Mh. Math. 111, 137—145 (1991)

Monatshefte für Mathematik

On the Hahn—Banach Extension Property in Hardy and Mixed Norm Spaces on the Unit Ball

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(Received 12 March 1990; in revised form 12 November 1990)

Abstract. For a nonempty set E of nonnegative integers let $H_E^{p,q,\alpha}$ and H_E^p be the closed linear span of

$$\{z_1^{\alpha_1} z_2^{\alpha_2} \dots z_n^{\alpha_n} \colon (\alpha_1, \alpha_2, \dots, \alpha_n) \in (Z_+)^n,\ \alpha_1 + \alpha_2 + \dots + \alpha_n \in E\}$$

in the mixed norm space $H^{p,q,\alpha}(B_n)$ and in the Hardy space $H^p(B_n)$, respectively. In this note we prove that the Hahn—Banach Extension Property (HBEP) of $H_E^{p,q,\alpha}$ is independent of q. As an application, we show that if $0 < p < 1$ and $H_E^{p,q,\alpha}$ or H_E^p has HBEP then E must be thick in the sense that if $E = \{m_n \colon n = 1, 2, \dots\}$, where $m_1 < m_2 < \dots$, then $m_n \leqslant cn$ for some constant c. This result is an extension over those obtained in [2] and [4].

1. Introduction

Let B_n denote the unit ball in C^n, $n \geqslant 1$, S_n its boundary, σ_n the positive rotation invariant measure on S_n, with $\sigma_n(S_n) = 1$. By $H(B_n)$ we denote the class of all functions holomorphic in B_n.

The Hardy space H^p, $0 < p < \infty$, is defined on B_n by

$$H^p = H^p(B_n) = \{f \in H(B_n) \colon \|f\|_p < \infty\},$$

where

$$\|f\|_p = \sup_{0<r<1} M_p(r,f), \quad M_p(r,f) = \left\{\int_{S_n} |f(r\eta)|^p \, d\sigma_n(\eta)\right\}^{1/p}.$$

If $0 < p, q, \alpha < \infty$, define

$$H^{p,q,\alpha} = H^{p,q,\alpha}(B_n) = \{f \in H(B_n) \colon \|f\|_{p,q,\alpha} < \infty\}$$

where

$$\|f\|_{p,q,\alpha} = \left(\int_0^1 (1-r)^{q\alpha-1} M_p(r,f)^q \, dr\right)^{1/q}.$$

For a nonempty set E of nonnegative integers we let

$$H_E = H_E(B_n) = \{f \in H(B_n): f_k \equiv 0,\ k \notin E\},$$

where $f_k(z)$ is the homogeneous polynomial of degree k in the Taylor expansion $f(z) = \sum_{k=0}^{\infty} f_k(z)$.

In this paper we consider the Hahn–Banach Extension Property (HBEP) of the closed subspaces $H_E^{p,q,\alpha} = H^{p,q,\alpha} \cap H_E(B_n)$ and $H_E^p = H^p \cap H_E(B_n)$ of the spaces $H^{p,q,\alpha}$ and H^p, respectively. We recall that $H_E^{p,q,\alpha}$ (resp. H_E^p) has HBEP if every continuous linear functional on $H_E^{p,q,\alpha}$ (resp. H_E^p) can be extended to a continuous linear functional on $H^{p,q,\alpha}$ (resp. H^p).

Our main result is the following theorem which shows that HBEP of $H_E^{p,q,\alpha}$ is independent of q.

Theorem 1. *Let p, q, s, α be positive real numbers. Then $H_E^{p,q,\alpha}$ has* HBEP *if and only if $H_E^{p,s,\alpha}$ has* HBEP.

If $1 \leqslant p, q < \infty$, $H^{p,q,\alpha}$ is a Banach space. Therefore, the following result is an immediate consequence of Theorem 1 and the Hahn–Banach theorem.

Corollary. *If $1 \leqslant p < \infty$, $0 < q < 1$, then $H_E^{p,q,\alpha}$ has* HBEP *for any subset E of nonnegative integers.*

As a further application of Theorem 1 we prove

Theorem 2. *Let $0 < p < 1$ and $E = \{m_k\colon k = 1, 2, \ldots\}$, where $m_1 < m_2 < \ldots$. If* i) *$H_E^{p,q,\alpha}$ has* HBEP, *or* ii) *H_E^p has* HBEP, *then there is a constant $C > 0$ such that $m_k \leqslant Ck$, $k = 1, 2, \ldots$.*

The one variable case $0 < q < p < 1$ of Theorem 2 (i) follows from the case $0 < p \leqslant q < \infty$, that was proved in [2], and from Theorem 1. For the special case $n = 1$ Theorem 2 (ii) is due to N. KALTON and D. TRAUTMAN [4]. The rest of Theorem 2 (a several variables version) will be proved in Section 4.

To show that $H^p(B_n)$, $0 < p < 1$, and $H^{q,q,n/p-(n/q)}(B_n)$, $0 < p < q < 1$, are not locally convex, SHI JI-HUAI [9] constructed closed subspaces of $H^p(B_n)$ and $H^{q,q,n/p-(n/q)}(B_n)$ that fail HBEP. It follows from Theorem 2 that H_E^p and $H_E^{p,q,\alpha}$ fail HBEP if, for example, $E = \{n^2\colon n = 1, 2, \ldots\}$, for any $0 < p < 1$, $0 < q < \infty$, $0 < \alpha < \infty$.

Throughout this paper we will use the convention of denoting by C

any positive constant which is independent of the relevant parameters in the expression in which it occurs. The value of C may change from one occurrence to the next.

We will use the notation $A \cong B$ to mean $C^{-1}A \leqslant B \leqslant CA$.

2. Preliminaries

Let $g(z) = \sum_{k=0}^{\infty} \hat{g}(k) z^k$ be holomorphic in the unit disc B_1 and $f(z) = \sum_{k=0}^{\infty} f_k(z)$ holomorphic in the unit ball B_n. We define

$$(g \times f)(z) = \sum_{k=0}^{\infty} \hat{g}(k) f_k(z), \quad z \in B_n.$$

If $f \in H(B_1)$ we write $g * f$ instead of $g \times f$.

In [3] we have proved that if w_n, $n = 0, 1, 2, \ldots$, are polynomials defined by $w_0(z) = 1 + z$, $w_n(z) = \sum_{k=2^{n-1}}^{2^{n+1}} \varphi\left(\frac{k}{2^{n-1}}\right) z^k$, $z \in B_1$, $n = 1, 2, \ldots$, where $\varphi(t) = \omega(t/2) - \omega(t)$, and ω: $R \to R$ is any infinitely differentiable function satisfying

$$0 \leqslant \omega(t) \leqslant 1 \quad \text{and} \quad \omega(t) = \begin{cases} 1, & t \leqslant 1 \\ 0, & t \geqslant 2 \end{cases}$$

then

$$f = \sum_{n=0}^{\infty} w_n * f, \quad \text{for all} \quad f \in H(B_1),$$

and

$$\|w_n * f\|_p \leqslant C \|f\|_p, \quad f \in H^p(B_1), \quad 0 < p \leqslant 1.$$

It follows immediately from the representation

$$(w_n * f)(z) = \frac{1}{2\pi} \int_0^{2\pi} w_n(e^{it}) f(z e^{-it}) \, dt, \quad f \in H(B_1),$$

that if $1 \leqslant p < \infty$, then $\|w_n * f\|_p \leqslant \|w_n\|_1 \|f\|_p$, $f \in H^p(B_1)$. In [3] we also proved that $\|w_n\|_1 \leqslant C$, $n = 0, 1, 2, \ldots$, where C is a constant independent of n. Thus, if $1 \leqslant p < \infty$, then $\|w_n * f\|_p \leqslant C \|f\|_p$ for $f \in H^p(B_1)$. Now it follows by a slice integration that

$$\|w_n \times f\|_p \leqslant C \|f\|_p, \quad f \in H^p(B_n), \quad 0 < p < \infty. \tag{2.1}$$

Since $\sum_{n=0}^{\infty} \hat{w}_n(k) = 1$, $k = 0, 1, 2, \ldots$, we have $f = \sum_{n=0}^{\infty} w_n \times f$ for any polynomial f. From this it follows easily that

$$f(z) = \sum_{n=0}^{\infty} (w_n \times f)(z), \quad z \in B_n, \quad \text{for all} \quad f \in H(B_n). \tag{2.2}$$

If $f(z) = \sum_{k=n}^{m} \hat{f}(k) z^k$, $z \in B_1$, $0 \leqslant n \leqslant m$, then

$$r^m \|f\|_p \leqslant M_p(r, f) \leqslant r^n \|f\|_p \quad \text{(see [5])}.$$

From this it follows by slice integration that if $g(z) = \sum_{k=n}^{m} g_k(z)$, $0 \leqslant \leqslant n \leqslant m$, where g_k are homogeneous polynomials of degree k, then

$$r^m \|g\|_p \leqslant M_p(r, g) \leqslant r^n \|g\|_p, \quad 0 < p < \infty. \tag{2.3}$$

Lemma 2.1. ([5], [6]). *A measurable function F: $(0, 1) \to (0, \infty)$ satisfying*

$$\sup_{n \geqslant 0} |b_n| r^{2^n} \leqslant (1 - r)^{(1/q) - \alpha} F(r) \leqslant \sum_{n=0}^{\infty} |b_n| r^{2^n}, \quad \alpha > 0,$$

belongs to $L^q(0, 1)$, $0 < q \leqslant \infty$, if and only if $\{2^{-n\alpha} b_n\}$ belongs to the sequence space l^q.

Lemma 2.2. *Let $0 < p, q, \alpha < \infty$. A function $f \in H(B_n)$ belongs to $H^{p,q,\alpha}(B_n)$ if and only if the sequence $\{2^{-n\alpha} \|w_n \times f\|_p\}$ belongs to l^q.*

Proof. Without loss of generality we may suppose $f(0) = 0$. Since

$$\sup_{n \geqslant 0} \|w_n \times f\|_p r^{2^{n+1}} \leqslant C M_p(r, f), \quad \text{by (2.1) and (2.3)},$$

we have $\{2^{-n\alpha} \|w_n \times f\|_p\} \in l^q$, by Lemma 2.1.

Conversely, if $0 < p \leqslant 1$, then using (2.2), (2.3) and Lemma 2.1 we find that

$$\|f\|_{p,q,\alpha}^q \leqslant C \int_0^1 \left(\sum_{n=1}^{\infty} \|w_n \times f\|_p^p r^{2^{n-1}} \right)^{q/p} (1 - r)^{q\alpha - 1} dr \leqslant$$
$$\leqslant C \|\{2^{-n\alpha} \|w_n \times f\|_p\}\|_{l^q}.$$

If $1 \leqslant p < \infty$, then using (2.2), Minkowski's inequality and (2.3) we obtain

$$\|f\|_{p,q,\alpha}^q \leqslant C \int_0^1 (1-r)^{q\alpha-1} \left(\sum_{n=1}^{\infty} \|w_n \times f\|_p r^{2^n-1} \right)^q dr \leqslant$$
$$\leqslant C \|\{2^{-n\alpha} \|w_n \times f\|_p\}\|_{l^q}, \text{ by Lemma 2.1.}$$

Let $f(z) = \sum_{|\alpha|=0}^{\infty} f_\alpha z^\alpha$ and $g(z) = \sum_{|\alpha|=0}^{\infty} g_\alpha z^\alpha$ be holomorphic in B_n. We define

$$(f \circ g)(z) = \sum_{k=0}^{\infty} \left(\sum_{|\alpha|=k} f_\alpha g_\alpha \right) z^k, \quad z \in B_1.$$

Let $H^\infty = H^\infty(B_1)$ be the space of holomorphic bounded functions in B_1 with the sup-norm. A function $g \in H(B_n)$ is said to be a multiplier from $H^{p,q,\alpha}(B_n)$ to H^∞ if the map $f \to f \circ g$ acts as a bounded linear operator from $H^{p,q,\alpha}(B_n)$ to H^∞. The set of all such multipliers will be denoted by $(H^{p,q,\alpha})^*$. Analogously, we define $(H_E^{p,q,\alpha})^*$.

It is easily seen that $(H^{p,q,\alpha})^*$ is a quasi-normed space with the quasi-norm

$$\|g\|_{(H^{p,q,\alpha})^*} = \|g\|_{p,q,\alpha;\infty} = \sup\{\|f \circ g\|_\infty : f \in H^{p,q,\alpha}, \|f\|_{p,q,\alpha} \leqslant 1\}.$$

The next lemma shows that the dual of $H^{p,q,\alpha}$ (denoted by $(H^{p,q,\alpha})'$) may be identified with the space $(H^{p,q,\alpha})^*$.

Lemma 2.3. *If $g \in (H^{p,q,\alpha})^*$ and if we define $\lambda_g(f) = \lim_{r \to 1} (f \circ g)(r)$, $f \in H^{p,q,\alpha}$, then $\lambda_g \in (H^{p,q,\alpha})'$ and $\|\lambda_g\| = \|g\|_{p,q,\alpha;\infty}$.*

Conversely, given $\lambda \in (H^{p,q,\alpha})'$, then there is a unique $g \in (H^{p,q,\alpha})^*$ such that $\lambda_g = \lambda$. Also, $\|g\|_{p,q,\alpha;\infty} \leqslant C \|\lambda\|$.

Proof. Define λ_z, $z \in B_1$, by $\lambda_z(f) = (f \circ g)(z)$, $f \in H^{p,q,\alpha}$. Then $\{\lambda_z : z \in B_1\}$ is a bounded subset of $(H^{p,q,\alpha})'$. On the other hand if f is a holomorphic polynomial in n-variables and $a \in S_1$ then the limit of $\lambda_z(f)$, as $z \to a$, exists. From this we conclude that the above limit exists for all $f \in H^{p,q,\alpha}$. Thus, λ_g is well defined. It is easily seen that $\lambda_g \in (H^{p,q,\alpha})'$ and $\|\lambda_g\| = \|g\|_{p,q,\alpha;\infty}$.

Conversely, define $g_\alpha = \lambda(z^\alpha)$, $\alpha \in (Z_+)^n$. Since $(f \circ g)(z) = \lambda(f_z)$, $z \in B_1$, for all $f \in H^{p,q,\alpha}$, where $f_z(w) = f(zw)$, $w \in B_n$, and $\lim_{r \to 1} \lambda(f_r) = \lambda(f_1)$, we have $\lambda_g = \lambda$. The function $g \in (H^{p,q,\alpha})^*$ because $\|(f \circ g)(z)\| = |\lambda(f_z)| \leqslant \|\lambda\| \|f\|_{p,q,\alpha}$, for $z \in B_1$. The uniqueness of g is obvious.

As a consequence of (2.3) we have $\|w_n \times f\|_{p,q,\alpha} \cong 2^{-n\alpha}\|w_n \times f\|_p$, $f \in H(B_n)$. Using this and Lemma 2.2 we find that

$$\|f\|_{p,s,\alpha} \cong \|\{\|w_n \times f\|_{p,q,\alpha}\}\|_{l^s}. \tag{2.4}$$

Lemma 2.4. *Let $g \in H(B_n)$. Then*

$$\|g\|_{p,s,\alpha;\infty} \cong \|\{\|w_n \times g\|_{p,q,\alpha;\infty}\}\|_{l^{s'}},$$

where $s^{-1} + (s')^{-1} = 1$, if $1 < s < \infty$, and $s' = \infty$, if $0 < s \leqslant 1$.

Proof. Define $P_n = w_{n-1} + w_n + w_{n+1}$, $n \geqslant 0$ $(w_{-1} = 0)$. Since

$$w_n * w_k = 0, \quad \text{for} \quad |n-k| \geqslant 2, \tag{2.5}$$

we have

$$P_n * w_n = w_n, \quad n \geqslant 0, \tag{2.6}$$

and

$$w_n * P_k = 0, \quad \text{if} \quad |k-n| \geqslant 3. \tag{2.7}$$

Let $\|f\|_{p,s,\alpha} < \infty$ and $\|\{\|w_n \times g\|_{p,q,\alpha;\infty}\}\|_{l^{s'}} < \infty$. If $z \in B_1$, then using (2.2) and (2.6) we get

$$\begin{aligned}
|(f \circ g)(z)| &= \left|\sum_{n=0}^{\infty} (w_n * (f \circ g))(z)\right| = \\
&= \left|\sum_{n=0}^{\infty} ((P_n * w_n) * (f \circ g))(z)\right| = \\
&= \left|\sum_{n=0}^{\infty} ((P_n \times f) \circ (w_n \times g))(z)\right| \leqslant \\
&\leqslant \sum_{n=0}^{\infty} \|P_n \times f\|_{p,q,\alpha} \|w_n \times g\|_{p,q,\alpha;\infty} \leqslant \\
&\leqslant \|\{\|P_n \times f\|_{p,q,\alpha}\}\|_{l^s} \|\{\|w_n \times g\|_{p,q,\alpha;\infty}\}\|_{l^{s'}}
\end{aligned}$$

by Hölder's inequality. It is easily seen that $\|\{\|P_n \times f\|_{p,q,\alpha}\}\|_{l^s} \cong \cong \|f\|_{p,s,\alpha}$, by (2.4). Thus, we have proved that $\|g\|_{p,s,\alpha;\infty} \leqslant \leqslant C\|\{\|w_n \times g\|_{p,q,\alpha;\infty}\}\|_{l^{s'}}$.

Conversely, let $\|g\|_{p,s,\alpha;\infty} < \infty$. Fix $0 < \varepsilon < 1$. Since $\|w_n \times g\|_{p,q,\alpha;\infty} < < \infty$, $n = 0, 1, 2, \ldots$, for each n there exists f_n, $\|f_n\|_{p,q,\alpha} = 1$, so that

$$((w_n \times g) \circ f_n)(1) = \|(w_n \times g) \circ f_n\|_\infty \geqslant \varepsilon \|w_n \times g\|_{p,q,\alpha;\infty}. \tag{2.8}$$

If $\{b_k\} \in l^s$, $b_k \geqslant 0$, using (2.4), (2.5) and (2.1) we conclude

$$\left\| \sum_{k=0}^{\infty} w_k \times b_k f_k \right\|_{p,s,\alpha} \cong \left\| \left\{ \left\| w_n \times \sum_{k=0}^{\infty} w_k \times b_k f_k \right\|_{p,q,\alpha} \right\} \right\|_{l^s} \leqslant C \|\{b_k\}\|_{l^s},$$

since $\|f_n\|_{p,q,\alpha} = 1$, for all $n \geqslant 0$.

By hypotheses $\|g\|_{p,s,\alpha;\infty} < \infty$. Therefore

$$\left\| \sum_{n=0}^{\infty} w_n \times b_n f_n \circ g \right\|_{\infty} \leqslant C \|g\|_{p,s,\alpha;\infty} \|\{b_n\}\|_{l^s}. \tag{2.9}$$

On the other hand, $w_n \times b_n f_n \circ g = b_n (w_n \times g) \circ f_n$, and by (2.8)

$$\begin{aligned} \left\| \sum_{n=0}^{\infty} (w_n \times b_n f_n) \circ g \right\|_{\infty} &= \sum_{n=0}^{\infty} b_n \|(w_n \times g) \circ f_n\|_{\infty} \geqslant \\ &\geqslant \varepsilon \sum_{n=0}^{\infty} b_n \|w_n \times g\|_{p,q,\alpha;\infty}. \end{aligned} \tag{2.10}$$

Combining (2.9) and (2.10) we obtain

$$\|\{\|w_n \times g\|_{p,q,\alpha;\infty}\}\|_{l^{s'}} \leqslant C \|g\|_{p,s,\alpha;\infty}.$$

In the same way we prove that if $g \in H_E(B_n)$ then

$$\|g\|_{(H_E^{p,s,\alpha})^*} \cong \|\{\|w_n \times g\|_{(H_E^{p,q,\alpha})^*}\}\|_{l^{s'}}. \tag{2.11}$$

3. Proof of Theorem 1

Set $X = H^{p,s,\alpha}$, $Y = H_E^{p,s,\alpha}$, $A = H^{p,q,\alpha}$, $B = H_E^{p,q,\alpha}$. Suppose now that B (as a subspace of A) has HBEP. Let $g \in Y^*$. By Lemma 2.3 it is sufficient to prove that there exists a $G \in X^*$ such that $G \circ f = g \circ f$, for all $f \in Y$.

Since $w_n \times g \in B^*$, $n \geqslant 0$, and B has HBEP, there are functions $g_n \in H(B_n)$ so that

$$g_n \circ f = w_n \times g \circ f, \quad f \in B \tag{3.1}$$

and

$$\|g_n\|_{p,q,\alpha;\infty} \leqslant C \|w_n \times g\|_{p,q,\alpha;\infty}, \tag{3.2}$$

where C is a positive constant independent of n (a consequence of the open mapping theorem).

Define the function G by $G(z) = \sum_{n=0}^{\infty} (P_n \times g_n)(z)$, $z \in B_n$. One can easily show that $G \in H(B_n)$. We claim that G satisfies the conditions cited above.

Using Lemma 2.4 and equations (2.7), (2.1), (3.2) and (2.11) we obtain

$$\begin{aligned}\|G\|_{p,s,\alpha;\infty} &\cong \|\{\|w_n \times G\|_{p,q,\alpha;\infty}\}\|_{l^{s'}} = \\ &= \left\|\left\{\left\|w_n \times \sum_{k=0}^{\infty} P_k \times g_k\right\|_{p,q,\alpha;\infty}\right\}\right\|_{l^{s'}} \leqslant \\ &\leqslant C\,\|\{\|w_n \times g_n\|_{p,q,\alpha;\infty}\}\|_{l^{s'}} \leqslant C\,\|\{\|g_k\|_{p,q,\alpha;\infty}\}\|_{l^{s'}} \leqslant \\ &\leqslant C\,\|\{\|w_n \times g\|_{p,q,\alpha;\infty}\}\|_{l^{s'}} \cong C\,\|g\|_{Y^*} < \infty.\end{aligned}$$

It follows that $G \in X^*$.

From (3.1), (2.6) and (2.1) it follows that if $f \in Y$, then

$$\begin{aligned}G \circ f &= \sum_{n=0}^{\infty} (P_n \times g_n) \circ f = \sum_{n=0}^{\infty} P_n * (g_n \circ f) = \\ &= \sum_{n=0}^{\infty} P_n * ((w_n \times g) \circ f) = \sum_{n=0}^{\infty} w_n * (g \circ f) = g \circ f.\end{aligned}$$

This completes the proof of Theorem 1.

4. Proof of Theorem 2

i) Note that we proved the case $n = 1$ in the Introduction. Thus, to finish the proof of Theorem 2 (i), by Theorem 1, it is sufficient to show that if $H_E^{p,p,\alpha}(B_n)$, $n > 1$, has HBEP than $H_E^{p,p,\alpha+(n-1)/p}(B_1)$ has HBEP.

Let $\psi \in (H_E^{p,p,\alpha+(n-1)/p}(B_1))'$. Define $\varphi(g) = \psi(\varrho g)$, $g \in H_E^{p,p,\alpha}(B_n)$, where ϱ is a restriction operator defined by $\varrho g(z) = g(z, 0, \ldots, 0)$, $z \in B_1$, $g \in H(B_n)$. The proof of Lemma 2.2 ([1]) shows that ϱ is a bounded transformation from $H_E^{p,p,\alpha}(B_n)$ into $H_E^{p,p,\alpha+(n-1)/p}(B_1)$. Hence, $\varphi \in (H_E^{p,p,\alpha}(B_n))'$. Since $H_E^{p,p,\alpha}(B_n)$ has HBEP, there exists $\Phi \in (H^{p,p,\alpha}(B_n))'$ such that $\Phi(g) = \varphi(g)$, for all $g \in H_E^{p,p,\alpha}(B_n)$.

Now define $\Psi(f) = \Phi(\tau f)$, $f \in H^{p,p,\alpha+(n-1)/p}(B_1)$, where τ is an extension operator defined by $\tau f(z_1, \ldots, z_n) = f(z_1)$, $(z_1, \ldots, z_n) \in B_n$, $f \in H(B_1)$.

It follows from Fubini's theorem that $\|\tau f\|_{p,p,\alpha} = \|f\|_{p,p,\alpha+(n-1)/p}$ (see [8], pp. 127—128). Thus, the extension τ is a linear isometry of $H^{p,p,\alpha+(n-1)/p}(B_1)$ into $H^{p,p,\alpha}(B_n)$. Hence, $\Psi \in (H^{p,p,\alpha+(n-1)/p}(B_1))'$. If $f \in H_E^{p,p,\alpha+(n-1)/p}(B_1)$, then $\Psi(f) = \Phi(\tau f) = \varphi(\tau f) = \psi(\varrho\tau f) = \psi(f)$, since $\tau f \in H_E^{p,p,\alpha}(B_n)$ and Φ is an extension of φ from $H_E^{p,p,\alpha}(B_n)$ to $H^{p,p,\alpha}(B_n)$.

ii) The proof is the same as in i), since ϱ is a bounded transformation from $H_E^p(B_n)$ into $H_E^{p,\,p,\,(n-1)/p}(B_1)$ and τ is a linear isometry of $H^{p,\,p,\,(n-1)/p}(B_1)$ into $H^p(B_n)$.

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