*Foundations of Physics, Vol. 12, No. 11, 1982*

# On the Direct Observability of Quantum Waves

**F. Selleri**[1]

*Received June 9, 1982*

*Fundamental experiments on the dual nature of atomic entities (photons, electrons, neutrons, etc.) can be interpreted in terms of "empty" waves not carrying energy and momentum. Similar points of view were advanced in famous papers by Einstein, de Broglie, Bohr, and Born. Recent proposals could lead to experimental tests of this idea, using low intensity photon beams, thanks to modern experimental apparatuses.*

## 1. EINSTEIN'S "GESPENSTERFELDER"

Einstein's discovery of the equivalence between mass and energy[(1)] could have brought about a great unification of the scientific world view. Since energy and momentum are always conserved, while mass can be created and destroyed, the old idea that matter as an irreducible substance of the world had to be replaced by the idea that everything is really made of energy-momentum. This allowed one to understand in an unified way, at least in principle, matter itself, light, electromagnetic and gravitational fields, and so on, briefly all of the objects of physical investigation. If this unification in practice did not take place it was for good reasons, starting from the fact that Einstein himself in the year 1905 was forced to admit the existence of something real (in the sense that it was assumed to propagate causally in space and time), but containing neither matter nor energy-momentum. This was a necessary consequence of his dualistic picture of the electromagnetic radiation.[(2)]

Difficulties for the classical theory of radiation were found in 1902 by Lenard and in 1903 by Ladenburg in the photoelectric effect. As a conse-

[1] Istituto di Fisica, Università di Bari, INFN, Sezione di Bari.

quence, the idea began to be accepted that the energy was not uniformly distributed on the wavefront. In a lecture at Yale University in 1903, J. J. Thomson made reference to "bright specks on a dark background" for a correct representation of the wave front.[3]

Also Einstein started to work on the same idea. He thought that one could not exclude the appearance of new phenomena for instantaneous values of the fields, like those entering in the absorption and emission processes. His basic idea was that the energy of an electromagnetic wave with frequency $\nu$ was concentrated in very small regions (quanta or particles of radiation), each with energy $h\nu$, where $h$ is Planck's constant. This was the birth of the wave-particle dualism. The particles carried the energy and other physical attributes. The waves constituted an extended entity which surrounded the particles. The presence of the particles was essential for understanding the photoelectric effect, while the presence of the wave allowed one to understand interference and diffraction.

Einstein wrote an important paper on dualism in 1917.[4] In it, he considered a gas of molecules interacting through the emission and the absorption of electromagnetic radiation; assumed for the molecules the existence of discrete energy states capable of transforming into each other by emission or absorption of electromagnetic radiation; assumed that the possible processes were those of stimulated emission under the influence of the radiation field and of absorption of the same nature, and also of emission "without excitation from external causes"; finally assumed that the different energy states were found in the molecular gas with a frequency deduced from the canonical distribution of states of statistical mechanics. Among the results obtained was a very simple derivation of Planck's formula which is still repeated today in many textbooks. But Einstein considered as more important another result which is instead practically forgotten.

When radiation is emitted or absorbed by a molecule it imparts to the latter a momentum (except if one is dealing with a spherically symmetric wave). The maximum momentum is obtained when all of the energy $E$ is emitted in one direction and in this case it equals $E/c$. Einstein could prove that the experimentally established Maxwell distribution of molecular velocities was obtained only under the hypothesis that all energy was emitted in (or absorbed from) a unique direction in every individual radiation-matter interaction process.

In the final paragraph he stressed that the following points could be considered as "fairly certainly proved": "If a radiation bundle has the effect that a molecule struck by it absorbs or emits a quantity of energy $h\nu$ in the form of radiation (ingoing radiation), then a momentum $h\nu/c$ is always transferred to the molecule. For an absorption of energy, this takes place in the direction of propagation of the radiation bundle, for an emission in the opposite direction." Einstein considered the previous result as the *most important conclusion of his paper*, obviously because it threw light on the nature of the electromagnetic radiation.

A more spectacular proof of the directional character of quanta was given by Compton in 1923.[5] He found that in the scattering of X-rays from matter a change of frequency was observed and he could obtain a theoretical formula in complete agreement with his experimental data. Compton wrote: "The present theory depends essentially upon the assumption that each electron which is effective in the scattering, scatters a complete quantum. It involves also the hypothesis that the quanta of radiation are received from definite directions and are scattered in definite directions. The experimental support of the theory indicates very convincingly that a radiation quantum carries with it directed momentum as well as energy."

Compton's experiment *suggested* that corpuscles were indeed present in the electromagnetic radiation. Much stronger evidence was obtained in 1925 by Compton and Simon[6] who observed recoil electrons from X-ray scattering in a cloud chamber. If an unobserved corpuscle was scattered, one could calculate from energy and momentum conservation the direction along which it had to move. This corpuscle could make a second scattering on an electron and the resulting electron track should have its starting point on the calculated trajectory of the unseen corpuscle. Compton and Simon observed 18 such double scatterings, which was, within statistical errors, what they expected.

Discussing this experiment, Compton wrote two years later: "... we can find no interpretation of the scattering except in terms of the deflection of corpuscles or photons of radiation."[7]

Einstein's objective dualism associated wave and corpuscles in such a way that the particle properties, energy and momentum ($E$ and $\mathbf{p}$), and the wave properties, frequency and wave number vector ($\nu$ and $\mathbf{k}$), were related by

$$E = h\nu; \qquad \mathbf{p} = h\mathbf{k}$$

As is well-known, de Broglie extended these relations to material particles (electrons, neutrons, etc.), so that they appear as a very fundamental property of nature.

A problem coming immediately to mind within Einstein's philosophy is the following: If the localized particle carries all the energy and momentum, in what sense can the wave be considered real? This problem was felt so acutely by Einstein that he referred to these waves as *Gespensterfelder* (ghost fields): An object without energy and momentum is in fact unable to exert a pressure when hitting other bodies, which means that it does not have that quality which makes us call real a normal object. Still the equations of quantum mechanics describe this wave as propagating in space and time.

The difficulties associated with the conception of an "empty wave" (here and in the following these two words are taken as being equivalent to "wave propagating in space and time but devoid of energy-momentum") have led many people to discard the idea as a scientific impossibility: If the wave is empty, it cannot induce changes into physical objects; hence it cannot be observed; therefore it does not make any sense to postulate its existence.

This was probably one of the reasons for the rejection of Einstein's dualism by the large majority of the physicists, a rejection which displeased Einstein so much that he wrote: "I must look like an ostrich hiding always his head in the relativistic sand for not having to face the ugly quanta."[8] The idea of the empty wave was however not only Einstein's. As will be seen in the second and third section of this paper, it was entertained in different forms also by de Broglie, Bohr, and Born, and it can be said to belong to the line of thought of the Copenhagen school, even if Heisenberg preferred a "withdrawal into the mathematical scheme" to the ambiguous formulation of dualism contained in the complementarity principle. In the fourth section it will be seen that fundamental experiments on the nature of dualism can naturally be interpreted in terms of an empty wave, leaving aside for a moment the problem of the empirical meaning of such an entity. The latter problem is however the central one, and it will be shown in the last section that it is not inconceivable that its solution could be possible using modern technological devices in the study of low intensity photon beams.

## 2. WAVE-PARTICLE DUALISM ACCORDING TO DE BROGLIE

A full exposition of the fundamental ideas from which Louis de Broglie's formulation of dualism arose is beyond the scope of the present paper and is contained in other contributions to the de Broglie issue of *Foundations of Physics.*

For our purposes we recall a 1977 paper,[9] in which de Broglie discussed the Pfleegor–Mandel experiment,[10] in which interferences between the beams emitted by two different lasers were observed under conditions in which the probability for having more than one quantum of energy $h\nu$ outside the lasers was negligibly small. He expressed his ideas in the following way: "In agreement with classical conceptions, for me a particle is a very small object which is constantly localized in space and a wave is a physical process which propagates in space... ." Later be added: "The particle is a very small region of high concentration of energy which is embodied in the wave in which it constitutes some kind of singularity generally in motion."

With these ideas the explanation of Pfleegor and Mandel's observations becomes elementary. Laser 1 and Laser 2 emit continuously two undulatory beams which cross each other at region *R* where they interfere, giving rise to interference maxima (shown as black stripes in Fig. 1). If a particle is emitted from one of the two lasers (arrow in Fig. 1) it will be guided by the wave to reach with greater probability the regions in *R* where the wave amplitude is larger. Collecting many photons on a screen *S* across the region *R*, the interference figure produced by the wave will be revealed by the photon distribution. In this way, the role of the particles is only to allow the detection of the interference from the observation of their statistical distribution in the region where the interference exists.

The many problems posed by this interpretation concerning Heisenberg's uncertainty relations, the indistinguishable nature of photons, the arbitrary normalization of a probabilistic wave, and so on, are satisfactorily solved by de Broglie in his "theory of the double solution."(11) There remains, of course, the fundamental problem of the nature of the directly unobservable wave. On this point such a revolutionary thinker as de Broglie

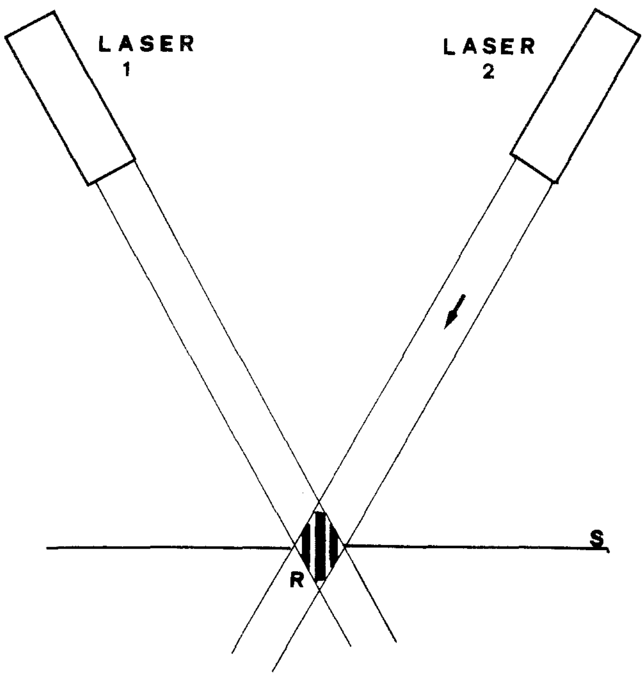


Fig. 1. Set-up of the Pfleegor–Mandel experiment.

prefers to be rather conservative and to assume that *almost* all energy-momentum is associated with the particle, but that a very small fraction of it, so small to have escaped all observations to the present, is, so to speak, smeared on the wave. de Broglie believes that this idea can explain the cosmological red shift without invoking any Doppler effect: The condition for observing the light from a distant galaxy is that its particles (photons) do not undergo any absorption while traveling from the source to our instruments. Even if this condition is satisfied, argues de Broglie, it is very likely that the *wave* was slightly absorbed by interstellar matter which is known to be present in space. This should then result in a small energy loss for the dual (wave + particle) system and therefore in a slight reddening of the light.

Since a longer path described implies a larger number of interactions with interstellar matter, one thus reaches the conclusion that the red shift should increase with distance, as observed.

A proposal which might *in principle* allow laboratory tests of the idea that a small fraction of energy-momentum is associated with the wave has been advanced by de Broglie.[12] Diffraction of a monochromatic light beam through a slit is known to lead to a figure with a central maximum and some lateral secondary maxima. By using suitable optical devices (Fig. 2), it is possible to eliminate the secondary maxima. The name usually give to this phenomenon is *apodization*, at least in the French literature. A plate of variable thickness $P$ is placed in front of a hole made on a screen $S$: The light illuminating the screen gives rise to a variable intensity in the circular region of the hole and this results in the lack of secondary maxima of the diffraction pattern. To explain this phenomenon with very low intensity

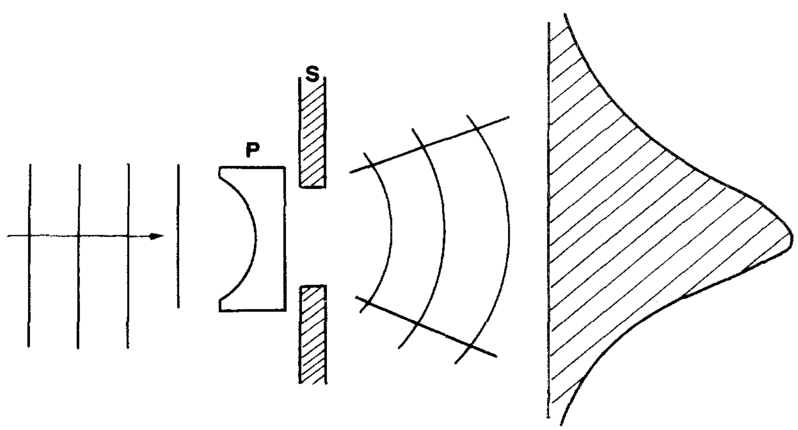


Fig. 2. The "apodization" experiment: The optical device $P$ is built in such a way that single-slit diffraction does not contain secondary maxima.

incident light ("one photon at a time") one needs to assume that the wave associated with every photon undergoes a partial absorption in the plate $P$. In de Broglie's picture this means that the energy of the dual system "wave + particle" should diminish slightly, and the challenge was therefore posed to find a way to reveal this change.

We believe that a feasible experiment could be the following. The energy of the 14.4 kev $\gamma$-ray emitted by (nuclear) $^{57}Fe$ has been measured with a precision of something like $10^{-15}$ (fractional error) by using the Mössbauer effect. This was achieved, in particular, by Pound and Rebka,[13] who could measure the "apparent weight of photons" in the earth's gravitational field. An energy of 14.4 kev corresponds to a wave-length of about 1Å. Detailed study of radiation quanta in the angstrom range of wave lengths was initiated by Bonse and Hart[14] using powerful X-ray interferometers. These researches enabled one to know with good precision the coherence length of the wave packets, their transversal dimensions, and so on. Using this knowledge, it should therefore be easy to conceive and build a device which necesarily absorbs part of the wave of a $\gamma$-ray traversing it. Measurements based on the Mössbauer effect on $\gamma$-rays which have traversed such a partial absorber could therefore reveal a measurable energy loss, if it exists.

This test appears interesting also because of the following argument. If quantum waves carried some energy-momentum, it would be natural to think that this is the case also for massive particles. In the case of a nonrelativistic neutron, one would therefore expect a Hamiltonian operator of the type

$$H = -\frac{\hbar^2}{2m}\nabla^2 + V + F(|\psi|) \tag{1}$$

where $\nabla^2$ is the Laplacian operator, $V$ the potential energy term, and $F(|\psi|)$ the energy term associated with the wave amplitude $|\psi|$. Such a Hamiltonian would lead to a generalized (nonlinear) Schrödinger equation of the type

$$i\hbar\frac{\partial\psi}{\partial t} = -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi + F(|\psi|)\psi \tag{2}$$

This equation was studied by Bialynicki-Birula and Mycielski[15] and shown to have many interesting properties irrespective on the exact form of the functional $F(|\psi|)$. One particularly appealing form of $F$ is, however,

$$F(|\psi|) = -b\log(a|\psi|^2) \tag{3}$$

where $b$ has dimensions of energy. Shimony[16] proposed to study Eq. (2) with $F$ given by (3) using low intensity neutron beams and neutron interferometers. The best upper limit on $b$ has been reported by Zeilinger at the Perugia conference[17] and is

$$b \lesssim 10^{-15}\ eV \simeq 1.6 \times 10^{-27}\ \text{erg}$$

Since the logarithmic factor is expected to be of the order of one, this is also a measure of an upper limit of the energy associated with the wave

$$|F(|\psi|)| \lesssim 1.6 \times 10^{-27}\ \text{erg}$$

Now the studied neutrons have a de Broglie wavelength of about 1 Å, which corresponds to a kinetic energy

$$T \simeq 1.3 \cdot 10^{-13}\ \text{erg}$$

It follows then that the upper limit for $|F|$ represents about $10^{-14}\,T$, *if* one assumes that $F$ has the form (3) and that the logarithmic factor is not larger than unitary. In view of the theoretical uncertainties and of the fact that the Mössbauer effect allows one to measure fractional energy losses smaller than $10^{-15}$, it seems therefore interesting to carry out the proposed experiment on the partial absorption of photonic wavepackets.

## 3. BOHR, FROM CORRESPONDENCE TO COMPLEMENTARITY

Niels Bohr treated the electromagnetic field in purely undulatory terms until 1926: For example, the 1918 paper on the correspondence principle used classical electrodynamics, not only in the region of high quantum numbers, but also for low and intermediate quantum numbers.[18] In this way, intensities, polarizations, and selection rules for the emitted radiation could be calculated.

The success of these calculations helps in understanding why Bohr was not willing to accept Einstein's picture of the electromagnetic field (with the energy carried by small particles). Also the famous 1924 paper by Bohr, Kramers, and Slater[19] (BKS) treated the electromagnetic field as continuous everywhere but "virtual." This field was introduced in the theory by means of the correspondence principle and was considered coupled to the "virtual harmonic oscillators," which one used then for calculating atomic transitions. The problem of the actual existence in space and time of this virtual field is not discussed explicitly in this work, even though its reality is somewhat implied by the very fact that one introduces it in the theoretical frame. What is certain, however, is that the BKS field does not carry energy and momentum: "... we abandon on the other hand any attempt at a causal connection between the transitions in distant atoms, and especially a direct application of the principles of conservation of energy and momentum, so characteristic of the classical theories." Bohr's virtual waves are therefore essentially the same thing as Einstein's "ghost" waves and coincide with what we call "empty" waves in the present paper.

Bohr, Kramers, and Slater claimed that energy conservation would still be valid as a *statistical* concept.

A consequence of this theory is that no time correlation between atomic events are predicted: consider a gas of identical atoms, half of them in the state $E_1$ and half in the state $E_2$. Einstein's description would say that if an atom $A$ goes from $E_2$ to $E_1$ it emits a photon which travels in space and eventually makes a second atom $B$ go from $E_1$ to $E_2$: there is clearly a time relation between the events $A$ and $B$ calculable from the $AB$ distance, and the speed of light. In the BKS theory, however, no such relation can exist: No energy quanta are propagated in space, there is only a continuous field that determines only a *transition probability* for individual atoms.

It was precisely this point which led Bothe and Geiger,[20] and Compton and Simon[21] to make experiments which disproved the BKS theory. These authors used the Compton scattering of X -rays which in this theory would only be a continuous beam of virtual radiation with short wavelength. Thus, in place of atom $A$ making a transition between two stationary states, we have an electron suddenly scattered (according to a probability law similar to the atomic one) and in place of atom $B$ we have the discharge of a counter due to ionization processes. Bothe and Geiger found a very sharp time correlation between electron scattering and $X$-ray absorption, thereby excluding the possibility that the BKS theory could provide a correct description of the physical world.

These experiments pushed Bohr to reconsider his totally negative opinion toward the wave-particle dualism. The principle of complementarity was his peculiar way to accomodate in his world picture the dualism.

Bohr's complementarity can be introduced in the following way.[22] The experimenter lives in a macroscopic world and conceptions typical of this condition such as causality and space-time are deeply rooted in all human beings. But it is not necessary and, in fact, according to Bohr, it is not true that even conceptions of such a general nature have an unlimited applicability in the study of microphenomena. The key for a correct understanding of this fundamental point is the existence of the quantum of action $h$.

Let us examine the experimental meaning of causality and of space-time. Causality for Bohr is synonymous with processes taking place according to well defined rules. These rules are in practice those of energy-momentum conservation. Therefore, an experimenter wishing to check the rigorous validity of the causality law must perform infinitely precise measurements of energy and momentum. The relations $\Delta E = 0 = \Delta p$ imply, however, because of Heisenberg's relations,

$$\Delta t = \infty = \Delta x$$

which mean that absolutely no localization in time and in space is produced during the measurement. Complete lack of localization means for Bohr that space and time in practice do not exist. (It should be noted that here, as in many other places, Bohr makes an important concession to the positivistic philosophy assuming that what is not observed does not exist.) Therefore one concludes that observation of causality forbids an observation of space and time.

Conversely one way wish to observe space-time correlations: an ideal measurement would now imply $\Delta x = 0$, whence $\Delta p = \infty$. This means that an arbitrarily large momentum is exchanged between apparatus and atomic system. Furthermore, such an exchange is *in principle* impossible to determine "if the measuring apparatus has to serve its purpose," as Bohr shows in detail in several concrete situations. In these conditions it is obviously impossible to verify the law of causality (that is to check energy-momentum conservation). Thus one concludes that observation of space and time precludes an experimental control of the validity of the law of causality. Therefore, it does not make any sense to talk about such a law when one performs space-time observations.

Bohr concludes from these considerations that there is a *complementary relation* (rigid mutual exclusion) between space-time coordination and causality: they can never be used simultaneously.

A similar conclusion is obtained when one considers the conceptions of particle and wave. When studying causality (e.g., in the photoelectric effect or in Compton effect), one finds that the conservation of energy and momentum "finds its adequate expression just in the light quantum idea put forward by Einstein." Conversely, the conceptual instrument suitable for predicting the possible points $P'$ in which a localization of the observed system will manifest itself, after a previous localization in $P$, is the wave function, whose squared modulus gives the probability density to observe the system in a different point $P'$. It is naturally impossible to know in which point the localization will manifest itself. Therefore, the evolution appears as intrinsically stochastic, which is the same as saying once more that causality does not hold. It should furthermore be stressed that one is here talking an undulatory language. There is also a mutual exclusion between particle and wave, which must be considered complementary descriptions of atomic systems.

It should be stressed that the wave which represents one of the complementary aspects of atomic entities is a daughter of the BKS virtual wave and conceptually very near to the probability wave which Born introduced in 1926 in quantum mechanics. In fact, as we stated above, the wave is applicable in studying space-time localizations. Since this excludes the possibility to check the causality law (synonymous with energy-momentum conservation) one is faced once more, as in the BKS paper, with a wave that does not generate energy conserving transitions. However, this wave can modify the probabilities of atomic transitions.

It should also be noticed that the well-known interpretation of quantum waves in probabilistic terms, proposed by Max Born in 1926[(23)] and now universally accepted, was essentially a reformulation and a generalization of the idea of Bohr, Kramers, and Slater. This continuity was stressed particularly by Heisenberg, who wrote[(24)]: "The probability wave of Bohr, Kramers, and Slater ... was a quantitative version of the old concept *potentia* in Aristotelian philosophy. It introduced something standing in the middle between the idea of an event and the actual event, a strange kind of physical reality just in the middle between possibility and reality. Later, when the mathematical framework of quantum theory was fixed, Born took up this idea of the probability wave and gave a clear definition of the mathematical quantity in the formalism ...." In this way, the "virtual" waves of BKS became the usual wave functions of quantum theory.

One often finds in textbooks that quantum waves for Max Born were *only* a mathematical expedient for calculating probabilities. This is not completely correct, however, since Born wrote[(25)]: "The question of whether the waves are something 'real' or a function to describe and predict phenomena in a convenient way is a matter of taste. I personally like to regard a probability wave, even in $3N$-dimensional space, as a real thing, certainly as more than a tool for mathematical calculations .... Quite generally, how could we rely on probability predictions if by this notion we do not refer to something real and objective?"

Coming back to Bohr's use of empty waves, it is important to realize that probably the Danish physicist also associated some reality with these waves. An illuminating episode took place in 1957 when the Soviet physicist Fock went to Copenhagen[(26)] and presented Bohr with a paper in which complementarily was criticized in four different ways: 1) one should insist on the fact that the $\psi$ function of quantum mechanics represents *something real*; 2) the presence of precise mathematical laws is equivalent to a certain type of causality; 3) limitations in understanding come only from the use of a classical language; 4) no "uncontrollable interaction" between apparatus and system takes place during measurements.

After reading the paper, it is known that Bohr agreed on these four points. This acceptance of Fock's ideas, which is reflected in the papers written by Bohr in the last part of his life, is important for our purposes because of the first point. It shows that, at least after 1957, Bohr had a picture about quantum waves not different from that of "virtual" waves, which he had discussed in the BKS paper and introduced later in his complementary principle. Heisenberg wrote in 1958: "... the concept of complementarity introduced by Bohr into the interpretation of quantum theory has encouraged the physicists to use an ambiguous rather than an unambiguous language, to use the classical concepts in a somewhat vague manner in conformity with the principle of uncertainty, to apply alternatively different classical concepts which would lead to contradictions if used simultaneously. In this way one speaks about electronic orbits, about matter waves and charge density, about energy and momentum, etc., always conscious of the fact that these concepts have only a very limited range of applicability.

"When this vague and unsystematic use of the language leads into difficulties, the physicist has to withdraw into the mathematical scheme and its unambiguous correlation with the experimental facts."[27]

It is interesting to notice that this withdrawal "into the mathematical scheme" has become a mass phenomenon in contemporary theoretical physics and that Bohr's complementarity and the other forms of dualism are half forgotten.

Heisenberg made great discoveries which have shaped the theoretical physics of the present century. His line is however fully characterized by a great fascination for the mathematical language coupled to a philosophically negative attitude towards the material world.

Strong statements of a philosophical nature were made by Heisenberg in his paper on the uncertainty relations.[28] Just as an example, we recall that the validity of these relations still allows one to calculate position and momentum in the past with any desired accuracy. About this fact Heisenberg wrote in 1930: "Then for these past times $\Delta p \, \Delta q$ is smaller than the usual limiting value, but this knowledge of the past is of a purely speculative character, since it can never ... be used as an initial condition in any calculation of the future progress of the electron and thus cannot be subjected to experimental verification. It is a matter of personal belief whether such a calculation concerning the past history of the electron can be ascribed any physical reality or not." Heisenberg's "personal belief," which has become the dominant point of view today, was certainly that one should refrain as much as possible from talking about physical reality.

Admiring Heisenberg's discoveries does not necessarily mean that one should accept his *philosophy*.

The set of ideas developed by Einstein, de Broglie, Bohr, and Born could finally prove to be more fruitful than an eternal withdrawal into the mathematical scheme.

## 4. SOME IMPORTANT EXPERIMENTAL FACTS

The review of some recent proposals which could conceivably give empirical meaning to the empty waves of Einstein and Bohr is left for the last section. Here we will rather check that the idea is not already excluded by existing experimental evidence by considering new and old experiments on self-interference (neutron interferometry[29] and the Jánossy-Naray experiment[30] on optical photons) and some researches on stimulated emission (Blake–Scarl experiment[31] and atomic radiation in presence of mirrors[32]). The latter could actually provide an *indirect* evidence in favor of the empty wave idea.

The first neutron interferometer was operated in 1974 by Rauch, Treimer, and Bonse at the Austrian Nuclear Institute in Vienna.[33] The interferometer was built starting from a single silicon crystal completely free of dislocations and other defects in the regular atomic structure. For example, one can start from a cylindrical crystal about eight centimeters long and five centimeters in diameter, and cut away part of it, leaving three semicircular "ears" connected by the remainder of the cylinder.

When a beam of neutrons strikes the first ear in the interferometer at an angle $\theta$ from the normal to the surface (Bragg angle) of 20–30° it is scattered by planes of atoms *perpendicular* to the face of the crystal. This kind of scattering gives rise to two beams: a transmitted one at the Bragg angle $\theta$ and a diffracted one at the same angle but on the opposite side of the scattering planes. In other words, the emerging beams form a $V$ whose vertex lies in the first ear. At the second ear each of these beams are again Laue scattered, the four emerging beams forming a $W$ whose vertexes lie in the second ear. The geometry of the apparatus is such that the two external beams of this $W$ do not have any further interaction with the interferometer (Fig. 3). The important events happen when the two internal beams of the $W$ converge at the same point on the face of the third ear: Each of them is again Laue scattered and gives rise to a $V$ coming out of the third ear. The two $V$'s are however spacially superimposed in such a way that each of the two beams which emerge from the third ear results from the sum of the transmitted component of one beam and the diffracted component of the other beam.

Let $\psi_R$ and $\psi_L$ be the two wave functions describing the beams which arrive on the *third* ear from right and from left, respectively. At the third ear each wave is split in a transmitted ($T$) and a diffracted ($D$) wave: $\psi_R$ gives

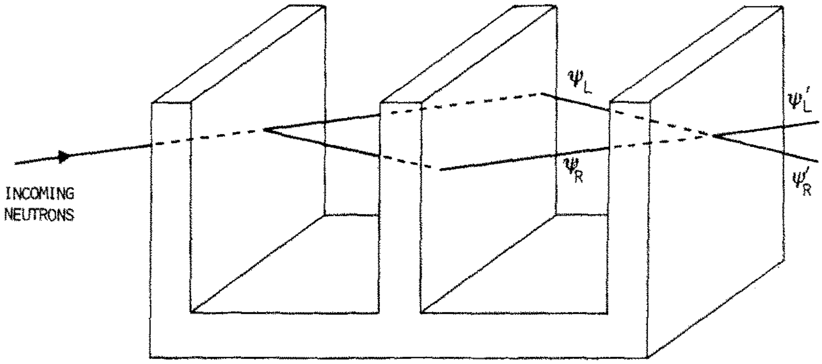


Fig. 3. Neutron paths within a neutron interferometer. Not shown are the paths emerging from the second "ear" and leading outside the interferometer.

rise to $\psi_R^T$ and $\psi_R^D$, while $\psi_L$ gives rise to $\psi_L^T$ and $\psi_L^D$. The coherence of this phenomenon guarantees that $\psi_R$, $\psi_R^T$, and $\psi_R^D$ have the same phase and that the same holds for $\psi_L$, $\psi_L^T$, and $\psi_L^D$.

Now, in the $V$ which emerges from the third ear one has two waves, which we call $\psi_R$ and $\psi_L$ and which result from the physical spatial superposition (and therefore the algebraic sum) of two different waves

$$\begin{cases} \psi_R' = \psi_L^T + \psi_R^D \\ \psi_L' = \psi_R^T + \psi_L^D \end{cases}$$

With some simplyfying assumptions, it is possible to show that

$$\begin{cases} |\psi_R'|^2 = \frac{1}{2}[|\psi_R|^2 + |\psi_L|^2 + 2\,|\psi_R| \cdot |\psi_L| \cdot \cos\alpha] \\ |\psi_L'|^2 = \frac{1}{2}[|\psi_R|^2 + |\psi_L|^2 - 2\,|\psi_R| \cdot |\psi_L| \cdot \cos\alpha] \end{cases}$$

where $\alpha$ is the phase difference between an $R$ and an $L$ wave. From the previous result one can see that probability is conserved and furthermore that the squared modulus of each beam emerging from the neutron interferometer depends on the relative phase of the right and left components. If this phase is known, one can check if the famous probability law of Born is really valid: In fact $\psi_R'$ and $\psi_L'$ are spatially divided, and the probability of a single neutron choosing the left or the right should be equal to $|\psi_L'|^2$ and $|\psi_R'|^2$, respectively. Repeating the experiment many times one should have emerging neutron *fluxes* proportional to $|\psi_L'|^2$ and $|\psi_R'|^2$. Practical experiments have been done with monochromatic neutron beams emerging from a nuclear reactor. Typically these have a flux of about 100 neutrons per second with an average time spacing of $10^{-2}$ sec. From a reactor thermal neutrons normally emerge with velocity of the order of $10^5$ cm/sec: Every neutron therefore crosses the ten centimeter long interferometer in about $10^{-4}$ sec. The probability of having two neutrons in the apparatus is therefore of the order of $10^{-2}$. Obviously this allows one to conclude that self-interference is observed in these experiments. The experiments performed by using the most diverse physical means for varying the relative phase $\alpha$ (phase-shifting materials, magnetic fields, gravitational field of earth, earth rotation, etc.) have shown a very good consistency with Eq. (1). Thus Born's statistical postulate (particle distribution given by $|\psi|^2$) is confirmed. Moreover every neutron interferes with itself (just as in the double slit experiment exposed in textbooks). Thus we conclude that in the case of every single neutron something propagates over both the possible paths within the interferometer.

This conclusion is apparently contradicted by a second experiment which one could do by putting neutron counters immediately behind the first ear: As is well-known, since such experiments have been performed many times with photons and electrons, every neutron would be revealed only by a single counter. Neutron counters are in last analysis sensitive to incoming energy momentum: We can conclude that energy momentum (and therefore mass) take always only one of the two paths which are possible when crossing an interferometer ear. On the other path, one thus concludes, a wave propagates which does not carry energy and momentum.

This wave, devoid of energy-momentum, which we extracted with a naïve phenomenological analysis of neutron interferometric experiments, resembles obviously very much the empty wave of Einstein, de Broglie, Bohr, and Born discussed in the previous sections; the two will tentatively be taken to be the same thing.

The first observations of self-interference in experimentally certain conditions were made by L. Jánossy at the Central Research Institute of Physics in Budapest in the fifties. The final experiment used an interferometer of the Michelson type and monochromatic photons coming from the 5461 Å (green) line of mercury.[30] The experiment had the following features (Fig. 4):

**Arm length of about 14m.** Since a photon can be considered to be a packet of length comparable with the coherence length of a spectral line, in the case of the green mercury line one would obtain longitudinal dimensions of about 1m. If the dimensions of the interferometer were smaller than the "size" of the photon, the interference phenomenon might be accounted for by assuming that the photon "floods" the whole of the arrangement simultaneously and thus comes under the influence of the two mirrors at the ends of the arms. This picture cannot work if the arm length is much larger than the photon coherence length, as 14m obviously is.

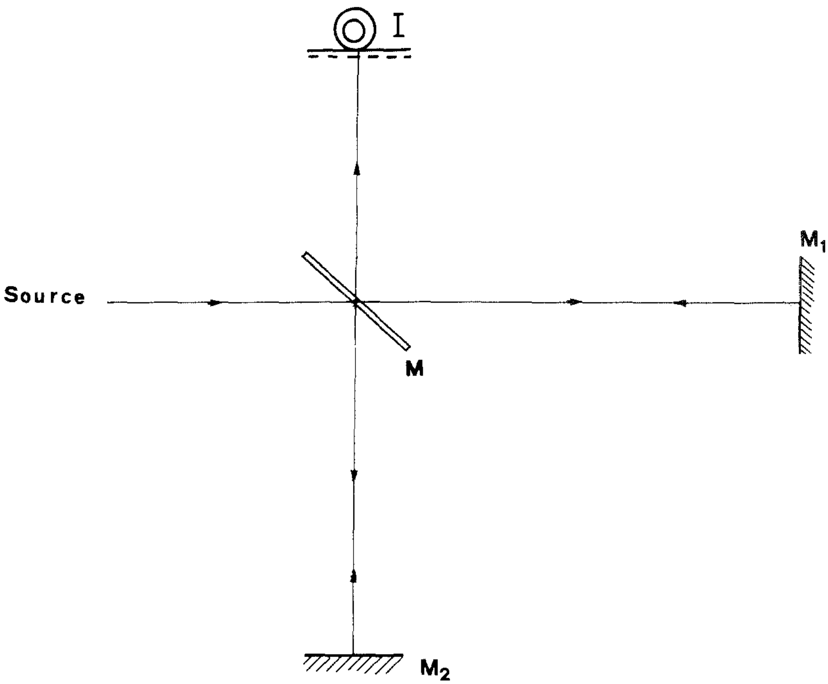


Fig. 4. Set-up of the Jánossy-Naray experiment. $M$ is a semitransparent mirror, $M_1$ and $M_2$ are normal mirrors, and $I$ is an interference detecting device.

**Photon intensity as low as $10^4$ photons/sec.** A 30m long apparatus of the type used by Jánossy and Naray is crossed by a photon in $10^{-7}$ sec. An intensity lower than $10^7$ photons/sec is therefore required in order to insure that only one photon is present in the apparatus in any given time.

**Underground tunnel cut in rock.** In order to achieve the necessary stability, the apparatus was built in an underground tunnel cut in rock 30m under the surface of the earth. Even the presence and activity of a human being in the tunnel could disturb the regular functioning of the apparatus. The problem was solved by using remote control, observing the interference pattern from outside by the help of a periscope.

These experimental precautions allowed Jánossy and Naray to reach important conclusions. Their paper ends with the following statement: "As the result of our measurements, we have therefore to conclude that *interference phenomena are perfectly normal even at such low intensities where at one time on average less than one photon is to be found in the arrangement;*

*this is true even if the dimensions of the arrangement greatly exceed the coherence length of the photons giving rise to the patterns.*"

An attempt at understanding this results in space-time terms would naturally assume that when the photon wave packet arrives at the semitransparent mirror it is split into two parts which follow different trajectories, are reflected by different mirrors, and recombine in the region where the interference pattern is observed. This possibility seems however to be excluded by another experiment, just as in the case of neutrons.

If one sends a low intensity photon beam on a semitransparent mirror and puts on the two outgoing trajectories two photomultipliers, coincidences should be observed if the energy-momentum of the incoming photon is split in two parts. Accurate measurements were performed by Clauser[(34)] and, as a by-product of a more complex experiment, by Mandel and Dagenais,[(35)] but no coincidences were seen. Also for photons, therefore, the existing experimental evidence does not contradict the idea that a quantum system is composed of an empty wave and a localized structure carrying energy, and momentum, and that a semitransparent mirror splits only the empty wave, while energy and momentum make a well defined (albeit random) choice between the two trajectories. If this picture is correct, it would imply that in the Jánossy-Naray experiment empty waves propagate on one of the two trajectories while normal (energetic) photons propagate on the other one. These empty waves, if they exist, obviously lack the most fundamental qualities which make us call real any phenomenon: energy and momentum. This does not mean, however, that they are in principle unobservable since *they could still manifest their presence by changing the transition probability of excited systems.* In fact, no energy needs to be provided to an excited atom $A^*$ in order to make it decay. The energy momentum balance can be fully satisfied from the energy contained in $A^*$ when the decay $A^* \to A +$ photon is considered. Recent experiments seem indeed to be consistent with this picture.

Blake and Scarl[(31)] have investigated laser light amplification in a laser gain tube. A schematic representation of their apparatus is shown in Fig. 5: A beam of light generated by on He–Ne laser (L) at a wavelength of 6328 Å crosses an He–Ne laser gain tube (LGT in fig. 5) capable of emitting at the same wavelength. The gain, defined as the ratio of the intensity of the laser beam with the discharge on, divided by the intensity with the discharge off (after the amplifier tube spontaneous emission had been subtracted) was $1.30 \pm 0.02$. At those low amplifications the gain is known to be linear for the intensity levels used in this experiment: Physically, one can say that in the first approximation it is possible to neglect the further amplification of the light generated in LGT by the laser beam. Under these conditions one might expect the stimulating photon and the stimulated photon to be related

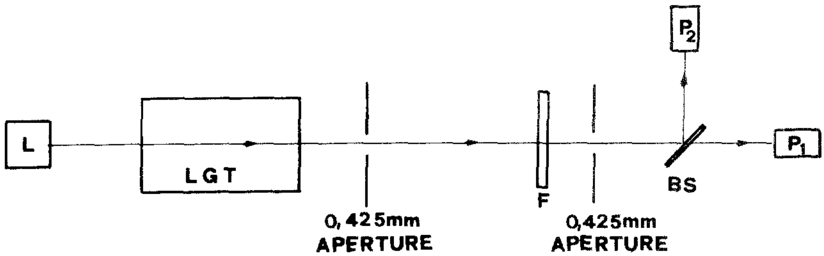


Fig. 5. Schematic representation of the Blake–Scarl apparatus: $L$ is a He–Ne laser and LGT and He–Ne laser gain tube. The beamsplitter BS sends light to the phototubes $P_1$ and $P_2$.

in time, *if the process of stimulated emission is generated by the energy momentum of the e.m. field,* since these physical quantities are always detected as localized in space and time. The amplified laser light, together with the unavoidable spontaneous emission from the LGT, crossed an angular spatial filter consisting of two 0.425 mm pinholes and an interference filter $F$ which allowed only light with wavelength $(6328 \pm 5)$ Å to pass. A beamsplitter (BS) and two photomultipliers $(P_1, P_2)$ were used as the two photon detection system.

If the coincidence rate is defined as the number per unit time of photon detections by $P_1$ *and* $P_2$ having a time difference less than 1 nanosecond, Blake and Scarl found that, after background subtraction, the coincidence rate was the same, *independently on the intensity of the laser light.* This result was found in four experimental runs in which the ratios of amplified laser light to amplifier tube chaotic light in the same mode were 0, 0.5, 1, and 2. Blake and Scarl could therefore conclude that "no correlations between incident and induced photons seem to be necessary in order to explain the results."

This result was considered surprising by the authors who stated that "Stimulated emission illustrates one of the apparent difficulties of current photon descriptions of the electromagnetic field."

A natural explanation can be given in terms of the empty wave. Suppose a photon is a localized entity carrying energy momentum and that it is accompanied by an extended empty wave. Suppose that the stimulated emission is due to the action of this wave. For laser light the wave has an infinitely long coherence length and has a constant probability of generating a stimulated photon. This is produced independently of the presence and position of photons in the stimulating wave. Thus no time correlation is expected between primary and secondary photons, as observed.

Another phenomenon whose existence is consistent with the empty wave idea is the modification of fluorescence lifetimes in presence of mirrors. This observation shows that spontaneous emission of radiation has the interesting property of being affected by the source atom's local environment. This is hard to understand with different pictures of the wave-particle dualism, and Milonni and Knight[32] voiced the difficulty of many people by writing: "*The experimental observation that fluorescence lifetimes depend upon the atom's surroundings was apparently met with some surprise: how could the spontaneous emission of a photon be affected by the local environment, since the atom can only "see" its surroundings by emitting a photon in the first place?*" In the following discussion Milonni and Knight conclude that the radiated field (or the photon "wave function") contains all the information needed for changing the atomic emission probability, provided only a time $t = 2l/c$ has elapsed since the atom entered the region where the mirror is placed, at a distance $l$ from it.

## 5. SOME EXPERIMENTS ON EMPTY WAVES

The problem of the empirical meaning of the idea that empty waves exist in space and time independently of our observations was discussed in two papers,[36] in which the general solution was indicated in the following way: The wave could reveal that it is real, independently of the *localized* energy and momentum which sometimes is associated with it, if it gives rise to changes in the transition probabilities of the systems with which it interacts.

It is well-known that all the known elementary entities (photons, electrons, neutrons, etc.) have a fundamental dualistic nature. Therefore, for each of them there is an associated wave, and a blind guess could be that all *waves* are of the same nature, even though they are described by very different mathematical structures within quantum mechanics. One could therefore try to see whether the waves associated with a beam of neutrinos (which, being very weakly interacting particles, can be viewed as passing through a piece of matter without any transfer of energy-momentum) are by any chance capable of modifying transition probabilities of unstable systems. A beam of neutrinos traverses a piece of matter in which unstable entities (nuclei excited atoms or molecules) are contained. The life time of these entities are measured under such conditions and then compared to the life time of the same entities in the absence of any traversing beam. If a difference is observed, its only logical explanation could be that it is due to the action of the wave. Naturally one would try to ensure that the transition frequency of the unstable systems equals one of the frequencies available in the neutrino beam.

Another possible way to approach the problem[36] is to consider a source emitting photons that can reach, with a definite probability density, a certain region $R$ of a screen on which they are absorbed. Within $R$ there is an unstable (e.g., fluorescent) substance $\sigma$ having a suitably long life time and a transition frequency equal to that of the incoming photons.

Photons will arrive as chunks of energy momentum randomly at different points on $R$, but the waves associated with them will always impinge on all of $R$, including therefore the unstable substance $\sigma$. It is thinkable that the wave, when it interacts with the atoms of $\sigma$, will change their life time.

The first proposal to study stimulated emissions due to a wave which for certain has no associated energy-momentum[37] considered the double-slit experiment.

The incoming photon flux is assumed to be so low that, if two counters are put just behind the slits and one of them counts a photon at time $t_0$, the probability that the other one detects a photon in the interval $(t_0 - \Delta t, t_0 + \Delta t)$ is negligible. The proposed experiment should then have a counter $C$ behind the first slit and a fluorescent substance $\sigma$ behind the second slit. Fluorescent emissions by $\sigma$ are detected only in time intervals $2\Delta t$ surrounding the instants at which $C$ has counted an incoming photon, with the hope that the detection rate in such intervals (during which one imagine $\sigma$ as continuously crossed by an empty wave) will be different from the normal one.

An apparatus with interesting properties was proposed by Szczepanski.[38] It consists of a light source $S$ which emits monochromatic photons split by the semitransparent mirror BS (Fig. 6). One part of the beam is detected by $D_1$, the second part by $D_2$. In front of $D_2$ a two-level atom $A$ is placed with an excitation energy equal to the energy of the photons emitted by $S$. One measures the delayed coincidences of the

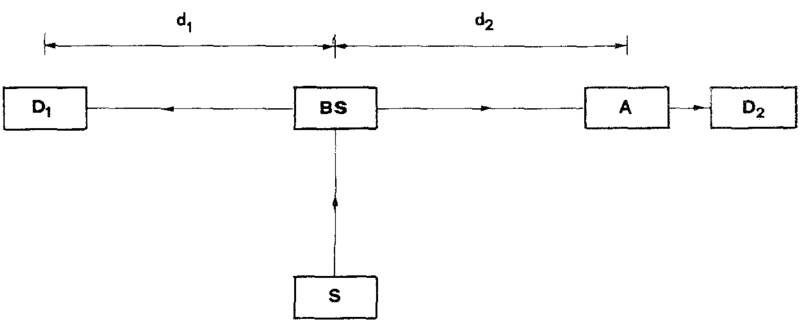


Fig. 6. The experiment proposed by Szczepanski: $S$ is a light source, BS a beam splitter, and $D_1$ and $D_2$ are two photon detectors. $A$ is a laser gain tube.

recordings of $D_1$ and $D_2$ whose respective distances from BS, $d_1$ and $d_2$, are variable.

Szczepanski considered this experiment's main interest to lie in the possibility of distinguishing the classical description of the photons, resurrected by Jaynes,[39] from their quantum description. He stressed that, from both the classical and the quantum points of view, the wave packet emitted by $S$ is split by BS, and the situation is similar in both theories as long as $D_1$ does not record the photon. After it does, the quantum and the classical descriptions differ dramatically. Classically, the detection of the wave packet by $D_1$ does not affect in any way the second part of the wave packet, i.e., the radiation propagating in the direction of $D_2$.

Quantum-mechanically, the recording of the photon by $D_1$ causes, according to Szczepanski, an immediate vanishing of the inducing field propagations toward $D_2$ because of the reduction of the photon's wave packet. Thus the packet emitted by $S$ is able to induce an emission from $A$ only until it has *not* been detected by $D_1$. The dependence of the (delayed) coincidence rate upon the distances $d_1$ and $d_2$ is then the main information the experiment is supposed to yield.

We note that the classical description was already excluded by the 1974 experiment of Clauser[34] and that the reduction of the wave packet is only one of the several parallel formulations of the quantum measurement theory. In the so-called "statistical interpretation" of quantum mechanics[40] the reduction of the wave packet does not take place. In all cases Szczepanski's experiment can indeed allow one to check whether the reduction of the wave packet takes place: If it does, as many people seem to believe, the $D_1 D_2$ coincidences should disappear as soon as $d_2 > d_1$.

The importance of Szczepanski's idea for the verification of an active role of the empty waves in photon interactions was stressed by the present author.[41]

A very low intensity photon beam emitted by the source $S$ (Fig. 7) is split by the beam splitter $M$ into two orthogonal beams. On the reflected beam a phototube $P_1$ is located, and one concentrates attention on events in which $P_1$ does reveal a photon arrival. For such events no energy-momentum can propagate in the transmitted beam since no coincidences above the casual background would be revealed by a second phototube eventually put on this beam (and not shown in Fig. 7). However, the transmitted beam crosses a laser gain tube (LGT), where the eventual wave packet devoid of energy and momentum has a chance to reveal its existence by generating a zero energy transfer stimulated emission. The emitted photon could than be detected by the phototube $P_2$ put behind LGT. In this way $P_1 P_2$ coincidences would reveal the propagation of a zero-energy undulatory phenomenon transmitted by $M$. The space-time propagation of this entiry could be studied

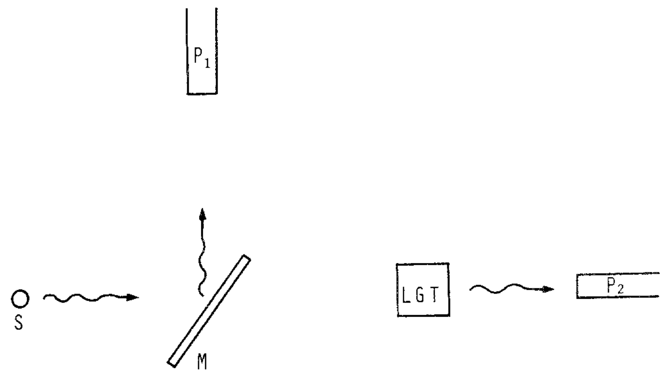


Fig. 7. Experiment proposed by the author for the detection of empty waves: $M$ is a beam splitter, LGT a laser gain tube, and $P_1$ and $P_2$ are two photomultipliers.

by checking that $P_1 P_2$ coincidences disappear whenever an obstacle is put before LGT in the transmitted beam.

It is remarkable that such an experiment can be done with a feasible development of the most sophisticated optical devices.[42]

A different suggestion for the detection of empty waves was advanced in Ref. 43 and proposes to use superfluorescence (SF), that is, the coherent decay of many atoms predicted by Dicke[44] and studied experimentally by different groups.[45] Since SF is a phenomenon similar to the so-called laser effect, the coherent decay of something like $10^8$ atoms being triggered by a single spontaneous emission due to quantum fluctuations of the electromagnetic field in vacuum, the natural idea would be to see whether an empty wave could give rise to SF pulses. The basic apparatus would be different from the one shown in Fig. 7 only because the LGT would be replaced by a superfluorescent atomic cesium pencil.

A research program for detecting observable differences between the Copenhagen and the statistical interpretation of quantum mechanics has been started by Garuccio and Vigier.[46] In a letter in collaboration with K. Popper[47] they proposed an apparatus which was conceived for observing interferences of the Mandel–Pfleegor type between two light beams emitted from two independent lasers under conditions in which it is very likely that no more than one single photon can be present in the experimental area at any given time. The main idea was that of adding a device capable of revealing by which laser every photon that appears in the interference pattern was emitted. This proposal has provoked a complex and heated discussion, which has led Garuccio, Rapisarda, and Vigier[48] to present a new experimental set-up that clearly manages to bypass all the objections raised. This setup, shown in Fig. 8, uses a very low intensity *incoherent* source of

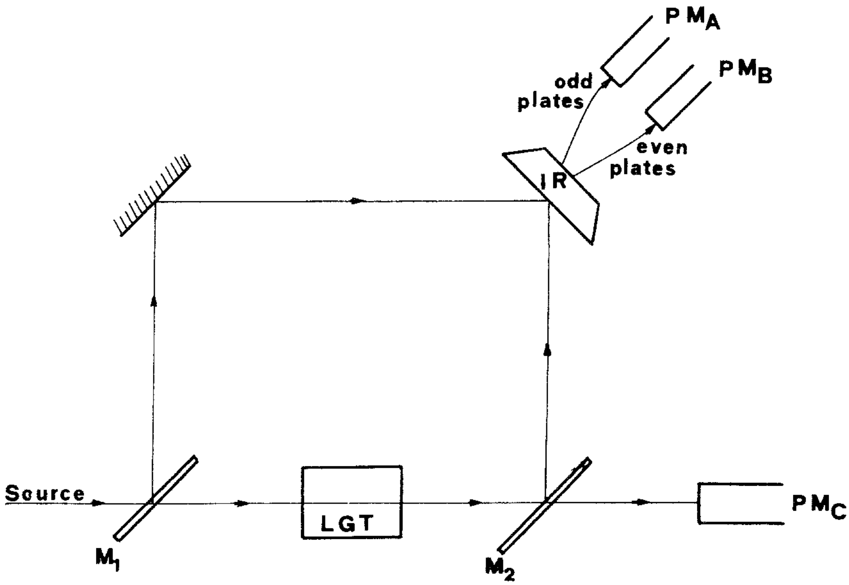


Fig. 8. Set-up of the Garuccio–Rapisarda–Vigier experiment. $M_1$ and $M_2$ are semitransparent mirrors; $PM_A$, $PM_B$, $PM_C$, are photomultipliers; and IR is an interference detecting, apparatus.

monochromatic photons. These photons are made to cross a semitransparent mirror $M_1$: on the transmitted path a laser gain tube LGT is inserted to effect amplification. Behind the LGT a second semitransparent mirror $M_2$ splits the beam again into two parts, and one considers only those cases in which a phototube $PM_c$ put on the $M_2$-transmitted part counts one photon.

This proposal depends on an idea opposite to that discussed in Ref. 41 namely on the assumption that the experiment proposed there has been performed and *has failed* (in the sense, at least, that a reasonably low upper limit has been set on the photon production rate from incoming empty waves).

If this were the case one could conclude that when a photon has been detected by $PM_c$ no energetic photon is propagating on the $M_1$-reflected path. The photon detected by $PM_c$ is therefore necessarily connected with a situation where the single photon emitted by the source has been transmitted by $M_1$ and has crossed LGT. If it has generated an amplification it will happen rather often (33% of the cases, if only *two* photons come out of LGT) that the photon revealed by $PM_c$ is accompanied by a twin photon which is reflected by $M_2$ and goes, following a certain optical trajectory, to an interference detecting system (IR), where it is superimposed on the $M_1$-

reflected beam. Interferences should show up in IR if an empty wave has propagated in the $M_1$-reflected beam. Garuccio, Rapisarda, and Vigier propose to use in IR a detector devised by Pfeegor and Mandel[10] and built with a stack of thin glass plates, each of which has a thickness corresponding to a half fringe width. The plates are cut and arranged so that any photon falling on the odd plates is fed to one photomultiplier, $PM_A$, while photons falling on the even plates are fed to the other, $PM_B$. Detection of differences in the coincidence rates

$$PM_A \wedge PM_C \qquad \text{vs} \qquad PM_B \wedge PM_C$$

would constitute evidence, under the stated conditions, that something not carrying energy-momentum is propagating in the $M_1$-reflected trajectory.

A necessary consequence of this idea is that an obstacle put in the latter trajectory should destroy the interference.

In order that the effect be observable it is obviously necessary that every single act of amplification does not give rise to a random change of the phase of the wave packet crossing LGT. This condition is known to be satisfied in those cases in which the amplifying gas volume has dimensions small compared with a radiation wavelength.[44] In general, a detailed calculation is needed in order to know the size of the phase shifting effects.

It is remarkable that the latter experiment can be carried out with the same apparatus[42] that is used for the experiment proposed in Ref. 41. The working of the apparatus is based on the Doppler-free $n$ photon transition theory developed by Vasilenko, Chebotayev, and Shishaev,[49] discussed by Cagnac *et al.*[50] and verified experimentally by Grynberg *et al.*[51]

The amplification of a photon (or of an empty wave) is revealed by a final emission of a fluorescent photon. The spontaneous emission is more or less isotropic and contributes very little to the over-all transition in the fixed direction of the stimulating beam.

## ACKNOWLEDGMENTS

The author is deeply grateful to prof. L. de Broglie for early correspondence on the nature of the wave-particle dualism. He also wishes to thank prof. A. Gozzini and prof. H. Rauch for very stimulating discussions.

## REFERENCES

1. A. Einstein, *Annalen der Phys.* **18**, 639 (1905).
2. A. Einstein, *Annalen der Phys.* **17**, 132 (1905).

3. M. Jammer, *The Conceptual Development of Quantum Mechanics* (McGraw-Hill, New York, 1966).
4. A. Einstein, *Phys. Zs.* **18**, 121 (1917).
5. A. H. Compton, *Phys. Rev.* **21**, 483 (1923).
6. A. H. Compton and A. Simon, *Phys. Rev.* **25**, 306 (1925).
7. A. H. Compton, Nobel Lecture (1927).
8. L. de Broglie, *Ann. Fond. L. de Broglie*, **4**, 13 (1979).
9. L. de Broglie, *Ann. Fond. L. de Broglie*, **2**, 1 (1977).
10. R. L. Pfleegor and L. Mandel, *Phys. Rev.* **159**, 1084 (1967).
11. L. de Broglie, *Une tentative d'interprétation causale et non-linéaire de la mécanique ondulatoire: la théorie de la double solution* (Gauthier-Villars, Paris, 1956).
12. L. de Broglie, *La réinterprétation de la mécanique ondulatoire* (Gauthier-Villars, Paris, 1971).
13. R. V. Pound and G. A. Rebka, Jr., *Phys. Rev. Lett.* **4**, 337 (1960).
14. U. Bonse and M. Hart, *Zeits. f. Phys.* **194**, 1 (1966).
15. I. Bialynicki-Birula and J. Micielski, *Ann. Phys.* **100**, 62 (1976).
16. A. Shimony, *Phys. Rev.* **A20**, 394 (1979).
17. A. Zeilinger, in *Proceedings of the Symposium on Wave-Particle Dualism* (in honor of the 90th birthday of Louis de Broglie), Perugia, Italy, April 1982 (D. Reidel, Dordrecht, 1983).
18. N. Bohr, *Kgl. Danske Vid. Selsk.*, 8 Raekke IV, 1 (1918).
19. N. Bohr, H. A. Kramers, and J. C. Slater, *Phil. Mag.* **47**, 785 (1924).
20. W. Bothe and H. Geiger, *Zeits. f. Phys.* **32**, 639 (1925).
21. A. H. Compton and A. Simon, *Phys. Rev.* **26**, 289 (1925).
22. N. Bohr, *Atomic Theory and the description of Nature* (University Press, Cambridge, 1961).
23. M. Born, *Zeits. f. Phys.* **38**, 803 (1926).
24. W. Heisenberg, *Physics and Philosophy* (Harper, New York, 1962), p. 41.
25. M. Born, *Natural Philosophy of Cause and Chance* (Dover, New York, 1964), p. 105.
26. M. E. Omelyanovskij, V. A. Fock, *et al.*, *L'interpretazione materialistica della meccanica quantistica* (a cura di S. Tagliagambe, Feltrinelli, Milano, 1972).
27. W. Heisenberg, *loc. cit.*, p. 179.
28. W. Heisenberg, *Zeits. f. Phys.* **43**, 172 (1927).
29. U. Bonse and H. Rauch, eds., *Neutron Interferometry* (Clarendon Press, Oxford, 1979).
30. L. Jánossy and Zs. Naray, *Suppl. Nuovo Cim.* **9**, 588 (1958).
31. G. D. Blake and D. Scarl, *Phys. Rev.* **A19**, 1948 (1979).
32. P. W. Milonni and P. L. Knight, *Opt. Comm.* **9**, 119 (1973).
33. H. Rauch, W. Treimer, and U. Bonse, *Phys. Lett.* **47A**, 369 (1974).
34. J. F. Clauser, *Phys. Rev.* **D9**, 853 (1974).
35. M. Dagenais and L. Mandel, *Phys. Rev.* **A18**, 2217 (1978).
36. F. Selleri, *Lettere Nuovo Cim.* **1**, 908 (1969); F. Selleri, in Rendiconti della Scuola Internazionale di Fisica "Enrico Fermi," IL Corso: *Fondamenti di Meccanica Quantistica*, B. d'Espagnat, ed. (Academic Press, New York, 1971).
37. F. Selleri, paper presented at the INFN Meeting on *Fundamental Quantum Problems* (Frascati, 1974).
38. A. Szczepanski, *Found. Phys.* **6**, 427 (1976).
39. E. T. Jaynes, in *Third Rochester Conference on Coherence and Quantum Optics*, L. Mandel and E. Wolf, eds. (Plenum Press, New York, 1973), p. 35.
40. L. Ballantine, *Rev. Mod. Phys.* **42**, 358 (1970).
41. F. Selleri, *Ann. Fond. L. de Broglie* **7**, 45 (1982). This paper contains a wider discussion of the historical positions on dualism.

42. A. Gozzini, in *Proceedings of the Symposium on Wave-Particle Dualism* (in honor of Louis de Broglie's 90th birthday), Perugia, Italy, April 1982 (D. Reidel, Dordrecht, 1983).
43. F. Selleri and J.-P. Vigier, in *Old and New Questions in Physics, Cosmology, Philosophy and Theoretical Biology: Essays in Honor of Wolfgang Yourgrau*, Alwyn van der Merwe, ed. (Plenum Press, New York, 1982).
44. R. H. Dicke, *Phys. Rev.* **93**, 99 (1954).
45. Q. H. F. Vrehen, in *Proceedings of the Fourth Int. Conf. Rottach-Egern*, June, 1979, H. Walther and K. W. Rothe, eds. (Springer-Verlag, Heidelberg, 1979).
46. A. Garuccio and J. P. Vigier, *Found. Phys.* **10**, 797 (1980).
47. A. Garuccio, K. Popper, and J.-P. Vigier, *Phys. Lett.* **86A**, 397 (1981).
48. A. Garuccio, V. Rapisarda, and J.-P. Vigier, *Phys. Lett.* **90A**, 17 (1982).
49. L. S. Vasilenko, V. P. Chebotayev, and A. V. Shishaev, *JETP, Lett.* **12**, 113 (1970).
50. B. Cagnac, G. Grynberg, and F. Biraben, *Journ. de Phys.* **34**, 845 (1973).
51. G. Grynberg, F. Biraben, M. Bassini, and B. Cagnac, *Phys. Rev. Lett.* **37**, 283 (1976).