Current partition in multiprobe conductors in the presence of slowly oscillating external potentials

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Abstract. In the presence of a static potential drop a carrier stream incident at a contact of the sample is partitioned into the other contacts according to the transmission probabilities of the sample. The bare response to oscillating potentials, on the other hand, violates current conservation due to the piling up of unscreened charges in the sample, and has to be modified by taking the induced screening potential into account. We present a novel derivation of the conductance response to oscillating external chemical potentials, find the response to an arbitrary internal potential in terms of functional derivatives with respect to the local potential of the scattering matrix of the conductor, and determine the screening potential for slowly oscillating potentials from the condition of local charge neutrality. We find that the current partitioning depends on ratios of local densities of states which reflect the injection and emission properties of the contacts of the sample.

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I. Introduction

In a mesoscopic conductor with unequal static chemical potentials μ_α at the contacts $\alpha = 1, 2, 3, \ldots$ a carrier stream incident in probe β is partitioned into the differing probes of the conductor according to the total transmission probabilities $T_{\alpha\beta}$ of the conductor. The current at probe α is determined by [1]

$$I_\alpha = \sum_\beta g_{\alpha\beta} (\mu_\beta / e), \tag{1}$$

where at $kT = 0$, $g_{\alpha\alpha} = (e^2/h)(M_\alpha - R_{\alpha\alpha})$ and $g_{\alpha\beta} = -(e^2/h) T_{\alpha\beta}$ for $\alpha \neq \beta$. Here M_α is the total number of channels below energy E in contact α and $R_{\alpha\alpha}$ is the total reflection probability for carriers incident in contact α. The current is conserved since $M_\alpha = R_{\alpha\alpha} + \sum_\beta T_{\alpha\beta} = R_{\alpha\alpha}$

$+ \sum_\alpha T_{\alpha\beta}$. The transmission and reflection probabilities depend on the equilibrium electro-static potential. The actual potential distribution inside the sample which arises in the presence of transport is irrelevant [1, 2]. In contrast in the presence of oscillating voltages applied to the contacts of the sample the partitioning of currents is in part determined by the *internal* potential distribution which unscreened charges induce. This internal potential in turn generates currents in the contacts of the sample. We present a solution of this problem for a metallic conductor (in an insulating environment). The solution invokes three steps: First we determine the current and density response to external potentials which act only on the carriers in the contacts [3]. In a second step the internal potential is calculated using Thomas-Fermi screening of the excess carriers injected into the sample as a consequence of the external perturbation. In a third step we find the current induced into the contacts by the internal potential.

Both ac-theories and ac-experiments are of considerable current interest. We can refer the reader only to a very few recent works [4–7]. Typically ac-theories treat the response to a field which is presumed to be known everywhere inside the sample. In the most simple cases the electric field is even taken to be uniform throughout the sample. The discussion presented below dispells the notion that an ac-response can be discussed assuming the field to be known. What counts is the response to the actual, sample-specific self-consistent potential. Compared to formal response theory [7] our results are simple and provide physical insight.

First we present a direct and simple calculation of the current reponse to external chemical potential oscillations: in earlier work [3] the external response was obtained with respect to preturbations specified in terms of electrical and magnetic potentials. The external response was found by evaluating a linear response kernel. The derivation given below avoids any appeal to formal linear response theory. Second we derive a novel result which gives the response to an arbitrary internal potential. For slowly oscillating potentials this response can be expressed in terms of functional derivatives with respect
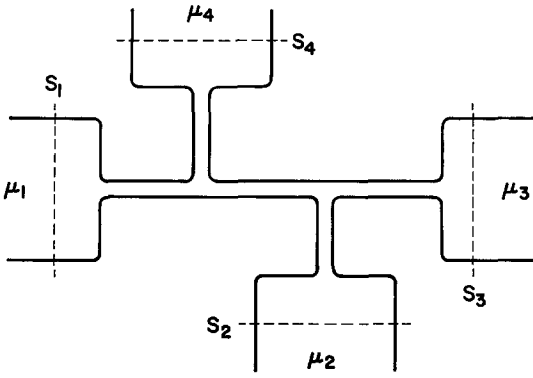
Fig. 1. Multi-probe conductor with wide metallic contacts at time-dependent chemical potentials μ_α. Currents are evaluated at the surfaces S_α located deep in the contacts

to the local potential of the scattering matrix of the conductor.

We consider a metallic conductor with an arbitrary number of contacts $\alpha = 1, 2, 3, \ldots$ as shown in Fig. 1. The contacts widen into massive metallic conductors which serve as equilibrium electron reservoirs. At these contacts the conductor is connected to an external macroscopic circuit. The external circuit imposes at each contact either a voltage (electro-chemical potential) or a current. The external circuit is described by a macroscopic admittance matrix. We are intersted in the relation between currents and voltages at the contact of the sample if one or more of the externally imposed currents or voltages are oscillating functions of time. For frequencies which are not too large the metallic conductor of interest here is in a locally charge neutral state at all times. Hence, like in the dc-case, the sum of all frequency-dependent currents must add up to zero. The crucial problem which must be solved is thus to find a *current conserving* relationship between the frequency-dependent currents and the frequency-dependent voltages (chemical potentials) at the contacts of the sample.

II. Response to external perturbation

We begin our analysis with the investigation of the response of the conductor to oscillating chemical *contact* potentials. Initially, we consider non-interacting electrons subject to an equilibrium confinement potential and (possibly) subject to an elastic impurity potential. At the contacts the conductor is connected to electron reservoirs at potentials $\mu_\alpha(t) = \mu_0 + \delta\mu_\alpha \cos(\omega t)$ where μ_0 is the equilibrium electro-chemical potential and $\delta\mu_\alpha$ is the modulation amplitude of the chemical potential at contact α. The modulation of the *chemical* potential gives rise to a perturbation $-\alpha^\dagger_{\alpha m} a_{\alpha m} \delta\mu_\alpha(t)$ in the (grand canonical) Hamiltonian of the system. Here $a_{\alpha m}$ annihilates a carrier in contact α in quantum channel m and $a^\dagger_{\alpha m}$ creates a carrier in contact α in quantum channel m. As a consequence of the modulation [8] of the potential the time evolution of the operator $a_{\alpha m}$ acquires an extra phase

$$\phi(t) = -\int dt\, \delta\mu_\alpha \cos(\omega t)/\hbar = -(\delta\mu_\alpha/\hbar\omega)\sin(\omega t)$$

and is given by

$$a^1_{\alpha m}(E, t) = a_{\alpha m}(E)\exp\left(-i(E_{\alpha m} - \mu_0)t/\hbar + \phi(t)\right).$$

We are interested in the linear response and can expand $\exp(-i\phi(t))$ to first order in $\delta\mu_\alpha$. In addition to an amplitude at E the time evolution of $a(E, t)$ now contains side-bands at energy $E + \hbar\omega$ and $E - \hbar\omega$. Alternatively we can say that the annihilation operator $a^1_{\alpha m}(E)$ at energy E in the presence of the potential modulation is a superposition of carriers which have been annihilated at energy E and remain unaffected by the presence of the modulation, by carriers which have been annihilated at energy $E + \hbar\omega$ and have lost a modulation quantum, and by carriers which have been annihilated at energy $E - \hbar\omega$ and have gained a modulation quantum,

$$a^1_{\alpha m}(E) = a_{\alpha m}(E) + a_{\alpha m}(E + \hbar\omega)(\delta\mu_\alpha/2\hbar\omega)$$

$$- a_{\alpha m}(E - \hbar\omega)(\delta\mu_\alpha/2\hbar\omega). \quad (2)$$

The annihilation operators used here obey the commutation rule $[a^\dagger_{\alpha m}(E), a_{\beta n}(E')]_+ = \delta_{\alpha\beta}\delta_{mn}\delta(E - E')$. These operators are normalized such that $(e/h)^{1/2} a_{\alpha m}(E)\, dE$ is a current amplitude [9]. The current incident in channel m in contact α is determined by $I_{\alpha m} = (e/h) \times (a^\dagger_{\alpha m}(t) a_{\alpha m}(t))$. The outgoing current in channel m in contact α is $I_{\alpha m} = -(e/h)(b^\dagger_{\alpha m}(t) b_{\alpha m}(t))$ where the operator b annihilates a carrier in the outgoing channel m in contact α. Using the Fourier transforms of the vectors $\mathbf{a}(t)$ and $\mathbf{b}(t)$ gives for the current operator [9]

$$I_\alpha(t) = \frac{e}{h}\int dE\, dE'\, [\mathbf{a}^\dagger_\alpha(E)\,\mathbf{a}_\alpha(E')$$

$$- \mathbf{b}^\dagger_\alpha(E)\,\mathbf{b}_\alpha(E')]\exp(i(E - E')t/\hbar). \quad (3)$$

The a and b operators are connected by a unitary transformation [9] which is (in matrix notation) the scattering matrix, $\mathbf{b}_\alpha = \sum_\beta \mathbf{s}_{\alpha\beta}\mathbf{a}_\beta$. To be definite we assume that each reservoir is intersected by a surface Ω_α and that the scattering matrices relate the incident current amplitudes on these surfaces to the out-going current amplitudes on these surfaces. To proceed we solve (2) for a, and use (3) to find the current. Invoking the standard equilibrium statistical assumptions [9] for the a^1 we find an equilibrium admittance matrix with elements [3] $g^e_{\alpha\beta}(\omega) \equiv e\langle\delta I_\alpha(\omega)\rangle/\delta\mu_\beta(\omega)$,

$$g^e_{\alpha\beta}(\omega) = \frac{e^2}{h}\int dE\, \mathrm{Tr}[\mathbf{1}_\alpha\,\delta_{\alpha\beta}$$

$$- \mathbf{s}^\dagger_{\alpha\beta}(E)\,\mathbf{s}_{\alpha\beta}(E + \hbar\omega)]\frac{f(E) - f(E + \hbar\omega)}{\hbar\omega}. \quad (4)$$

Equation (4) gives the currents at the sample contacts in response to the external (index e) chemical potentials. The unit matrix $\mathbf{1}_\alpha$ has dimensions equal to the number of quantum channels below the Fermi energy in lead α. We are interested in the low-frequency limit of (4). To first order in ω we find from (4),

$$g^e_{\alpha\beta}(\omega) = g_{\alpha\beta}(0) - i\omega e^2(dN_{\alpha\beta}/dE), \quad (5)$$

where we have introduced the temperature dependent density of states

$$\frac{dN_{\alpha\beta}}{dE} = \frac{1}{4\pi i} \int dE \left(-\frac{df}{dE}\right)$$

$$\times \mathrm{Tr}\left[\mathbf{s}_{\alpha\beta}^{\dagger}(E) \frac{d\mathbf{s}_{\alpha\beta}(E)}{dE} - \frac{d\mathbf{s}_{\alpha\beta}^{\dagger}(E)}{dE}\mathbf{s}_{\alpha\beta}(E)\right]. \quad (6)$$

In contrast to the zero-frequency case, where $\sum_{\beta} g_{\alpha\beta} = \sum_{\alpha} g_{\alpha\beta} = 0$ due to current conservation, the sum of the frequency-dependent conductances (4) is in general not zero. To linear order in ω we find

$$\sum_{\beta} g_{\alpha\beta}^{e}(\omega) = -i\omega e^2 (dN_{\alpha}/dE), \quad (7)$$

where $dN_{\alpha}/dE = \sum_{\beta} dN_{\alpha\beta}/dE$ is the density of states of carriers incident in all contacts and *emitted* at contact α. Similarly

$$\sum_{\alpha} g_{\alpha\beta}^{e}(\omega) = -i\omega e^2 d\bar{N}_{\beta}/dE \quad (8)$$

is determined by the total density of states $d\bar{N}_{\beta}/dE = \sum_{\alpha} dN_{\alpha\beta}/dE$ of carriers *injected* in probe β irrespective through which probe they leave the conductor. If no magnetic field is present $dN_{\alpha}/dE = d\bar{N}_{\alpha}/dE$. Because current is not conserved, (4) cannot describe the partitioning of currents correctly. The simple law $I_{\alpha}(\omega) = \sum_{\beta} g_{\alpha\beta}^{e}(\omega)\mu_{\beta}(\omega)/e$ is not invariant under a common shift in potential of all applied voltages. The additional charges brought into the conductor by the time-dependent external potential create an internal Coulomb potential $U_i(\mathbf{r}, t)$.

III. Response to internal potential

Next we wish to find the response to a given *internal* potential $U_i(\mathbf{r}, t)$. We assume that this potential is non-vanishing only inside the sample. We can Fourier transform this potential with respect to time and consider a perturbation of the form $v(\mathbf{r})(U_{+\omega}\exp(-i\omega t) + U_{-\omega} \times \exp(+i\omega t))$. Since the potential is real we have $U_{-\omega} = U_{+\omega}^*$. Again we consider the response to such a potential from a scattering point of view. Due to the oscillating internal potential a carrier incident with energy E can gain or loose modulation energy $\hbar\omega$ during reflection at the sample or during transmission through the sample. The amplitude of an outgoing wave is again a superposition of carriers created at energy E and at the side-band energies. Let us denote the scattering amplitude with which a carrier appears in an outgoing channel m in lead α at energy $E \pm \hbar\omega$ if it was incident at energy E in channel n in probe β by $s_{\pm,\alpha\beta mn}(E \pm \hbar\omega, E)eU_{+\omega}$. With this notation we find for the operator b which annihilates a carrier in an outgoing channel,

$$b_{\alpha m}(E) = \sum_{\beta n}(s_{\alpha\beta mn}(E)a_{\beta n}(E)$$

$$+ s_{-,\alpha\beta mn}(E, E+\hbar\omega)eU_{-\omega}a_{\beta n}(E+\hbar\omega)$$

$$+ s_{+,\alpha\beta mn}(E, E-\hbar\omega)eU_{+\omega}a_{\beta n}(E-\hbar\omega)). \quad (9)$$

Using (9) and (3) a simple calculation, similar to the one that leads to (4), gives for the current at probe α, $I_{\alpha}(\omega) = g_{\alpha}^i(\omega)U_{+\omega}$, a conductance $g_{\alpha}^i(\omega) = -(e^2/h)\sum_{\beta}\int dE\,\mathrm{Tr}\,\square$ with

$$\mathrm{Tr}\,\square \equiv \mathrm{Tr}[\mathbf{s}_{\alpha\beta}^{\dagger}(E)\mathbf{s}_{+,\alpha\beta}(E+\hbar\omega, E)f(E)$$

$$+ \mathbf{s}_{-,\alpha\beta}^{\dagger}(E, E+\hbar\omega)\mathbf{s}_{\alpha\beta}(E+\hbar\omega)f(E+\hbar\omega)]. \quad (10)$$

Particle conservation requires that all outgoing channels are completely filled if all incident channels at all energies are completely filled. The particular constraint due to unitarity of the scattering matrix which we need is,

$$\sum_{\beta}[\mathbf{s}_{-,\alpha\beta}^{\dagger}(E, E+\hbar\omega)\mathbf{s}_{\alpha\beta}(E+\hbar\omega)$$

$$+ \mathbf{s}_{\alpha\beta}^{\dagger}(E)\mathbf{s}_{+,\alpha\beta}(E+\hbar\omega, E)] = 0. \quad (11)$$

Using (11) gives for the internal conductance

$$g_{\alpha}^i(\omega) = -(e^2/h)\sum_{\beta}\int$$

$$\times dE\,\mathrm{Tr}[\mathbf{s}_{\alpha\beta}^{\dagger}(E)\mathbf{s}_{+,\alpha\beta}(E+\hbar\omega, E)]$$

$$\times(f(E) - f(E+\hbar\omega)). \quad (12)$$

In the low-frequency limit of interest here we can simplify (12). The scattering matrix $\mathbf{s}(E, U(\mathbf{r}))$ is a function of energy E and is a functional of the static equilibrium potential $U(\mathbf{r})$. For a potential $U(\mathbf{r}, t)$ that varies slowly with time we obtain the transition amplitudes $\mathbf{s}_{\pm}(E+\hbar\omega, E)$ by replacing the static potential in $\mathbf{s}$ by the dynamic potential [10]. Taking the functional derivative with respect to $U(\mathbf{r}, t)$ and expanding the result in terms of Fourier components with respect to time gives,

$$\lim_{\omega\to 0}\mathbf{s}_{\pm,\alpha\beta}(E\pm\hbar\omega, E)$$

$$= \int d^3r(\delta\mathbf{s}_{\alpha\beta}(E)/\delta eU(\mathbf{r}))v(\mathbf{r}). \quad (13)$$

Thus to first order in ω the conductance g_{α}^i is

$$g_{\alpha}^i(\omega) = ie^2\omega\int d^3r(dn(\alpha, \mathbf{r})/dE)v(\mathbf{r}). \quad (14)$$

Here we have introduced the density of states

$$dn(\alpha, \mathbf{r})/dE =$$

$$-\left(\frac{1}{4\pi i}\right)\int dE\left(-\frac{df}{dE}\right)\sum_{\beta}$$

$$\times\mathrm{Tr}\left[\mathbf{s}_{\alpha\beta}^{\dagger}(E)\left(\frac{\delta\mathbf{s}_{\alpha\beta}(E)}{\delta eU(\mathbf{r})}\right) - \left(\frac{\delta\mathbf{s}_{\alpha\beta}^{\dagger}(E)}{\delta eU(\mathbf{r})}\right)\mathbf{s}_{\alpha\beta}(E)\right] \quad (15)$$

of carriers at point **r** which are emitted by the conductor at probe α. To arrive at (14) and (15) we have again used (11). Equation (14) gives the response to an arbitrary internal potential. This result is useful only if we can actually determine the internal potential. Fortunately, since (14) is already proportional to ω it is sufficient to find the internal potential in quasi-static approximation.

IV. Self-consistent potential

The additional charge density at point **r** brought into the sample by the oscillating chemical potential at probe α is $(dn(\mathbf{r},\alpha)/dE)\,\delta\mu_\alpha$ where

$$dn(\mathbf{r},\alpha)/dE$$
$$=(1/h)\int dE(-df/dE)\sum_m (1/v_{\alpha m})|\Psi_{\alpha m}(\mathbf{r})|^2. \qquad (16)$$

Here $v_{\alpha m}$ is the channel velocity and $\Psi_{\alpha m}$ is a scattering state [1]. In the absence of a magnetic field [11] $dn(\alpha,\mathbf{r})/dE=dn(\mathbf{r},\alpha)/dE$. According to Thomas-Fermi the additional external charge gives rise to an induced potential $U_i(\mathbf{r})$ which in turn induces a density $n_i(\mathbf{r})=-(dn(\mathbf{r})/dE)eU_i(\mathbf{r})$ that compensates the additional external charge [1, 12, 13]

$$(dn(\mathbf{r},\beta)/dE)\,\delta\mu_\beta - (dn(\mathbf{r})/dE)eU_i(\mathbf{r})=0. \qquad (17)$$

The additional charge is screened by all available charges. The variation of the density with respect to the potential U is thus the sum of all densities given by (15): $(dn(\mathbf{r})/dE))=\sum_\alpha (dn(\alpha,\mathbf{r})/dE)$. Using the potential determined by (17) in (14) we find that the oscillating potential $\delta\mu_\beta$ gives rise to an induced current at probe α given by $g_\alpha^i(\omega)\,U_{+\omega}(\delta\mu_\beta)\equiv g_{\alpha\beta}^i(\omega)(\delta\mu_\beta/e)$ with a conductance determined by

$$g_{\alpha\beta}^i(\omega)=ie^2\omega\int d^3r(dn(\alpha,\mathbf{r})/dE)$$
$$\times(dE/dn(\mathbf{r}))(dn(\mathbf{r},\beta)/dE). \qquad (18)$$

Since it is the additional charges due to the oscillation of the chemical potential μ_β that via a screening consideration generate the internal potential, the current induced by the internal potential is ultimately also proportional to the oscillating chemical potential. Equation (18) states that the emission of carriers into probe α is proportional to the density of carriers injected at contact β weighted by the total density of states.

V. Low-frequency metallic current partition

The complete current response of the conductor with *interactions* (superscript *I*) in contact α due to an oscillating potential in contact β is given by the sum of the two conductances, $g^I=g^e+g^i$

$$g_{\alpha\beta}^I(\omega)=g_{\alpha\beta}(0)-i\omega e^2((dN_{\alpha\beta}/dE)$$
$$-\int d^3r(dn(\alpha,\mathbf{r})/dn(\mathbf{r}))(dn(\mathbf{r},\beta)/dE)). \qquad (19)$$

The interacting conductance (19) obeys current conservation, $\sum_\alpha g_{\alpha\beta}^I(\omega)=0$ and $\sum_\beta g_{\alpha\beta}^I(\omega)=0$. Taking the sum over α gives a volume integral which depends only on $(dn(\mathbf{r},\beta)/dE)$ but $\sum_\alpha (dN_{\alpha\beta}/dE)\equiv d\bar{N}_\beta/dE=\int d^3r(dn(\mathbf{r},\beta)/dE)$. Summing $g_{\alpha\beta}^I(\omega)$ over β leaves only $dn(\alpha,\mathbf{r})/dE$ under the volume integral and this is equal to dN_α/dE.

The external response (4) and (5), through the density of states, depends on the location of the cross-sections through the leads used to evaluate the current (see Fig. 1). The further out in the leads the current is evaluated the larger is the density of states. In contrast, the interacting conductance (19) is independent of the exact spatial location of these cross-sections as long as the cross-sections are sufficiently far in the leads. For a point **r** deep in lead α the total density $dn(\mathbf{r})$ is equal to the density $dn(\alpha,\mathbf{r})$ of carriers which from that point will be emitted into lead α. Hence for r deep inside reservoir α the last term in (19) is determined by $dn(\mathbf{r},\beta)$. But the carriers from contact β which reach this point will with high probability escape into contact α. Therefore, since $dn(\mathbf{r},\beta)\sim dn(\alpha,\beta)$ the second term counteracts any increase in $(dN_{\alpha\beta}/dE)$ arising from a change in the location of the cross-section. The density of states in the asymptotic (wide lead or) reservoir regions is completely screened.

VI. Simple example

We illustrate our main result (19) by solving a simple example. We consider a two probe conductor (a quantum dot [14]) with a long lived state described by a scattering matrix $s_{\alpha\beta}(E)=(\delta_{\alpha\beta}-i(\Gamma_\alpha\Gamma_\beta)^{1/2}/\Delta)\exp(i(\delta_\alpha+\delta_\beta))$ where $\Delta=E-E_r+i\Gamma/2$ is an abbreviation for the resonant pole at energy E_r, and Γ_1, Γ_2 are the decay widths into the left and right probe. $\Gamma=\Gamma_1+\Gamma_2$ is the total decay width. From (6) we find

$$\frac{dN_{\alpha\alpha}}{dE}=\left(\frac{1}{2\pi}\right)\frac{\Gamma_\alpha}{|\Delta|^2}\frac{(E-E_r)^2-(\Gamma/2)^2+\Gamma_\alpha(\Gamma/2)}{|\Delta|^2}, \qquad (20)$$

and for $\alpha\neq\beta$

$$\frac{dN_{\alpha\beta}}{dE}=\left(\frac{1}{2\pi}\right)\frac{\Gamma_\alpha}{|\Delta|^2}\frac{\Gamma_\beta(\Gamma/2)}{|\Delta|^2}. \qquad (21)$$

For the density inside the sample generated by the oscillating potential at contact α we find

$$\int d^3r(dn(\mathbf{r},\alpha)/dE)=(1/2\pi)(\Gamma_\alpha/|\Delta|^2). \qquad (22)$$

To calculate the variation of the scattering matrix with respect to U we assume that the main effect of an induced potential is a shift in the resonant energy such that $\Delta=(E-E_r-eU)+i\Gamma/2$ and all other coefficients of the s matrix remain unchanged. This leads to densities of states

$$\int d^3r(dn(\alpha,\mathbf{r})/dE)=(1/2\pi)(\Gamma_\alpha/|\Delta|^2). \qquad (23)$$
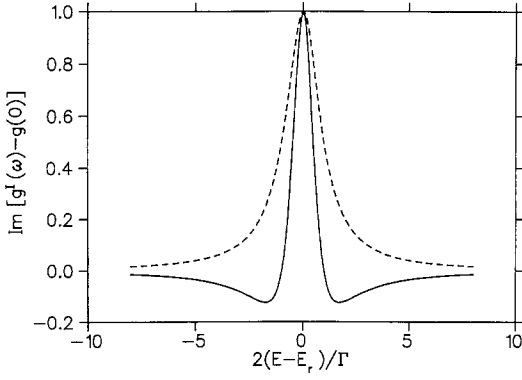
Fig. 2. Frequency-dependent part of the interacting conductance for a long lived state as a function of Fermi energy E (solid line) compared to the line-shape of a Breit-Wigner resonance (dashed line)

At $kT = 0$ this gives an induced conductance (18)

$$g_{\alpha\beta}^i = i\,(\omega e^2/2\pi)\,(\Gamma_\alpha \Gamma_\beta)/(\Gamma |\Delta|^2). \qquad (24)$$

For the total conductances of the interacting system we find $g^I \equiv g_{11}^I = g_{22}^I = -g_{12}^I = g_{21}^I$ with

$$g^I(\omega) = g(0) + i\left(\frac{\omega e^2}{2\pi\Gamma}\right)$$
$$\times \left(\frac{\Gamma_1 \Gamma_2}{|\Delta|^2}\right)\left(\frac{\Gamma^2/2 - |\Delta|^2}{|\Delta|^2}\right), \qquad (25)$$

where $g(0) = \int dE\,(-df/dE)\,T(E)$ is the dc conductance. The interacting system, in contrast to the non-interacting problem, is again described by a single admittance. The interesting energy dependence of (25) is depicted in Fig. 2 which shows $\mathrm{Im}\,[g^I(\omega) - g(0)]$, in units of $(e^2/h)(\hbar\omega/\Gamma)\,(\Gamma_1\Gamma_2/\Gamma^2)$. The solid line in Fig. 1 represents $(\Gamma^2/2 - |\Delta|^2)/|\Delta|^2$ and is compared with a Breit-Wigner line-shape $(\Gamma^2/4)/|\Delta|^2$. While the voltage follows the current for Fermi energies near resonance, the current follows the voltage for energies more than a half-width away from the resonance. Equation (25) can now be compared with calculations which assume the field to be uniform throughout the conductor [15].

VII. Discussion

Our discussion given here does not take into account unscreened long-range fields. As seen from this example the conductance of the interacting system vanishes (except for $E_F = E_r$) when the resonance is completely stable, i.e. when the decay widths of the long-lived state tend to zero. A real conductor exhibits an ac-conductance even in this case through capacitive coupling [3, 13]. The theory presented here is clearly most useful when the conductor is metallic everywhere and thus screens all fields everywhere efficiently.

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