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Heat and mass transfer by natural convection in a non-Darcy porous medium

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Summary. The Forchheimer free convection heat and mass transfer near a vertical surface embedded in a fluid saturated porous medium has been analyzed. A similarity solution is presented for constant wall temperature and concentration distributions with specified power function form ($Ax^{-1/2}$) of mass flux parameter. The effect of Grashof number (Gr), the buoyancy ratio (N), the Lewis number (Le) and the surface mass flux (f_w) on the nondimensional heat and mass transfer coefficients are presented.

Notation

A	real constant
c	inertial coefficient
D	concentration molecular diffusion
f	dimensionless stream function
g	gravitational acceleration in the x direction
Gr	the non-dimensional inertial parameter $\frac{c\sqrt{K}\,Kg\beta_T\theta_w}{\nu^2}$
K	permeability
k	thermal conductivity
Le	Lewis number $\frac{\alpha}{D}$
N	Buoyancy ratio parameter $\left(\frac{\beta_C\phi_w}{\beta_T\theta_w}\right)$
p	pressure
q	local heat flux
m	local mass flux
Ra_x	modified Rayleigh number $\left(\frac{Kg\beta_T\theta_w x}{\alpha\nu}\right)$
T	temperature
C	concentration
u, v	velocity components in x and y directions respectively
x, y	Cartesian coordinates

Greek symbols

α	effective thermal diffusivity
β_T	thermal expansion coeffcient
β_C	concentration expansion coefficient
θ	dimensionless temperature variable
ϕ	dimensionless concentration variable
η	similarity parameter
μ	viscosity
ν	kinematic viscosity

θ dimensionless temperature
ϱ fluid density

Subscripts

w evaluated at wall condition
∞ evaluated at the outer edge of the boundary layer

1 Introduction

The transport of heat and solutes by a fluid moving through a porous matrix is a phenomenon of great interest from both theory and application point of view. Heat transfer in the case of homogeneous fluid saturated porous media has been studied with relation to different applications like dynamics of hot underground springs, terrestrial heat flow through aquifer, hot fluid and ignition front displacements in reservoir engineering, heat exchange between soil and atmosphere, flow of moisture through porous industrial materials, heat exchanges with fluidized beds, etc. Mass transfer in isothermic conditions has been studied with applications to problems of mixing of fresh and salt waters in aquifers, miscible displacements in oil reservoirs, spreading of solutes in fluidized beds and crystal washers, salt leaching in soils, etc. Near the sea shores, prevention of spreading of saline and salt dissolution into the drinking water zones has become a serious problem of research. Bejan [1], and Nield and Bejan [2] have presented reviews on free convection heat transfer and free convection mass transfer mechanisms independently in porous media.

The study of the dynamics of hot and salty springs of a sea where the combined convection of heat and mass transfer is involved has been analysed by Dagan [3]. A systematic derivation of the governing equations with various types of approximations used in applications has been presented. This study also focusses the importance of solutal and thermal dispersion effects in homogeneous and isotropic porous media. Heat and mass transfer by free convection in a porous medium under boundary layer approximations has been studied by Bejan and Khair [4], Lai and Kulacki [5], Nakayama and Hossain [6], and Singh and Queeny [7]. A review of combined heat and mass transfer by free convection in porous medium is given in Trevisan and Bejan [8]. Very recently, Angirasa et al. [9] presented the analysis for combined heat and mass transfer by natural convection for aiding and opposing buoyancies in fluid saturated porous enclosure. All these studies are confined to Darcy flow only.

The inertial effects due to the solid obstructions at moderate and high flow velocities are accounted by considering the Forchheimer flow model in the porous structure [10]–[12]. Vafai and Tien [10]–[11] indicated the importance of the inertial effects on heat and mass transfer in porous media, where as Whitacker [12] presented the theoretical derivation of the Forchheimer flow model using the modern statistical averaging techniques. An account of pure heat and pure mass transfer with Forchheimer flow model is given in Nield and Bejan [2].

The aim of the present study is to analyse the effect of lateral mass flux on the free convection heat and mass transfer from a vertical wall in a fluid saturated porous medium with Forchheimer flow model. When Darcy law is used, similarity solution is possible for a class of wall temperature and concentration variations and corresponding mass flux variations. Because of the nonlinearity in the Forchheimer flux model, this generality is reduced to the isothermal wall temperature and concentration variation with fixed form of lateral mass flux $A/\sqrt{x}$. The results indicate a reduction in the Nusselt and Sherwood numbers in the Forchheimer flow region, and an increase in the Nusselt and Sherwood numbers as the mass flux parameter moves from the injection domain to the suction domain. The mass transfer coefficient increases with Lewis number to certain maximum value and then decreases with further increase in the Lewis number in case of fluid injection. This maximum value is observed to depend on the buoyancy ratio parameter and the intensity of the injection parameter.

2 Governing equations

Consider a vertical flat plate in a fluid saturated porous medium as shown in Fig. 1, x-axis is along the plate and y-axis is normal to it, $x = 0$ being the leading edge. The wall is at constant temperature T_w and concentration C_w, greater than the ambient temperature T_∞ and concentration C_∞, respectively. The wall may be impermeable $\big(v_w(x) = 0\big)$ or permeable with lateral mass flux $\big(v_w(x) = A/\sqrt{x}\big)$.

The governing equations for the flow, heat and mass transfer from the wall $y = 0$ into the fluid saturated porous medium $x \geq 0$ and $y > 0$ are given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0\,, \tag{1}$$

$$u + \frac{c\sqrt{K}}{\nu}\,u^2 = -\frac{K}{\mu}\left(\frac{\partial p}{\partial x} + \varrho g\right), \tag{2}$$

$$v + \frac{c\sqrt{K}}{\nu}\,v^2 = -\frac{K}{\mu}\left(\frac{\partial p}{\partial y}\right), \tag{3}$$

$$u\,\frac{\partial T}{\partial x} + v\,\frac{\partial T}{\partial y} = \alpha\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right), \tag{4}$$

$$u\,\frac{\partial C}{\partial x} + v\,\frac{\partial C}{\partial y} = D\left(\frac{\partial^2 C}{\partial x^2} + \frac{\partial^2 C}{\partial y^2}\right) \tag{5}$$

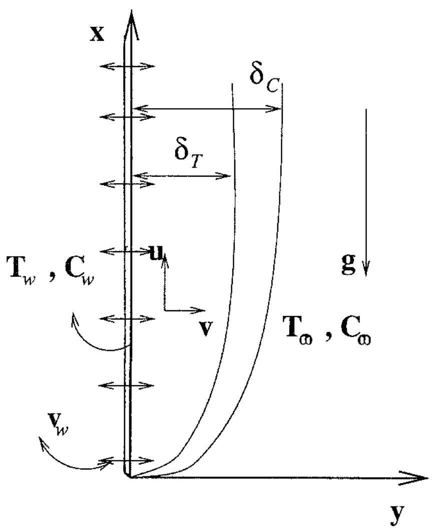

Fig. 1. Temperature and concentration boundary layers in natural convection near a vertical surface in a fluid saturated porous medium

along with the Boussinesq approximation

$$\varrho = \varrho_\infty[1 - \beta_T(T - T_\infty) - \beta_C(C - C_\infty)]\,. \tag{6}$$

The boundary conditions are as follows: for $x < 0$, we suppose that there is no wall [1] and for $x \geq 0$,

$$\begin{aligned} &y = 0 : v_w(x) = A/\sqrt{x}\,, \qquad T_w = \text{const.}\,, \qquad C_w = \text{const.} \\ &y \to \infty : u = 0\,, \qquad T \to T_\infty \qquad C \to C_\infty\,. \end{aligned} \tag{7}$$

Here x and y are the Cartesian coordinates, u and v are the averaged velocity components in x- and y-directions, respectively, T is the temperature, C is the concentration, ϱ is the density, p is the pressure, β_T is the coefficient of thermal expansion, β_C is the coefficient of concentration expansion, μ is the viscosity of the fluid, ν is the kinematic viscosity of the fluid, K is the permeability, c is an empirical constant, g is the acceleration due to gravity, α and D are the thermal and solutal diffusivities, respectively. The suffix w and ∞ indicate the conditions at the wall and at the outer edge of the boundary layer, respectively.

Under the boundary layer approximations and making use of the Boussinesq approximation, the Eq. (2)–(5) become

$$\frac{\partial u}{\partial y} + \frac{c\sqrt{K}}{\nu}\frac{\partial u^2}{\partial y} = \left(\frac{Kg\beta_T}{\nu}\right)\frac{\partial T}{\partial y} + \left(\frac{Kg\beta_C}{\nu}\right)\frac{\partial C}{\partial y}\,, \tag{8}$$

$$u\,\frac{\partial T}{\partial x} + v\,\frac{\partial T}{\partial y} = \alpha\,\frac{\partial^2 T}{\partial y^2}\,, \tag{9}$$

$$u\,\frac{\partial C}{\partial x} + v\,\frac{\partial C}{\partial y} = D\,\frac{\partial^2 C}{\partial y^2}\,. \tag{10}$$

Consider the following transformation:

$$\eta = \frac{y}{x}\sqrt{\mathrm{Ra}_x}\,, \tag{11}$$

$$u = \frac{\alpha}{x}\,\mathrm{Ra}_x f'(\eta)\,, \tag{12}$$

$$v = -\frac{\alpha}{2x}\sqrt{\mathrm{Ra}_x}\,[f - \eta f']\,, \tag{13}$$

$$\theta(\eta) = \frac{T - T_\infty}{T_w - T_\infty}\,, \tag{14}$$

$$\phi(\eta) = \frac{C - C_\infty}{C_w - C_\infty}\,, \tag{15}$$

where Ra_x is the modified Rayleigh number defined as $Kg\beta_T\theta_w x/(\alpha\nu), \theta_w = T_w - T_\infty$ and $\phi_w = C_w - C_\infty$. Now Eqs. (11)–(15) constitute the similarity transformation if it transforms the governing partial differential equations (8)–(10) along with Eq. (1) into ordinary differential equations, with x being eliminated from the ordinary differential equations as well as the boundary conditions. On the wall ($\eta = 0$) Eq. (13) can be written as

$$f_w = \frac{v_w(x)}{-\dfrac{\alpha}{2x}\sqrt{\mathrm{Ra}_x}} \tag{16}$$

and the form for $v_w(x)$ considered in Eq. (7) will make f_w free from x. There exist a wide choice for l for $v_w(x) = Ax^l$ in the Darcy media but the nonlinearity in the Forchheimer flow model reduces this into a single value. The negative power distribution for injection/suction will lead to infinite injection/suction at the leading edge, which is unrealistic, but the method of similarity solution will still give accurate results sufficiently far from the leading edge (see Cheng [13]).

The above similarity transformation reduces the governing equations into the following ordinary differential equations

$$f'' + 2\mathrm{Gr} f' f'' = \theta' + N\phi' , \tag{17}$$

$$\theta'' = -\frac{1}{2} f\theta' , \tag{18}$$

$$\phi'' = -\frac{1}{2} \mathrm{Le} f\phi' \tag{19}$$

and the boundary conditions Eq. (7) transform into

$$\begin{aligned} &\eta = 0 : f = f_w , \qquad \theta = 1 , \qquad \phi = 1 \\ &\eta \to \infty : f' = 0 , \qquad \theta = 0 , \qquad \phi = 0 . \end{aligned} \tag{20}$$

Here $\mathrm{Gr} = c\sqrt{K}\, Kg\beta_T\theta_w/\nu^2$ represents the inertial effects in the porous medium, $N = \beta_C\phi_w/(\beta_T\theta_w)$ is the buoyancy ratio, and the Lewis number $\mathrm{Le} = \alpha/D$ represents the diffusivity ratio. The mass flux parameter f_w is defined in Eq. (16). The mass flux parameter f_w is varied from –1.0 to 1.0. It is clear from the analysis that $f_w = 0$ corresponds to the impermeable surface, $f_w > 0$ corresponds to suction and $f_w < 0$ corresponds to injection of the fluid through the wall into the porous medium.

3 Results and discussion

The resulting ordinary differential equations with the boundary conditions are solved using the generalized techniques for solving ordinary differential equations. The results are observed upto the accuracy 5×10^{-6}.

The nondimensional velocity component in the x-direction $f'(\eta)$, temperature distribution $\theta(\eta)$ and concentration distribution $\phi(\eta)$ in the Darcy and non-Darcy regions for varying mass flux parameter are plotted in Figs. 2–4 fixing $N = 1, \mathrm{Le} = 2$. The results obtained for the wide range of parameters clearly indicate that the thermal and concentration boundary layers thicken as the mass flux parameter passes from the suction to injection domain. The increase in Gr reduces the intensity of the flow and increases the thermal and concentration boundary layer thicknesses.

The fundamental interest of heat and mass transfer study is to find out the heat and mass transfer coefficients in terms of the Nusselt and Sherwood numbers, respectively. The local heat and mass fluxes from the wall are given by

$$q = -k\,\frac{\partial T}{\partial y}_{y=0} \tag{21}$$

and

$$m = -D\,\frac{\partial C}{\partial y}_{y=0} , \tag{22}$$

where k is the thermal conductivity of the porous medium and D is the molecular diffusivity of the fluid. Nondimensional heat transfer coefficient can be written as

$$\frac{\mathrm{Nu}}{\sqrt{\mathrm{Ra}_x}} = -\theta'(0) \tag{23}$$

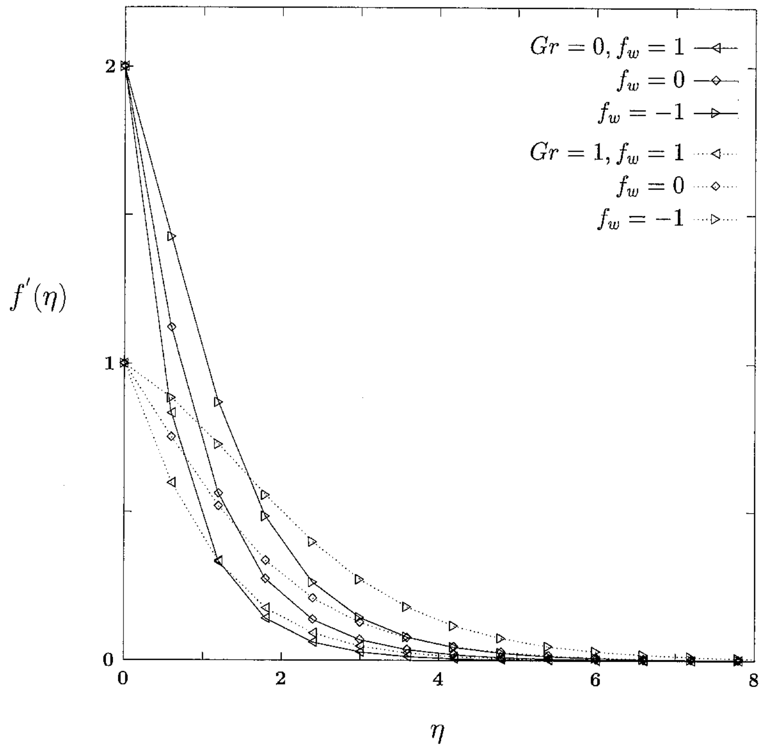

Fig. 2. The effect of inertial and mass flux parameters on the nondimensional vertical velocity component when $N = 1$ and $\mathrm{Le} = 2$

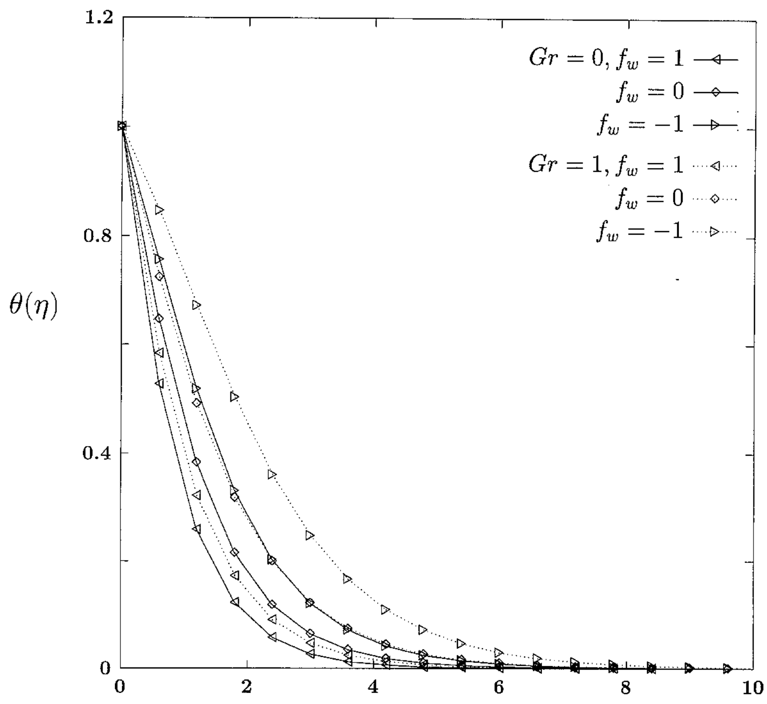

Fig. 3. The effect of inertial and mass flux parameters on the nondimensional temperature distribution, $N = 1$ and $\mathrm{Le} = 2$

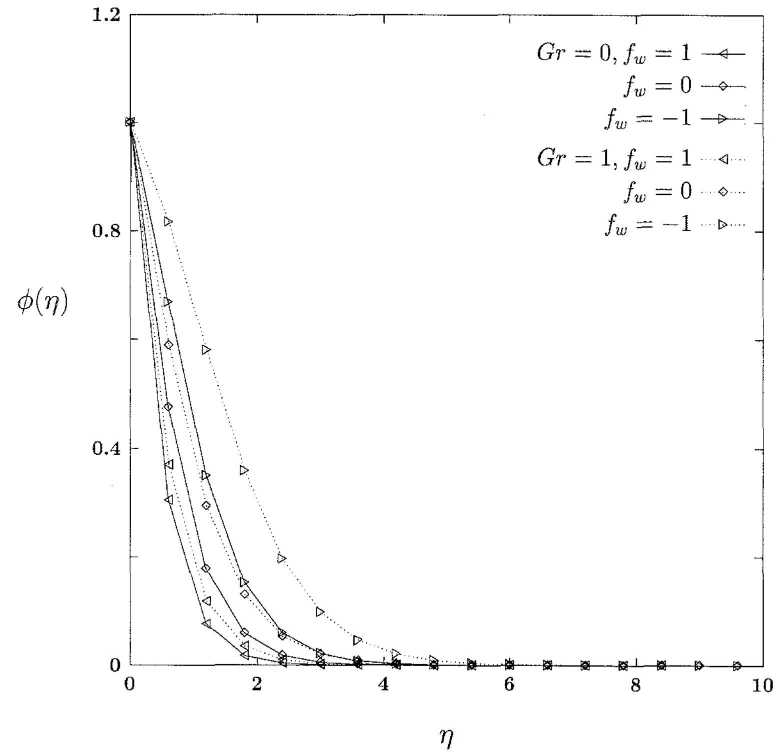

Fig. 4. The effect of inertial and mass flux parameters on the nondimensional concentration distribution, $N = 1$ and Le $= 2$

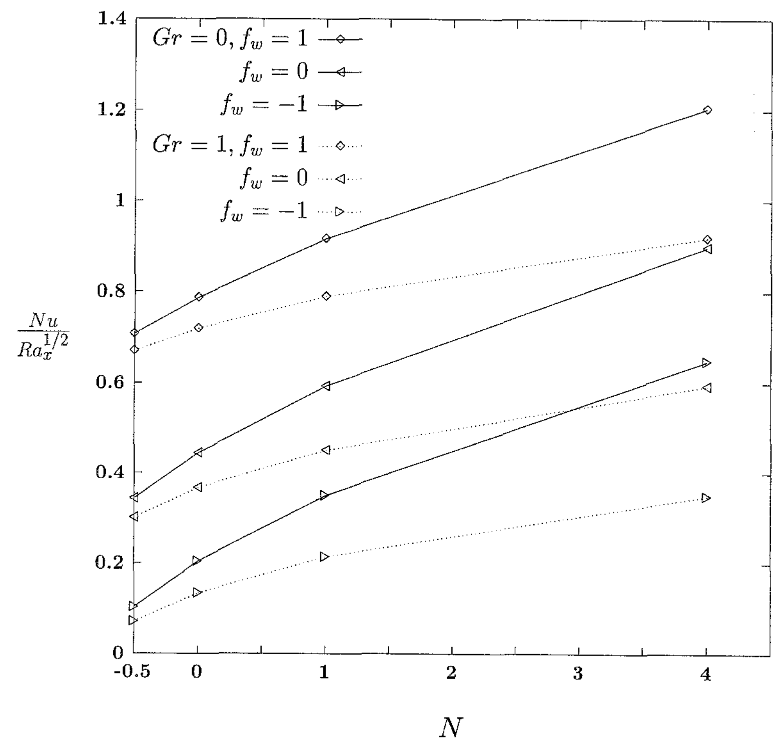

Fig. 5. The effect of buoyancy ratio on the Nusselt number when Le $= 2$ in the Darcy and Forchheimer flow regions

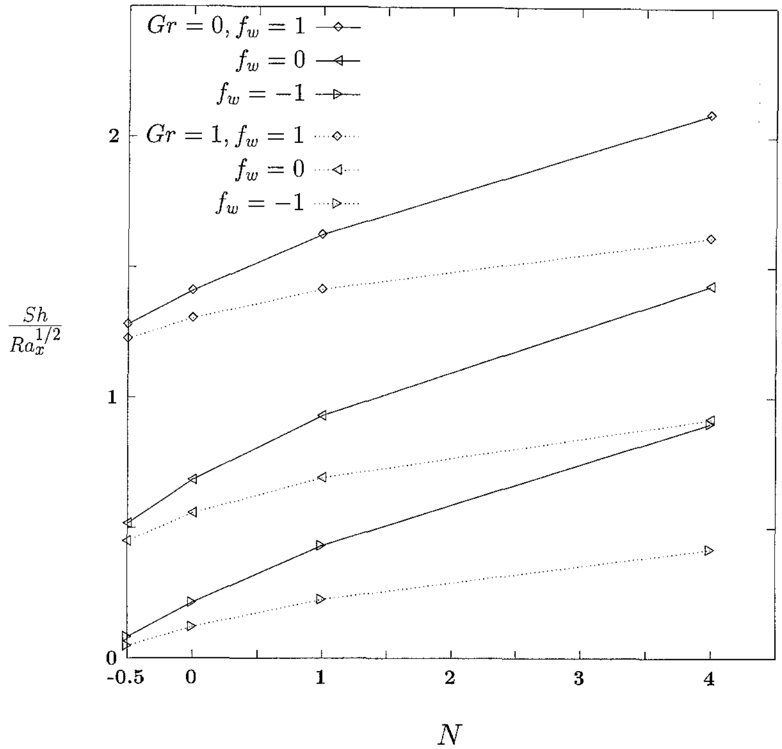

Fig. 6. The effect of buoyancy ratio on the Sherwood number when Le = 2

and the nondimensional mass transfer coefficient will be

$$\frac{\mathrm{Sh}}{\sqrt{\mathrm{Ra}_x}} = -\phi'(0)\,. \tag{24}$$

The Nusselt and Sherwood number results are plotted against N, Le and f_w in Figs. 5–11. The impermeable wall results for Darcy case coincide with those in Bejan and Khair [4]. The Nusselt and Sherwood numbers are observed to increase as the value of the buoyancy ratio parameter increases. In the present study, N is varied between –1 and 4 (–1 is excluded for reasons mentioned in Lai and Kulacki [5]). In accordance with Lai and Kulacki [5], we obtained the results for Le > 0 in the range $-1 < N < 0$.

Figures 5–6 depict the enhanced heat and mass transfer results with the buoyancy ratio N. It is clearly seen that the inertial effects reduce the heat and mass transfer coefficients. Also it becomes vivid that for higher values of the buoyancy ratio, the effect of mass flux is predominant.

The surface mass flux results for varying $-1 < N \leq 4, 0.1 \leq \mathrm{Le} \leq 500$ are considered in the present study. Figures 7–8 represent $\mathrm{Nu}/\sqrt{\mathrm{Ra}_x}$ and $\mathrm{Sh}/\sqrt{\mathrm{Ra}_x}$ results for $N = 1$, and varying Gr and Le. The increase in the Lewis number decreases the heat transfer coefficient, where as it increases the mass transfer coefficient. The inertial effects reduce the heat and mass transfer coefficients. It is evident from these figures that the fluid suction increases heat and mass transfer results.

With fixed Gr and N, as Le increases, the Nusselt number decreases for all f_w where as the Sherwood number increases for $f_w \geq 0$. The effect of Le on heat and mass transfer coefficients is plotted in Figs. 9–10 fixing N at 1 (the point where the heat and mass buoyancy effects are of equal order magnitude). For the impermeable wall, a uniform difference in the Sherwood

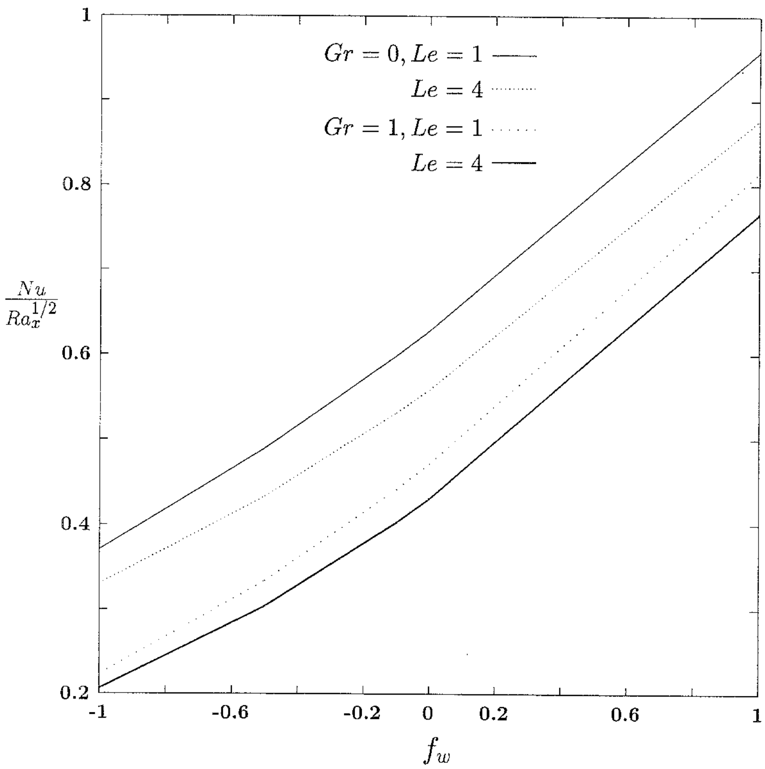

Fig. 7. The effect of mass flux on the Nusselt number results when $N = 1$

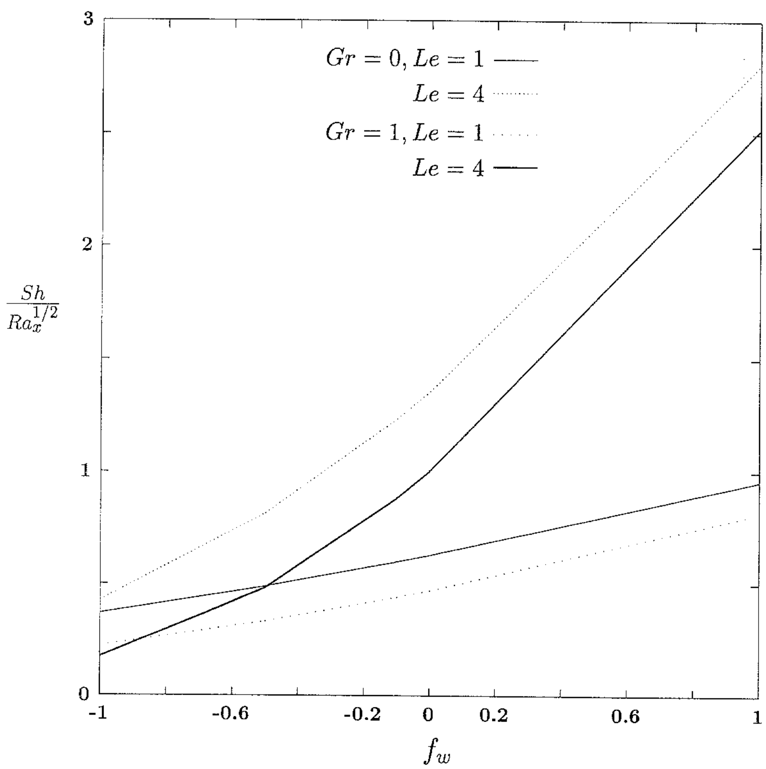

Fig. 8. The effect of mass flux on the Sherwood number when $N = 1$

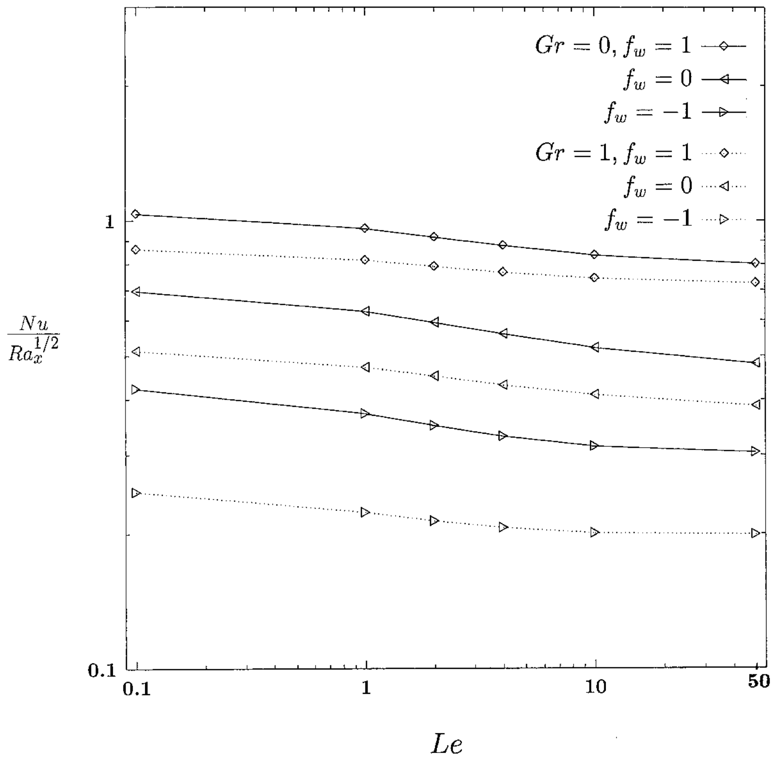

Fig. 9. The effect of Lewis number on the Nusselt number when $N = 1$, for suction, injection and impermeable wall cases

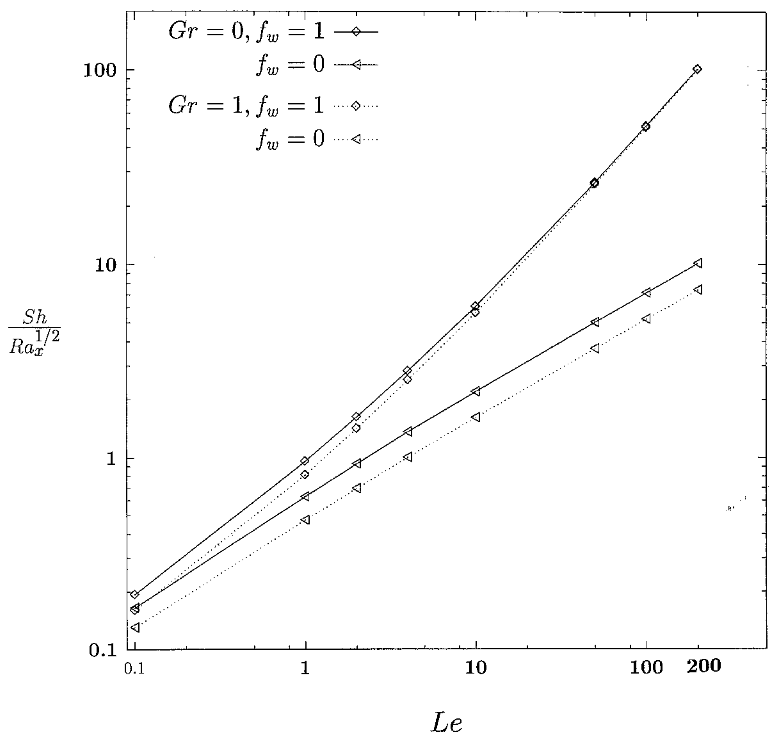

Fig. 10. The effect of Lewis number on the Sherwood number when $N = 1$, in cases of impermeable wall and suction through the wall

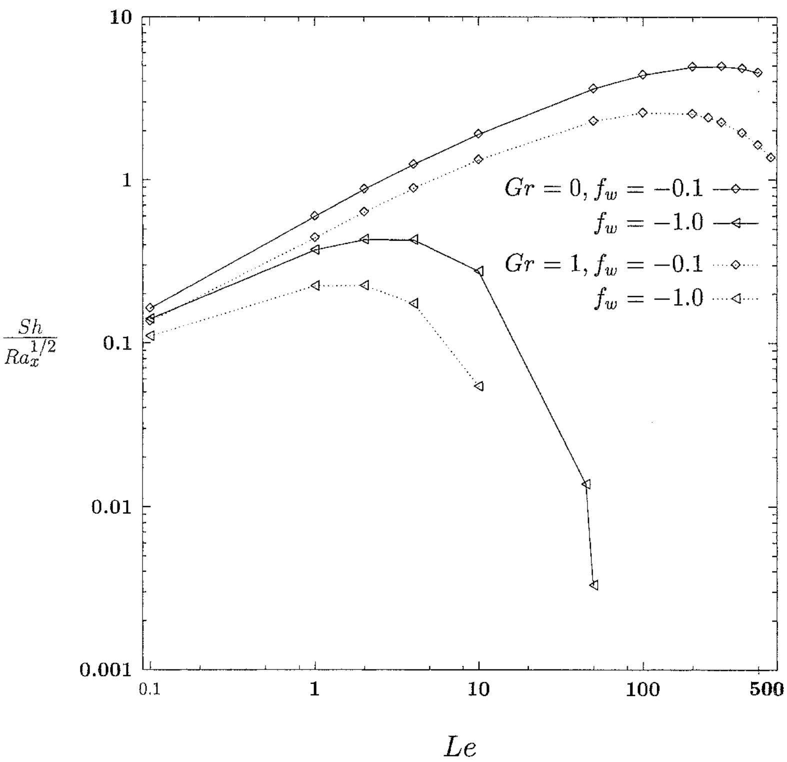

Fig. 11. The effect of Lewis number on the Sherwood number results when $N = 1$, in the case of fluid injection into the porous medium

number results is seen in the Darcy and Forchheimer regions as Le increases where as in the case of suction, the effect of Gr is observed to diminish at large Le. Due to the removal of the fluid through the impermeable wall, the thickness of the thermal and concentration boundary layers reduce and lead to enhanced heat and mass transfer.

When fluid injection through the wall into the fluid saturated porous medium is considered, the results for wide range of Gr and N considered in the present study indicate that the Sherwood number increases with Le up to certain maximum value and then decreases as Le is increased further. This maximum value depends on the buoyancy ratio and the inertial parameter. This phenomenon is clearly seen from Fig. 11. For $\mathrm{Gr} = 0$ and $f_w < 0$ the present study leads to the study of power function form of the injection with constant wall temperature and concentration case considered in Lai and Kulacki [5]. The present results are consistant with those given in Lai and Kulacki [5] for varying N and f_w. But their study did not mention about the effect of Lewis number on mass transfer coefficient. The reason for the peculiar behavior might be because of the diminishing influence of the Lewis number to enhance the mass transfer coefficient at larger injection values where the concentration boundary layer becomes very thick (similarity results indicated that the concentration profile attains its value at the outer edge of the boundary layer at around 25). This feature becomes very clear from the curves for $f_w = -0.1$ and $f_w = -1.0$ in both the cases of $\mathrm{Gr} = 0$ and 1.

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