sigma=\frac{2\varepsilon_1+\varepsilon_2}{4\pi R^3}\mathbf{A}_1\mathbf{n} \tag{1.4}$$

For $\mathbf{A}_1$, we obtain from (1.2) the relaxation equation

$$d\mathbf{A}_1/dt=[\mathbf{\Omega}\times\mathbf{A}_1]-\tau^{-1}(\mathbf{A}_1-\mathbf{A}_{10}),\quad \mathbf{A}_{10}=3(\varepsilon_1{}^\circ\gamma_2{}^\circ-\varepsilon_2{}^\circ\gamma_1{}^\circ)\mathbf{E}_0R^3,\quad \tau=(2\varepsilon_1+\varepsilon_2)/4\pi(2\gamma_1+\gamma_2) \tag{1.5}$$

Here, τ is the relaxation time of the surface charge density in a suspension of spherical particles.

The equations of the macroscopic field are obtained by surface averaging of the microscopic fields (1.3). The averaging is made for the case of weakly concentrated systems, when the volume fraction of the inclusions is small. The averaging over an area with normal $\mathbf{n}$ = (0, 0, 1) and area S with allowance for the circumstance that the part of the averaging surface $S_2=\sum_i\pi(R^2-z_i{}^2)$ (the summation is over particles at distance $|z_i|\leqslant R$ from the averaging surface) passes through the solid phase, and the part $S-S_2$ through the liquid phase, gives for the intensity $S\mathbf{n}\langle\mathbf{E}\rangle=\int\mathbf{n}\mathbf{E}dS$ of the macroscopic field

$$S\langle\mathbf{E}\rangle\mathbf{n}=-S_2\mathbf{B}\mathbf{n}+(S-S_2)\left[\mathbf{E}_0\mathbf{n}-\int\limits_{|z|\geqslant R}^{\infty}dz\int\limits_0^{\infty}2\pi\rho\,d\rho n(\mathbf{n}\nabla)\frac{\mathbf{A}\mathbf{r}}{r^3}-\int\limits_{|z|\leqslant R}dz\int\limits_{\sqrt{R^2-z^2}}^{\infty}2\pi\rho\,d\rho n(\mathbf{n}\nabla)\frac{\mathbf{A}\mathbf{r}}{r^3}\right]$$

Here, n is the number of particles of the suspension in unit volume, and the z and ρ are the coordinates of a cylindrical coordinate system with polar axis at the considered point of the surface $S-S_2$ directed along the z axis.

Calculations with allowance for (1.3) in the case of a spatially uniform distribution of particles, when $S=S\varphi$, gives to terms of first order in φ

$$\mathbf{n}\langle\mathbf{E}\rangle=\mathbf{n}\mathbf{E}_0-3\varphi\mathbf{A}\mathbf{n}/R^3 \tag{1.6}$$

Similar relations hold for the other components of the field intensity. Averaging of the displacement gives

$$S\langle\mathbf{D}^*\rangle=\int\varepsilon\mathbf{E}\,dS,\quad \langle\mathbf{D}^*\rangle=\varepsilon_1\mathbf{E}_0+(\varepsilon_2-\varepsilon_1)\varphi\mathbf{E}_0-(2\varepsilon_1+\varepsilon_2)\varphi\mathbf{A}/R^3 \tag{1.7}$$

For the macroscopic field $\langle\mathbf{D}^*\rangle$ Gauss's theorem holds:

$$\int\limits_S\langle\mathbf{D}^*\rangle\mathbf{n}\,dS=4\pi q$$

where q is the free charge in the volume V surrounded by the surface S.

Because, in accordance with (1.4), the total free charge on the inclusions is zero, the contribution to q due to particles within V is partial. Calculation with allowance for (1.4) gives

$$q=-V\operatorname{div}[(2\varepsilon_1+\varepsilon_2)\varphi\mathbf{A}_1/4\pi R^3]$$

It follows that the equation div $\mathbf{D}$ = 0 in the absence of external charges in the dielectric suspension is satisfied by $\langle\mathbf{D}\rangle=\langle\mathbf{D}^*\rangle+(2\varepsilon_1+\varepsilon_2)\varphi\mathbf{A}_1/R^3$, which plays the part of the displacement in the macroscopic equations. From (1.6) and (1.7) we then obtain

$$\langle\mathbf{D}\rangle=\varepsilon_{12}{}^\infty\langle\mathbf{E}\rangle+4\pi\mathbf{P},\quad \varepsilon_{12}{}^\infty=\varepsilon_1(1+3\varphi(\varepsilon_2{}^\circ-\varepsilon_1{}^\circ)) \tag{1.8}$$

Here, $\varepsilon_{12}^{\infty}$ is the instantaneous permittivity of the suspension, and $\mathbf{P}=3\varepsilon_1\varphi\mathbf{A}_1/4\pi R^3$.

For $\mathbf{P}$ there follows from (1.5) the relaxation equation

$$d\mathbf{P}/dt=[\boldsymbol{\Omega}\times\mathbf{P}]-\tau^{-1}(\mathbf{P}-\mathbf{P}_0),\quad \mathbf{P}_0=\varkappa_0\langle\mathbf{E}\rangle,\quad \varkappa_0=9\varepsilon_1\varphi(\varepsilon_1{}^\circ\gamma_2{}^\circ-\varepsilon_2{}^\circ\gamma_1{}^\circ)/4\pi \tag{1.9}$$

Here, $\varkappa_0$ is the inertial part of the dielectric susceptibility of a suspension of particles of conductivity

Note that in the case when the field frequency satisfies $\omega \ll \tau^{-1}$ and there is no rotation of the particles, $\mathbf{P}=\mathbf{P}_0$ and $\langle\mathbf{D}\rangle=\varepsilon_{12}{}^\circ\langle\mathbf{E}\rangle$, where $\varepsilon_{12}{}^\circ=\varepsilon_{12}{}^\infty+4\pi\varkappa_0$ is the known expression for the dielectric susceptibility of a suspension of particles of conductivity γ_2 in a medium with conductivity γ_1 [8].

Equations (1.8) and (1.9) are the basis of the electrostatics of weakly conducting dielectric suspensions and are analogous to the corresponding equations of the model of media which can be magnetized with internal rotation [4-6]. However, the case of dielectric suspensions has a number of important differences. First, in contrast to magnetic liquids, $\varkappa_0$ may be negative ($\gamma_2{}^\circ<\varepsilon_2{}^\circ\gamma_1{}^\circ/\varepsilon_1{}^\circ$ is the case of weakly conducting particles); second, the corresponding relaxation time may be significantly longer than in magnetic liquids.

Thus, for a suspension of nonconducting particles with $\varepsilon_2 = 3.75$ (quartz) in a medium with $\varepsilon_1 = 2.1$ (transformer oil) and conductivity $10^{-11}\ \Omega^{-1}\ \mathrm{cm}^{-1}$ we obtain the relaxation time $\tau = 3.5\cdot10^{-2}$ sec, which is appreciably longer than the relaxation time of the magnetization of a magnetic liquid based on a comparatively viscous carrying medium with $\eta = 1$ p (mineral oil), which is $4.5\cdot10^{-4}$ sec [9].

2. The hydrodynamic equations of magnetic and polarizable media [10] obtained in the case of nonequilibrium polarization of the medium show that when one is describing this class of motions the stresses due to body couples must also be taken into account as well as the ordinary viscous and electric stresses. The complete system of equations of motion and the field of an incompressible polarizable liquid with allowance for the relations (1.8) and (1.9) obtained in the first section has the form (the ponderomotive forces due to the instantaneous dielectric susceptibility are potential forces and are therefore included in the pressure)

$$\begin{gathered}\rho d\mathbf{v}/dt=-\nabla p+\eta\Delta\mathbf{v}+(\mathbf{P}\boldsymbol{\nabla})\mathbf{E}+{}^1/{}_2\operatorname{rot}[\mathbf{P}\times\mathbf{E}]\\ \operatorname{div}\mathbf{D}=0,\quad \operatorname{rot}\mathbf{E}=0,\quad \mathbf{D}=\varepsilon_{12}{}^\infty\mathbf{E}+4\pi\mathbf{P},\quad d\mathbf{P}/dt=[\boldsymbol{\Omega}\times\mathbf{P}]-\tau^{-1}(\mathbf{P}-\mathbf{P}_0)\end{gathered} \tag{2.1}$$

The system of equations (2.1) is closed by the equation of the balance of the couples acting on a particle:

$$-\alpha(\boldsymbol{\Omega}-\boldsymbol{\Omega}_0)+[\mathbf{P}\times\mathbf{E}]=0,\quad \boldsymbol{\Omega}_0={}^1/{}_2\operatorname{rot}\mathbf{v} \tag{2.2}$$

Here, α is the coefficient of rotational friction of the particles, and in the case of spherical particles is equal to $8\pi\eta R^3 n$.

On the basis of the equations of motion and the field (2.1)-(2.2) we can consider the rotational motion of a cylinder in an unbounded suspension in the presence of a homogeneous electric field at right angles to the axis of the cylinder. The problem is solved in a cylindrical coordinate system with axis parallel to the cylinder axis. The case of a single-phase liquid corresponds to the assumption $\mathbf{D}=\varepsilon_{12}{}^\circ\mathbf{E}$.

The perturbation of the homogeneous field by a cylinder of radius R_1 rotating with angular velocity $\boldsymbol{\Omega}_1=(0,\ 0,\ \Omega_1)$ is found by solving the Laplace equations for the potential of the electric field under the boundary conditions (1.1)-(1.2) for the case of cylindrical symmetry. In the region exterior to the cylinder, the potential of the electric field is given by

$$\psi_1=-\mathbf{Er}+2\mathbf{K}_0\mathbf{r}/\varepsilon_{12}{}^\circ r^2+2\mathbf{K}_1\mathbf{r}/\varepsilon_{12}{}^\circ r^2,\quad \mathbf{K}_0=\varkappa_1{}^\infty R_1{}^2\mathbf{E},\quad \varkappa_1{}^\infty={}^1/{}_2\varepsilon_{12}{}^\circ(\varepsilon-\varepsilon_{12}{}^\circ)/(\varepsilon+\varepsilon_{12}{}^\circ) \tag{2.3}$$

Here, ε and γ are the permittivity and conductivity of the cylinder, respectively.

For $\mathbf{K}_1$, we have the relaxation equation

$$\partial\mathbf{K}_1/\partial t=[\boldsymbol{\Omega}_1\times\mathbf{K}_1]-\tau_1{}^{-1}(\mathbf{K}_1-\mathbf{K}_{10}),\quad \tau_1=(\varepsilon+\varepsilon_{12}{}^\circ)/4\pi(\gamma+\gamma_{12}{}^\circ) \tag{2.4}$$

$$\mathbf{K}_{10}=(\varkappa_1^\circ-\varkappa_1^\infty)R_1^2\mathbf{E},\ \varkappa_1^\circ-\varkappa_1^\infty=\varepsilon_{12}^\circ(\varepsilon_{12}^\circ\gamma-\varepsilon\gamma_{12})/(\varepsilon+\varepsilon_{12}^\circ)(\gamma+\gamma_{12}) \tag{2.4}$$

Integrating the electric stresses $T_{ik}={}^1/_4\pi(E_iD_k-{}^1/_2E^2\delta_{ik})$ over a distant cylindrical surface, we obtain for the moment of the electric forces acting in the stationary case on unit length of the cylinder [taking into account (2.3) and (2.4)] the expression [1]

$$M_+=[\mathbf{K}_1\times\mathbf{E}]_z=-\frac{\tau_1\Omega_1}{1+(\tau_1\Omega_1)^2}(\varkappa_1^\circ-\varkappa_1^\infty)R_1^2E^2 \tag{2.5}$$

The equation of the balance of the moments of the electric and viscous forces acting on the cylinder when (2.5) is taken into account, $M_+-4\pi\eta R_1^2\Omega_1=0$, shows that the state of rest of a cylinder with $\varkappa_1^\circ<0$ is unstable at field intensities $E^2\geqslant E_C^2=-4\pi\eta/\tau_1(\varkappa_1^\circ-\varkappa_1^\infty)$.

For the angular velocity of stationary rotation of the cylinder when $E \geqslant E_C$ we obtain

$$\tau_1\Omega_1=\sqrt{(E/E_C)^2-1} \tag{2.6}$$

The linearity of the dependence (2.6) of Ω_1 on E for $E \gg E_C$ corresponds to the experimental results for the case of single-phase liquids [2].

The features of the phenomenon of spontaneous rotation of a cylinder in a dielectric suspension can be described on the basis of the model (2.1)-(2.2) when rheo-electric phenomena [3] are ignored, i.e., under the assumption that the perturbation of the electric field by the cylinder has the form (2.3)

$$\mathbf{E}_d=-\nabla\psi_d,\quad \psi_d=2\mathbf{K}\mathbf{r}/\varepsilon_{12}^\circ r^2,\quad \mathbf{K}=\mathbf{K}_0+\mathbf{K}_1$$

Further, we find the solution of (2.1) and (2.2) for the case of motion of the dielectric suspension in the electric field $\mathbf{E}_d$ of a rotating cylinder. The solution is found for $\varkappa_0\tau E_d^2\alpha^{-1}<1$, when the nonlinear term in the polarization equation can be ignored. In this case, the relaxation equation for the polarization for $\tau\Omega_1 < 1$ gives

$$\mathbf{P}=\mathbf{P}_0+\mathbf{P}_b,\quad \mathbf{P}_0=\varkappa_0(\mathbf{E}_d+\mathbf{E}_b),\quad \mathbf{P}_b=-\tau\varkappa_0(\mathbf{v}\nabla)\mathbf{E}_d+\tau\varkappa_0[\mathbf{\Omega}_0\times\mathbf{E}_d] \tag{2.7}$$

The equation of motion (2.1) with allowance for (2.7) takes the form

$$-\nabla(p-{}^1/_2\mathbf{P}_b\mathbf{E}_d)+\eta\Delta\mathbf{v}-{}^1/_2\{[\mathbf{E}_d\times\operatorname{rot}\mathbf{P}_b]+\mathbf{E}_d\operatorname{div}\mathbf{P}_b\}=0 \tag{2.8}$$

With allowance for the azimuthal symmetry of the problem $\mathbf{v}=(0, f(r), 0)$, the relations (2.7) and (2.8) for the azimuthal velocity of the dielectric suspension give the differential equation

$$(r^6+ar^2)d^2f/dr^2+(r^5-3ar)df/dr+(-r^4+3a)f=0 \tag{2.9}$$

Here, the parameter $a/R_1^4=\tau\varkappa_0K^2/[\varepsilon_{12}^\circ]^2\eta R_1^4$ measures the ratio of the rotational and shear viscosities, and is negative in the case of nonconducting inclusions.

Equation (2.9) is solved for no-slip boundary conditions on the cylinder, $f(R_1)=\Omega_1R_1$, and the absence of motion at infinity. The change of the independent variable $r^4=|a|z$ reduces Eq. (2.9) to the form

$$16z^2(z-1)d^2f/dz^2+16z^2df/dz-(3+z)f=0$$

The general solution of this equation is the function

$$f=C_1z^{1/4}+C_2z^{1/4}\ln[(\sqrt{z}-1)/(\sqrt{z}+1)]$$

which can be written with allowance for the boundary conditions in the form

$$f=\frac{\Omega_1r\ln[(1-|a|^{1/2}/r^2)/(1+|a|^{1/2}/r^2)]}{\ln[(1-|a|^{1/2}/R_1^2)/(1+|a|^{1/2}/R_1^2)]} \tag{2.10}$$

With allowance for the asymptotic behavior of (2.10) as $r \to \infty$, for the moment of the viscous forces on the cylinder in the presence of internal rotations we obtain

$$M_-=\frac{8\pi\eta|a|^{1/2}\Omega_1}{\ln[(1-|a|^{1/2}/R_1^2)/(1+|a|^{1/2}/R_1^2)]} \tag{2.11}$$

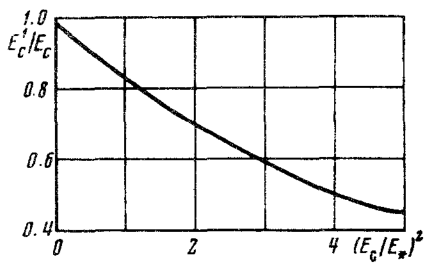

Fig. 1

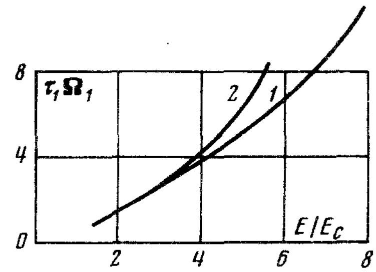

Fig. 2

On the basis of (2.5) and (2.11) we can investigate the influence of internal rotations on the stability of rest and the angular velocity of spontaneous rotation of a dielectric cylinder. In the case $\Omega_1 \to 0$, we have $\mathbf{K}=\mathbf{K}_0+\mathbf{K}_{10}=\varkappa_1{}^\circ R_1{}^2\mathbf{E}$, which for the determination of the threshold at which the stability of rest of the cylinder is lost gives the equation

$$\frac{E_c{}^1}{E_*}=-\frac{2(E_c/E_*)^2}{\ln[(1-E_c{}^1/E_*)/(1+E_c{}^1/E_*)]}, \quad E_*{}^2=\frac{\eta[\varepsilon_{12}{}^\circ]^2}{\tau|\varkappa_0|[\varkappa_1{}^\circ]^2} \tag{2.12}$$

As is shown by the relation (2.12), the interaction between the polarization and the motion reduces the instability threshold of the cylinder in the field. Graphically, E_C^1/E_C is shown as a function of the parameter $(E_C/E_*)^2$, which is determined by the properties of the suspension, in Fig. 1. The reduction of the instability threshold for the dielectric cylinder can be clearly represented as follows: the nonconducting particles of the suspension, which are polarized in the dipole field of the cylinder, are carried as a result of the drag of the medium by the cylinder to a different point of space, where, due to the finite relaxation time of the polarization of the particles, the field of the cylinder causes them to rotate with respect to the carrier medium, this producing a moment of the viscous forces in the direction of rotation of the cylinder.

It should be noted that the destabilization of the state of rest of the cylinder by the nonconducting particles is not determined by the specific properties of the field E_d but is due to the rotational instability of the state of rest of the particles with $\varkappa_0<0$ themselves in electric fields with field intensity $E^2\geqslant E_p{}^2=-\alpha/\tau\varkappa_0$.

As an example of such a situation when instability of the particles leads to instability of the medium as a whole, we can give the example of a coaxial cylindrical condenser with a narrow relative gap and inner electrode which is not fixed. In this case, analysis of the stability of the combined system of equations (2.1) and the equation of rotational motion of the inner cylinder shows that the system becomes unstable against rotations of the electrode which is not fixed when $E^2\geqslant E_p{}^2/(1+\alpha/4\eta)$. The rotations of the particles of the suspension then occur in one direction.

In the limit of high velocities of the cylinder $(\Omega_1\tau_1\gg 1)$, when the retarded part of the polarization K_1 of the cylinder is small, we can take for the parameter $|a|/R_1{}^4$, because of (2.3), the value $\tau|\varkappa_0|[\varkappa_1{}^\infty]^2E^2/\eta[\varepsilon_{12}{}^\circ]^2$, which for the rate of rotation of the cylinder when $E \gg E_C^1$ gives on the basis of the balance for the couples and with allowance for (2.5) and (2.11)

$$\tau_1\Omega_1=\sqrt{-\frac{1}{2}\left(\frac{E_*}{E_c}\right)^2\frac{1}{b}\frac{E}{E_*}\ln\left[\frac{(1-bE/E_*)}{(1+bE/E_*)}\right]-1}, \quad b^2=\frac{[\varkappa_1{}^\infty]^2}{[\varkappa_1{}^\circ]^2} \tag{2.13}$$

The dependence (2.13) for b = 0.176, which corresponds to an ebonite cylinder with $\varepsilon = 3$ and $\gamma = 0$ in a medium with $\varepsilon_1 = 2.1$ (transformer oil), is shown in Fig. 2. Curves 1 and 2 correspond to the values $(E_C/E_*)^2 = 0.5$ and 1.0. As is shown in the figure, the function (2.13), in contrast to (2.6), is nonlinear, which corresponds to the experimental results of the investigation of the spontaneous rotation of a cylinder in a dielectric suspension [11].

3. We now estimate the part played by rheo-electric phenomena in the spontaneous rotation of a dielectric cylinder in a field. The perturbation of the field by the motion

of the dielectric suspension can be taken into account by the method of perturbations. For $|a|^{1/2}/R_1^2 \ll 1$, the expansion (2.10) in the zeroth order in $|a|^{1/2}/R_1^2$ gives

$$f=\Omega_1 R_1^2/r \tag{3.1}$$

The perturbation of the polarization of the medium introduced by motion with the velocity field (3.1) is in accordance with (2.7)

$$\mathbf{P}_b=\left(\frac{4\tau\varkappa_0\Omega_1 R_1^4 K}{\varepsilon_{12}^\circ r^4}\sin\varphi,\ -\frac{4\tau\varkappa_0\Omega_1 R_1^2 K}{\varepsilon_{12}^\circ r^4}\cos\varphi,\ 0\right),\quad \mathbf{P}_*=\varkappa_0\mathbf{E}_b+\mathbf{P}_b \tag{3.2}$$

Here, the angle φ is measured from the direction of the dipole moment $\mathbf{K}$ of the dielectric cylinder.

The perturbation of the field in the stationary case can be found by solving the problem

$$\Delta\psi_{1b}=-4\pi\,\mathrm{div}\,\mathbf{P}_b/\varepsilon_{12}^\circ,\quad \Delta\psi_{2b}=0 \tag{3.3}$$

$$D_{1bn}-D_{2bn}=4\pi\sigma_b,\quad \Omega_1\partial\sigma_b/\partial\varphi=-j_{1bn}+j_{2bn} \tag{3.4}$$

The density of the macroscopic current in the dielectric suspension can be found by the method of averaging described in the first section. A corresponding calculation gives

$$\mathbf{j}=\gamma_1\mathbf{D}/\varepsilon_1+[(2\varepsilon_1+\varepsilon_2)/3\varepsilon_1]\partial\mathbf{P}/\partial t \tag{3.5}$$

We note that (3.5) in the stationary case yields results [12] which show that in a dielectric suspension there is an analog of the Hall effect in a shear flow.

Solution of the problem (3.3)-(3.4) with allowance for (3.2) and (3.5) for the moment of the electric forces acting on the cylinder and due to the rheo-electric effect gives

$$M_{+b}=-4\pi\tau\varkappa_0\Omega_1[\varkappa_1^\circ]^2E^2R_1^2/[\varepsilon_{12}^\circ]^2$$

It can be seen that in the case $\varkappa_0<0$ the perturbation of the field due to the rheo-electric effect has, like internal rotation, a destabilizing influence on the state of rest of the dielectric cylinder. At small $|a|^{1/2}/R_1^2$, it follows from (2.11) that

$$M_-=\Omega_1(-4\pi\eta R_1^2+4\pi\tau|\varkappa_0|[\varkappa_1^\circ]^2E^2R_1^2/3[\varepsilon_{12}^\circ]^2)$$

For the threshold of instability of rest of the cylinder when simultaneous allowance is made for internal rotation and the rheo-electric effect we obtain

$$[E_C^1]^2=E_C^2/(1+4E_C^2/3E_*^2)$$

Finally, it should be noted that the problem considered above is analogous to the problem of the translational motion of an ion in a polar liquid [10] with allowance for the dielectric friction due to the finiteness of the polarization relaxation time. We also mention that in [10] the coefficient in front of the dielectric friction term is incorrect; the correct value 17/420, which also agrees with [13], is given in the author's dissertation.*

On the basis of the obtained results it can be concluded that the model of a dielectric suspension which takes into account internal rotation and finiteness of the time of relaxation of the inter-phase density of free charge describes some qualitative features of the phenomenon of spontaneous rotation of a dielectric cylinder in a dielectric suspension. The proposed system of equations of motion and the field leads to a class of problems of the mechanics of polarizable media in which the field equations and the equations of motion are interconnected, the coupling being due to transport of the polarization of the medium by the translational and rotational motions of the particles of the suspension.

LITERATURE CITED

1. J. B. Melcher and G. I. Taylor, "Electrohydrodynamics: a review of the role of interfacial shear stresses," in: Ann. Rev. Fluid Mech., Vol. 1, Palo. Alto Calif. Annual Revs. (1969).

*A. O. Tsebers, "Intrinsic rotation of particles in the hydrodynamics of magnetized and polarized media." Author's Abstract of Candidate's Dissertation [in Russian], Riga (1976).

2. Yu. S. Karpov, V. A. Krasnoperov, Yu. T. Okunev, and V. V. Pasynkov, "On the motion of a dielectric in an electric field," in: Physics of Dielectrics [in Russian], Izd. AN SSSR (1960).
3. Z. P. Shul'man, Yu. F. Deinega, R. G. Gorodkin, and A. D. Matsepuro, The Electro-rheological Effect [in Russian], Nauka i Tekhnika, Minsk (1972).
4. V. M. Suyazov, "Motion of a ferromagnetic suspension in rotating homogeneous magnetic fields," Magn. Gidrodin., No. 4 (1976).
5. M. I. Shliomis, "Effective viscosity of magnetic suspensions," Zh. Eksp. Teor. Fiz., 61, 6 (1971).
6. A. O. Tsebers, "Ferrohydrodynamics as the hydrodynamics of a system with internal degrees of freedom," in: Physical Properties and Hydrodynamics of Disperse Ferromagnets [in Russian], Sverdlovsk (1977).
7. R. I. Nigmatulin, Fundamentals of the Mechanics of Heterogeneous Media [in Russian], Nauka, Moscow (1978).
8. É. Ya. Blum, M. V. Zake, U. I. Ivanov, and Yu. A. Mikhailov, Heat and Mass Transfer in an Electromagnetic Field [in Russian], Zinatne, Riga (1967).
9. M. M. Maiorov and A. O. Tsebers, "Relaxation of magnetic birefringence and dichromism of sols of ferromagnets," Kolloidn. Zh., 39, No. 6 (1977).
10. A. O. Tsebers, "Flow of dipole liquids in external fields," Magn. Gidrodin., No. 4 (1974).
11. Z. P. Shul'man, R. G. Gorodkin, and V. A. Kuz'min, "On some factors influencing the rotation of a dielectric in electrorheological disperse systems," Inzh.-Fiz. Zh., 23, No. 5 (1972).
12. A. O. Tsebers, "Some features of transport phenomena in suspensions with internal rotation," Prikl. Mat. Mekh., 42, No. 4 (1978).
13. J. Hubbard and L. Onsager, "Dielectric dispersion and dielectric friction in electrolyte solutions I," J. Chem. Phys., 67, No. 11 (1977).

INITIAL STAGE OF REFLECTION OF A PLANE SHOCK WAVE FROM A CYLINDER, SPHERE, AND ELLIPSOID OF REVOLUTION

N. I. Nosenko, N. N. Sysoev, and F. V. Shugaev UDC 533.6.011.72

The stage of regular reflection of a plane shock wave from a blunt body (cylinder, sphere, and ellipsoid of revolution) is considered. At the point of intersection of the reflected shock wave and the surface of the body, analytic expressions are found for the derivative of the Mach number of the wave with respect to the time, the curvature of the wave, the normal derivatives of the density and the pressure, and the derivative of the Mach number along the wave front. It is shown that the flow has a singularity at $\alpha = \alpha_* < \alpha_{**}$ (α_{**} is the limiting angle [1] of regular reflection of a shock wave from a rigid surface). The distribution of the parameters in the region between the reflected shock wave and the surface of the body is found up to terms of third order in the time. The density distribution behind the reflected shock wave was measured experimentally, and also the shape of the reflected wave at different instants of time.

The reflection of a plane shock wave from blunt bodies was investigated experimentally in [2-4]. In [4, 5], semi-empirical expressions were proposed for the motion of the reflected shock wave on the symmetry axis. In [6, 7], the results are given of a numerical calculation of the initial stage of reflection of a shock wave from a sphere.

Below, using the conditions of consistency of second and third order [8-10], we obtain analytic expressions for various characteristic quantities of the flow behind a shock wave reflected by a blunt body. We make an expansion in the time t to terms of third order in the neighborhood of the point of intersection of the reflected wave and

---

Moscow. Translated from Izvestiya Akademii Nauk SSSR, Mekhanika Zhidkosti i Gaza, No. 2, pp. 94-100, March-April, 1980. Original article submitted January 3, 1979.