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The systematics of charged particle emission for 14 MeV neutron induced reaction cross sections

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Abstract. The systematics of cross sections for 14 MeV neutron induced (n, p), (n, d), (n, t), $(n, {}^3\mathrm{He})$ and (n, α) reactions are studied within a simple evaporation model. Using few parameters, global prescriptions are obtained giving the cross sections in terms of the parameter $(N-Z+1)/A$ instead of the generally used $(N-Z)/A$ parameter. Improved fits to the data are expected as has been shown for the case of the (n, p) cross section.

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The study of the systematic behaviour of the cross-section of the (n,p) reaction around 14 MeV incident energy [1–7] as well as other neutron-charged particle reactions such as the (n,α), (n,t) and $(n,{}^3\mathrm{He})$ reactions [1,7–9] has received much attention in recent years. Several approximate formulae describing these systematics were obtained by fits to the experimental data over a wide range of nuclei. Most of these expressions show a dependence of the cross section on the relative neutron excess $(N-Z)/A$ of the target nucleus.

Using the evaporation model, we showed [5, 6] that the (n,p) cross section, as is reasonably expected, is directly related to the reaction Q-value, the Coulomb barrier and the nuclear temperature (a level density information) and moreover we found that the dependence on Q leads to a dependence on the parameter $\rho=(N-Z+1)/A$ instead of the generally used parameter $(N-Z)/A$.

This can be generalized to other neutron-charged particle cross sections and is presented in the following. To show this, we write the (n,a) cross section for the emission of the charged particle a in terms of the decay widths as:

$$\sigma(n,a)=\sigma_R \frac{\Gamma_a}{\Gamma_{\mathrm{tot}}} \tag{1}$$

where σ_R is the reaction cross section, Γ_α the charged particle decay width and Γ_{tot} the total decay width given by:

$$\Gamma_{\mathrm{tot}}=\Gamma_n+\Gamma_p+\Gamma_\alpha+\cdots \tag{2}$$

and which can be well approximated at the energy of 14 MeV by:

$$\Gamma_{\mathrm{tot}}\simeq\Gamma_n. \tag{3}$$

The decay width Γ_a for the emission of particle a in the process $C\to a+A$ is related to the emission rate of that process by

$$\Gamma_\alpha=h\int\left(\frac{\mathrm{d}^2 W}{\mathrm{d}\varepsilon_a \mathrm{d}t}\right)\mathrm{d}\varepsilon_a \tag{4}$$

where ε_a is the energy of the emitted particle.

The emission rate $\left(\frac{\mathrm{d}^2 W}{\mathrm{d}\varepsilon_a \mathrm{d}t}\right)$ is conveniently obtained from the Weisskopf evaporation model [10] in the form

$$\frac{\mathrm{d}^2 W}{\mathrm{d}\varepsilon_a \mathrm{d}t}=\frac{(2I_a+1)}{2\pi^2 h} k_a^2 \sigma_a \frac{\omega_A(E_A^*)}{\omega_C(E_C^*)} \tag{5}$$

where ε_a, I_a, k_a are the kinetic energy, the spin and the wave number of the particle a respectively, σ_a is the compound nucleus formation cross section and $\omega_A(E_A^*)$ is the level density of the residual nucleus A at an excitation energy E_A^* and similarly for $\omega_C(E_C^*)$.

The level density ratio in (5) is well approximated [10] by:

$$\frac{\omega_A(E_A^*)}{\omega_C(E_C^*)}\simeq\exp\left(S_A(E_A^*)-S_C(E_C^*)\right) \tag{6}$$

where $S_A(E_A^*)$ is the entropy of the nuclear system A at energy E_A^* and similarly for $S_C(E_C^*)$. Using the thermodynamical relation

$$\frac{\mathrm{d}S}{\mathrm{d}E}=\frac{1}{T} \tag{7}$$

we get:

$$S_A(E_A^*)-S_C(E_C^*)\simeq\frac{E_A^*-E_C^*}{T}=-\frac{(\varepsilon_a+B_a)}{T} \tag{8}$$

where T is the nuclear temperature and B_a is the separation energy of particle a from the compound nucleus. Putting this together (5) then becomes:

$$\frac{d^2 W}{d\varepsilon_a dt}\simeq\frac{(2I_a+1)}{2\pi^2 h}k_a^2\sigma_a\exp\left(-\frac{\varepsilon_a+B_a}{T}\right). \tag{9}$$

At the energy considered here ($\sim$14 MeV) the cross section σ_a can be taken as:

$$\sigma_a\simeq\pi R^2 \qquad \text{for neutrons}$$

$$\sigma_\alpha\simeq\begin{cases}\pi R^2\left(1-\frac{V_a}{\varepsilon_a}\right) & \varepsilon_a>V_a\\ 0 & \varepsilon_a<V_a\end{cases} \qquad \text{for charged particles} \tag{10}$$

where R is the nuclear radius and V_a the Coulomb barrier seen by a entering the nucleus A.

The decay widths for neutrons and charged particle a can now be calculated by carrying the integration in (4) and using (9) with (10). This gives

$$\begin{aligned}\Gamma_n&\simeq\frac{2m_n R^2}{\pi\hbar 2}T^2\exp\left(-\frac{B_n}{T}\right)\\ \Gamma_a&\simeq\frac{(2I_a+1)m_a R^2}{\pi\hbar^2}T^2\exp\left(-\frac{B_a+V_a}{T}\right).\end{aligned} \tag{11}$$

This gives finally:

$$\sigma(n,a)\simeq\frac{(2I_a+1)}{2}\frac{m_a}{m_n}\sigma_R\exp\left(\frac{Q-V_a}{T}\right) \tag{12}$$

where Q is the reaction Q-value given by

$$Q=Q(n,a)=B_n-B_a. \tag{13}$$

This expression (12), although only approximate, shows clearly the dependence of the (n,a) cross section on the reaction Q-value, the Coulomb barrier encountered by the outgoing protons and the nuclear temperature. The empirical dependence of the cross section on the parameter $(N-Z+1)/A$ arises simply from that on the Q-value as is shown below.

In the following we consider the reactions (n,p), (n,d), (n,t), $(n,{}^3\mathrm{He})$ and (n,α). Using the semi-empirical mass formula, the energy Q may be written as:

$$\begin{aligned}Q\simeq&-\alpha(A-A_R)+\beta(A^{2/3}-A_R^{2/3})\\ &+\gamma\left\{\frac{(A-2Z)^2}{A}-\frac{(A_R-2Z_R)^2}{A_R}\right\}\\ &+\varepsilon\left(\frac{Z^2}{A^{1/3}}-\frac{Z_R^2}{A_R^{1/3}}\right)+\delta(A,Z)-\delta(A_R,Z_R)\end{aligned} \tag{14}$$

where the coefficients α, β, γ, ε and δ are those of the semi-empirical mass formula and where the subscript R refers to the residual nucleus. The contributions from the terms proportional to α and β may be omitted since to a first approximation they correspond to a constant value added to $Q(n,a)$.

If we consider only the term proportional to γ i.e. the term arising from the difference between the symmetry energy of the target and residual nuclei then we find that it reasonable to attempt to parametrize the (n,a) cross section not in terms of $(N-Z)/A$ but in terms of $(N-Z+1)/A$. A possible parametrization is thus:

$$\sigma(n,a)=\beta(A^{1/3}+1)^2\exp\left\{\alpha_1\left(\frac{N-Z+1}{A}\right)+\alpha_2\left(\frac{1}{A}\right)\right\} \tag{15}$$

where α_1,α_2 and $_\beta$ are parameters (they are distinct from the parameters of the preceeding equations) with $\alpha_2=0$ for the (n,p) and $(n,{}^3\mathrm{He})$ reactions and where we used

$$\sigma_R\simeq\pi r_0^2(A^{1/3}+1)^2.$$

If we further take into account the contributions from the difference between the parity terms, the term proportional to δ, then we obtain the following relation with one additional parameter α_3:

$$\sigma(n,a)=\beta(A^{1/3}+1)^2\exp\left\{\alpha_1\left(\frac{N-Z+1}{A}\right)+\alpha_2\left(\frac{1}{A}\right)+\alpha_3\left(\frac{1}{A^{1/2}}\right)\right\} \tag{16}$$

where $\alpha_2=0$ for (n,p) and $(n,{}^3\mathrm{He})$ and where α_3 is such that for A odd ($\alpha_3=0$) and for A even ($\alpha_3>0$ if Z is odd and $\alpha_3<0$ if Z is even).

Finally, for certain nuclei the (n,p) cross section is quite large and the approximation used in (3) may not be very accurate. In this case and for cross sections other than the (n,p) one it may be preferable to use the following approximation instead of (3)

$$\Gamma_{\mathrm{tot}}\simeq\Gamma_n+\Gamma_p. \tag{17}$$

This leads to an additional correction of the previous formulae and gives the following expression for the (n,a) cross section.

$$\sigma(n,a)=\{\sigma(n,a)\}_0\left\{\frac{\sigma_R}{\sigma_R+\{\sigma(n,p)\}_0}\right\} \tag{18}$$

where the subscript zero in the cross section $\{\sigma(n,a)\}_0$ indicates the cross section obtained through one of the previous global prescriptions (15) or (16).

Using the available data in the 14 MeV region for 150 nuclei ranging from ${}^{40}\mathrm{Ca}$ to ${}^{209}\mathrm{Bi}$ the following relation was obtained [5,6] from (15):

$$\sigma(n,p)=107.98(A^{1/3}+1)^2\times\exp\left\{-36.749\left(\frac{N-Z+1}{\mathrm{A}}\right)\right\} \tag{19}$$

where $\sigma(n, p)$ is expressed in millibarns. The χ^2 value obtained in this case was 4.65 considerably better than other formulae using more parameters. The above expressions can be used to attempt similar fit to the data for other charged particle reactions and also for the (n, p) cross section using the new formulation of these global formulae. Similar expressions have been used in the past to fit these cross sections. However, the present global expressions if used successfully as was done for the (n, p) case have the advantage of being transparent as to the origin of the various parameters. This can be very important as was shown in the case of (n, p) reaction where the use of the new parameter $\left(\frac{N-Z+1}{A}\right)$ instead of the usual $\left(\frac{N-Z}{A}\right)$ made a significant difference to the quality of the fits.

The present expressions can therefore be used to give a rough but rapid estimate of the (n, a) cross sections at about 14.5 MeV if no experimental information is available. Also, it can be used to determine for which of the nuclides the experimental data is probably incorrect and thus needs to be measured more accurately in the future. Examples of such nuclides for the (n, p) reaction are ^{64}Ni, ^{100}Ru, ^{165}Ho, ^{208}Pb and ^{209}Bi [7]. Some examples of such difficult or previously inaccurate measurements that have been made recently may be found in [11, 12], and recent measurements of (n, p) cross sections for Sm isotopes [13] have shown that global formulae such as (19) can be used to obtain a reliable estimate of the cross section of interest.

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