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The Effect of Particle Strength on the Compression of Crushed Aggregate

By

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With 8 Figures

Summary — Zusammenfassung — Résumé

The Effect of Particle Strength on the Compression of Crushed Aggregate. The paper examines the variables controlling the compression of rock aggregates, and presents experimental data on the behaviour of limestone, dolerite and sandstone samples in size ranges up to 19.0 mm.

The results suggest that while the initial void ratio of the aggregate determines the degree of compressive strain potentially available, the average aggregate dimension (d_m) and intrinsic compressive strength (S_c) determine the compressive stress-strain characteristics in the form,

$$\varepsilon \propto \ln \frac{\sigma d_m{}^{1/2}}{S_c}$$

Einige Faktoren, die die Kompressibilität von Aggregaten bestimmen. Diese Arbeit untersucht die Variablen, die die Kompressibilität von Gesteinsaggregaten bestimmen, und liefert experimentelle Angaben über das Verhalten von Kalkstein-, Dolerit- und Sandsteinproben bis zu 19,0 mm Korngröße.

Die Ergebnisse weisen darauf hin, daß, während der anfängliche relative Porenraum des Aggregats den Grad der potentiell vorhandenen Kompressionsspannung bestimmt, der durchschnittliche Aggregatsdurchmesser (d_m) und die eigentliche Druckfestigkeit (S_c) die Kompressibilitäts-Spannungs-Verformungscharakteristik laut der Formel

$$\varepsilon \propto \ln \frac{\sigma d_m{}^{1/2}}{S_c}$$

bestimmen.

Influence de la résistance des particules sur la compression d'un agrégat broyé. Les auteurs examinent les paramètres qui affectent la compressibilité des agrégats de roches et présentent des résultats expérimentaux obtenus avec des échantillons de calcaire, de dolérite et de grès, sur l'influence du diamètre des grains en-dessous de 19 mm.

Les résultats suggèrent que l'indice des vides initial impose le taux de la déformation à la compression dont on dispose, cependant que le diamètre des grains (d_m) et la résistance à la compression (S_c) définissen les caractéristiques de la courbe de compressibilité.

Key Words — Stichwörter — Mots cléfs

Aggregate — Aggregat — Agrégat. Compressibility — Kompressibilität — Compressibilité. Compressive strength — Druckfestigkeit — résistance à la compression. Particle size — Körnung — taille des particules. Rockfill — Steinpackung — enrochement. Rock Mechanics — Felsmechanik — mécanique des roches. Void Ratio — Relativer Porenraum — indice des vides.

Symbols — Symbole — Symbols

Strain — Verformung — déformation. Stress — Spannung — contrainte. Average particle size — Korndurchmesser — diamètre des grains. Compressive strength — Druckfestigkeit — résistance à la compression. Void ratio — relativer Porenraum — indice des vides. Initial void ratio — anfänglicher relativer Porenraum — indice des vides initial. Final void ratio — endrelativer Porenraum — indice des vides final. Angularity number — Eckigkeit — angulairité. Uniformity coefficient — Ungleichförmigkeitsgrad — coefficient d'uniformité. Deformation — Verformung — déformation. Force — Kraft — force. Modulus of elasticity — Elastizitätsmodul — module d'élasticité. Poisson's ratio — Poisson-Zahl — coefficient de Poisson.

1. Introduction

It is well known that the compressibility and shear resistance of an aggregate of rock particles subjected to low applied stresses depends principally on the shear or frictional resistance between individual particles and the arrangement of particles in the aggregate (see for instance Boughton, 1970; Lowe, 1964; Marachi, Chan, and Seed, 1972; Wilkins, 1970). Thus when an aggregate is loaded, compression occurs as a result of interparticle slip and the resultant rearrangement of particles.

However, in the case of aggregates subjected to high applied stresses in confinement — a situation which may occur in some rockfills and highway subgrades and in aggregates used in foundations or as mine or tunnel backfills — this concept is difficult to justify. Although initially compression may be controlled by low pressure mechanisms the major deformational mechanism will undoubtedly be the failure of individual particles — in which case compression of the aggregate could be defined in terms of particle strength, rather than interparticle friction.

The present paper examines this approach through a series of compression tests on aggregates of varying strength and size.

2. Experimental

A test programme was designed to investigate on a laboratory scale the effect of some of the fundamental variables affecting aggregate compressibility, and to appraise the possibility of predicting ultimate compressibility from a knowledge of the rock properties. The programme comprised

Table 1. Rock Aggregate Properties

Rock	Size Range mm	Average Size mm	Before compression e	A	U	After compression e	A	U
Dolerite	19.0—12.7	15.9	.84	13	1.2	.40	—	6.5
	12.7— 9.5	11.1	.83	12	1.2	.39	—	4.6
	6.4— 4.7	5.6	.83	12	1.2	.39	—	3.5
	3.2— 2.4	2.8	.80	11	1.2	.40	—	2.6
Limestone	19.0—12.7	15.9	.84	13	1.2	.37	—	10
	12.7— 9.5	11.1	.84	13	1.2	.38	—	7.7
	9.5— 6.4	8.0	.81	11.5	1.2	.37	—	8.1
	6.4— 4.7	5.6	.80	11	1.2	.38	—	8.1
	4.7— 3.2	4.0	.80	11	1.2	.38	—	6.4
	3.2— 2.4	2.7	.79	11	1.2	.38	—	5.7
Sandstone	19.0—12.7	15.9	.80	11	1.2	.26	—	41
	12.7— 9.5	11.1	.74	9	1.2	.20	—	29
	9.5— 6.4	8.0	.76	10	1.2	.23	—	26
	6.4— 4.7	5.6	.74	9	1.2	.21	—	22
	4.7— 3.2	4.0	.76	10	1.2	.24	—	16
	3.2— 2.4	2.7	.75	9.5	1.2	.23	—	10

three series of confined axial compression tests on samples contained in a 150 mm diameter, 100 mm deep steel cylinder. Each series consisted of at least four tests on oven-dried samples (24 hours at 110° C) in the size ranges listed in Table 1, in which a pressure of 20 N/mm² was applied in increments of 2 N/mm² at 1 minute intervals. Similar tests were carried out on saturated samples (24 hours vacuum) in the 9.5—12.7mm size range.

Principal initial variables were the rock material compressive strength (S_c) and particle size (d_m). The independent test variable was the applied stress (σ) and the measured variable was axial compression (equivalent to volumetric compression under the test conditions). The initial void ratio range was between 0.74 and 0.84 (Table 1).

Table 2. Rock Material Properties

Rock	(1) S. G.	(2) Porosity %	(3) Void ratio	(4) Compressive strength Dry N/mm²	(5) Saturated N/mm²	(6) (5)/(4) %
Dolerite	2.98	0.17	0.02	132.5	125.4	95
Limestone	2.68	0.42	0.04	84.5	76.8	90
Sandstone	2.23	14.80	0.17	25.4	17.0	66

The rocks used in the tests (Table 2) were:

(a) *Whin Sill Dolerite* — medium-grained crushed quartz dolerite forming strong angular fragments.

(b) *Great Limestone* — fine-grained slightly-dolomitised limestone crushed to form fairly strong angular fragments.

(c) *Springwell Sandstone* — weak coarse-grained sandstone comprising 56% quartz in a 12% kaolinite, 26% illite matrix. This crushed aggregate was weak and less angular than the other two aggregates.

The rocks were selected to provide, in turn, a weak, a strong and a very strong aggregate. They were crushed in a jaw crusher with a low reduction ratio to eliminate as far as possible flaky or elongated particles and to obtain a reasonably consistent initial void ratio. Classification tests on the aggregates included void ratio (e) and angularity number (A = porosity — 33%) determined as laid down in B. S. 812, 1967, and uniformity coefficient, U (U. S. Bureau of Reclamation, 1968). This latter is the ratio between D_{60} and D_{10} size fractions and indicates the initial grading and the subsequent degree of degradation. Figs. 1 and 2 summarise the deformational response of different strength and particle size aggregates to increasing levels of stress. Fig. 1 records compressive stress-strain characteristics of the largest and smallest size fraction of each rock type. Intermediate size fractions perform in a manner between the curves. Fig. 2 differentiates between the dry and saturated behaviour of aggregates in the 9.5—12.7mm size range.

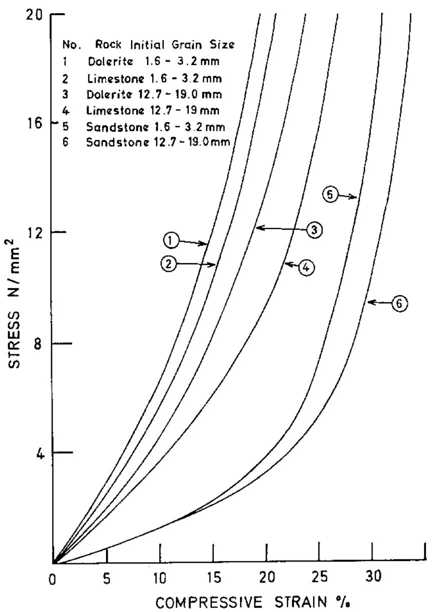

Fig. 1. Compression tests on different aggregate size samples
Kompressionsversuche der Aggregatsproben
Essais de compression des échantillons d'agrégats

3. Proposed Deformation Mechanisms

The simplest empirical approach is to adopt an average dimension (d_m) or surface area (A_m) for the particles in the aggregate, and thence to deduce the number of particle contacts and/or the force transmitting area normal to the applied stress in the aggregate. Marsal (1963) develops this quite elegantly through a surface concentration factor (n_s), equal to the number of particles per unit area intersected by a given plane in the aggregate. n_s is

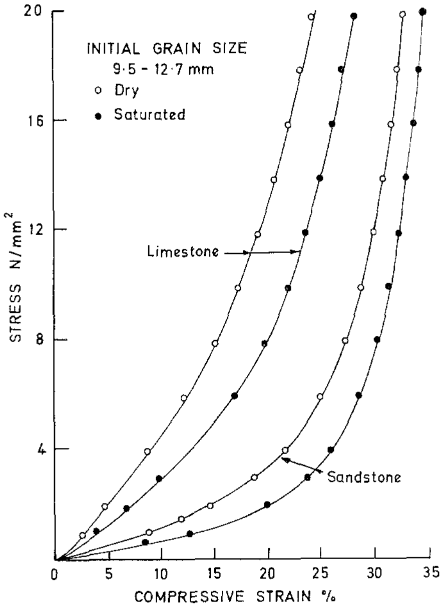

Fig. 2. Compression tests on dry and immersed samples
Kompressionsversuche der trockenen Probe und der Probe unter Wasser
Essais de compression des échantillons secs et immergés

linked by Marsal to the void ratio (e) of the aggregate through the equation.

$$n_s = \frac{1}{d_m^2}\left[\frac{6}{\pi r_v (1+e)}\right]^{2/3} \tag{1}$$

where r_v is a dimensionless factor equal to 1 for spherical particles and reducing to 0.5 for very angular particles. The average force transmitted by each particle will therefore be given by:

$$F = \frac{\sigma}{n_s} = \frac{\sigma d_m^2 [r_v (1+e)]^{2/3}}{1.54} \tag{2}$$

Hobbs (1962) and Millard *et al.* (1955) have shown that the force required to fracture an irregular piece of rock is roughly proportional to the square root of its volume (V). This means that the ratio $F/\sqrt{V}$ is a constant directly proportional to the crushing strength of the rock (S_c). In the case of an aggregate, therefore, deformation will occur when:

$$\frac{kF}{\sqrt{V}} = \frac{1.38\,kF}{d_m^{3/2}} > S_c \tag{3}$$

or substituting in Eq. (2), when

$$k' \frac{\sigma d_m^{1/2}}{S_c} [r_v (1+e)]^{2/3} < 1 \tag{4}$$

where k' is a constant.

For rocks of uniform aggregate shape and void ratio (the present experimental situation) this deformation condition may be re-written as:

$$k'' \frac{\sigma d_m^{1/2}}{S_c} < 1 \tag{5}$$

Discrete particle failure will be evidenced by compression of the aggregate associated with a reduction in particle size. Thus, as compression proceeds, proportionately higher stresses will be required to maintain the conditions for continuing compression. If d_m is taken as the initial average particle size it is probable, therefore, that the compressive strain resulting from progressive particle fracture may be described by an expression of the general form:

$$\varepsilon \propto \left[\frac{\sigma d_m^{1/2}}{S_c}\right]^f \tag{6}$$

Eq. (6) assumes a uniform initial void ratio and shape factor. In practice, the void ratio of aggregates can vary from well over unity for loose single-size aggregates, to less than 0.2 for well-compacted graded material. The effect of such variations will obviously be significant in determining the *overall* magnitude of volumetric strain.

A further shortcoming with respect to Eq. (6) lies in the inherent assumption that there are only material constraints on progressive compression. In fact, the reducing void ratio during compression serves to reduce the rate of increase in compression. After an initial phase, therefore, the exponent (f) should reduce and the relationship may be more accurately represented by a logarithmic function.

4. Discussion of Results

The information in Fig. 3 shows that there is an irregular relationship between compressive strain and initial aggregate size, but there is clearly a greater facility for compression with increasing grain size. By smoothing the data, a power law relationship can be suggested between the degree of compressive strain and the initial aggregate size, the influence of the latter becoming less pronounced (lower d_m exponent) as the compression increases. This is again a reflection of the very rapid equalization of a changing void ratio at increasingly higher pressures — a rapid degradation of larger aggregate and a correspondingly reduced breakdown of the (originally) smaller pieces.

In order to investigate the influence of intrinsic aggregate strength, it is necessary to examine a suite of rocks encompassing a single size range of aggregate. Plotting stress against compression in Fig. 4 for a power law

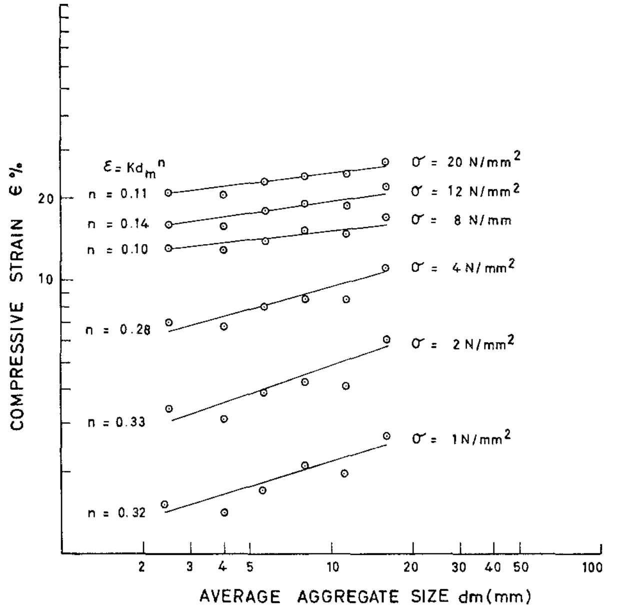

Fig. 3. Effect of aggregate size on the compression of limestone
Einfluß des Korndurchmessers auf die Kompressibilität von Kalkstein
Effet du diamètre des particules sur la compressibilité d'un calcaire

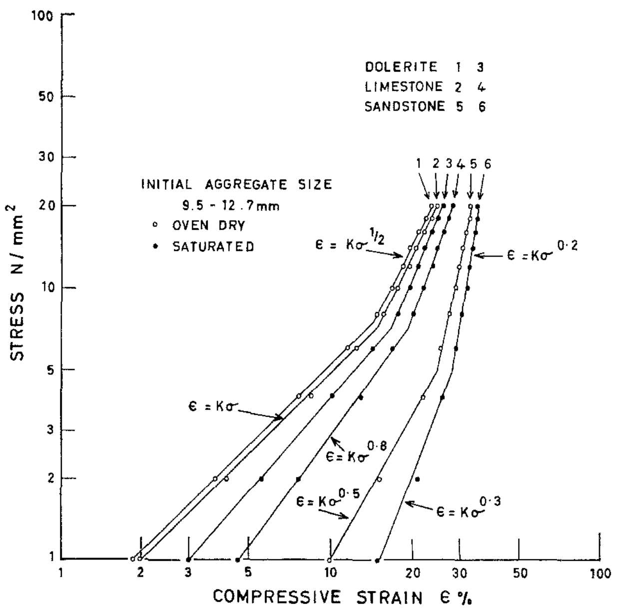

Fig. 4. Effect of water immersion on compression
Einfluß des Eintauchens auf die Kompressibilität
Effet de l'immersion sur la compressibilité

relationship, the discontinuities on the curves — occurring at stress levels of between 5 and 8 N/mm² — represent the accelerated onset of a 'strain hardening' condition associated with a critical void ratio. This same criticality at the 5—8 N/mm² stress level is also demonstrated on the slopes of the graphs in Fig. 3.

The effect of saturation is most easily discerned by reference to the dimensionless plots on Fig. 5. Terzaghi (1960) has noted a reduction in rock strength as a result of saturation and this certainly applies to the weaker sandstone in Fig. 5. However, the increase in compressibility of the stronger rocks exceeds that attributable solely to a reduction in strength.

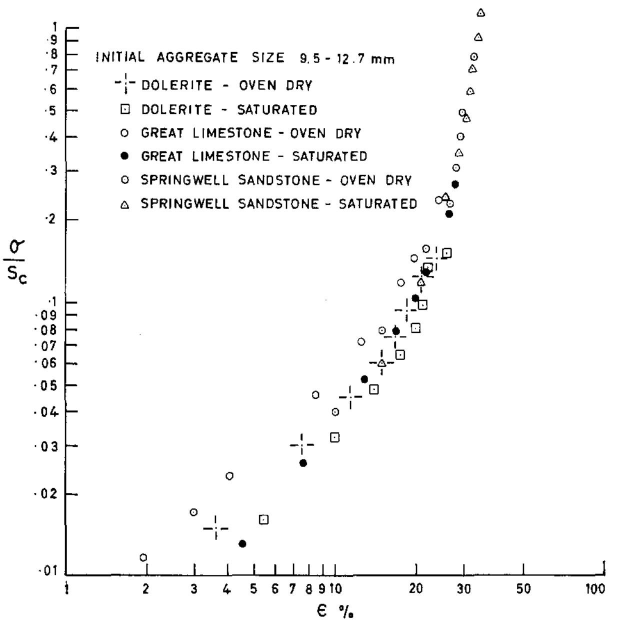

Fig. 5. ε vs $\frac{\sigma}{S_c}$ (dry and saturated rocks)

(Trockenes und gesättigtes Gestein)

(Roches séches et saturées)

Since the samples were drained under test and the rate of compression was low, a probable explanation must be that either the water acts as a rock-surface lubricant, so reducing the coefficient of friction, or that the effect of saturation is selectively to weaken the surface irregularities rather more than the body of each aggregate particle. Whilst the former has been disproved by Terzaghi's experimentation the latter would appear to be a more logical explanation, since any surface angularities created by crushing would contain a multiplicity of microfractures offering a high water penetration facility. There are some similarities between this concept and the work of

Rehbinder (1948) who showed that in general the surface hardness of rocks is affected more by wetting than is their compressive strength.

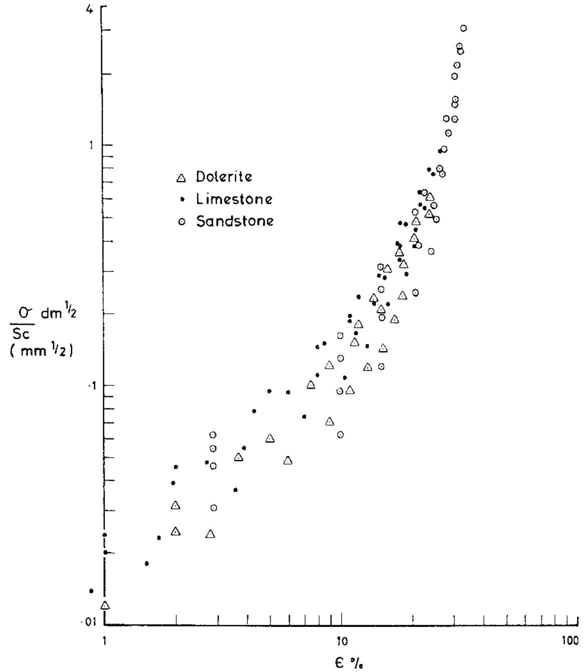

Fig. 6. ε vs $\frac{\sigma\, d_m{}^{1/2}}{S_c}$ (power law form)

(Potenzgesetz-Maßstab)

(échelle de loi faisant intervenir une puissance)

Data on dry rocks are collected in Fig. 6 in the form suggested by Eq. (6). This equation is satisfied only up to a strain value of about 10 %, throughout which range the d_m exponent is approximately unity, vis:

$$\varepsilon_{<0.1} \propto \frac{\sigma\, d_m{}^{1/2}}{S_c} \tag{7}$$

At higher compressions, the exponent f reduces progressively, the earlier relationship degenerating to one of logarithmic form (see Fig. 7):

$$\varepsilon_{>0.1} \propto \ln\left[\frac{\sigma\, d_m{}^{1/2}}{S_c}\right] \tag{8}$$

Eqs. 7, 8 are satisfied by a limited range of aggregates of similar *initial* void ratio, shape and grading. It is necessary to test the present aggregate behaviour against previous examples taken from the literature (as plotted in Fig. 8) before accepting the present arguments for further development. Although there is a general similarity in the form of the curves there are differences in the steepness of their quasi-linear portions, in the stress/strain amplitudes at which this linearity develops, and in the overall amplitudes of the curves. Amplitude differences at higher pressures and aggregate deformations may be explained by differences in the initial void ratio restricting the overall degree of compression. Similarly, differences in the low pressure — low strain range down to compressive strains of about 1—2% could be attributed to interparticle locking resulting from the non-destructive re-arrangement of particles due to the collapse of loosely-packed zones.

The particular significance of these observations lies less in the use of any experimental results to develop a specific design approach than in the emphasis placed on *aggregate dimension* and *material strength* as controlling factors in fundamental aggregate compressibility. Obviously, the effect of aggregate bulk properties — principally the initial void ratio e_0 together with the relative density of the aggregate will be a significant factor in any quantitative approach. The principal effect of void ratio is best summarised

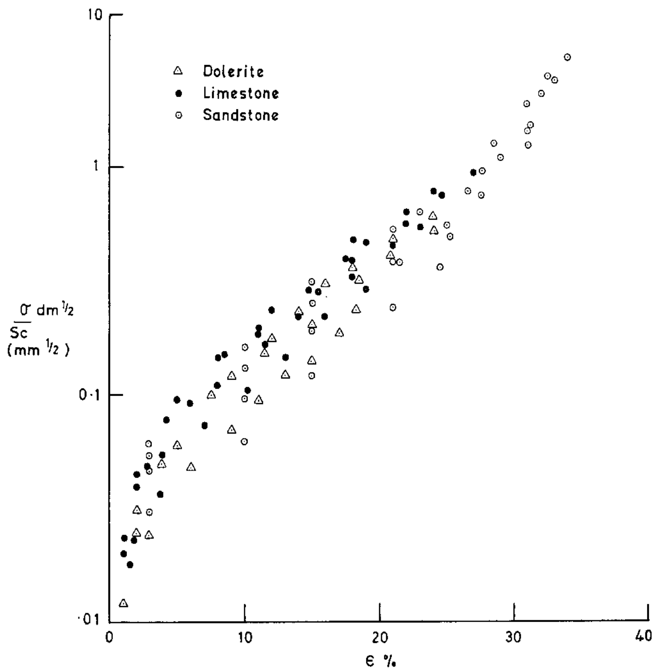

Fig. 7. ε vs $\frac{\sigma\, d_m{}^{1/2}}{S_c}$ (exponential form)

(Exponentieller Maßstab)

(échelle exponentielle)

by the work of Kjaernsli and Sande (1963). A basic conclusion which may be drawn from their work on aggregates of different void ratios is that e_0 specifies the *degree* of potential compression available in an aggregate.

It is relevant here to return to the basic curves in Fig. 1. In the case of the sandstone, the final void ratio at an applied stress of 20 N/mm² is obviously approaching a minimum value (Table 1), the curves being virtually asymptotic to the stress axis. With the limestone and dolerite, there is

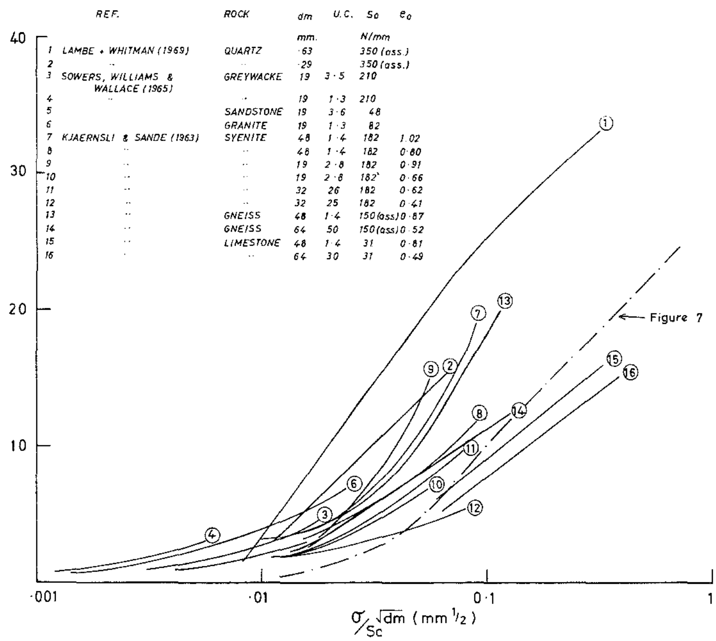

Fig. 8. Comparison of test data with data obtained by other workers
Vergleich der experimentellen Angaben mit den allgemeinen Angaben
Comparaison des données expérimentales avec des données générales

a facility for further compression at higher stresses. The differences result from the relative strengths of the rock materials in aggregates of similar initial void ratio.

It may be concluded, therefore, that the initial void ratio determines the degree of compressive strain potentially available, and that the material properties of the aggregate — particularly strength — control the rate per unit increment of pressure at which the minimum void ratio is approached.

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