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The Surface Delta Interaction in the Transuranic Nuclei*

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The Surface Delta Interaction (SDI) is employed to calculate the ernergy of gamma-vibrational and the $K=0$, 1, 2, and 3 Octupole vibrational band heads in the even-mass Transuranic nuclei. The quasi-particle Random Phase (RPA) and the quasi-particle Tamm Dancov approximations (TDA) are utilized to solve the Hamiltonian. The resulting energies and gamma transitions are compared with the Pairing plus Quadrupole (PQF) or Octupole (POF) force model and the available experimental data. The agreement between theory and experiment is satisfatory. Only the $K=0$, 1, and 2 Octupole bands are collective. The $K=3$ negative parity state is practicaly a pure two quasi-particle state.

1. Introduction

The Surface Delta Interaction (SDI) was proposed by GREEN and MOSZKOWSKI[1] on the grounds that it able to reproduce the strong seniority zero coupling and the deformation producing Quadrupole force. Recently, FAESSLER and PLASTINO[2] showed that the SDI simulates the ^1S and ^3S phase shifts of the two nucleon scattering for the many body problem. This simple force has been successfully applied to different nuclear structure problems[3-9] within the last 2 years. The SDI seems to be as good or even better than an effective interaction in double closed shell nuclei[4], in single closed shell nuclei[3] and in the deformed nuclei of the (s, d)-shell[5]. The application[6-9] to the strongly deformed Rare Earth

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[1] GREEN, I. M., and S. A. MOSZKOWSKI: Phys. Rev. **139**, B 790 (1965).

[2] FAESSLER, A., and A. PLASTINO: N. C. **47** B, 297 (1967).

[3] PLASTINO, A., R. ARVIEU, and S. A. MOSZKOWSKI: Phys. Rev. **145**, 873 (1966).

[4] LE TOURNEUX, J., and J. M. EISENBERG: Nuclear Phys. **85**, 119 (1966).

[5] CLAUDEMANS, P. W. M., B. H. WILDENTHAL, and J. B. MCGRORY: Phys. Letters **21**, 427 (1966).

[6] FAESSLER, A., A. PLASTINO, and S. A. MOSZKOWSKI: To be published in Phys. Rev. — PLASTINO, A., A. FAESSLER, and S. A. MOSZKOWSKI: Bull Am. Phys. Soc. **11**, 320 (1966).

[7] FAESSLER, A., and A. PLASTINO: Nuclear Phys. A **94**, 580 (1967).

[8] FAESSLER, A., and A. PLASTINO: To be published in Phys. Rev.

[9] FAESSLER, A., and A. PLASTINO: Nuovo Cimento **48** B, 429 (1967).

nuclei shows that the SDI is equivalent to the conventional Pairing plus Quadrupole (PQF) or Octupole (POF) force. But the PQF and the POF have three free parameters while the SDI has only one coupling constant. (If one considers only the ^1S scattering. This is the only important mode of interaction in heavy nuclei.)

Here we want to apply the SDI to the strongly deformed Transuranic nuclei and compare these results with the ones obtained using the PQF or POF model and with the available experimental data.

In section 2 we shall shortly review the method for the solution of the Hamiltonian. In section 3 the single particle energies and the parameters of the Nilsson model are given. In section 4 we present and discuss our results. More explicitly, the γ-vibrational energies and the $K=0$, 1, 2, and 3 Octupole band heads are calculated. Futhermore, the $E1$, $E2$, and $E3$ transitions from the ground state into the 1^- $K=0$ Octupole vibrational band head, the 2^+ γ-vibrational state, and the 3^-, $K=0$, 1, 2, 3 Octupole vibrational levels are given. The Hamiltonian is solved using the Quasi-particle Random Phase approximation (RPA). These results are partially compared with values calculated employing the Quasi-particle Tamm Dancov approximation (TDA). The most astonishing result is that all the Octupole vibrational bands are collective with the exception of the $K=3^-$ band. The $E3$-transitions into this band are smaller than the corresponding ones into the other bands by a factor of about 0.01.

In section 5 we summerize the results.

2. Theory

We describe the nucleus with the Hamiltonian

$$H=\sum_{\alpha} \varepsilon_\alpha c_\alpha^\dagger c_\alpha+\tfrac{1}{4} \sum_{\alpha,\beta,\gamma,\delta} V^A_{\alpha\beta;\gamma\delta} c_\alpha^\dagger c_\beta^\dagger c_\delta c_\gamma . \tag{1}$$

The interaction is chosen to be the Surface Delta Interaction[1,2] (SDI):

$$V(1,2)=-4\pi F(u_0 R_0)^{-4} \delta(r_1-R_0)\delta(r_2-R_0)\delta(\theta_{12}) . \tag{2}$$

The basis $|\alpha\rangle$, $|\beta\rangle$, ... are Nilsson functions. The symbols c_α^+ and c_β are creation and anihilation operators for these states. The expression $V^A_{\alpha\beta;\gamma\delta}$ denotes the antisymmetrized matrix element of the SDI within the basis of the Nilsson functions. The ε_α's are single particle energies, while u_0 is the radial amplitude at the nuclear radius R_0 ($u(R_0)\equiv u_0$). We assume that this amplitude is constant. Calculations of radial eigenfunctions for a Saxon-Woods potential have shown[10,11] that this is a fair approximation. The constant F is the only free coupling parameter.

[10] BLOMQUIST, J., and S. WAHLBORN: Arkiv Fysik **16**, 545 (1960).

[11] FAESSLER, A., and R. K. SHELINE: Phys. Rev. **148**, 1003 (1966).

It has been shown[3,6] that the monopole part of the particle-particle element of the force (2) is state independent. One has therefore for this part of the interaction a simple pairing force. The factor 4π in Eq. (2) is chosen in such a way that a fit of the odd-even mass differences gives the same numerical value for the pairing parameter G and for the coupling constant F. The approximation to solve the Hamiltonian (1) either by the Random Phase approximation (RPA) or using the Tamm-Dancov approximation (TDA) has been described in references 6 and 7. The excited states are given in the RPA by the expression:

$$B^{i\dagger}_{K\pi}\,|\,\Psi_0\rangle=\sum_{\alpha\geqq\beta;\,\alpha>0}\{\xi^{i\,K\pi}_{\alpha\beta}\,A^{K\pi\,\dagger}_{\alpha\beta}-\eta^{i\,K\pi}_{\alpha\beta}\,A^{K\pi}_{\alpha\beta}\}\,|\,\Psi_0\rangle\,. \tag{3}$$

The sum runs over all $\Omega_\alpha\geqq\Omega_\beta$ with the projection of the angular momentum along the intrinsic axis $\Omega_\alpha>0$ and $K=\Omega_\alpha\pm\Omega_\beta$. The ket $|\,\Psi_0\rangle$ is the physical ground state. The operator $B^{i\dagger}_{K\pi}$ is the so called quasi-boson operator. It is a linear combination of two-quasi-particle creation $A^\dagger$ and anihilation operators A.

$$\begin{aligned}A^{K\pi\,\dagger}_{\alpha\beta}&=(1-\delta_{\alpha;\,\beta})\,2^{-\frac{1}{2}}\,[\alpha^\dagger_\alpha\alpha^\dagger_\beta+\alpha^\dagger_{\bar\alpha}\alpha^\dagger_{\bar\beta}]_{K\pi}\,,\\ A^{K\pi\,\dagger}_{\alpha\bar\beta}&=[2(1+\delta_{\alpha;\,\beta})]^{-\frac{1}{2}}\,[\alpha^\dagger_\alpha\alpha^\dagger_{\bar\beta}-\alpha^\dagger_{\bar\alpha}\alpha^\dagger_\beta]_{K\pi}\,.\end{aligned} \tag{4}$$

The bar indicates the time reversed state

$$|\,\bar\alpha\rangle=T\,|\,\alpha\rangle=(-)^{\frac{1}{2}-\Omega}\,|\,-\alpha\rangle \tag{5}$$

with a negative projection Ω of the angular momentum along the intrinsic axis. The two-quasi-particle states (4) are symmetrized under time reversal. The quantum number $K=\Omega_\alpha\pm\Omega_\beta$ is their projection onto the symmetry axis and π is the parity. The connection between the quasi-particle and particle operators is given by the Bogolyubov-Valatin transformation (see for example references [12,13]):

$$\alpha^\dagger_\alpha=u_\alpha\,c^\dagger_\alpha-v_a\,c_{\bar\alpha}\,. \tag{6}$$

We utilize the convention that the coefficients u and v are independent on the sign of Ω_α, but that

$$T\,|\,\bar\alpha\rangle=T^2\,|\,\alpha\rangle=-|\,\alpha\rangle\,. \tag{7}$$

The mixing coefficients ξ and η are obtained by diagonalizing a non-symmetric matrix. Then the superscript i distinguishes solutions for different eigenvalues.

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[12] Kisslinger, L. S., and R. A. Sorenson: Kgl. Danske Videnskab. Selskab, Mat.-fys. Medd. **32**, No. 9 (1960).

[13] Soloviev, V. G.: Nuclear Phys. **69**, 1 (1965). — Soloviev, V. G.: Atomic Energy Review **3**, 117 (1965). — Bes, D.: Nuclear Phys. **49**, 544 (1963).

If one utilizes the level scheme 2 of SOLOVIEV[13] as a basis for the Transuranic nuclei, one has to diagonalize matrices with a dimension larger than 500. This is a nummerically nonfeasible problem, if the matrix is not separable. We enforce separability using the approximations which are also employed in the PQF- and the POF-model: One neglects the exchange term and takes only the leading term in the Slater expansion for the particle-hole matrix element. (One utilizes for the γ-vibrations only the Quadrupole term $\lambda=2$ and for the Octupole vibrations only the octupole term $\lambda=3$.) In this approximation the particle-hole element (and the whole matrix) are seperable

$$V_{\alpha\bar{\delta};\bar{\beta}\gamma}\approx -F D^{\lambda K}_{\alpha\beta} * D^{\lambda K}_{\gamma\delta}. \tag{8}$$

The expressions $D^{\lambda K}_{\alpha\beta}$ are given by FAESSLER, PLASTINO and MOSZKOWSKI[6] for the SDI and the PQF or the POF model. We can now write the secular equation

$$F^{-1}=\sum_{\alpha\geqq\beta;\,\alpha>0}\{D^{\lambda K}_{\alpha\beta}\}^2\{[E_{\alpha\beta}-\omega]^{-1}+[E_{\alpha\beta}+\omega]^{-1}\} \tag{9}$$

and the mixing coefficients

$$\begin{aligned}\xi^{K\pi}_{\alpha\beta}&=N_{\lambda K}D^{\lambda K}_{\alpha\beta}[E_{\alpha\beta}-\omega]^{-1}\\ \eta^{K\pi}_{\alpha\beta}&=N_{\lambda K}D^{\lambda K}_{\alpha\beta}[E_{\alpha\beta}+\omega]^{-1}.\end{aligned} \tag{10}$$

The normalisation constant $N_{\lambda K}$ is given by the condition:

$$\sum_{\alpha\geqq\beta;\,\alpha>0}\{\xi^{K\pi^2}_{\alpha\beta}-\eta^{K\pi^2}_{\alpha\beta}\}=1. \tag{11}$$

The expressions for the TDA are the same. One has only to put $[E_{\alpha\beta}+\omega]^{-1}$ equal to zero in the Eqs. (3), (9), (10), and (11).

3. The Parameters

The level scheme 2 of SOLOVIEV[13-15] has been employed. It contains 43 proton levels and 45 neutron levels.

We have only changed slightly the energies of 5/2+[633] and the 1/2+[631] states for the neutrons into 0.725 $\hbar\omega$ and 0.965 $\hbar\omega$ respectively. These changes are suggested by experimental compilations of energy levels. For the Nilsson functions we utilized the spin-orbit parameter $\kappa=0.05$. The strength for the l^2-term has been chosen $\mu=0.45$, apart from the following values: $\mu_{N=1}(\text{protons})=\mu_{N=5}(\text{protons})=0.55$ and $\mu_{N=7}$ (neutrons) $=0.40$.

[14] SOLOVIEV, V. G., P. VOGEL, and A. A. KORNEICHUK: Izvest. Akad. Nauk S.S.S.R., Ser. Fiz. **28**, 1599 (1964).

[15] VOGEL, P.: Dubna publication E 1703 (1964). — SOLOVIEV, V. G., and P. VOGEL: Phys. Letters **6**, 126 (1963).

In Table 1 the coupling parameter for the SDI and the three parameters for the PQF- and POF-model are listed for the RPA and the TDA. The parameters are fitted by seven independent sets of data. The fit of $A \cdot F_p$, $A \cdot F_N$ for the SDI and $A \cdot G_p$, $A \cdot G_N$ for the pairing force by the odd-even proton and neutron mass differences yields for both models

Table 1. *Coupling parameters for the Transuranic nuclei*

The coupling parameter of the Surface Delta Interaction (SDI) and the parameters of the Pairing, the Quadrupole, and the Octupole force are fitted by independent data groups: The coupling constant F of the SDI was adapted by the odd-even mass differences of the protons and neutrons (F_p, F_N), the γ-vibrational band head (F_2), and the octupole vibrations with different K-values (F_{3K}). Although these values are derived from totaly different data, the coupling constant is within $\pm 27\%$ the same. This is not the case for the Pairing plus Quadrupole (PQF) or Octupole (POF) force. Here one finds three esentially independent parameters. The fits for the TDA show a much more wider spread for the coupling constant F. This reflects the poorer quality of this approximation.

		QRPA	QTDA
SDI	AF_P [MeV]	28—29	28—29
SDI	AF_N [MeV]	26—27	26—27
SDI	AF_2 [MeV]	29.4	53.5
SDI	AF_{30} [MeV]	33.5	59—64
SDI	AF_{31} [MeV]	18.5—20	—
SDI	AF_{32} [MeV]	18.5—20	—
SDI	AF_{33} [MeV]	18.5—20	—
PQF POF	AG_p [MeV]	28—29	28—29
PQF POF	AG_N [MeV]	26—27	26—27
PQF	k_2	9.65	15.6
POF	k_{30}	1.06	1.06—1.16
POF	k_{31}	0.60—0.67	—
POF	k_{32}	0.60—0.67	—
POF	k_{33}	0.60—0.67	—

the same parameters. This is due to the fact that, in the deformed nuclei, the odd-even mass difference depends mainly on the monopole part of the particle-particle element. This is state independent for the SDI, as for a pairing force. The value $A \cdot F_2$ and the quadrupole coupling constant for the PQF

$$\chi_2 = k_2 A^{-\frac{4}{3}} \hbar\omega \tag{12}$$

are adapted by the γ-vibrations. In the same way the parameters $A \cdot F_{3K}$ and

$$\chi_{3K} = k_{3K} A^{-\frac{4}{3}} \hbar\omega \tag{13}$$

for the POF in the Octupole vibrational band with the angular momentum projection K on the symmetry axes are fitted by the octupole vibrations with the projection K. If the SDI is a good force the independently fitted values F_p, F_N, F_2, and F_{3K} have to be nearly the same. This is fullfilled for the RPA. The spread for the TDA is much more larger. This is due to the poorer quality of this approximation. For the PQF- and the POF-model the pairing, the quadrupole, and the octupole constants have no connection with each other.

4. Results

The Figs. 1 and 2 show the energies of the γ-vibrational band heads in the Transuranic nuclei. A comparison exhibits as good results for the Surface Delta Interaction (SDI) as for the Pairing plus Quadrupole force (PQF). In Fig. 2 one has, for the Quasi-particle Tann Dancov approximation (TDA), to enlarge the coupling parameter F of the SDI from $A \cdot F = 29.4$ MeV, value used for the Quasi-particle Random Phase approximation (RPA), to $A \cdot F = 53.5$ MeV. Similarly one has to encrease

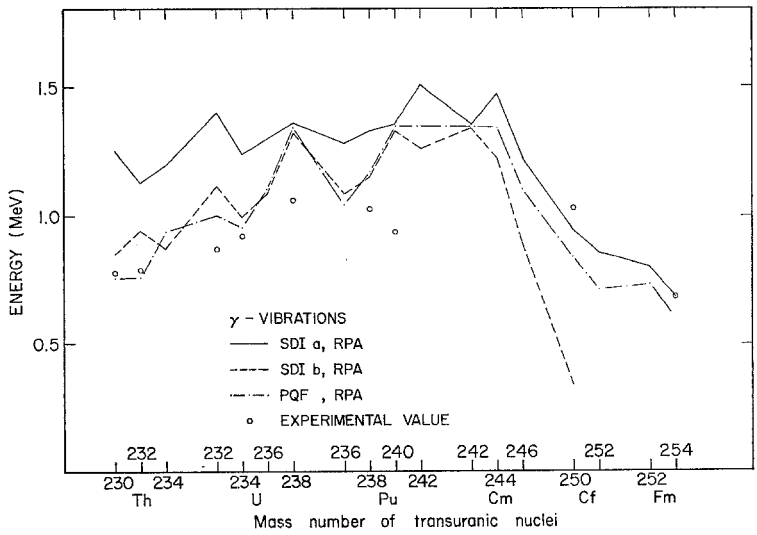

Fig. 1. The γ-vibrational energies are shown for the Surface Delta Interaction (SDI) and the Pairing plus Quadrupole model (PQF). The coupling parameters for the SDIa and the PQF are listed in Table 1. The coupling constant for the dashed line (SDIb) is AF = 33 [MeV]. The theoretical energies are all calculated in the Qusasi-particle Random Phase approximation (RPA)

the Quadrupole constant from 9.65 to 15.6 as displayed in Table 1. This renormalisation simulates the contributions from the back-going graphs which have been neglected in the TDA.

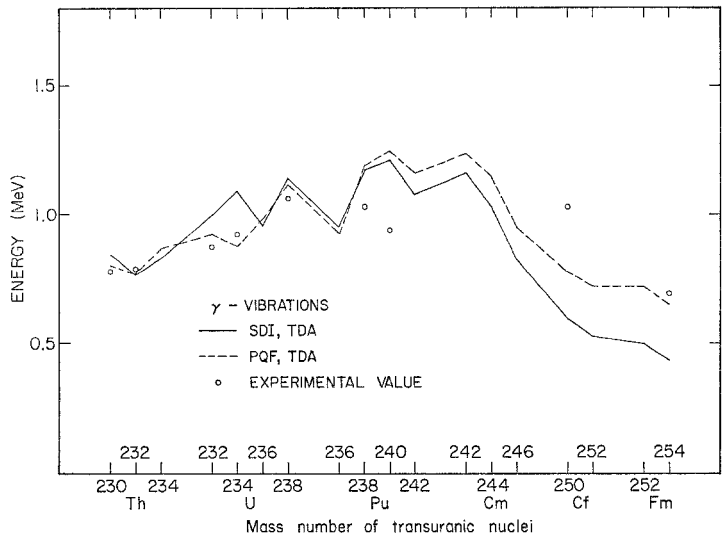

Fig. 2. The same results as in Fig. 1 are shown calculated with the Quasi-particle Tamm Dancov approximation (TDA). The coupling parameters F of the SDI and k_2 of the PQF are given in Table 1

Tables 2 and 3 exhibit the $E2$-transition in Single Particle Units

$$B(E\lambda; 0\to\lambda)=\frac{2\lambda+1}{4\pi}\left(\frac{3}{3+\lambda}R_0^{\lambda}\right)^2 [e^2\,\mathrm{cm}^{2\lambda}]. \tag{14}$$

The symbol R_0 is the nuclear radius. We adopted the expression $R_0 = 1.2\,A^{\frac{1}{3}}$ [fm]. The TDA, in Fig. 2, displays a less collective behavior of the γ-vibrational band than the one shown for the RPA in Fig. 1. This is a well known effect. It is due to the neglection of the back-going graphs. This yields the quasi-particle vacuum as the ground state. A lot of correlations are not taken into account in this way. The effective charge was chosen to be $e_{\mathrm{eff}}=Z/A$. This value is given by a selfconsistent argument of MOTTELSON[16] for nuclei far away from closed shells. This argument is only valid for $E2$-transitions. But we have taken the same value also for $E3$-transitions due to the lack of any theoretical argument. The effective charge for $E1$-transitions is uniquely given by momentum

[16] MOTTELSON, B. R.: The many body problem, p. 283. New York: John Wiley & Sons 1958.

Table 2. *Transitions in the RPA*

The electric transition probabilities $B(E2; 0_g^+ \to 2_\lambda^+)$, $B(E3; 0_g^+ \to 3_{K=0})$, and $B(E1; 0_g^+ \to 1_{K=0})$ from the ground state into the γ-vibrational band head and into states in the $K=0$ octupole vibrational band are listed for the SDI and the Pairing plus Quadrupole (PQF) or Octupole force. These are compared with the few available experimental data (Exp.). The transition probabilities are given in Single Particle Units (S.P.U.). The theoretical values are calculated in the Quasi-particle Random Phase approximation (RPA). The parameter of the SDI is compared to Table 1 enhanced to the value $AF_2=33$ [MeV] for the $E2$-transitions as for SDIb in Fig. 1. For the $E3$- and the $E1$-transition we chose the same value $AF_{30}=33.5$ [MeV] as in Table 1. But the mass parameter was taken to have always the value $A=232$ as for SDIb in Fig. 3. The effective charge for the $E2$- and $E3$-transitions is $e_{\text{eff}}=Z/A$. The effective charge for the $E1$-transitions is uniquely given by momentum conservation as the value $e_{\text{eff}}=-Z/A$. The experimental data and also these of Table 3 are taken from ref. [18].

Nuclei		$B_\gamma(E2)$ [S.P.U.]			$B_{K=0}(E3)$ [S.P.U.]			$B_{K=0}(E1)$ [S.P.U.]		
Z	A	SDI	PQF	Exp.	SDI	POF	Exp.	SDI	POF	Exp.
$_{90}$Th	230	6.1	5.7		46	31		.065	.22	
	232	5.6	4.8	3	34	23	12	.046	.16	
	234	4.8	3.7		32	23		.041	.15	
$_{92}$U	232	3.5	3.4		38	27		.058	.23	
	234	3.1	2.9		30	21		.043	.17	
	236	2.7	2.3		29	21		.039	.16	
	238	2.1	0.8	2	29	22	21	.036	.17	
$_{94}$Pu	236	2.4	2.2		25	19		.060	.20	
	238	2.1	1.8		24	18		.055	.19	
	240	2.1	0.16		24	18		.052	.20	
	242	2.7			22	17		.037	.17	
$_{96}$Cm	242	0.85	2.0		20	17		.084	.25	
	244	2.8	0.18		19	16		.065	.22	
	246	3.7	2.0		18	16		.046	.18	
$_{98}$Cf	250	—	3.6		10	11		.051	.17	
	252	—	4.4		10	11		.044	.17	
$_{100}$Fm	252	—	4.3		4.0	4.7		.040	.11	
	254	—	5.4		4.0	5.2		.036	.12	

conservation [17]. It has the value $e_{\text{eff}}=-Z/A$. The results for the $K=0$ Octupole vibrational band are displayed in Figs. 3 and 4 and in Tables 2 and 3. Table 1 shows also for this case the renormalisation of the coupling constants when one goes from the RPA to the TDA. It seems also that the TDA is not able to reproduce the variation of the $K=0$ Octupole

[17] GOLDHABER, M., and J. WENESER: Ann. Rev. Nuclear Sci. **5**, 1 (1955).

[18] ELBEK, B.: Determination of nuclear transition probabilities by Coulomb excitation. Copenhagen: Ejnar Munksgaards Forlag 1963.

Table 3. *Transitions in the TDA*

The same transitions as in Table 2 are compared with the experimental results. But the Quasi-particle Tamm Dancov approximation (QTDA) was employed for the theoretical values. The coupling constants are listed in Table 1.

Nuclei		$B(E2)$ [S.P.U.]			$B(E3)$ [S.P.U.]			$B(E1)$ [S.P.U.]		
Z	A	SDI	PQF	Exp.	SDI	POF	Exp.	SDI	POF	Exp.
$_{90}$Th	230	1.6	1.4		5.6	5.1		.0035	.028	
	232	1.4	1.3	3	5.9	5.3	12	.0033	.028	
	234	1.4	1.3		6.0	5.4		.0031	.028	
$_{92}$U	232	1.3	1.2		5.5	5.0		.0035	.032	
	234	1.1	1.0		5.7	5.2		.0033	.032	
	236	1.1	1.0		5.9	5.4		.0030	.033	
	238	1.4	1.2	2	5.9	5.3	21	.0026	.032	
$_{94}$Pu	236	1.0	0.92		5.5	5.0		.0064	.044	
	238	1.0	0.90		5.6	5.2		.0061	.044	
	240	1.2	1.0		5.6	5.2		.0056	.043	
	242	1.2	1.0		5.5	5.2		.0040	.039	
$_{96}$Cm	242	1.3	1.0		5.5	5.0		.013	.062	
	244	1.2	1.1		5.6	5.1		.011	.058	
	246	1.2	1.0		5.7	5.2		.0079	.052	
$_{98}$Cf	250	1.3	1.1		5.3	4.8		.018	.072	
	252	1.3	1.2		5.1	4.6		.015	.065	
$_{100}$Fm	252	1.4	1.2		4.5	4.4		.037	.10	
	254	1.4	1.3		4.3	4.2		.032	.093	

vibrational energy at the beginning of the Transuranic nuclei. The RPA yields too high energies at the end of the region.

Figs. 5, 6, and 7 and Table 4 display the energies and the $E3$-transitions $B(E3; 0^+ g \rightarrow 3^-, K)$ for the $K=1$, 2, and 3 Octupole vibrational bands.

Table 4 exhibits the fact that the $E3$-transitions from the ground state into the $K=1$ and $K=2$ Octupole vibrational bands are almost as collective as the transitions into the $K=0$ band. A comparision with Table 2 shows an enhancement of about a factor four for the gamma-ray transitions into the $K=0$ band over the gamma-rays into the $K=1$ and $K=2$ bands. This enlargement of the $K=0$ transitions is due to the greater number of two quasi-particle combinations with $K=0$ at the beginning of the Transuranic region. The $K=0$, 1, 2 transitions into the Octupole bands have at the end of the region (in the Fm-isotopes) about the same intensities. This result does not agree with a statement of VOGEL and SOLOVIEV[14,15], that only the gamma transitions into the $K=0$ Octupole band display a collective behavior.

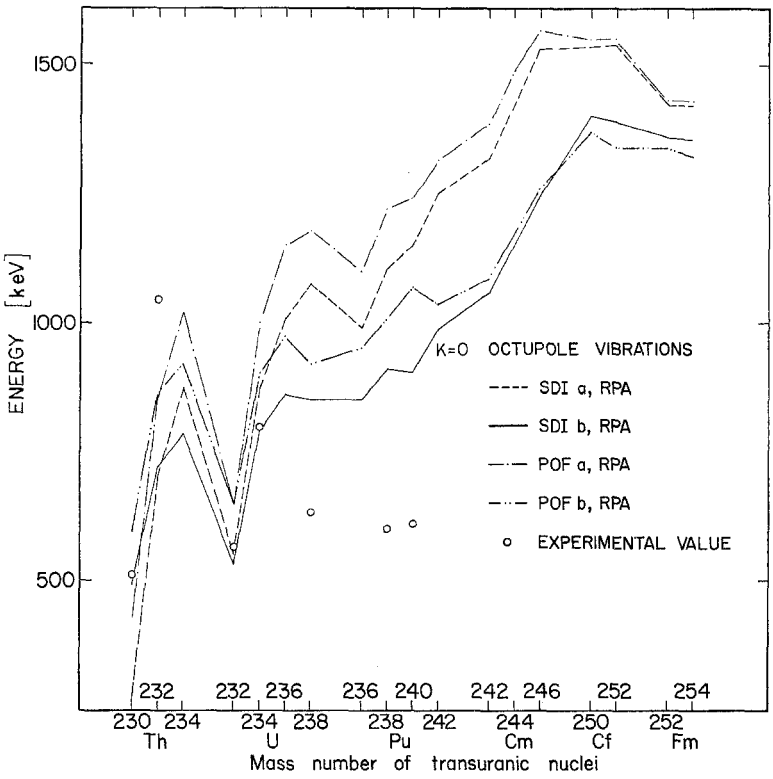

Fig. 3. The energies of the $I\pi=1^-$, $K=0$ band heads are shown. The parameters for SDIa and POFa are listed in Table 1. The curve SDIb is calculated with the coupling parameter AF=33.5 [MeV] as for SDIa. But the mass parameter A was chosen to be $A=232$ for the whole Transuranic nuclei. The theoretical points for POFb are calculated with the Octupole constant $k_{30}=1.06\ (A^{4/3}/A_0^{4/3})$. The value $A_0=232$ was chosen for POFb. This means one has for the mass number $A=232$ identical values as for POFa

5. Conclusion

The Surface Delta Interaction (SDI) is capable to describe the non-rotational states in the Transuranic nuclei. It was previously shown[2] that this simple force can be derived from a realistic nucleon-nucleon force which fits the two-body data. It has only one parameter F for heavy nuclei, where one has predominantly singlet-S scattering due to the general Pauli-principle. This single coupling parameter has been adapted by four independent groups of data in the Transuranic nuclei. The odd-even mass differences, the γ-vibrational energies, the $K=0$ Octupole vibrational band heads, and the collective $K=0$ negative parity

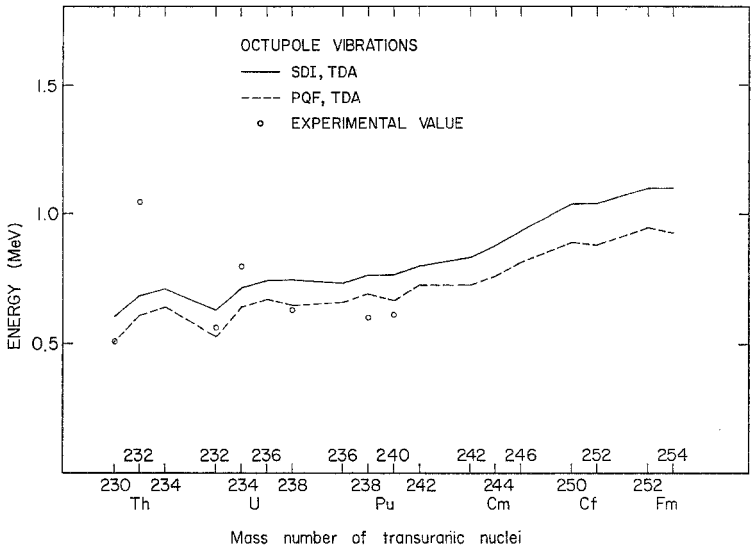

Fig. 4. The energies of the $K=0$ Octupole vibrational band heads are shown calculated by the Quasi-particle Tamm Dancov approximation (TDA). The coupling constants for the SDI and the POF given in Table 1

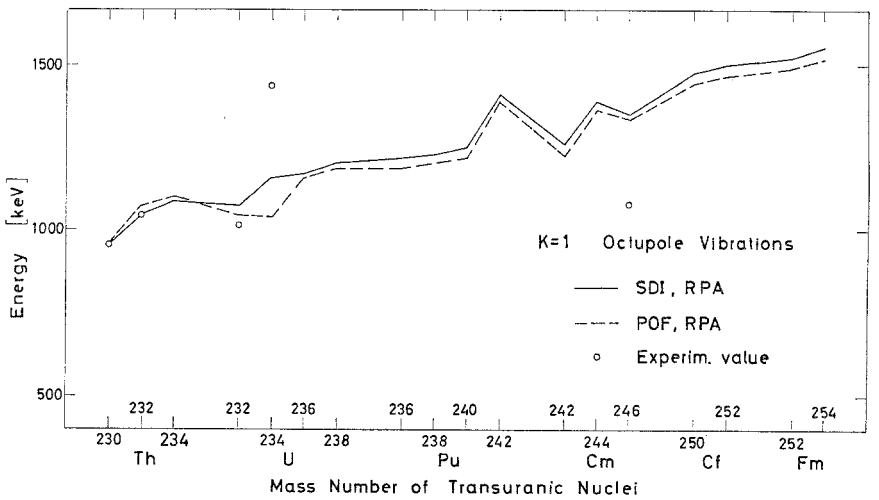

Fig. 5. The energies of the $K=1$ Octupole vibrational band head are compared with the available experimental data. The theoretical values for the SDI and the POF are calculated with the RPA. The coupling constants are recorded in Table 1

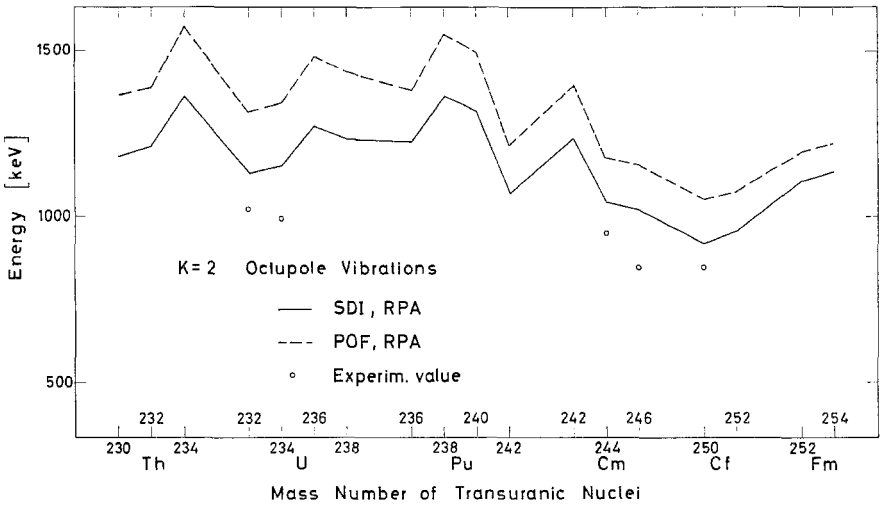

Fig. 6. The energies of the $K=2$ Octupole vibrational band head are compared with the available experimental data. The values are calculated with the SDI and POF in the RPA. The coupling constants are given in Table 1. They are the same as for the $K=1$ Octupole band

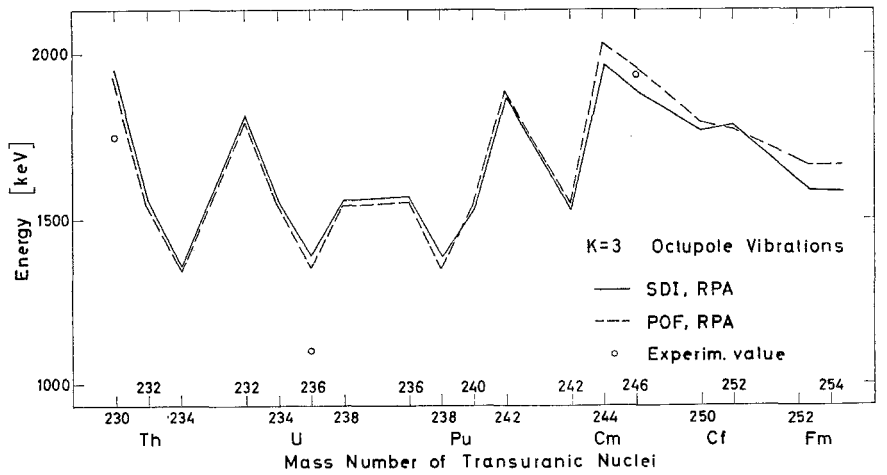

Fig. 7. The theoretical $K=3$ Octupole vibrational band heads are compared with the experimental data. The same parameters as for the $K=1$ and $K=2$ Octupole states are employed

Table 4. *Transitions in the RPA*

The transition probabilities $B(E3; 0_g^+ \to 3_K^-)$ between the ground state $|0_g^+\rangle$ and the 3^- state in the $K=1$, 2 and 3 octupole vibrational band are tabulated in Single Particle Units for the SDI and POF-model. The values are calculated within the RPA. Experimental data were not available to the authors.

Nuclei		$B_{K=1}(E3)$ [S.P.U.]			$B_{K=2}(E3)$ [S.P.U.]			$B_{K=3}(E3)$ [S.P.U.]		
Z	A	SDI	POF	Exp.	SDI	POF	Exp.	SDI	POF	Exp.
$_{90}$Th	230	13.0	10.8		7.3	3.5		0.022	0.011	
	232	10.0	7.8		7.0	3.2		0.010	0.004	
	234	8.6	6.7		7.4	3.7		0.010	0.004	
$_{92}$U	232	10.1	8.7		7.4	3.7		0.110	0.057	
	234	7.4	6.1		7.1	3.5		0.011	0.004	
	236	6.5	5.3		7.0	3.6		0.011	0.004	
	238	5.8	4.8		7.3	3.7		0.011	0.004	
$_{94}$Pu	236	5.3	4.7		6.2	3.1		0.011	0.004	
	238	4.9	4.3		6.5	3.4		0.012	0.004	
	240	4.2	3.8		6.6	3.5		0.012	0.004	
	242	5.1	4.5		6.4	3.3		0.347	0.107	
$_{96}$Cm	242	3.4	3.2		5.8	3.2		0.013	0.004	
	244	3.9	3.7		6.1	3.4		1.981	0.903	
	246	2.9	2.8		6.2	3.6		2.050	1.104	
$_{98}$Cf	250	2.4	2.5		5.9	3.7		0.924	0.036	
	252	2.8	2.9		5.3	3.4		0.397	0.026	
$_{100}$Fm	252	1.5	2.0		4.5	3.1		2.136	1.244	
	254	1.9	3.5		4.1	2.9		2.110	1.206	

bands. These fits yields (see Table 1) within $\pm 26\%$ the same value. This parameter agrees with the fits in the Rare Earth region and a selfconsistent theoretical calculation[6]. On the other side the Pairing plus Quadrupole (PQF) or Octupole (POF) force needs three essentially independent parameters to give the energies of these states.

The most striking result of this work follows from the gamma-ray transitions, which are displayed in Tables 2, 3, and 4. All the vibrational bands exhibit a collective nature. This is true also for the $K=1$ and the $K=2$ Octupole vibrational bands. The transitions are of the order of a few Single Particle Units. Only the intensities into the $K=3$ Octupole band are of the order of a single particle Nilsson transition.

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