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Life time of soft multipole excitations

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Abstract. Decay of soft multipole excitations in halo nuclei is studied in comparison with the potential resonance state. Although the soft excitation has a sharp peak just above the particle threshold and carries extremely large transition strength, the decay rate looks much faster than that expected for a resonance state. Consequently, the half life is shown to be several orders of magnitude shorter than what one naively expects from the uncertainty principle. It is shown also that the soft excitations accumulate large transition strength as a non-resonant single-particle excitation, but not as particle-hole collective excitations like giant resonances.

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1. Introduction

Multipole excitations in halo nuclei have been recently one of the most intriguing subjects and they have been shown to be quite different from those of stable nuclei because of the unique rote played by the loosely bound nucleons in these excitations. It has been pointed out by various theoretical studies [1–3] that the dipole excitation in halo nuclei has a large transition strength exhausting a few percent of the energy-weighted sum rule value (EWSR) at very low excitation energy ($E_x \sim 1$ Me V). This characteristic excitation is called "soft" dipole mode. It has been shown that the monopole and quadrupole response functions in halo nuclei also exhibit sharp peaks near the threshold energy of the breakup reactions with significant portions of the EWSR values [4, 5]. Recently, the existence of soft dipole excitations at very low excitation energies have been experimentally observed in ^{11}Li[6, 7] and ^{11}Be [8] by Coulomb break-up reactions. Although this dipole peak exhausts only a few percent of the EWSR value, the transition strength itself is as large as that of the giant resonance because the excitation energy of the soft dipole state is extremely low.

The origin and the properties of giant resonances are by now well understood. They are described as coherent superpositions of many 1-particle 1-hole ($1p$-$1h$) states across the major shells in terms of the shell model. Their excitation energies are then characterized by the energy difference between the major shells modified by the collective shift due to the residual particle-hole (p–h) interaction. In the isovector dipole case this shift is upwards. On the other hand, it is still debated how the soft excitations accumulate large transition strengths at very low energies near the threshold. The question is whether the soft excitation is collective or single-particle. If the latter possibility dominates, it should be clarified further if one is dealing with a resonance state of the potential-model type, or not. The aim of this paper is to study these questions concerning multipole excitations in halo nuclei. We especially discuss the origin of the large transition strengths in low energy peaks. We will discuss also the life time of the soft excitations by taking Fourier transforms of the strength distributions.

2. Strength distributions

As a simple model, the point-like cluster model has been often used to discuss the excitation modes in halo nuclei like ^{11}Li[1]. This wave function has been successfully applied to structure and reaction problems of halo nuclei, despite the fact that the model oversimplifies the correlations among the halo nucleons. The point-like dineutron wave function is expressed as

$$|\phi_{\text{cluster}}\rangle = N_0\sqrt{2\gamma}\,\frac{e^{-\gamma r}}{r}\,Y_{00}(\hat{r}), \tag{1}$$

where γ is related to the separation energy S of the dineutron and to its reduced mass $\mu : \gamma^2 = 2\mu S/\hbar^2$. This form corresponds to the wave function outside the potential well of an $l = 0$ bound state in a square well of radius R[1]. Consequently, the factor N_0 is equal to

$\exp(\gamma R)/\sqrt{1+\gamma R}$ if the separation energy S is small enough compared to the potential depth. One can adopt a similar radial dependence for the halo single-particle wave function ϕ_h in ^{11}Be since the interior of the wave function does not contribute significantly to the present discussions. In the plane wave approximation (PWA), the final state is given by the wave function

$$|\phi_E; \mathrm{lm}\rangle = \sqrt{\frac{2\mu k}{\hbar^2 \pi}}\, j_l(kr)\, Y_{\mathrm{lm}}(\hat{r}), \tag{2}$$

where $k^2 = 2\mu E$.
The transition amplitude is calculated as

$$T_\lambda(E) = \int \mathrm{d}\mathbf{r}\, \phi_E^*(\mathbf{r})\, \mathbf{O}(\lambda)\, \phi_i(\mathbf{r}), \tag{3}$$

where ϕ_i is either ϕ_{cluster} or ϕ_h, and the transition operator is defined as

$$\mathbf{O}(\lambda) = \begin{cases} r^2 & \lambda = 0 \\ r^\lambda Y_{\lambda\mu} & \lambda \neq 0 \end{cases}, \tag{4}$$

in the long wave length limit. The transition strength of excitation in the continuum is then expressed as

$$\frac{\mathrm{d}B(E\lambda)}{\mathrm{d}\omega} = \int \mathrm{d}E\, \delta(\omega - (E - E_h))\, |T_\lambda|^2 . \tag{5}$$

In the PWA the strength function can be obtained analytically in the form

$$\frac{\mathrm{d}B(E\lambda)}{\mathrm{d}\omega} = \begin{cases} N_0^2 \dfrac{\alpha^2}{\pi} \left(\dfrac{\hbar^2}{2\mu}\right)^\lambda \dfrac{2\lambda+1}{4\pi} (\lambda!)^2 \dfrac{2^{2\lambda+1} S^{1/2} (\omega - S)^{(2\lambda+1)/2}}{\omega^{2\lambda+2}} & (\lambda \neq 0) \\ N_0^2 \dfrac{\alpha^2}{\pi} \left(\dfrac{\hbar^2}{2\mu}\right)^2 8 \dfrac{(\omega - S)^{1/2} S^{1/2} (4S - \omega)^2}{\omega^6} & (\lambda = 0) \end{cases}, \tag{6}$$

where α is the reduced charge which takes into account the recoil effect of the core with respect to the di-neutron or the neutron [9]. The value for α in the dipole case is $(N_2 Z_1 - N_1 Z_2)/(A_1 + A_2)$ where $N_2(N_1)$ and $Z_2(Z_1)$ are neutron and proton numbers of the core (halo), respectively, and $A = Z + N$. The energy dependence of the PWA response is shown in Figs. 1–3 for the monopole, dipole and quadrupole cases for the operators (4). The curves have been calculated for different choices of separation energy S. Each strength distribution has a sharp peak near the particle threshold in the case of the small separation energy $S = 0.5$ MeV. On the other hand, in the case of well-bound nucleons, the strength functions show a smooth energy dependence and no clear peak. The integrated energy-weighted strength of Eq. (6) for the dipole operator coincides exactly with the sum rule value of the so-called molecular vibration [10].

The dipole transition strength (6) in ^{11}Be is compared with the experimental data of Nakamura et al. [8] in Fig. 4. The calculation has been done with $N_0^2 = 1.8$ which corresponds to a typical choice $S = 0.5$ MeV, $R = 3.4$ fm. The calculated curve shows a good agreement with the data not only near the threshold but also out to the tail region. The peak energy is determined by the separation energy, and one has

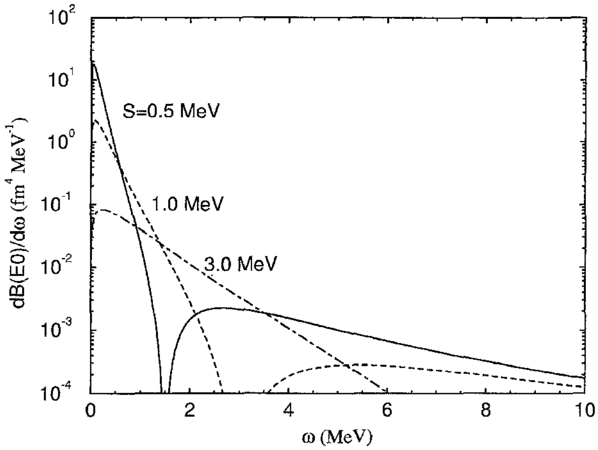

Fig. 1. Monopole strength distribution calculated in PWA as a function of the excitation energy (referred to the particle threshold). Results for different choices of separation energy S are shown

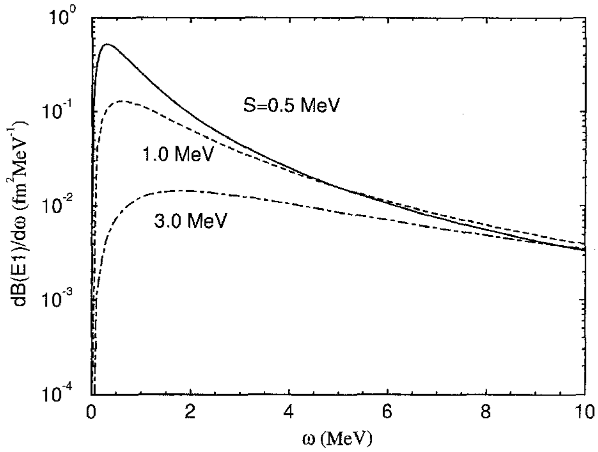

Fig. 2. Same as Fig. 1, for dipole strength distribution

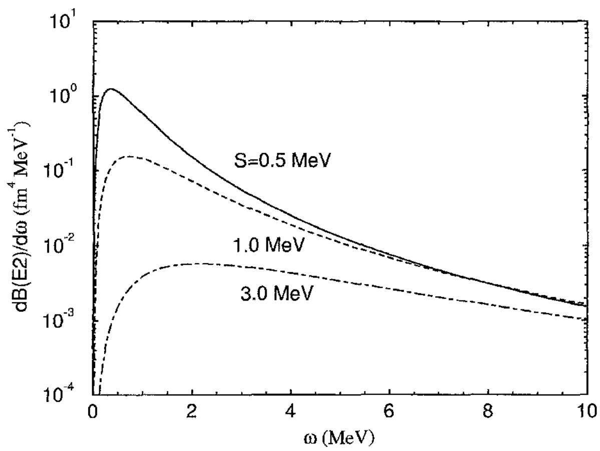

Fig. 3. Same as Fig. 1, for quadrupole strength distribution

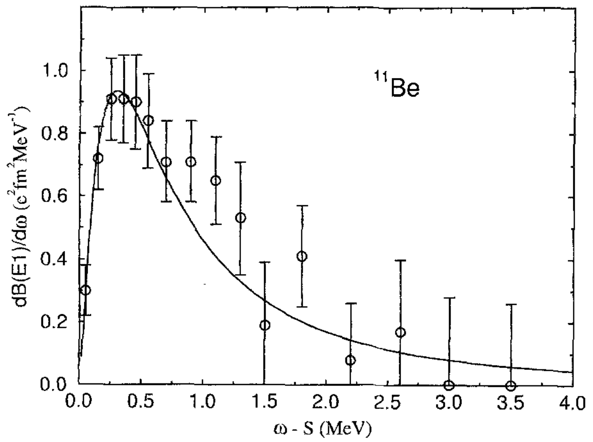

Fig. 4. Calculated and experimental dipole strength distribution in ^{11}Be. The data are from Ref. [8].

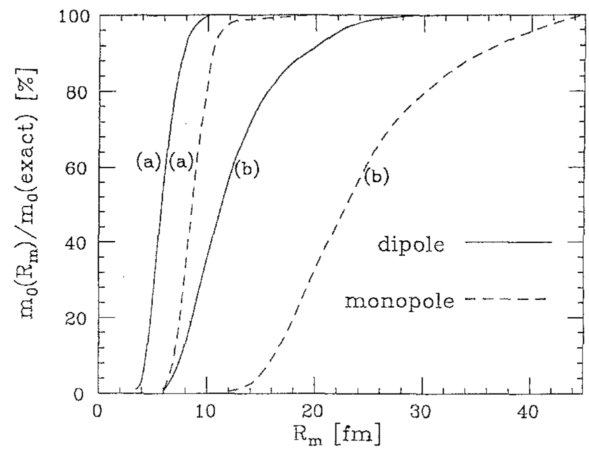

Fig. 5. Ratio of the calculated sum rule value $m_0(R_m)$ to $m_0(exact)$ for dipole and monopole responses. The value $m_0(R_m)$ is obtained by carrying out integrals up to the cutoff radius R_m, while the value $m_0(exact)$ is calculated by integrating analytically up to infinity. (*a*) and (*b*) refer to well-bound and halo case, respectively

$\omega_{\text{peak}} = 1.8\,S$. The experimental peak energy also shows a good agreement with the value predicted by the model.

We now discuss the very large strengths of the soft excitations. The peaks in the strength functions do not necessarily mean the existence of resonances. A historical example is the observed peak in the photodisintegration cross section of deuteron [11]. The cross section has a peak at $E_x \simeq 4$ MeV, which is well reproduced by Eq. (6) with appropriate parameters $\alpha = 1/2$ and $S = 2.23$ MeV for the deuteron. The deuteron does not, however, have any bound or resonant excited state in this energy region which can be accessed by the dipole operator. Thus, the peak is hardly considered as any resonance, but it is due to the large spatial distribution of the deuteron wave function. The same mechanism could explain the appearance of peaks at low excitation energies in halo nuclei as is shown in Figs. 1–3.

A key issue of the present mechanism of having a peak at low energy is the large spatial distribution of halo neutrons. In order to see this, let us examine the convergence problem of the sum rule value of the transition strength in halo nuclei. For a given hamiltonian, the single-particle wave function is often obtained by solving the differential equation in coordinate space. In stable nuclei, enough numerical accuracy can be reached by taking the radial extent of the coordinate space to be $3 \sim 4$ times larger than the size of the nucleus. This is, however, not the case for halo nuclei. As was pointed out in Ref. [2], the dipole response is drastically changed when the model space is extended from 12.5 fm to 40 fm. We show in Fig. 5 the sum value of the transition strength $m_0(R_m) \equiv \int_S^{\omega_c} d\omega dB(E\lambda)/d\omega$ for a given cut-off radius R_m (i.e., T_λ of Eq. (3) is calculated numerically up to a radius R_m) and compare it with the exact analytical result. The maximum energy $(\omega_c - S)$ is taken as 10 and 3 MeV for the well bound case ((a), $S = 3$ MeV) and the halo case ((b), $S = 0.5$ MeV), respectively. The dipole sum rules are drawn by the solid curves, while the monopole ones are shown by the dashed curves. In the well-bound nucleus (a), we can obtain almost 100% of the exact values by integrating up to 12 fm. On the contrary, in the halo nucleus, the integration up to 12 fm gives about 60% of the exact value for the dipole response, and only 1% for the monopole one. In order to obtain reasonable convergence in halo nuclei, one has to integrate the wave functions up to 30 fm and 45 fm for the dipole and the monopole transitions, respectively. These values are more than 10 times large than the size of the core part of the halo nucleus. Figure 5 gives also interesting information on the effective reaction distance in the halo nucleus. The dipole transition takes place mostly in the distance between 10 and 20 fm, while the monopole transition occurs between 20 and 40 fm. This anomalously large reaction distance will have strong influence on other observables like the Coulomb dissociation cross sections, the fusion cross sections and the transfer reactions [12, 13].

So far we have discussed the transition strength by using the PWA. The distortion effects due to the mean field potential were examined in Ref. [16] where the initial and final states were calculated consistently in the same one-body potential. Since the plane wave function (2) is not orthogonal to the hole or dineutron wave function, the PWA transition strength is overestimated in most cases compared with that of the distorted wave approximation (DWA). However, the multipole response distributions in both DWA and PWA have clear peaks just above the particle threshold in the case of halo neutrons. Namely, the peak appears at the same excitation energy in the two calculations although the peak height of the DWA is about 50% lower than that of PWA due to distortion effects.

3. Life time and widths

Let us now examine the time development of the soft states. It is calculated by the Fourier transform of the transition amplitude $T_\lambda(\omega)$:

$$f(t) = \int d\omega e^{i(\omega - \omega_0)t} T_\lambda(\omega), \tag{7}$$

where ω_0 is the peak energy of the response. In the case of a potential resonance, the energy dependence of the transition amplitude is given by a function $T_{res}(\omega) = (\omega - \omega_0 - i\Lambda/2)^{-1}$ where Λ is the width of the resonance. The shape of the resonance corresponds to a Lorentzian distribution. The life time will be determined from the decay rate

$$P(t) = |f(t)|^2, \quad (8)$$

which is $\exp(-\Lambda t)$ for the resonant state with the Lorentzian shape. Then, the mean life of the resonance:

$$T_{mean} \equiv \int_0^\infty tP(t)dt / \int_0^\infty P(t)dt, \quad (9)$$

becomes $T_{mean} = 1/\Lambda$ which is often associated with the uncertainty principle, though the connection is not strictly correct since the uncertainty principle should be applied to the root mean square deviations. In the case of a Lorentzian distribution, T_{mean} is related to the half life $T_{1/2}$ by the familiar relationship $T_{1/2} = T_{mean} \ln 2$.

Following this scheme, we have performed the Fourier transform of the transition amplitude (3) of the soft excitation. It should be noticed that the energy integration in Eq. (7) converges very slowly due to a low power of the energy dependence $\sim\omega^{-5/4}$ at very large excitation energy. We take the maximum energy ω_{max} of the integration large enough to have $P(t)$ with 3 significant digits. In Fig. 6 is shown the probability function $P(t)$ for a soft mode in comparison with the corresponding function for a resonance state having the same full width at half maximum (FWHM) $\Lambda = 0.91$ MeV. Clearly, the behaviours of the two curves will lead to rather different values of half life times. In Table 1 we show the values of $T_{1/2}$ (defined as $P(T_{1/2}) = 0.5$) and T_{mean} calculated for different cases of soft excitations corresponding to different values of the FWHM Λ. It must be pointed out that the values of $P(t)$ at small times, and hence the values of $T_{1/2}$ are much dependent on the cut-off energy ω_{max} in the energy integral (7) while T_{mean} is relatively insensitive to ω_{max}. With our present choice ($\omega_{max} \sim 10^5$ MeV) we already see that $T_{1/2}$ is several orders of magnitude smaller than T_{mean}, in strong contrast with the resonance state case. Actually, $T_{1/2}$ becomes extremely small if the energy integration is carried out up to infinity, in the halo case. On the other hand, T_{mean} is not much different from the half life of a potential resonance having the same FWHM, $T^{res}_{1/2} = \ln 2/\Lambda$. The values of T_{mean} shown in Table 1 were calculated with an upper limit t_{max} for the integrals of Eq. (9), this limit being defined by $P(t_{max}) = 0.001$. Figure 6 shows that T_{mean} is affected by a small, long-living component of the soft excitation as can be seen in the tail of the function $P(t)$.

The soft dipole states have been observed in recent measurements of the Coulomb break-up reactions of ^{11}Li and ^{11}Be as a sharp peak with a width $\Lambda \sim 1$ MeV in the spectra near the particle threshold of each nucleus. The peak energies and the widths in both nuclei are well-reproduced by the predicted distributions (6) using the empirical separation energies. If it is a resonance state, the half life time is given by $T_{1/2} \sim 1/\Lambda$. The medium energy projectile with $E_{lab} \sim 70$ MeV is allowed a flight path of 100 fm during this decay time. This flight length is large enough to escape from the Coulomb field of a heavy target like ^{208}Pb. On the other hand, if the peak is due to a halo effect, the excited state of ^{11}Li decays immediately at ^{9}Li and two neutrons. In this case, the core of the projectile alone is affected substantially by the Coulomb field, while the neutron fragments move independently with no Coulomb effect. The observed post-acceleration of ^{9}Li[7, 8] in the break-up process of ^{11}Li might be considered as evidence of the non-resonant nature of the soft excitation.

Table 1. The full width at half maximum Λ, half-life $T_{1/2}$ and mean-life T_{mean} of soft dipole peak with different sepration energies (see text)

S (MeV)	Λ (MeV)	$T_{1/2}/\hbar$ (MeV^{-1})	$T_{mean}/\hbar$ (MeV^{-1})
0.5	0.91	0.0063	0.58
1.0	1.81	0.0031	0.27
3.0	5.44	0.00095	0.095

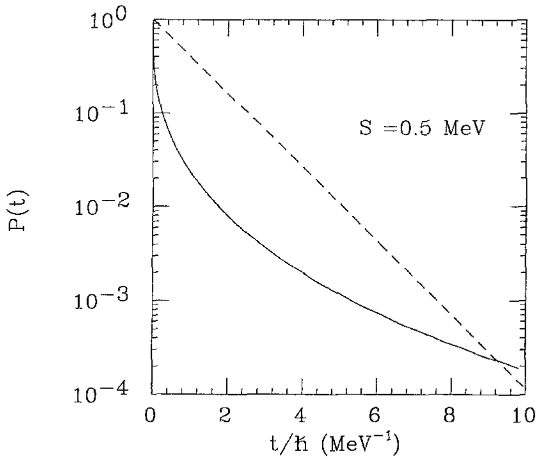

Fig. 6. The probability function $P(t)$ for a soft mode corresponding to a separation energy $S = 0.5$ MeV (*solid curve*), and for a resonance state having the same FWHM (*dashed curve*)

4. Concluding remarks

It is known that the pairing and the p–h correlations are crucial for proper description of collective excitations, for example, the first 2^+ states in spherical nuclei or giant resonances. In Ref. [2], it was found that the pairing correlation enhances the dipole transition strength in ^{11}Li by about 20–30 percent, but the energy and the width of the peak are not changed much. The effect of the pairing correlation is relatively small on the soft excitations in comparison with the enhancement shown in Figs. 1–3. Moreover, the p–h correlations have only a minor influence on the soft excitation due to the small overlap between the halo and the well bound single-particle wave functions [4]. Thus, it is concluded that the soft mode of excitation is a new type of independent particle excitation having a narrow width and a large transition strength; it is not caused by a coherent superposition of *p–h* configurations like in collective states nor by a resonant behavior of unbound single-particle states, but it is due to the large spatial extension of the bound single-particle states.

One could expect similar threshold behavior for any loosely-bound systems, e.g., in hypernuclei. It is likely that a Σ-hyperon occupies an orbit with small separation energy since the mean field potential of Σ-hypernuclei is very shallow for hyperons [14, 15]. It would, however, be a very difficult challenge for experimentalists to observe such "soft excitations" in Σ-hypernuclei because of the large decay width of Σ-hyperon itself.

In summary, we have studied the multipole transition strength in a typical halo nucleus and its decay time. The model is simple enough to be treated analytically, but it contains the main features of loosely bound systems. It is shown that the sharp single peak just above the particle threshold is dominantly created by the large spatial distribution of halo neutrons with an enhancement of a few orders of magnitude compared to that of the well bound nucleus. The pairing and RPA correlations play minor roles for this enhancement of the soft modes in halo nuclei. The half life time is calculated by using the energy distributions of the dipole strength. It is shown that the life time of the soft excitation is much shorter than that of a potential resonance of comparable width. The observed post-acceleration effect of the charged fragments in the Coulomb breakup of ^{11}Li and ^{11}Be might be an indication of this short life time of the soft excitation mode.

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