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The Effect of Financing Decisions on the Choice of Manufacturing Technologies

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Abstract. This paper demonstrates the importance of jointly considering financing and technology choices when making manufacturing investments. We show that considerable value can be added to investments through financing decisions, and that the gains due to financing are sensitive to technology choice. A model of financing and technology choice is presented that considers differences in cost structure and product flexibility, and applies it to an example involving flexible manufacturing systems (FMSs). Three main results emerge. First, optimal financing decisions are different for different technologies and the choice of technology can change when financing and technology decisions are made simultaneously. Second, if one technology's fixed and variable costs are lower or its initial investment higher than another technology's the former has higher value added due to financing. Since empirical data shows that FMS and conventional technologies have this pattern, ignoring the benefits of debt financing leads to undervaluation of new technology. Third, product flexibility can add considerable value through its effect on financing decisions because product flexibility reduces variability of cash flows. A major conclusion is that financing and technology choice are long-term strategic decisions that should be made jointly. Firms that make these decisions separately, not considering the effect of one on the other, may make suboptimal technology decisions.

Key Words: technology, investments, finance, justification.

1. Introduction

This paper demonstrates the importance of jointly considering financing and technology choices when evaluating major manufacturing investments. By financing decisions we mean the mix of debt and equity used to finance the investment. We show that considerable value can be added through financing and the gains are sensitive to the choice of technology. Major investments requiring substantial amounts of capital are studied. These investments might be for a plant or expensive production equipment such as flexible manufacturing systems (FMSs), robotics, or computer-integrated manufacturing (CIM).

There are many factors that differentiate technologies. This paper focuses on technology's cost structure, product flexibility, and lifetime. When comparing technologies it is important to consider the effect of these factors on the cash flows and financing. We argue that new manufacturing technologies, such as FMSs, have very different characteristics than conventional systems such as labor-intensive job shops and automated but inflexible equipment. These differences change the way a firm finances investments and make new technologies more valuable.

This paper develops a model of financing and technology choice that considers differences in technology, corporate tax advantage of debt financing, costs of bankruptcy, and probability of bankruptcy. Our analysis results in three major conclusions. First, optimal financing decisions are different for different technologies, and the choice of technology can change when financing and technology decisions are made simultaneously. Second, if one technology's fixed and variable costs are lower or its initial investment higher than another technology's, the former has a higher value added due to financing. Empirical data shows that FMS and conventional technologies can have this pattern. Hence, ignoring the benefits of debt financing may lead to undervaluation of FMS. Third, product flexibility can add considerable value through its effect on financing decisions because product flexibility reduces the variability of cash flows.

The major managerial implication of this work is that financing and technology choice are long-term strategic decisions that should be made jointly. We show that firms that make these decisions separately, not considering the effect of one on the other, may make suboptimal decisions. Firms that delegate technology decisions to operations managers who choose technology ignoring its effect on financing, may not select the technology with the highest value. A conclusion is that general managers who desire to maximize firm value must understand technology, financing, and their interaction.

Several other results are derived. Not surprisingly, the value of a technology rises with its expected lifetime. It is also shown that the value of an incremental investment increases with diversification. This is because the value added by financing opportunities rises with diversification of cash flows.

The rest of the paper is organized as follows. Section 2 briefly reviews the literature on the economic evaluation of manufacturing investments in new technologies. It concludes with a review of the literature on the costs and benefits of debt financing, and the interaction between financing and investment decisions. Section 3 presents the model and discusses the effect of cost structure and financing on the value of technology. Section 4 presents examples to illustrate the main results of the paper. The concluding section summarizes, and suggests directions for future research.

2. Literature review on the economic evaluation of investments in new technologies

Many researchers have suggested improvements in the framework and models for evaluating manufacturing investments in new technologies. Most have focused on the interaction of technology with the firm's costs. Gaimon (1985) formulated models for the optimal acquisition strategies of automated technology considering the impact of automation on labor productivity. Lederer and Singhal (1988) showed how the appropriate discount rate for evaluating a technology depends on the technology's cost structure. Porteus (1985), Karmarkar and Kekre (1987), Vander Veen and Jordan (1989), and Keller and Noori (1988) studied interactions among technology investments, inventories, and setup reductions. Another major stream of research examined flexibility and how it affects profitability; see for example, Hutchinson and Holland (1982), Fine and Li (1988), Graves (1988), and Fine and Freund (1990). Finally, several papers have considered the option structure of investments; see for example, Monahan and Smunt (1989) and Andreou (1990). Burstein (1988) studied how modular adoption of FMS systems affects investment strategies. However, no papers have addressed how the interaction between technology and financing decisions affects technology choice.

2.1. Literature review on the determinants of financing policies

The source of the interrelationship between the economic evaluation of investments and financing is the interaction between the costs and benefits of debt financing. Modigliani and Miller (1958), under restrictive conditions, demonstrated that the market value of the firm is independent of the amount and kind of debt and equity used. This result followed from the assumption that capital markets are perfect, including that there are no taxes. Many authors have studied how financing policies would be affected when this assumption is relaxed. Modigliani and Miller (1963) showed that when interest payments are tax deductible, the firm's value rises as it uses more debt. However, this implied that firms should be financed entirely by debt, which is inconsistent with observed practice.

It has been noted that the corporate tax advantage of debt is offset by direct and indirect costs of bankruptcy, and the probability of bankruptcy. These costs reduce the tax advantage of debt financing and encourage firms to use both debt and equity. Direct costs include the out-of-pocket costs associated with the administrative expenses of bankruptcy, such as fees to lawyers, trustees, appraisers, etc.[1] Indirect costs include lost sales; lost profits; lost tax shields and credits; losses due to restrictions on a firm's production, investment, and financing decisions; increased costs due to renegotiation of contracts of employees and suppliers; and the loss in value from liquidation of the firm's assets below its economic value. Kraus and Litzenberger (1973), Scott (1976), Brennan and Schwartz (1978), Kim (1978), Turnbull (1979), and DeAngelo and Masulis (1980), among others, consider the issue of optimal capital structure by trading off the advantage of interest tax shields against various bankruptcy costs while considering the probability of bankruptcy.

Jensen and Meckling (1976) argue that while tax advantages and bankruptcy cost tradeoffs provide a theory of optimal capital structure, this theory is incomplete since it cannot explain the use of debt when interest payments were not tax deductible. They use the agency theory framework to study the effect of conflicts of interest among managers, bondholders, and stockholders on investment and financing decisions, and the benefits of bondholders monitoring managers. Their analysis helps explain various aspects of the capital structure problem including why an optimal capital structure exists and why debt agreements include restrictive covenants (also see Myers (1977) and Smith and Warner (1979)).

Several authors explore how investment decisions are affected by financing decisions. Hite (1977) develops a single-period investment model with the debt-related tax shields and shows that the optimal level of capital and labor and the output are affected by the amount of risk-free debt issued by the firm. Dotan and Ravid (1985) present a single-period model for determining capacity as a function of debt used. These two papers consider how investment choices are affected by changes in financing, but do not seek joint optima of investment and financing decisions.

Dammon and Senbet (1988) study the joint choice of investments and financing. They show how loss of tax shields in bankruptcy affects choice of production rate and financing in a single-period model. The technology is very simple, for example, the production rate does not affect operating costs. The effect of financial parameters, such as depreciation and tax rate, on financing decisions is studied, but not the effect of these parameters on technology choice.

3. A model for technology and financing decisions

This paper presents a multiperiod model of simultaneous financing and technology choice that more realistically considers the characteristics of manufacturing technologies. Our model is different from previous work in three ways. First, the cost structure assumed for investments is more detailed. Technology investments require an initial cash investment and incur fixed operating costs and per unit variable costs. Most previous work assumes an initial investment and per unit variable costs but ignores fixed operating expense. Fixed operating costs should be considered because manufacturing technologies have large fixed operating costs. Second, we consider a finite multiperiod model instead of the single-period approach of previous work. This allows analysis of the effect of the lifetime of an investment on its value. Third, the multiperiod approach introduces significant bankruptcy costs not present in a single-period model. In the event of bankruptcy, the firm loses the cash flows and tax credits expected to be generated in future periods. Stockholders declare bankruptcy when the value of their claims from continuing operations is negative. At the same time, bondholders claims may have a positive expected value if the firm continues. When stockholders declare bankruptcy, the value of the bondholders' claims can decrease. This conflict of interest between stockholders and bondholders introduces agency costs.

The following assumptions are made:

1. The firm can exist for N periods, denoted $t = 1, \ldots, N$. The firm manufactures and sells a single product at a known fixed price, P, in each period. The product's demand in each period, $\tilde{D}$, is assumed to be stochastic, stationary, independent, and normally distributed with mean $\bar{D}$ and standard deviation σ_D.
2. The firm chooses and invests in a technology at the beginning of period 1. After the investment, the firm does not change its technology. The cost structure of the technology is as follows. At the beginning of period $t = 1$, there is an initial investment of I. Each period the firm incurs fixed operating costs of F, and for every unit produced and sold it incurs a variable cost of C. Fixed and variable costs are paid at the end of each period. The demand for the product is revealed at the end of each period. Production is instantaneous and the quantity produced equals the quantity demanded. Revenues are realized at the end of each period. The technology is depreciated for taxes over N periods using straight-line depreciation method, so that at the end of period t, the book value of the asset is $((N - t)I)/(N)$.
3. The firm finances the initial investment, I, by issuing debt and equity. The terms of debt are that bondholders will receive payments equal to R at the end of each period for N periods. The firm issues debt only at the beginning of period $t = 1$. The payment R is assumed constant for all periods and is not changed.

4. If at the end of any period the firm is unable to pay bondholders the amount R, the firm is declared bankrupt. At bankruptcy, the assets of the firm are sold to meet obligations. It is assumed that the firm pays its obligations in the following order:
   1. Unit variable costs paid to workers and suppliers
   2. Fixed operating costs paid to managers and salaried personnel
   3. Interest and principal payments to bondholders
   4. Any remaining cash flows are paid to stockholders
5. If the firm goes bankrupt in period t, the liquidation value of the assets is a fraction γ of the book value of the assets, $(\gamma(N - t)I)/(N)$, with $\gamma < 1$. The assets cannot be sold unless the firm goes bankrupt.
6. Corporate profits are taxed at a constant rate, τ, and INT_t, the interest payments made by the firm to the bondholders in period t, are deductible for corporate tax purposes.
7. The risk-free rate of return, r_f, is the same for all periods. Investors are assumed to be risk neutral. Personal taxes of stockholders and bondholders are ignored.

Many of the above assumptions can be generalized. Assumption 1 can be generalized to many products by aggregating the cash flows for multiple products and also many investments. This is done in section 4.1, where product flexibility is discussed and in section 4.3, where the incremental value of technology to a diversified firm is considered. The assumption that the demand distribution is stationary and independent simplifies computation of cash flows and the value of debt and equity. It can be generalized at some cost of computational complexity, without qualitatively changing the results of the paper.

The importance of assumption 2 is that the firm selects a technology, utilizes it, and does not change it over a fixed period. Given that the development of a new technology can take years and the initial investment is large, the assumption that technology is utilized for at least a fixed period is a good first approximation.[2] The cost structure described is the one used by Lederer and Singhal (1988) to contrast the costs of FMSs and conventional technologies. The timing of costs and revenues are easily generalized. The assumption that production is instantaneous and equals demand can be generalized by allowing the firm to hold enough inventory to almost always satisfy demand. The effect of holding inventory is that the firm has higher per period fixed cost and higher liquidation value. Both of these factors can be easily added to the model. Any other tax depreciation schedule can be used instead of straight line.

Assumption 3 reflects restrictive covenants typically found in debt contracts which restrict firms from issuing additional debt unless some minimum financial ratios are maintained (see Smith and Warner (1979)). Assuming that the firm cannot issue any additional debt is a good first approximation which accords to the reality that firms do not issue bonds very often.[3] In fact, it is possible to generalize the model of this paper to allow early payment of debt or additional borrowing in each period. This change will add to the complexity of the model, but not change the basic trade-offs demonstrated here. We note that although issuing additional debt is forbidden, our model allows issuing additional equity when it is in the interest of shareholders to avoid bankruptcy.[4] The assumption that the firm can issue only straight debt and equity is very common in much of the literature, and consistent with the fact that in practice straight debt and equity are the primary methods of raising capital.[5]

Assumption 4 reflects the usual priority conventions in the event of bankruptcy. Assumption 5 reflects that the liquidation value of technology is some fraction of its book value. This can be generalized to assume that liquidation value is any arbitrary function of its time in service. Therefore, the book value of an investment and its liquidation value need not be related. We also assume that the technology cannot be sold except in the event of bankruptcy; in many debt contracts assets purchased with borrowed funds are held as security and cannot be sold without permission. Assumption 6 reflects current tax codes.

Assumption 7 about the risk-free rate and risk neutrality simplifies the analysis and avoids confounding issues of risk aversion with financing. Ignoring personal taxes similarly simplifies the model. All of these assumptions could be generalized with some cost of computational complexity, but basic results which follow will be unchanged. These assumptions are standard in the literature (see Hite (1977), Dotan and Ravid (1985), and Dammon and Senbet (1988)).

Next, these assumptions are used to derive an expression for the value of equity and debt for a given technology and fixed promised payment to the bondholders.

3.1. *The value of equity*

Let VE_t and VD_t be the value of equity and debt, respectively, at the beginning of period t. VE_t is a function of the expected cash flows to be received at the end of period t and all following periods. The cash flow available for distribution to the stockholders in period t is:

$$\tilde{X}_t = \text{Revenue} - \text{Costs} - \text{Taxes} - \text{Payment to Bondholders, or, using our notation,}$$

$$\tilde{X}_t = P\tilde{D} - (C\tilde{D} + F) - \tau[(P - C)\,\tilde{D} - F - \frac{I}{N} - INT_t] - R. \tag{1}$$

Simplifying, $\tilde{X}_t$ can be written as:

$$\tilde{X}_t = (1 - \tau)[(P - C)\tilde{D} - F] + \tau\frac{I}{N} + \tau INT_t - R. \tag{2}$$

The first term, $(1 - \tau)[(P - C)\tilde{D} - F]$, is the net operating income ignoring tax shields. The second term, $\tau(I/N)$, and the third term, τINT_t, are the tax shields from depreciation and interest payments, respectively. The final term, R, is the promised payment to bondholders.

$\tilde{X}_t > 0$ means that the firm can meet all its obligations during period t, and distributes $\tilde{X}_t$ as dividends to stockholders. It is optimal for the firm to distribute $\tilde{X}_t$ as dividends because if it does not and the firm goes bankrupt in the future, then the stockholders may partially (or fully) lose their claims on these funds. Hence, at the end of period t, the value of stockholders' equity is $\tilde{X}_t + VE_{t+1}$, the sum of the dividends received in current period and the value of the equity in the next period.

On the other hand when $\tilde{X}_t < 0$, the firm is unable to meet all its obligations from the cash flows generated in the current period. In such a situation, stockholders have two options: (a) They can contribute additional cash; or (b) they can declare bankruptcy. In evaluating thse two options, stockholders trade off the value of contributing additional cash in period t against the expected cash flows they hope to receive in future periods. If the first option is taken, stockholders contribute additional cash equal to $-\tilde{X}_t$. Clearly it is optimal for the stockholders to contribute additional cash if $\tilde{X}_t + VE_{t+1} \geq 0$. The value of stockholders' claim at the end of period t is then equal to $\tilde{X}_t + VE_{t+1}$.

But when $\tilde{X}_t + VE_{t+1} < 0$, it is optimal for the stockholders to declare bankruptcy instead of contributing additional cash. The probability of bankruptcy in period t can be expressed in terms of a critical demand level, a_t, such that if the demand in period t is less than a_t, it is optimal for the stockholders to declare bankruptcy. Using equation (2) and the condition that bankruptcy occurs when $\tilde{X}_t + VE_{t+1} < 0$, the value of a_t is:

$$a_t = \frac{R - \tau I/N - \tau INT_t - VE_{t+1}}{(1 - \tau)(P - C)} + \frac{F}{(P - C)}. \tag{3}$$

The following observations can be made about the probability of bankruptcy from equation (3). First, a_t, and hence the probability of bankruptcy, is an increasing function of the fixed costs per period, F, and the promised payment to bondholders, R. Second, tax credits on depreciation, $\tau(I/N)$, and on interest payments, τINT_t, and the value of equity next period, VE_{t+1}, reduce the value of a_t. Thus, an increase in the tax credits available to the firm and/or an increase in VE_{t+1}, reduces the probability of bankruptcy. Intuitively, stockholders have less of an incentive to declare bankruptcy when tax credits and VE_{t+1} are high since, in the event of bankruptcy, the tax credits are lost and VE_{t+1} equals zero. Third, since the interest payment, INT_t, and the next period's equity value VE_{t+1} decrease over time, a_t increases over time. Hence, the probability of bankruptcy increases over time.

In the event of bankruptcy the firm loses all its tax credits. The liquidated value of the firm at the end of period t, $\tilde{L}_t$, after the assets are sold and the fixed and variable costs are paid is

$$\tilde{L}_t = (P - C)\tilde{D} - F + \frac{\gamma(N - t)\,I}{N}. \tag{4}$$

The bondholders must be paid before stockholders. If the firm goes bankrupt in period t, the amount due to the bondholders at the end of period t, BD_t, is equal to R plus the value of the outstanding principal at the beginning of period $t + 1$.

The value of the outstanding principal at the beginning of period $t + 1$ can be computed as follows: suppose for a promised payment of R in each period, bondholders are willing to contribute VD_1, the market value of debt at the beginning of period $t = 1$ (the next section shows how VD_1 is determined). Given VD_1 and R one can determine the implied interest rate i^* demanded by the bondholders by solving the following equation:

$$VD_1 = \frac{R}{i^*}\left[1 - \frac{1}{(1 + i^*)^N}\right]. \tag{5}$$

Equation (5) states that the market value of debt is the present value of equal payments of R over N periods, discounted at a rate equal to i^*. Once i^* is found, the outstanding principal at the beginning of period $t + 1$ is the present value of the remaining $N - t$ payments of R discounted at i^*. The interest and principal due to the bondholders at the end of period t are:

$$BD_t = R + \frac{R}{i^*}\left[1 - \frac{1}{(1 + i^*)^{(N-t)}}\right], \tag{6}$$

where the second term on the right-hand side of equation (6) is the outstanding principal of the loan at the beginning of period $t + 1$. We note that the interest payment in period t, INT_t, is the product of the implied interest rate i^* and the principal outstanding at the beginning of period t.

In the event of bankruptcy, stockholders will receive $\tilde{L}_t - BD_t$ if $\tilde{L}_t > BD_t$, otherwise they receive nothing. Let b_t, be the demand level such that $\tilde{L}_t = BD_t$. Then using equation (4), b_t can be written as:

$$b_t = \frac{BD_t + F - \gamma(N - t)I/N}{(P - C)}. \tag{7}$$

Hence, in the event of bankruptcy, stockholders will receive $\tilde{L}_t - BD_t$ if demand during period t is less than a_t but greater than b_t.

Let $\widetilde{YE}_t$ be the uncertain value of the equity at the end of period t. Then $\widetilde{YE}_t$ can be written as:

$$\widetilde{YE}_t = \begin{cases} \tilde{L}_t - BD_t, & \text{if } b_t \le \tilde{D} < a_t \\ \tilde{X}_t + VE_{t+1}, & \text{if } \tilde{D} \ge a_t \end{cases} \tag{8}$$

The value of equity at the beginning of period t, VE_t is the expected value of $\widetilde{YE}_t$ discounted at the risk-free rate of return:

$$VE_t = \frac{E(\widetilde{YE}_t)}{(1 + r_f)}\ t = 1, \ldots, N. \tag{9}$$

where $E(\ )$ is the expectation operator. Appendix A gives the expression for $E(\widetilde{YE}_t)$ and shows the function dependence of VE_t on VE_{t+1}. Note that VE_{N+1} is equal to zero because the firm ceases to exist at the end of period $t = N$. The value of equity in each period can now be found by backward recursion of VE_N, VE_{N-1}, ..., VE_1 if VD_1 is known.

3.2. *The value of debt*

If the firm is not bankrupt at the end of period t, the value of debt by bondholders is $R + VD_{t+1}$: the sum of the payment received at the end of t and the value of debt in the next period. If the firm is bankrupt, the amount due to bondholders is BD_t. When the liquidated value of the firm, $\tilde{L}_t$, exceeds or equals BD_t, bondholders are paid the amount due. But, when the liquidating value of the firm is less than the amount due, bondholders receive the liquidating value. Let c_t, be the demand level such that $\tilde{L}_t = 0$. Then using equation (4), c_t can be written as:

$$c_t = \frac{F - \gamma(N - t)I/N}{(P - C)}. \tag{10}$$

Hence, bondholders will receive the liquidating value of the firm if demand during period t is less than b_t but greater than c_t.

Let $\widetilde{YD}_t$ be the uncertain value of the debt at the end of period t. Then $\widetilde{YD}_t$ can be written as:

$$\widetilde{YD}_t = \begin{cases} \tilde{L}_t & \text{if } c_t \leq \tilde{D} < b_t \\ BD_t & \text{if } b_t \leq \tilde{D} < a_t \\ R + VD_{t+1} & \text{if } \tilde{D} \geq a_t. \end{cases} \tag{11}$$

The value of debt at the beginning of period t, VD_t is the expected value of $\widetilde{YD}_t$ discounted at the risk-free rate of return:

$$VD_t = \frac{E(\widetilde{YD}_t)}{(1 + r_f)}\ t = 1, \ldots, N. \tag{12}$$

The expression for $E(\widetilde{YD}_t)$ is given in Appendix A and shows the functional dependence of VD_t on VD_{t+1}. Note that VD_{N+1} is equal to zero because the firm's debt obligation is over at the end of period N. The value of debt in each period can now be found by backward recursion of $VD_N, VD_{N-1}, \ldots, VD_1$ if $VE_N, VE_{N-1}, \ldots, VE_1$ are known.

We now have a system of equations in variables $VE_1, VE_2, \ldots, VE_N$ and $VD_1, VD_2, \ldots, VD_N$ given by equations (9) and (12), respectively. Appendix B uses the Brouwer fixed point theorem to show that a solution to the system of equations exists if and only if a solution to the fixed point problem $\phi(VD_1) = VD_1$ exists. Sufficient conditions for the solution to be unique are also stated. Appendix B describes an algorithm to calculate the value of debt and equity. For sample problems, the algorithm converges rapidly.

3.3 The value of technology

Suppose the firm has chosen its technology and has decided to offer debt with promised payments of R. At time $t = 1$, bondholders contribute the market value of debt, VD_1, in exchange for promised payments of R. Hence, the stockholders contribute $I - VD_1$, the difference between the initial investment and the amount received from bondholders. The value of technology to the stockholders is just the difference between the stockholders claim (VE_1) and their original contribution ($I - VD_1$):

$$VT = VE_1 - (I - VD_1) = VE_1 + VD_1 - I. \tag{13}$$

We refer to the expression in (13) as the value of the technology, VT, which is the net present value (NPV) of the technology, i.e., the difference between present value and initial investment.

Managers of the firm work for the stockholders and, therefore, their objective is to choose the technology and the mix of debt and equity that maximize VT. Holding technology fixed, the optimal financing decision from the stockholders' perspective is made by maximizing VT with respect to R. Let $VT(R)$ be the value of technology as a function of R, and let R^* be the promised payment, that maximizes $VT(R)$. The optimal decision from the stockholders perspective is to choose the technology with the highest $VT(R^*)$.

A technology can be parameterized by its cost structure, (I, F, C). The next two theorems analyze the effect of cost structure on the value of technology and the firm's optimal financing decision. These theorems demonstrate results under sufficient conditions that can be weakened. Each of the conditions is defined with respect to a fixed cost structure and a fixed value of R.

Condition A: $VE_{t+1} \geq (1 - \tau)\gamma I \dfrac{(N - t)}{N} + R - \tau\dfrac{I}{N} - \tau INT_t$ for all $t = 1, \ldots, N$

Condition B: $\dfrac{\tau}{(1 + r_f)^{N+2}} \dfrac{N(N + 1)}{2} \left(\dfrac{R}{VD_1}\right)^2 < 1$

Condition C: $a_t < \bar{D} + \sigma_D$, for all $t = 1, \ldots, N$

Condition D: $\dfrac{\tau}{(1 + r_f)^{N+2}} \dfrac{N(N + 1)}{2} \left(\dfrac{R}{VD_1}\right)^2 << 1$

Condition E: $\dfrac{d^2VT\,(R)}{dR^2} < 0$

Condition A implies that if bankruptcy occurs, there are no funds left to pay bondholders. In particular, for (8) and (11) cash flows occurring when $D < a_t$ are zero. Condition B implies that the firm expects to avoid bankruptcy for at least a few periods because the ratio of R to VD_1 is a fraction less than 1. Condition C implies that the probability of bankruptcy is less than 84% for each period, and this condition is consistent with condition B. Condition D is a stronger version of condition B; the firm expects to avoid bankruptcy for several periods. Condition E suggests that locally, the value of technology is a strictly concave function of the promised payment to bondholders. Conditions A, B, C, and D are not very strong. For realistic problems, these conditions hold at R^*. Numerical examples performed in the next section satisfy the conditions for all values of R less than R^*. Condition E is expected to hold at R^*.

Theorem 3.1: *(a) Suppose conditions A and B hold, then,*

$$\frac{dVT}{dC} < 0 \qquad \text{(14a)}$$

$$\frac{dVT}{dF} < 0 \tag{14b}$$

$$\frac{dVT}{dI} > -1 \tag{14c}$$

(b) Further, suppose that R^*, the promised payment to bondholders that maximizes the value of technology, is greater than zero, and at R^* Conditions C, D and E hold. Then,

$$\frac{dR^*}{dC} < 0 \tag{15a}$$

$$\frac{dR^*}{dF} < 0 \tag{15b}$$

$$\frac{dR^*}{dI} > 0 \tag{15c}$$

Proof. See Appendix C.

The first part of the theorem in (14a) reports that as the variable production cost per unit increases, the value of the technology declines. Result (14b) makes a similar statement about per period fixed cost. These results are expected because as the firm earns more from operations, the value of technology rises. However, matters are complex with financing because operations cash flow and interest tax shields are inversely related. For example, as F decreases, the interest rate, and, therefore, the interest payment, decreases. Under conditions A and B, the decrease in the value of the interest tax shields does not exceed the gains due to lower costs. Result (14c) states that as I increases, the value of the technology plus the initial investment ($VT + I$) rises. This is because an increase in investment raises the value of the depreciation tax shield. Under conditions A and B, the value of the interest tax shield falls, but not enough to offset the rise in the depreciation tax shield. Another way to state (14c) is that the derivative of $VT + I$ with respect to I is positive.

Results (14a) and (14b) imply that the derivative of $VT + I$ with respect to C and the derivative of $VT + I$ with respect to F are both negative. A corollary result easily follows: Consider a fixed value of R and two technologies such that the first has lower variable production cost, lower per period fixed cost, and higher initial investment than the second. Then, the first has higher value of technology plus the initial investment, ($VT + I$).

The second part of the theorem reports that if C falls, or F falls or I rises, then R^* rises. Intuitively, as cash flow generated by operations and depreciation tax shields rise, the probability of bankruptcy falls and the firm can increase its interest tax shields by raising its promised payments to bondholders. This result may not hold if conditions C, D, and E do not hold.

Different technologies have distinct values for two reasons: production costs differ and financing opportunities differ. Theorem 3.1 showed how changes in production cost affect the value of technology. The next results shows how financing opportunities affect the value of technology.

To measure the effect of financing on a technology's value, the valued added by financing, V^*_{add}, is defined as the difference between the value of the technology at the optimal bondholder payment and the value of technology with no debt: $V^*_{\text{add}} = VT(R^*) - VT(0)$. The next theorem shows that the value added by financing increases as variable cost per unit decreases, per period fixed cost increases, or the initial investment increases.

Theorem 3.2. *Consider two technologies, with different cost structures,* (I^i, F^i, C^i), $i = 1, 2$, *such that* $I^1 \geq I^2$, $F^1 \leq F^2$, *and* $C^1 \leq C^2$. *For* $i = 1, 2$, *let* R^{i*} *be the optimal payment to bondholders for technology i, let* V^{i*}_{add} *be the value added by financing for technology i and let* $VT^i(R)$ *be the value of technology i for promised payment R. If conditions A, C, and D hold for technologies* 1 *and* 2 *and all R such that* $0 \leq R \leq R^{2*}$, *then,*

$$V^{1*}_{\text{add}} > VT^1(R^{2*}) - VT^1(0) > V^{2*}_{\text{add}} > 0.$$

Proof. See Appendix C.

The theorem concludes that the value added by financing is positive. Holding the promised payment to bondholders constant, as production cost declines or the initial investment increases, the interest tax shield adds value to the firm. This occurs because the probability of bankruptcy falls and the interest tax shield is valuable for more periods. Also, theorem 3.2 shows that with a favorable cost structure, the optimal promisd payment to the bondholders can be increased. This raises the firm's value further due to additional interest tax shields.

The general conclusion of theorems 3.1 and 3.2 is that changes in cost structure can increase the value of technology for two reasons: operations cash flows increase because of lower costs or higher tax shields, and the value added by financing increases as well. The theorems help to clarify the economic benefits of modern production technology. The next section uses these results to analyze the effect of FMS's cost structure and flexibility on its value.

4. Optimal financing and technology choice: Examples

This section presents several examples of how cost structure affects financing and the value added by financing. The examples are based on data inferred from several empirical studies reported in Lederer and Singhal (1988). That paper presents a case study and summarizes published studies by Hartley (1983), Hollingum (1983), Sloggy (1984), Kaplan (1986), Jaikumar (1986), and Palframan (1987) on the economic evaluation of an FMS. The studies compare conventional, labor-intensive job shops with FMS systems designed to produce the same mix of parts. Lederer and Singhal (1988) conclude that the studies provide evidence that FMSs have a lower fixed operating cost per year (ignoring depreciation), and lower variable cost per unit, but require higher initial investment than conventional systems.

This conclusion is somewhat surprising. It is to be expected that variable costs are lower for less labor-intensive automated technology. What is unexpected is that FMSs have lower fixed cost per period. This is because the number of machines, number of operations, number of setups, number of support staff, and floor space all decrease. This observation is not definitive for all FMSs, but is a pattern observed in these studies. If these conclusions are correct, C and F are lower and I is higher for FMS compared with conventional systems. If some technical conditions hold, theorems 3.1 and 3.2 imply that R^* and V^*_{add} will be larger for FMS, so the firm can offer bondholders a larger promised payment and the value added by financing is larger for FMS than conventional technology. This implies that if financing opportunities are ignored, traditional economic evaluation procedures undervalue FMS.

This section presents a comparative economic evaluation of two technologies, labeled "FMS" and "Conventional" with respective cost structures described in table 1. These cost data are a variation of the data found in Lederer and Singhal (1988) for a system that fabricated 2,000 different metal parts. In that study the conventional technology was a labor-intensive job shop made up of numerous stand-alone machines. The FMS utilized new technologies such as laser metal cutting, robotics, automated material handling, and direct computer control. Consistent with our conjectures on cost, the per unit variable cost and the annual fixed operating cost are chosen to be lower for FMS than conventional technology, and the FMS's initial investment is much larger. The table also reports expected annual demand and the standard deviation of demand. The selling price of parts is assumed to be \$12.00 per unit.

In addition the following are assumed: (1) In the event of bankruptcy, the liquidation value of the technology is 50.0% of the book value (i.e, $\gamma = 0.5$); (2) the useful life of both the technologies is 10 years; (3) the corporate tax rate is 50.0% (i.e., $\tau = 0.5$); and (4) the risk-free rate of return, r_f, is 6.0%.

The optimal value of each technology was found by varying the value of R. For each value of R, the values of debt and equity that solve the system of equations in (9) and (12) was obtained by using the algorithm described in Appendix B. Summary results are presented in table 2 and in figures 1 and 2.

Table 1. Assumptions about the cost structure of conventional technology and the flexible manufacturing system.

Comparison	Conventional Technology	Flexible Manufacturing System
Variable cost/part	\$ 3.68	\$ 3.0
Annual fixed operating costs (excluding depreciation)	\$ 4.16 million	\$ 3.2 million
Initial investment	\$ 1.5 million	\$ 10.0 million

Expected annual demand for parts: 544,000/year
Standard deviation of demand: 136,000/year
Selling price per part: \$12.0

Table 2. Financing and technology choice using data of table 1.

	Conventional Technology	Flexible Manufacturing System
Value of technology with no debt ($R = 0$)	\$ 0.5 million	\$ −0.08 million
Optimal annual payment, R^*, to bondholders	\$ 0.19 million	\$1.2 million
Value of technology at R^*	\$ 0.65 million	\$ 1.0 million
Value added by financing at R^*	\$ 0.15 million	\$ 1.08 million
Value of debt at R^*	\$ 1.03 million	\$ 8.28 million
Interest rate on debt at R^*	12.6%	7.4%
$\frac{VD_1}{VD_1 + VE_1}$ at R^*	0.48	0.75
Probability of Bankruptcy at R^*	0.80	0.56

Choice of technology without debt financing: conventional technology
Choice of technology with debt financing: FMS

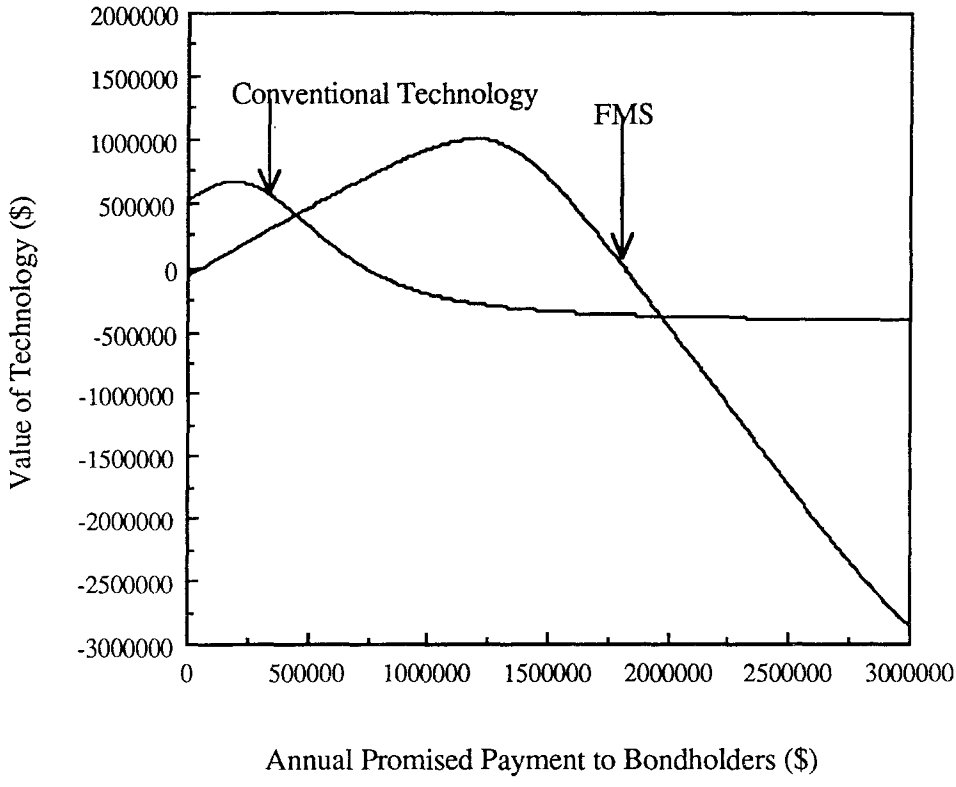

Figure 1. The behavior of the value of technology as a function of the annual promised payment to the bondholders, for the examples of table 1.

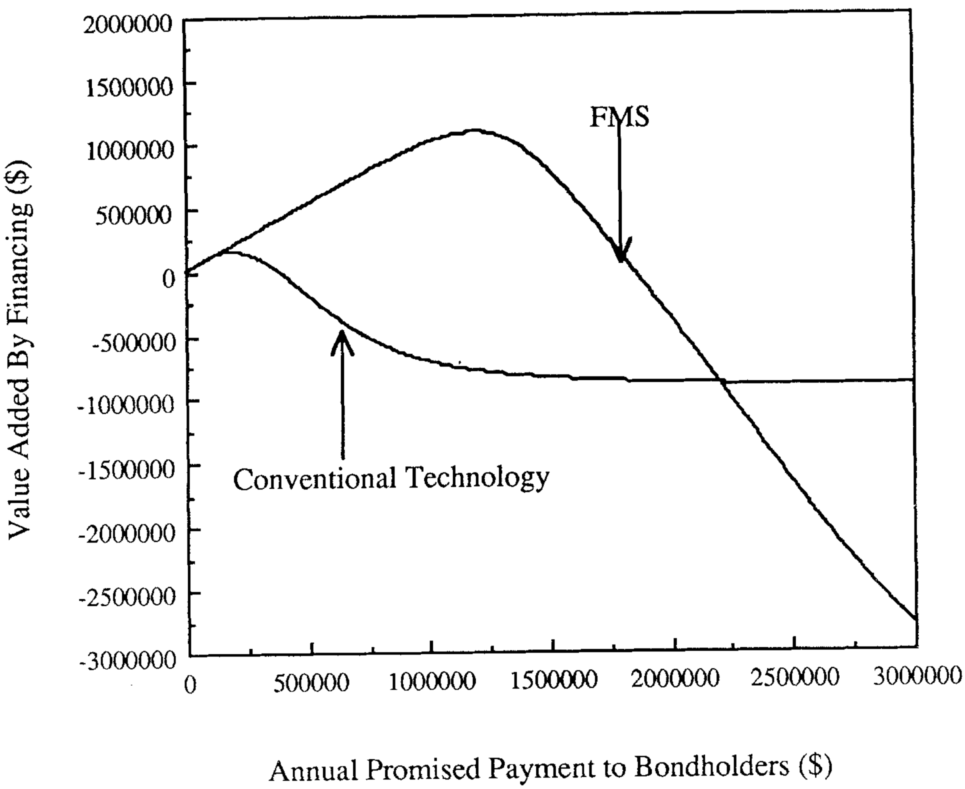

Figure 2. The behavior of the value added by financing as a function of the annual promised payment to the bondholders, for the examples of table 1.

Table 2 gives the value of the FMS and conventional technology without debt financing and with debt financing. It can be shown that conditions A, B, C, and D hold for all $R \leq R^*$. With no debt ($R = 0$), the FMS has a negative value and the conventional technology a positive value; \$−0.08 million versus \$0.5 million. At the optimal financing levels, the value of the FMS is \$1.0 million and the value of the conventional technology is \$0.65 million. As predicted by theorem 3.1, if $R = 0$, the value of technology plus the initial investment is larger for FMS than for conventional technology; −\$.08 million + \$10 million versus \$0.5 million + \$1.5 million. Consistent with theorem 3.1, the optimal promised payments to bondholders are higher for FMS than for conventional technology; \$1.2 million/year versus \$0.19 million/year. As expected from theorem 3.2, the value added by financing is larger for FMS than for conventional technology; \$1.08 versus \$.15 million. What is unexpected is the size of the difference.

The value of debt at optimal financing levels is greater for FMS than for conventional technology; \$8.28 million versus \$1.03 million. This means that bondholders are willing to pay more for their claims for the FMS. Furthermore, the interest rate is lower for the FMS than the conventional technology; 7.4% versus 12.6%. A common measure of the relative use of debt and equity is the ratio of the market value of the debt to the sum of the market value of debt and equity $(VD_1)/(VD_1 + VE_1)$. In this example the ratio is 0.75 for FMS and 0.48 for the conventional technology. Thus technologies with different cost structures have different financing mixes. Note that debtholders contribute a significant portion of the capital required. Shareholders need to contribute only $(I - VD_1)$, which equals \$1.72 million for the FMS and \$0.47 million of the conventional technology.

The probability of bankruptcy is higher with conventional technology than with FMS; 0.80 versus 0.56. Using equation (3) it is possible to calculate the probability of the firm going bankrupt in each period. While the probability of bankruptcy is high for both technologies, bankruptcy for FMS is more likely to occur in periods closer to the final period. The expected life of the firm using FMS is 8.96 years and for conventional technology it is 7.06 years.

Finally, note that technology choice changes when financing is considered. If financing is not considered, the optimal choice is to invest in conventional technology. On the other hand, if financing is considered, the optimal choice is to invest in FMS.[6]

Figure 1 depicts the behavior of the value of the conventional technology and FMS as a function of R, the annual promised payments to the bondholders. The value of each technology is a quasi-concave single-peaked function of R and has a finite nonzero optimal value of R. Note that for very large values of R, the value of technology declines and approaches an asymptotic value. This is because for very large values of R, the firm is almost certain to go bankrupt in the first period.[7]

Figure 2 depicts the behavior of value added by financing for conventional technology and FMS as a function of R. The figure dramatically shows that the value added by financing is greater for FMS than for conventional technology because of differences in the cost structure.[8]

4.1. *Effect of product flexibility on the value of technology*

Manufacturing technologies that allow firms to produce a portfolio of products are said to be product flexible. Product flexibility can be beneficial for three reasons. First, it can allow firms to increase profits by raising output or changing product mix to sell products with larger per unit contribution. Second, it can enable firms to introduce new products quickly, without incurring additional investment. Third, it can lower the variability of revenues faced by the firm because of the pooling effect of selling products that are not perfectly correlated. Lower variability of revenues decreases the probability of bankruptcy and the value of debt. Lower variability of revenues also allows a higher amount of debt financing, that is, an increase in the promised payment to bondholders. This, in turn, increases the value added due to financing.

Some qualitative insights about the third aspect of product flexibility can be obtained from our model by looking at the behavior of the value of the technology as the variability of demand changes. For this purpose, we hypothesize a production technology that can produce m different products. Each product has independent and identically distributed demand with mean of $\bar{D}/m$ units and standard deviation of σ_D/m. The selling price per unit, P, and the variable cost per unit produced, C, are constant and same for all the products. The technology to produce the m products requires an initial investment, I, and per period fixed cost F. Note that although the expected total demand per period, contribution margin, investment, and fixed cost are independent of the number of products, the variance of total demand declines as $1/m$ with the number of products.

For the FMS example of table 1, figure 3 shows that the value of the technology at the optimal debt financing increases with a fall in the variance of demand. This happens because the probability of bankruptcy is a decreasing function of demand variability, and the firm is likely to last longer. Furthermore, because the probability of bankruptcy is lower, the firm can raise more money from bondholders at more favorable terms. These factors together increase the value of the technology as the variance of demand decreases. For example, when the firm produces 81 products ($\sigma_D = 15{,}111$), the value of the FMS technology is 57.0% higher than when the firm produces a single product ($\sigma_D = 136{,}000$), even though the expected revenues in each period are identical for both cases. It seems that product flexibility can substantially increase the value of the technology merely by reducing the revenue uncertainty and, thereby, increasing the value added by financing.

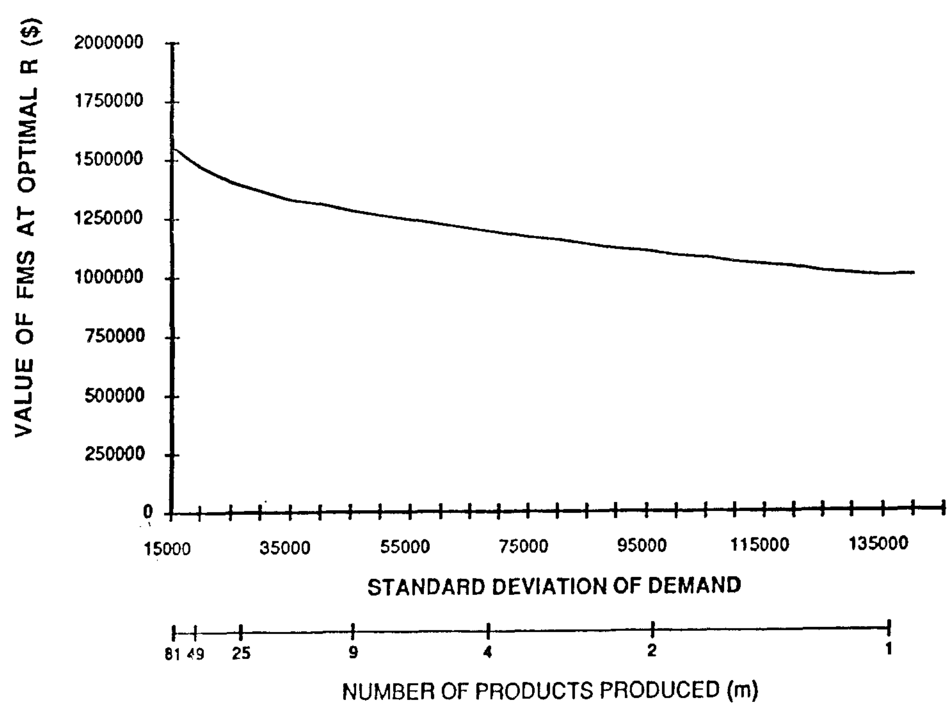

Figure 3. The behavior of the value of the FMS at optimal debt financing when the number of products, and thus the variability of total demand is varied. The FMS example is from table 1.

4.2. *Effect of the lifetime of investments on the value of technology*

The length of time an investment is expected to be in use will affect the value of the investment and the way it is financed. The lifetime assumed in cash flow calculations is a function of the way the technology wears out and how long the products it produces are expected to remain viable in the market. There is often an interaction between technology and products that affects an investment's useful life. For example, a product flexible technology can have an extended life compared with a less flexible technology because it can be used for future products.

All other things held constant, the longer the lifetime of an investment, the higher its value, and the larger the value added due to financing. This is because longer life yields more periods from which to collect cash flows. This raises the value of both debt and equity as larger future cash flows are forecast. Stockholders have more incentives to keep the firm from bankruptcy, lowering the probability of bankruptcy. This increases the value of debt, and encourages the firm to issue more debt, raising the value added by financing.

For example, table 3 reports the effect of different lifetimes (L), on the investment decision for the FMS described in table 1. Table 3 shows that as L increases, the value of the technology (VT_1^*) at optimal financing levels increases, as does the value added by financing (V_{add}^*). Note that the value added by financing increases by a factor of 9 as the lifetime increases from 5 to 25 years (a factor of 5). The table also reports the capital structure, measured by the ratio of the market value of the debt to the sum of the market value of debt and equity $(VD_1)/(VD_1 + VE_1)$, as a function of L. As expected, the ratio increases as L increases because the firm uses relatively more debt financing as the lifetime increases.

4.3. *Diversified firms and the value of an incremental investment in technology*

Typically, firms produce a number of products and make new investments to expand or supplement existing activities. The value of an incremental investment has two sources: the net present value of the cash flow from the investment plus additional financing opportunities provided by the new cash flow. The first source of value is almost always considered in investment decisions, but the second is not, especially when technology decisions and financing decisions are made separately.

Table 3. The effect of different lifetimes (L) on the value of the technology (VT_1^*), value added by financing (V_{add}^*), and the capital structure as measured by the ratio of the market value of the debt to the sum of the market value of debt and equity $(VD_1)/(VD_1 + VE_1)$. The values given in the table are for the optimal level of promised payment, R^*. The data used is the FMS example from table 1.

L (years)	VT_1^* ($ millions)	V_{add}^* ($ millions)	$\frac{VD_1}{VD_1 + VE_1}$
5	-1.82	0.395	0.697
10	1.0	1.08	0.753
15	3.35	1.87	0.772
20	5.31	2.71	0.805
25	6.95	3.56	0.857

If the cash flow from the incremental investment is not perfectly correlated with existing cash flows, the variability of the firm's cash flow can be reduced. This is because a combination of investments with less than perfectly correlated cash flows can reduce average cash flow variability per investment. Reduction of cash flow variability allows the firm to increase its borrowing, and raise the value added by financing. Because of this effect, diversification of activities is valuable. A diversified firm values an incremental project more highly than an undiversified firm, because of the value added by financing.

We apply our model to illustrate this. Consider a firm that produces M products where each product has independent and normally distributed demand with mean $\bar{D}$ and standard deviation σ_D, and sells it for P per unit. Each product requires a different technology which costs I to acquire, and has variable production cost per unit of C and fixed operating costs per period of F. The value of these M technologies and optimal financing can be found using equations (9) and (12). The total project has an initial investment of MI, fixed operating cost per period of MF, variable cost per unit of C, selling price per unit of P, normally distributed demand with mean $M\bar{D}$ and standard deviation of $\sqrt{M}\sigma_D$. The firm considers acquiring an additional technology with the same cost structure as the existing ones to produce a new product. The product's selling price is P with normally distributed demand having mean $\bar{D}$ and standard deviation σ_D, that is independent of the other products. We are interested in the value of acquiring this additional $(M + 1)^{\text{th}}$ technology as a function of M.

Using the FMS example from table 1, table 4 gives the value of the incremental investment in as a function of M, the existing number of investments already made. The table also gives the optimal capital structure of the firm (the ratio of the market value of debt to the sum of the market value of debt and equity) with M and $M + 1$ investments. Two observations can be made. First, diversification increases the value of the incremental investment. The table shows that at the optimal level of debt financing, the value of an incremental investment is an increasing function of the number of existing investments, and is always larger than the value of the technology when evaluated separately ($1.0 million from table 2). This is because the increased diversification of cash flows allows an increase in optimal debt financing, and value added by financing. Without additional debt financing opportunities, the incremental value of technology would be constant.

Table 4. Value of an incremental investment, the capital structure of the firm as measured by the ratio of the market value of the debt to the sum of the market values of debt and equity $(VD_1)/(VD_1 + VE_1)$, the amount of debt raised from the incremental investment, when the degree of diversification (M) is varied. The values in the table are for optimal level of debt. The data used is the FMS example from table 1.

M	Value of in Incremental Investment ($ millions)	$\frac{VD_1}{VD_1 + VE_1}$ M	$\frac{VD_1}{VD_1 + VE_1}$ $M + 1$	Value of debt raised from Incremental Investment ($ millions)
1	1.20	0.7534	0.7945	9.35
9	1.38	0.8646	0.8680	10.23
25	1.58	0.8931	0.8952	10.98
49	1.75	0.9142	0.9142	10.75
64	1.84	0.9236	0.9236	10.95

Second, as M increases, the firm adds more debt to its capital structure. This is clear as the ratio of the market value of debt to the market values of debt plus equity increases as M increases down a column or as M increases across a row. When M is small, each incremental investment adds significantly more debt to the firm's capital structure. However, as M becomes very large, capital structure of an incremental investment becomes close to that of the firm as a whole.

In particular, the way an incremental investment is financed changes as the firm becomes more diversified. For example when $M = 1$, the \$10.0 million required for the FMS is raised with \$8.28 million in debt (see table 2) and the remainder is obtained from the shareholders which is equal to $(I - VD_1)$, or \$1.72 million. When $M = 64$, an incremental investment is financed entirely with debt, issuing \$10.95 million in debt. The firm actually retains \$0.95 million in cash in excess of investment needs.

5. Summary

This paper has developed a model for jointly considering financing and technology decisions. It shows that considerable value can be added to a firm through financing decisions, and that these gains are sensitive to the choice of technology. Cost structure, product flexibility, lifetime of the investment, and firm diversification are shown to be important factors in financing and technology choice. An organizational lesson flowing from the results is the importance of general management involvement in technology choice decisions. Considerable value can be lost, if technology decisions are made by operations managers that ignore the effect of technology on financing.

There are number of directions for future research. This paper focused on one type of flexibility, product flexibility. The effect of volume flexibility and other technology-related options on financing can be considered. Such options include the ability change technology, add additional products, or change the cost structure. Aspects of risk that have been ignored in this paper are appropriate research areas. For example, the effect of implementation risk on financing and technology choice can be studied.

Acknowledgments

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Appendix A. Expressions for $E(\widetilde{YE}_t)$ and $E(\widetilde{YD}_t)$

This appendix derives the expressions for $E(\widetilde{YE}_t)$ and $E(\widetilde{YD}_t)$ (see equations (9) and (12)). Using equations (2), (4), and (8), $E(\widetilde{YE}_t)$ and be expressed as:

$$E(YE_t) = \int_{a_t}^{\infty} \left[(1 - \tau)((P - C)D - F) + \tau\frac{I}{N} + \tau INT_t - R + VE_{t+1}\right] g(D)dD$$

$$+ \int_{b_t}^{a_t} \left[(P - C)D - F + \frac{\gamma(N - t)\, I}{N} - BD_t\right] g(D)dD, \qquad (16)$$

where $g(\ )$ is the probability density function of the demand. For normally distributed demand, the partial mean of demand, $\tilde{D}$, is:

$$\int_{-\infty}^{X} Dg(D)dD = \bar{D}G(X) - \sigma_D^2 g(X), \tag{17}$$

where $\bar{D}$ is the expected value of demand; σ_D^2 is the variance of demand; and $G(\ )$ is the cumulative distribution function of demand. Using the result of equation (17), $E(\widetilde{YE}_t)$ is:

$$\begin{aligned} E(\widetilde{YE}_t) = & \left[(1-\tau)((P-C)\bar{D} - F) + \tau\frac{I}{N} + \tau INT_t - R + VE_{t+1}\right][1 - G(a_t)] \\ & + (1-\tau)(P-C)\ \sigma_D^2\ g(a_t) \\ & + [(P-C)D - F + \frac{\gamma(N-t)\ I}{N} - BD_t][G(a_t) - G(b_t)] \\ & - (P-C)\ \sigma_D^2\ [g(a_t) - g(b_t)], \end{aligned} \tag{18}$$

where a_t and b_t are given by equations (3) and (7), respectively.

Using equations (4) and (11), $E(\widetilde{YE}_t)$ can be expressed as:

$$\begin{aligned} E(\widetilde{YD}_t) = & \int_{a_t}^{\infty} [R + VD_{t+1}]g(D)dD + \int_{b_t}^{a_t} BD_t g(D)dD \\ & + \int_{c_t}^{b_t} \left[(P-C)D - F + \frac{\gamma(N-t)\ I}{N}\right] g(D)dD, \end{aligned} \tag{19}$$

Using the result of equation (17), $E(\widetilde{YD}_t)$ is given by:

$$\begin{aligned} E(\widetilde{YD}_t) = & [R + VD_{t+1}][1 - G(a_t)] + BD_t[G(a_t) - G(b_t)] \\ & + \left[(P-C)D - F + \frac{\gamma(N-t)\ I}{N}\right][G(b_t) - G(c_t)] \\ & - (P-C)\ \sigma_D^2\ [g(b_t) - g(c_t)], \end{aligned} \tag{20}$$

where a_t, b_t and c_t are given by equations (3), (7), and (10), respectively.

Appendix B. Existence, uniqueness of a solution, and a solution algorithm

This appendix shows that a solution to equations (9) and (12) exists for all $R \geq 0$. We can write the function in (9) as $VE_t[VD_1]$ and the function in (12) for $t = 1$ as $VD_1[VE_1, VE_2, \ldots, VE_N]$.

$VE_t[VD_1]$, $t = 1, \ldots, N$ are continuous functions of VD_1, and $VD_1[VE_1, VE_2, \ldots, VE_N]$ is a continuous function of $(VE_1, VE_2, \ldots, VE_N)$. VD_1 is always nonnegative and bounded from above by $(R/r_f)/(1 - 1/(1 + r_f)^N$. Therefore $VD_1 \in [0, (R/r_f)(1 - 1/(1 + r_f)^N)]$.

The composite function $VD_1[VE_1[VD_1], \ldots, VE_N[VD_1]]$ is continuous and maps the interval $[0, (R/r_f)(1 - /(1 + r_f)^N)]$ into itself. By the Brouwer fixed point theorem, the mapping $VD_1[VE_1[VD_1], \ldots, VE_N[VD_1]]$ has a fixed point: $VD_1[VE_1[VD_1^*], \ldots, VE_N[VD_1^*]] = VD_1^*$.

$(VE_1^*, \ldots, V_N^*, VD_1^*)$ solves (9) and (12) if and only if VD_1^* is a fixed point. Therefore at least one solution $(VE_1^*, \ldots, VE_N^*, VD_1^*)$ to equations (9) and (12) exists. If there is a unique fixed point then the solution is also unique. If

$$\phi(VD_1) = VD_1[VE_1[VD_1], \ldots, VE_N[VD_1]] - VD_1.$$

there is a unique fixed point iff $\phi(VD_1) = 0$ has a unique solution on the set $VD_1 \in [0, (R/r_f)(1 - 1/(1 + r_f)^N)]$. $\phi(VD_1) = 0$ has a unique solution if $\phi(VD_1)$ is one to one on the interval $[0,(R/r_f)(1 - 1/(1 + r_f)^N)]$. A condition sufficient for this to be true is that $\phi'(VD_1) < 0$ for all $VD_1 \in [0,(R/r_f)(1 - 1/(1 + r_f)^N)]$.

Description of the solution algorithm

For any fixed $R > 0$, an approximate solution to the systems of equations in (9) and (12) can be computed to any degree of precision $\epsilon > 0$ by the following algorithm. Let i^o be the old interest rate on the debt, i^c be the current interest rate on the debt, and s, $s > 0$, be the adjustment factor that increments i^o to i^c.

Step 1: Initialize the value of s and set $i^o = r_f$.
Step 2: Let $i^c = i^o + s$.
Step 3: Calculate the implied value of debt when interest rate equals i^c as:

$$VD_1^c = \frac{R}{i^c}\left(1 - \frac{1}{(1 + i^c)^N}\right).$$

Now by backward recursion, compute VE_t^c, $t = N, \ldots, 1$ using equation (9) with $VD_1 = VD_1^c$. Then by backward recursion on equation (12) compute VD_t^c, $t = N, \ldots, 1$ using VE_t^c, $t = 1, \ldots, N$. If $|VD_1 - VD_1^c| < \epsilon$ then stop. The resulting $(VE_1^c, \ldots, VE_N^c, VD_1^c, \ldots, VD_N^c)$ is the approximate solution. If $|VD_1 - VD_1^c| > \epsilon$ go to step 4.

Step 4: If $VD_1 < VD_1^c$, then the current interest rate is low. Set $i^o = i^c$, $i^c = i^c + s$, and go to step 3. If $VD_1 > VD_1^c$, then go to step 5.

Step 5: If $VD_1 > VD_1^c$, then the current interest rate is high. VD_1 equals VD_1^c for some interest rate in the interval (i^o, i^c). Set s equal to $s/2$ and $i^c = i^c - s$ and go to step 3.

Appendix C. Proofs of theorems 3.1 and 3.2

Proof of theorem 3.1

To show (14a) we demonstrate the result $dVT/dC_t < 0$ for all t where C_t is the variable cost per unit for period t. Then, $dVT/dC = \Sigma_{t=1}^{N} dVT/dC_t < 0$ and (14a) is proved. Choose $t = 2$. Let C be the variable cost for all periods, except $t = 2$ where the variable cost is C_2. Changes in C_2 affects VT in two ways: it changes the cost of production and the interest payments made: This implies that:

$$\frac{dVT}{dC_2} = \frac{\partial VT}{\partial C_2} + \sum_{t=1}^{N} \frac{\partial VT}{\partial INT_t} \frac{\partial INT_t}{\partial C_2}. \tag{21}$$

If condition A holds then VT can be expressed as:

$$VT = -I + E \begin{cases} \dfrac{(1-\tau)((P-C)\tilde{D}_1 - F) + \tau I/N + \tau INT_1 + VD_2 + VE_2}{(1+r_f)} & \text{if } \tilde{D}_1 \geq a_1 \\ 0 & \text{otherwise,} \end{cases} \tag{22}$$

and

$$VD_2 + VE_2 = E \begin{cases} \dfrac{(1-\tau)((P-C_2)\tilde{D}_2 - F) + \tau I/N + \tau INT_2 + VD_3 + VE_3}{(1+r_f)} & \text{if } \tilde{D}_2 \geq a_2 \\ 0 & \text{otherwise,} \end{cases} \tag{23}$$

where a_t is given by equation (3).

By direct computation from equations (22) and (23):

$$\frac{\partial VT}{\partial C_2} = \frac{1}{(1+r_f)} \left[\frac{(R + VD_2)\, g(a_1) \dfrac{\partial VE_2}{\partial C_2}}{(1-\tau)(P-C)} \right] - \frac{1}{(1+r_f)^2} \left[\bar{G}(a_1)(1-\tau) \int_{a_2}^{\infty} D_2 g(D_2) dD_2 \right]$$

$$- \frac{1}{(1+r_f)^2} \left[\bar{G}(a_1)(R + VD_3)\, g(a_2) \frac{a_2}{(P-C_2)} \right], \tag{24}$$

where $g(\;)$ is the probability density function of the demand, and $\bar{G}(\;)$ is the cumulative greater than distribution function of demand.

It must be that $\partial VE_2/\partial C_2 < 0$: the value of equity declines when variable cost rises. Thus all three terms in the right-hand side of equation (24) are negative so that $(\partial VT/\partial C_2) < 0$.

From equation (6) we have $INT_t = R[1 - (1/(1 + i^*)^{(N-t+1)})$. Differentiating with respect to i^*, we get $(\partial INT_t/\partial i^*) = R((N - t + 1)/(1 + i^*)^{(N-t+2)}$. Approximating i^* from equation (5) as $i^* = (R/VD_1)$, we have $(\partial i^*/\partial C_2) = -(R/(VD_1)^2)(\partial VD_1/\partial C_2)$. Therefore,

$$\frac{\partial INT_t}{\partial C_2} = \frac{\partial INT_t}{\partial i^*}\frac{\partial i^*}{\partial C_2} = -\left(\frac{R^2}{VD_1}\frac{(N - t + 1)}{(1 + i^*)^{(N-t+2)}}\frac{\partial VD_1}{\partial C_2}\right). \tag{25}$$

Now, $(\partial VT/\partial INT_t) \cong (\tau/(1 + R_f)^t)\,\bar{G}(a_1)\bar{G}(a_2)\ldots\bar{G}(a_t)$. Therefore,

$$\sum_{t=1}^{N}\frac{\partial VT}{\partial INT_t}\frac{\partial INT_t}{\partial C_2} = -\sum_{t=1}^{N}\left[\frac{\tau}{(1 + r_f)^t}\bar{G}(a_1)\bar{G}(a_2)\ldots\right.$$
$$\left.\bar{G}(a_t)\left(\frac{R}{VD_1}\right)^2\frac{(N - t + 1)}{(1 + i^*)^{(N-t+2)}}\frac{\partial VD_1}{\partial C_2}\right]. \tag{26}$$

The right-hand side of equation (26) is > 0, since $(\partial VD_1/\partial C_2) < 0$, but is less than

$$\frac{-\tau}{(1 + r_f)^{N+2}}\frac{N(N + 1)}{2}\left(\frac{R}{VD_1}\right)\frac{\partial VD_1}{\partial C_2}$$

But,

$$\frac{\partial VD_1}{\partial C_2} = \frac{1}{(1 + r_f)}\left[\frac{(R + VD_2)\,g(a_1)\frac{\partial VE_2}{\partial C_2}}{(1 - \tau)(P - C)}\right] - \frac{1}{(1 + r_f)^2}\left[\bar{G}(a_1)(R + VD_3)\,g(a_2)\frac{a_2}{P - C_2}\right] \tag{27}$$

which is identical to the sum of the first and third term of the right-hand side of equation (24). Therefore, by condition B:

$$\frac{dVT}{dC_2} = \frac{\partial VT}{\partial C_2} + \sum_{t=1}^{N}\frac{\partial VT}{\partial INT_t}\frac{\partial INT_t}{\partial C_2} < 0.$$

A similar argument can be repeated for all t, ($t = 2$ is an example of the general case, $t = 1$ and $t = N$ are much easier) and $(dVT/dC_t) < 0$ and (14a) is proved. A similar argument based upon the period-by-period changes in F and I can be used to show (14b) and (14c).

To show the second set of conclusions (15a), (15b), and (15c), a similar argument is used. Again we show (15a) first, by studying $(\partial^2 VT/\partial C_t \partial R)$. We choose $t = 2$ to demonstrate the idea. If condition D holds, then $(dVT/dC_2) \cong (\partial VT/\partial C_2)$. By direct computation we have:

$$\frac{\partial^2 VT}{\partial C_2 \partial R} = \frac{1}{(1 + r_f)}\frac{\left(1 + \frac{\partial VD_2}{\partial R}\right)g(a_1)\frac{\partial VE_2}{\partial C_2}}{(1 - \tau)(P - C)} \tag{28a}$$

$$+ \frac{1}{(1 + r_f)} \frac{\frac{\partial g(a_1)}{\partial D} \frac{\partial a_1}{\partial R} \frac{\partial VE_2}{\partial C_2} (R + VD_2)}{(1 - \tau)(P - C)} \quad \text{(28b)}$$

$$+ \frac{1}{(1 + r_f)} \frac{(R + VD_2)\, g(a_1) \frac{\partial^2 VE_2}{\partial C_2 \partial R}}{(1 - \tau)(P - C)} \quad \text{(28c)}$$

$$+ \frac{1}{(1 + r_f)^2}\, g(a_1) \frac{\partial a_1}{\partial R} (1 - \tau) \int_{a_2}^{\infty} Dg(D) dD \quad \text{(28d)}$$

$$+ \frac{1}{(1 + r_f)^2}\, G(a_1)\, (1 - \tau)\, a_2\, g(a_1) \frac{\partial a_2}{\partial R} \quad \text{(28e)}$$

$$- \frac{1}{(1 + r_f)^2}\, g(a_1) \frac{\partial a_1}{\partial R} (R + VD_3)\, g(a_2) \frac{a_2}{(P - C_2)} \quad \text{(28f)}$$

$$- \frac{1}{(1 + r_f)^2}\, \bar{G}(a_1) \left(1 + \frac{\partial VD_3}{\partial R}\right) g(a_2) \frac{a_2}{(P - C_2)} \quad \text{(28g)}$$

$$- \frac{1}{(1 + r_f)^2}\, \bar{G}(a_1)(R + VD_3) \frac{\partial g(a_2)}{\partial D} \frac{a_2}{(P - C_2)} \quad \text{(28h)}$$

$$- \frac{1}{(1 + r_f)^2}\, \bar{G}(a_1)(R + VD_3)\, g(a_2) \frac{\frac{\partial a_2}{\partial R}}{(P - C_2)}. \quad \text{(28i)}$$

All these terms are negative (condition C, $a_t < \bar{D} + \sigma_D$, is a sufficient condition for (28b) and (28h) to be negative) except for (28c), (28d), and (28e). However, it can be shown that (28c) is smaller in absolute value than (28b); (28d) − (28a) and (28e) − (28g) can be similarly paired. Therefore, $(\partial^2 VT_1(R^*)/\partial C_2\, \partial R) < 0$.

If R^* is an interior optimum for $VT(R)$, then $dVT(C_2, R^*)/dR = 0$. The implicit function theorem shows that since dVT/dR is a C^2 function of (C_2, R),

$$\frac{dR^*}{dC_2} = \frac{-\frac{\partial^2 VT(C_2, R^*)}{\partial C_2\, \partial R}}{\frac{\partial^2 VT(R^*)}{\partial^2 R}}.$$

By hypothesis the denominator is less than zero, so $dR^*/dC_2 < 0$. This argument can be repeated for all $t \neq 2$. Therefore, (15a) holds. In a similar fashion, (15b) and (15c) can be demonstrated.

Proof of theorem 3.2

We demonstrate the result for $C^1 < C^2$, $F^1 = F^2$, $I^1 = I^2$. In the proof of theorem 3.1 we showed that $\partial^2 VT(C_t, R)/\partial C_t \partial R < 0$ if conditions C, D, and E hold. This implies that $\partial^2 VT(C, R)/\partial C\, \partial R < 0$ as well. Then for any fixed R such that $0 \leq R \leq R^{2*}$,

$$\int_{C^1}^{C^2} \frac{\partial^2 VT(C, R)}{\partial C\, \partial R}\, dC = \frac{\partial VT(C^2, R)}{\partial R} - \frac{\partial VT(C^1, R)}{\partial R} \leq 0$$

Integrating again:

$$\int_0^{R^{2*}} \left[\frac{\partial VT(C^2, R)}{\partial R} - \frac{\partial VT(C^1, R)}{\partial R} \right] dR = VT(C^2, R^{2*}) - VT(C^1, R^{2*}) - [VT(C^2, 0) - VT(C^1, 0)]$$

$$= VT\,(C^2, R^{2*}) - VT\,(C^2, 0) - [VT\,(C^1, R^{2*}) - VT\,(C^1, 0)]$$

$$= V^{2*}_{\text{add}} - [VT\,(C^1, R^{2*}) - VT\,(C^1, 0)] < 0.$$

But by (15a) from theorem 3.1 we have $VT\,(C^1, R^{1*}) > VT\,(C^1, R^{2*})$. Thus

$$V^{1*}_{\text{add}} - V^{2*}_{\text{add}} > 0.$$

This result can be generalized for any $C^1 \leq C^2$, $F^1 \leq D^2$, and $I^1 \geq I^2$.

Notes

1. Baxter (1967) and Stanley and Girth (1971) report that these costs can be as much as 20% of the value of the firm, whereas Warner (1977) finds that for a sample of railroad firms the administrative expenses average about 1% of the market value of the firm prior to bankruptcy.
2. Note, that under this assumption, the valuation formulas developed in this paper can be used to explore the value of a fixed sequence of technology investments. Also, the effect of the option structure of technology on financing can be found. By using the valuation formulas developed here in backward recursion, the value of a sequence of technology choices that are conditioned upon future uncertain events can be found.
3. One explanation for this is the transaction costs involved in issuing debt.
4. It is important to note that the financial structure of the firm as measured by ratio of the market value of debt to the sum of the market values of debt and equity changes from period to period in the model. Therefore, the financial structure of the firm is not constant even with assumption 3. Also note, that a model that forbids additional borrowing gives a lower bound on the value of technology compared to the case when changes in debt financing are permitted.
5. Other types of hybrid securities such as convertible bonds and preferred equity shares can be used to finance investments. Inclusion of these hybrid securities can affect the associated agency costs. However, agency costs

cannot be completely eliminated, nor can the loss of interest rate and depreciation tax shields when bankruptcy occurs. Therefore, an optimal capital structure will continue to exist.

6. In computing the numerical results presented in this section, the tax rate, τ, is assumed to be 0.5. A reduction in tax rate will reduce both the optimal level of promised payment to the bondholders and the value added due to financing. However, given the differences in the cost structure of the technologies, both the optimal level of promised payment to the bondholders and the value added due to financing will be higher for the FMS than the conventional technology even with a lower tax rate. Furthermore, the value added from financing can still change the technology choice.
7. If the firm goes bankrupt in the first period, the value of the technology is simply the discounted value of the sum of the first period's pretax operating profits and the salvage value of the technology, less the initial investment, i.e.,

$$VT = \frac{(P - C)\bar{D} - F + \dfrac{\gamma I(N-1)}{N}}{(1 + r_f)} - I.$$

If I is much larger than the single-period pretax expected profits, the right-hand side of the above equation is approximately equal to

$$I\left(\frac{\gamma(N-1)}{N(1 + r_f)} - 1\right),$$

which is the loss to the firm for scrapping the technology after using it for a single period. The asymptotic behavior of the value of the conventional technology is evident from figure 1.

8. For large values of R, the value added by financing is smaller for FMS than conventional technology. If R is large enough so that the probability of bankruptcy in the first period is close to one for either technology, the value added by financing is $I(\gamma(N-1)/N(1 + r_f) - 1)$ *minus* the value of the technology with no debt. Although the value of the FMS technology with no debt is lower than that of the conventional technology ($-0.08 million versus $0.5 million), the FMS requires a much larger initial investment ($10.0 million versus $1.5 million). Therefore, the loss of scrapping the FMS at the end of the first period is much higher than that of the conventional technology. This is true despite the fact that the scrap value of the FMS is higher than that of the conventional technology. However, it is not optimal to issue so much debt that the probability of bankruptcy is close to one in the first period itself.

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