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Theory Absorption and the Testability of Economic Theory

By

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Introduction

Morgenstern writes as follows:

Nature does not care — so we assume — whether we penetrate her secrets and establish successful theories about her workings and apply these theories successfully in predictions. In the social sciences, the matter is more complicated and in the following fact lies one of the fundamental differences between these two types of theories: the kind of economic theory that is *known* to the participants in the economy has an effect on the economy itself, provided the participants can observe the economy, i. e., determine its present state.

However, the distribution of the kind of theory available, and the degree of its acceptance, will differ from one case to the other. This in turn will affect the working of the economy. There is thus a "back-coupling" or "feedback" between the theory and the object of the theory, an interrelation which is definitely lacking in the natural sciences.

In this area are great methodological problems worthy of careful analysis. I believe that the study of the degree of "theory absorption" by the members of the economy ... will make us all more modest in judging how far we have penetrated into the economic problems [35, p. 707].

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The purpose of this paper is to explicate Morgenstern's notion of theory absorption and to trace some of the implications of the concept for the testing of economic theory. The analysis proceeds as follows: first, an introduction to cognitivist inductive inference and two forms of cognitivist theory absorption are given. Second, the logic of theory maintenance is stated, detailing the differing influences of exact and inexact (or ambiguous) information. Third, the implications of the cognitivist account of theory absorption for the testing of economic theory (i. e., the theory of consumer behavior) are presented and discussed. We shall find that whereas exact information guarantees the maintenance of an absorbed theory, and thereby the possibility of testing that theory, inexact information is disruptive in an uncontrollable manner. The existence of that uncontrollability implies that certain tests of economic theory are impossible, and reinforces Morgenstern's recommendation for modesty on the part of economists (and social scientists generally)[1].

An Introduction to Cognitivist Inductive Inference

Morgenstern's comments relate to the acquisition and use of a (predictive) economic theory by an individual in resolving a practical decision problem. If, for example, the individual employs the expected utility model to resolve a problem with state space X and act space A, then his resolution of the problem is found by selecting $a \in A$ so as to

$$\max_{x \in X} \Sigma \phi (x) \, u \, [p \, (x, a)].$$

The economic theory he absorbs and employs includes the foregoing model and its consequences. Note that the outcome mapping p is an integral component of the absorbed theory[2]. If, however, the individual faces a (simple) consumer demand problem, then the absorbed economic theory consists of the Hicks-Slutsky model and its consequences. Any economic model and its consequences can be absorbed for the resolution of a practical problem. The issue of methodological interest here lies in the absorption of different models by the members of an economy where an experimenter is attempting to test the validity of a specific model or component of

[1] The cognitivist account of theory absorption advanced here also yields the conclusion that the traditional view of reduction is untenable within economics. See Dacey and Pitt [9].

[2] See Dacey [6] for a further discussion on this point.

a model. There are two issues present here. First, perhaps all the individuals in a group employ the same economic model, say the expected utility model, but use different components within that model, e. g., different outcome mappings. Then, even if all of the agents held the same (probabilistic) beliefs and the same tastes, their observable behavior could still be incompatible with the behavior predicted by an observer using a particular outcome mapping. Thus, variations in a component of an economic theory across individuals may render group testing impossible. A second, larger issue arises when the separate agents all employ different economic theories. For example, one individual may perceive a problem as one under uncertainty, while another may see it as a certainty problem. Any test of either theory based on observed behavior would be spurious at best. Perhaps these two issues can be put into sharper focus by the following paradigm. Consider two individuals in a group of agents facing a consumer demand problem. Both employ the (simplified) Lancaster model [27, 28], and Mr. i chooses a vector of goods x so as to

$$\max u^i (z)$$

subject to

$$z = Bx$$

$$p \cdot x = M^i$$

If both individuals maintain the Lancaster model, i. e., behave in accordance with the Lancaster model, then the experimenter can deduce and test certain propositions. Suppose, however, one agents holds $B = \mathrm{I}$, the identity matrix, and thus behaves in accordance with classical theory, whereas the other holds a matrix B which is unequal to I. The experimenter now has fouled observations, for there are testable propositions forthcoming from the traditional theory (wherein $B = \mathrm{I}$) that do not hold unconditionally for the Lancaster model. We shall return to the specifics of this issue in Section 3. For the present, however, one should note the difference between the absorption of a theory and of its components. If both individuals maintain a certainty theory but within that theory maintain different components, then testing is impossible. The maintenance of a theory *and* its components under the conditions of testing is necessary if the test is to be nontrivial.

An economic theory is here conceived of as a conditional generalization (i. e., a "for all x, if P of x, then Q of x" statement). The absorption of an economic theory thus consists of the (inductive) acceptance of a generalization. The individual is presumed to possess a view of the world, a *Weltanschauung*, which guides his acquisition and maintenance of beliefs. The logic of acquisition and maintenance is taken to be cognitivist inductive logic. There are, of course, alternative logics[3].

Inductive inference is traditionally separated into two views, the behavioralist and the cognitivist. The behavioralists argue that inference consists solely of the assignment of probabilities to (singular) hypotheses and that these probability assignments are to be employed in the resolution of practical decision problems[4]. The expected utility model of decision making adheres, implicitly to the behavioralist view. Cognitivists hold that rules of acceptance and rejection of hypotheses are indispensable and that knowledge consists of accepted hypotheses and that the collection of accepted hypotheses constitutes an individual's corpus of knowledge[5]. An individual's *Weltanschauung* consists of his factual and/or conceptual knowledge or assumptions concerning the world and includes his corpus of knowledge.

The two views of inductive inference are not incompatible. Indeed, the behavioralist view, as it applies to the standard expected utility maximization model of risk analysis and in employing a fixed and "known" outcome mapping, presupposes the cognitivist view on epistemological issues[6]. The following is a (brief) introduction to cognitivist inductive logic. It is based upon the presentation made in Niiniluoto and Tuomela [37].

Inductive logic provides an analysis of the relationship between an evidential statement e and a hypothesis g. Cognitivist inductive logic provides rules of acceptance — rules for selecting, on the basis of evidence, one hypothesis from among a class of alternative hypotheses. The cognitivist inductive logic considered here is a decision-theoretic logic.

Hempel [14, 15], Levi [31, 32], Hintikka [24] and other philosophers [16, 22] have applied decision-theoretic concepts to problems of scientific inference. Specifically, they have viewed the acceptance and rejection of scientific hypotheses as a process of maximizing epistemic utilities. Such utilities represent preferences over cognitive objectives of scientists, for example, *truth*, *information* (in the tech-

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[3] See, e. g., Churchman [5] and Michalos [35].

[4] Kihlstrom [25, 26] presents the economics of behavioralist inference within the context of the Lancaster model.

[5] On the notion of a corpus of knowledge see Levi [31].

[6] See Dacey [6] for the role of cognitivist inductive logic within the expected utility model.

nical sense of "amount of information"), *explanatory power,* and *simplicity*[7].

A cognitivist inductive logic is a pair $I = \langle P, U \rangle$, where P is an inductive probability measure defined on an algebra of the sentences of a language and U is an (expected) epistemic utility function defined in terms of P. In this paper we adopt the theory of inductive probability advanced by Hintikka [17, 18, 19] and employ an epistemic utility function introduced by Hintikka and Pietarinen [22].

The language upon which P is based is presumed to be a monadic, first-order language with k primitive predicates $R_1, R_2, \ldots, R_k$. Let $\lambda = \{R_1, \ldots, R_k\}$ and call the language L_λ[8]. The different kinds of individual objects that can be described in L_λ are specified by conjunctions of the form

$$Ct_j(x) = (\pm)\, R_1(x) \;\&\; (\pm)\, R_2(x) \;\&\; \ldots \;\&\; (\pm)\, R_k(x),$$

where $\pm$ may be replaced by negation or left blank. The Ct_j's are called *attributive constituents*; there are $K = 2^k$ attributive constituents in L_λ. The different kinds of possible worlds that can be described in L_λ are specified by conjunctions of the form

$$(\pm)\, \exists\, x\, [Ct_1(x)] \;\&\; (\pm)\, \exists\, [Ct_2(x)] \;\&\; \ldots \;\&\; (\pm)\, \exists\, x\, [Ct_K(x)],$$

called *constituents*, of which there are 2^K. A constituent can be rewritten to enumerate all the existing individuals, simultaneously announcing that other kinds of individuals do not exist; it then takes the form

$$C\pi = \exists\, x\, [Ct_{i_1}(x)] \;\&\; \exists\, x\, [Ct_{i_2}(x)] \;\&\; \ldots \;\&\; \exists\, x\, [Ct_{i_w}(x)] \;\&\; \forall\, x\, [Ct_{i_1}(x) \vee Ct_{i_2}(x) \vee \ldots \vee Ct_{i_w}(x)],$$

where "$\exists$", "$\forall$" and "v" denote "there exists", "for all" and (inclusive) "or", respectively. The role of constituents of most immediate interest is the following: any generalization in L_λ can be written as a finite disjunction of constituents. The disjunction is called the *distributive normal form* of the generalization. If a generalization has only one constituent in its distributive normal form, then it is called a *strong generalization*; all other generalizations are *weak.*

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7 See Niiniluoto and Tuomela [37], p. 64.

8 The grammar of L_λ is the usual predicate logic, see, e. g., Suppes [41]. We also presume that a semantics exists for L_λ so that the sentences of L are interpreted. See Przelecki [39].

The probability measure P is defined on an algebra of the sentences of L_λ. Like Carnap's earlier theory of logical probability [3, 4], the measure P is specified in terms of a continuum. Unlike Carnap's earlier theory, the measure P does not automatically assign the value 0 to a generalization. The continuum parameter α insures this result; as $\alpha \to \infty$, $P(C_k) \to 1$, and as $\alpha \to 0$, all constituents have equal *a priori* probabilities. Thus α can be seen as "an index of the strength of *a priori* consideration in inductive generalization" [19, p. 117] or similarly as "an index of caution" [20, p. 21]. Therefore, "in objective terms, α can be thought of as expressing the amount of disorder or irregularity that there probably exists in the universe as far as general laws are concerned, or in subjectivist terms, as representing the expectations of the investigator in regard to the amount of this disorder" [37, p. 25]. Throughout this paper α will be interpreted as an index of caution.

We will shortly employ a theory T to introduce a new concept R_0[9]. The introduction of R_0 expands the language L_λ to $L_{\lambda\, U\{R_0\}}$. T is a sentence in $L_{\lambda\, U\{R_0\}}$ which is a language with $k+1$ primitive concepts, $K' = 2^{k+1} = 2K$ attributive constituents, denoted Ct^r, and $2^{K'} = 2^{2K}$ constituents. We will discuss the inductive role of the theory T vis-a-vis an evidential statement e concerning the generalization g. Our discussion will involve the following:

- n: the number of individual objects reported upon by e;
- c: the number of Ct-predicates of L_λ exemplified by e;
- b: the number of Ct-predicates of L_λ which are empty by g $(0 < b \leqq K - c)$;
- b': the number of Ct^r-predicates of $L_{\lambda\, U\{R_0\}}$ which are empty by g but not by T $(0 \leqslant b' \leqslant 2b)$;
- r: the number of Ct^r-predicates of $L_{\lambda\, U\{R_0\}}$ which are empty by T.

Throughout this paper the (expected) epistemic utility function is specified as

$$U(g|e) = P(g|e) - P(g)$$

This form was introduced by Hintikka and Pietarinen [22] and measures the expected logical content of g relative to e[10].

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[9] Any number of new concepts can be so introduced. However, one new concept is sufficient for the present discussion.

[10] So defined, U is a measure of relevance [Definition: e is positively (negatively, ir-) relevant to h if and only if $P(h|e) > (<, =) P(h)$]. Alternative

The derivation of $U(g|e) = P(g|e) - (P\,g)$ is as follows. The *logical content* of a sentence g is the class of all sentences entailed by g[11]. A theorem of the probability calculus requires that if g_1 entails g_2, then $P(g_1) \leqq P(g_2)$[12]. Thus the size of the logical content of g and the prior probability of g vary inversely. A metric for logical content is then

$$\text{cont}(g) = 1 - P(g),$$

a simple inverse function of $P(g)$. Consider now a utility function u defined in terms of cont, as follows:

$$u(g) = \text{cont}(g)$$

when g is true, and

$$u(g) = -\text{cont}(\sim g)$$

when g is false (where "$\sim$" denotes "not"). Then, given evidence e, the conditional expected utility function $U(g|e)$ is

$$U(g|e) = P(g|e)\,\text{cont}(g) + P(\sim g|e)\,[-\text{cont}(\sim g)]$$

which reduces to

$$U(g|e) = P(g|e) - P(g).$$

If T is conjoined with the evidence e, then U gives rise to two variants, namely

$$U_1(g|e \,\&\, T) = P(g|e \,\&\, T) - P(g)$$

and

$$U_2(g|e \,\&\, T) = P(g|e \,\&\, T) - P(g|T).$$

The adoption of either U_1 or U_2 to the exclusion of the other reflects the individual's philosophical position. U_1 seems fitted to

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measures are $P(h|e)/P(h)$ and $\log[P(h|e)/P(h)]$. Further, we can introduce a "degree of boldness" q $(0 < q < 1)$, serving as a discount rate or weight on the prior probability $P(h)$. Then the conditional expected utility function can be formulated as $P(h|e) - qP(h)$, $P(h|e)/qP(h)$ or $\log[P(h_r e)/qP(h)]$. On relevance see Chapter IV of Carnap [3]. On boldness see Hilpinen [16], pp. 105—119 and Good [12]. See Hilpinen for alternative utility functions and their properties. Also see Dacey, et al. [8] for the application and analysis of five major utility functions to a specific decision problem.

11 See Carnap [3], pp. 405—409.

12 See Carnap [3], pp. 317.

the position of a (methodological) *instrumentalist*, whereas U_2 seems natural for a (scientific) *realist*[13].

The two inductive structures specified by U_1 and U_2 distinguish separate philosophies vis-a-vis the role of a *Weltanschauung* in the processing of potential additions to knowlegde. A (methodological) instrumentalist views a theory T merely as a device of convience in organizing data, i. e., a theory is a mere instrument. A (scientific) realist takes a theory literally, adhering to the view that the theory (more or less accurately) describes reality[14].

The desired explication of "theory absorption" can now be given. Let g be a theory (i. e., a generalization) and let $G(g)$ be the class of all those generalizations that compete with g[15]. Then g is *absorbed given e* if and only if

$$U(g|e) = \max_{h \varepsilon G(g)} U(h|e),$$

and g is *absorbed given e and T* if and only if

$$U_i(g|e \,\&\, T) = \max_{h \varepsilon G(g)} U_i(h|e \,\&\, T) \qquad (i=1, 2)$$

There are thus three variants of absorption: absorption on the basis of factual evidence alone and both instrumentalist and realist absorption on the basis of factual evidence and new (theoretical) conceptual evidence. There are only two forms of inductive logic. Expanding upon the earlier characterization of an inductive logic,

$l_1 = \langle P, U, U_1 \rangle$ is an instrumentalist logic of absorption

and

$l_2 = \langle P, U, U_2 \rangle$ is a realist logic of absorption

It is therefore more accurate to speak of instrumentalist absorption and realist absorption.

The Maintenance of an Absorbed Economic Theory

In this section we consider the role of a theory T in the maintenance of an economic theory under two conditions. First, T can

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13 See Niiniluoto and Tuomela [37], p. 70. On scientific realism and related philosophies see Hooker [24].

14 See Hooker [24].

15 Herein "competes" is left unformalized. For formalizations see Lehrer [29, 30]. Intuitively, we have "one sentence competes with a second if and only if the first does not logically imply the second" [29, p. 82]. Compare this with [30, pp. 197—198 and p. 201].

fail to introduce new concepts or it can do so exactly. Second, T can introduce a new concept inexactly, i. e., ambiguously. Exact and inexact concepts have different effects upon the maintenance of absorbed economic theories. The results of this section are formulated with the problems of an experimenter in mind.

The Role of Exact Concepts

The theory T insures the maintenance of the economic theory g_1 relative to the economic theory g_2 given the factual evidence e if (and only if) T blocks the reversal from g_1 to g_2. Reversal from g_1 to g_2 in the face of T takes place if and only if

$$U(g_1|e) > U(g_2|e), \tag{1}$$

and

$$U_i(g_1|e \,\&\, T) < U_i(g_2|e \,\&\, T), \tag{2}$$

where $i=1,2$. Any condition on T which blocks reversal, i. e., which is inconsistent with (1) and (2), insures the maintenance of g_1 over g_2. The following theorem provides the desired conditions.

Theorem 1[16]. Let g_i be any (strong or weak) generalization in $L_{\lambda \cup \{R_0\}}$.

The following conditions are sufficient for

$$P(g_i|e) = P(g_i|e \,\&\, T)$$

and for

$$P(g_i|e) - P(g_i) = P(g_i|e \,\&\, T) - P(g_i|T), \quad i=1,2.$$

(i) $\alpha \to \infty$.

(ii) α and n are large and $b_i = b_i'$.

(iii) T is an explicit definition of Ro.

In addition, the following conditions are sufficient for

$$P(g_i|e) = P(g_i|e \,\&\, T).$$

(iv) $n \to \infty$ and $\alpha \neq \infty$.

(v) $\alpha = n$ and $b_i = b_i'$.

For an instrumentalist, (1) becomes

$$P(g_1|e) - P(g_1) > P(g_2|e) - P(g_2) \tag{1'}$$

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[16] Niiniluoto and Tuomela [37], Theorem 6.49, p. 71.

and (2) becomes

$$P(g_1|e \,\&\, T) - P(g_1) < P(g_2|e \,\&\, T) - P(g_2). \tag{2'}$$

If any condition of Theorem 1 is satisfied by g_1 and g_2, then $P(g_1|e) = P(g_1|e \,\&\, T)$ and $P(g_2|e) = P(g_2|e \,\&\, T)$. These equalities, together with (1′) and (2′) imply a contradiction.

Similarly, for a realist, (1) remains

$$P(g_1|e) - P(g_1) > P(g_2|e) - P(g_2) \tag{1'}$$

but (2) becomes

$$P(g_1|e \,\&\, T) - P(g_1|T) < P(g_2|e \,\&\, T) - P(g_2|T). \tag{2'}$$

If g_1 and g_2 satisfy either (i), (ii) or (iii), then

$$P(g_1|e) - P(g_1) = P(g_1|e \,\&\, T) - P(g_1|T)$$

and

$$P(g_2|e) - P(g_2) = P(g_2|e \,\&\, T) - P(g_2|T).$$

These equalities, together with (1′) and (2′), imply a contradiction. Thus (i)—(v) of Theorem 1 constitute sufficient conditions on T to maintain g_1 as the absorbed economic theory for an instrumentalist, and (i)—(iii) serve the same role for a realist.

Consider the five conditions of Theorem 1 in turn. Condition (i) guarantees that $P(g_i|e) \to 0$ and $P(g_i|e \,\&\, T) \to 0$ for all i and thus maintains g_1 in a somewhat vacuous manner. Condition (ii) avoids such trivialities. If $b_i = b_i'$ for $i = 1, 2$, then T does not have observational consequences vis-a-vis g_1 and g_2. Recall that b_i is the number of Ct-predicates (i. e., possible objects) of L_λ which are empty (i. e., noninstantrated) by g_i and that b_i' is the number of Ct^r-predicates of $L_{\lambda \cup \{R_0\}}$ which are empty by g_i but not by T. Thus T, relative to g_i, does not exclude any new Ct^r-predicates, i. e., T has no observational consequences. The major component of (ii) is the requirement that α and n are large, i. e., that the individual is very cautious and that e reports upon a very large "sample".

Condition (iii) is the most useful to the experimenter. If T introduces a new concept (or concepts) R_0, if e reports upon R_0 and if R_0 is explicitly defined, i. e., is an exact concept with respect to λ, then reversal is blocked *for both instrumentalists and realists*. Information is the catalyst in a social science experiment. An experimenter can guarantee the maintenance of the theory under test if he is careful in the use of the catalyst. If all new concepts are explicitly defined (in terms of the initial concepts λ), then the experimental design is left intact. As we will soon discover, if a new concept is inexact, then the experimenter cannot guarantee the stability of his test. Herein lies the principal role of exact concepts. They maintain the individual's absorbed economic theory and hence the testability of that theory.

Condition (iv) is similar to (i). If $n \to \infty$, then $P(g_i|e) \to 1$ $P(g_i|e \,\&\, T) \to 1$ for all g_i compatible with $e \,\&\, T$. Thus (iv) maintains the absorption of g_1 relative to g_2 in a similarly vacuous way. Condition (v) is similar to (ii). Here, however, the individual's caution need only be balanced by the number of objects reported upon; neither α nor n need be large. Again, $b_i = b_i'$ requires that T have no observational consequence with respect to g_i.

The conditions of Theorem 1 are, of course, not exhaustive for maintaining g_1 over g_2. They are, however, indicative of the kinds of sufficient conditions one encounters in a cognitivist theory of absorption. Condition (iii), that T introduce R_0 exactly, is again the most useful of the five conditions from an experimenter's point of view.

The Role of Inexact Concepts

The concept "inexact" and its derivative froms have received repeated explication[17]. In this paper a concept is inexact if and only if it is introduced by a specific kind of theory T, a *piecewise definition*. A theory T in $L_{\lambda \cup \{R_0\}}$ is a *piecewise definition* of R_0 in terms of λ if and only if T logically implies a finite disjunction of explicit definitions of R_0 in terms of λ[18]. The sense of "inexact" is specified as follows: if T is a piecewise definition of R_0 in terms of λ, then there are statements $C_1, \ldots, C_n$ (in $L_{\lambda \cup \{R_0\}}$) such that

$$T \Rightarrow (C_1 \Rightarrow Df_1) \,\&$$
$$T \Rightarrow (C_2 \Rightarrow Df_2) \,\&$$
$$\ldots$$
$$T \Rightarrow (C_n \Rightarrow Df_n)$$

[17] On inexact and fuzzy concepts see Goguen [11] and Zadeh [45], respectively. On the effect of ambiguous (or fuzzy) concepts upon classical Bayesian decision making, see Gearing [10].

[18] On the methodological properties of piecewise definitions, see Niiniluoto and Tuomela [35], Chapter V, and especially Tuomela [43], Chapter IV. The latter provides the connection between definability and (econometric) identifiability. Hintikka and Tuomela [23] discuss piecewise definitions but call them conditional definitions.

where "⇒" denotes "if ..., then ...," and where each Df_i is a conjunction of explicit definitions of R_0 in terms of λ[19]. The conditions $C_1, \ldots, C_n$ may each describe a separate context, which in turn specifies an exact definition for R_0. Thus the notion of piecewise definition explicates, at least in part, the concept of a contextual definition[20]. Tuomela notes that "the methodological requirement of contextual definability might be formulated as follows: Even if one cannot define a concept explicitly, it may still be defined by means of the context in which it occurs, perhaps so that different contexts define (in some sense) the concept in different ways [42, pp. 94—95]. The inexactness of a piecewise defined concept arises out of the alternative meanings the concept can possess in different circumstances. Piecewise defined concepts are, like explicitly defined concepts, both eliminable and noncreative[21]. Thus piecewise definitions do not introduce new and unnecessary conceptual structures nor do they expand the underlying ontology of the language.

The role of inexact concepts in the maintenance of an economic theory is considerably more disruptive than that of exact concepts. As before, we proceed by specifying conditions which guarantee maintenance by blocking reversal as specified by Eqs. (1) and (2). The following theorem provides the desired sufficient conditions.

Theorem 2[22]. Let g_i be any (weak) generalization in $L_{\lambda \cup \{R_0\}}$, and let $T = T_1 \vee T_2$ be a piecewise definition of R_0 in terms of λ, with $t = T_1 \,\&\, T_2$.

(i) if t is incompatible with e, then $P(g_i|e) = P(g_i|e \,\&\, T)$.

(ii) If α and n are large, then $P(g_i|e) \geqq P(g_i|e \,\&\, T)$.

(iii) If t is incompatible with e, $b_i' < b_i$, and α is large, then $P(g_i|e) - P(g_i) < P(g_i|e \,\&\, T) - P(g_i|T)$.

(iv) If α and n are large and $b_i = b_i'$, then $P(g_i|e) - P(g_i) \approx P(g_i|e \,\&\, T) - P(g_i|T)$.

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[19] See Tuomela [43], pp. 75—76, for a discussion of this point.

[20] On contextual definitions see Pap [38] and Simon [40]. Also see Tuomela [43], Chapter IV.

[21] Tuomela [43], pp. 76—77. On eliminability, noncreativity and the theory of definitions see Suppes [41], Chapter 8.

[22] Niiniluoto and Tuomela [37], Theorems 6.53 and 6.63, p. 72 and p. 75, respectively.

For an instrumentalist, if g_1 and g_2 satisfy (i), then g_1 will be maintained. That is, (1′), (2′) and (i) imply a contradiction. Similarly, for a realist, if g_1 and g_2 satisfy (iv), then g_1 will be maintained[23]. Thus if t is incompatible with e, then an instrumentalist will maintain g_1 with respect to g_2 even though R_0 is inexactly specified. Likewise, if α and n are large and $b_i = b_i'$ ($i = 1$ and 2), i. e., if T has no observational consequences with respect to g_1 and g_2, then a realist will, in the face of an inexact concept, maintain g_1 over g_2.

Note, however, conditions (ii) and (iii). In (ii) the very conditions which guarantee nonreversal for a realist fail to do the same for an instrumentalist. Similarly, in (iii) the condition which guarantees nonreversal for a realist, when conjoined with $b_i' < b_i$ and when α is large, fails to block reversal for an instrumentalist. (Note that $b_i' < b_i$ requires that T lose observational consequences with respect to g_i). Thus the conditions which guarantee stability for a realist have no such effect on an instrumentalist, and conversely the conditions which guarantee stability for an instrumentalist have no such effect on a realist. Thus in an experiment involving an economy composed of both instrumentalists and realists, even if everyone begins with the same absorbed theory (i. e., g_1), the experimenter *cannot* guarantee the maintenance of the economic theory under test if inexact (i. e., piecewise defined) concepts are introduced or employed.

Morgenstern's initial comments concerned the impossibility of performing a test of a theory in an economy involving agents with separate views. His concern can be extended to the dynamics of experimental testing over an economy involving agents who initially hold the same economic theory but adhere to separate inductive logics. For if inexact concepts are employed, there is no single set of conditions which guarantees maintenance of the theory under test.

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[23] Two further assumptions are required. They are:

$$|[P(g_1|e) - P(g_1)] - [P(g_1|e \,\&\, T) - P(g_1|T)]| <$$
$$|[P(g_1|e) - P(g_1)] - [P(g_2|e) - P(g_2)]|$$

and

$$|[P(g_2|e \,\&\, T) - P(g_2)] - [P(g_2|e) - P(g_2)]| <$$
$$|[P(g_2|e \,\&\, T) - P(g_2)] - [P(g_1|e \,\&\, T) - P(g_1)]|.$$

where $|\cdot|$ denotes absolute value. Both of these assumptions are virtually guaranteed by the meaning of $\approx$.

The Testing of Consumer Behavior Theory

This section considers the application of the foregoing analysis to specific problems in testing the theory of consumer behavior. As is well known, the traditional Hicks-Slutsky theory is testable either directly, by observation on the Slutsky (substitution) term, or indirectly via its revealed preference formulation[24]. It is also well known that the traditional theory is a special case of Lancaster's theory. In Lancaster's simplified model the consumer selects an n-vector of commodities x so as to maximize his utility u of the m characteristics c where c is related to x via the (linear) household production technology B, i. e., $c=Bx$[25]. The maximization is constrained by the usual budget $p\cdot x=M$. Lancaster's theory is equivalent to the traditional theory if and only if $B=I$, the $n\times n$ identity matrix. The implications of the maximization problem for $B=I$ provide two major testable propositions. Both, as noted, concern the Slutsky (substitution) term $K_{ij}=\frac{2\,x_i}{2\,p_j}+x_i\frac{2\,x_j}{2\,M}$. The traditional theory (unconditionally) implies that (1) $K_{ij}=K_{ji}$, i. e., that the substitution terms are symmetric, and (2) $K_{ii}<0$, i. e., that the own substitution term is negative. Both components of K_{ij} are observable, and thus K_{ij} is observable and the above propositions about K_{ij} are testable.

If $B\neq I$, i. e., if the Lancaster model holds, then the unconditionality of the preceding proposition (2) is lost. Proposition (1), however, remains as an unconditional implication of the Lancaster theory. In this subsection we develop the consequences of the (simple) Lancaster model. The problem is to

$$\begin{aligned}&\text{maximize } u(c)\\&\text{subject to } c=Bx\\&\text{and } p\cdot x=M\end{aligned}$$

The Lagrangian for the problem is

$$G(x,m,B)=u(Bx)+\mu\,(p\cdot x-M).$$

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[24] For tests of the traditional theory see Battalio, et al., [1], [2]. Therein the revealed preference formulation of the theory is employed. On the connection between the revealed preference formulation and the standard theory see Uzawa [44].

[25] See Lancaster [27], pp. 136—137, for the simplified model.

The system of first order conditions is

$$
\begin{aligned}
G_\mu &= p_1x_1+p_2x_2+\ldots+p_nx_n-M=0\\
G_{x_1} &= \mu p_1+u_1b_{11}+u_2b_{21}+\ldots+u_mb_{m1}=0\\
&\vdots\\
G_{x_n} &= \mu p_n+u_1b_{1n}+u_2b_{2n}+\ldots+u_mb_{mn}=0
\end{aligned}
$$

where
$$u_i=\frac{2u}{2c_i}.$$

Presume that the usual second order conditions are satisfied, i. e., that

$$
\begin{vmatrix} G_{\mu\mu} & G_{\mu x_1} & \ldots & G_{\mu x_s}\\ \vdots & \vdots & & \vdots\\ G_{x_1\mu} & G_{x_s x_1} & \ldots & G_{x_s x_s}\end{vmatrix}
=
\begin{vmatrix} 0 & p_1 & \ldots & p_s\\ p_1 & u_{11} & \ldots & u_{1s}\\ \vdots & \vdots & & \vdots\\ p_s & u_{s1} & \ldots & u_{ss}\end{vmatrix}
\begin{matrix} >0 \text{ if } s \text{ is even}\\ \\ <0 \text{ if } s \text{ is odd}\end{matrix}
$$

where
$$G_{ij}=(b_{1i},\ldots,n_{mi})\begin{bmatrix} u_{11} & \ldots & u_{1m}\\ \vdots & & \vdots\\ u_{m1} & \ldots & u_{mm}\end{bmatrix}\begin{pmatrix} b_{ij}\\ \vdots\\ b_{mj}\end{pmatrix}$$

It is easy to show that if $u_{ij}=u_{ji}$ [as is presumed in the traditional (i. e., $B=I$) theory], then $G_{ij}=G_{ji}$.

The standard comparative statistics approach of differentiating the system of first order equations with respect first to M and then with respect to p_j provides the expected results that $\frac{2x_i}{2M}=\frac{A_i}{A}$ and $\frac{2x_i}{2p_j}=-x_j\frac{A_i}{A}-\mu\frac{A_{ij}}{A}$

where
$$A=\begin{vmatrix} 0 & p_1 & \ldots & p_n\\ p_1 & G_{11} & \ldots & G_{1n}\\ \vdots & \vdots & & \vdots\\ p_n & G_{n1} & \ldots & G_{nn}\end{vmatrix}$$

$$A_i=\begin{vmatrix} p_1 & G_{11} & \ldots & G_{1\,i-1} & G_{1\,i+1} & \ldots & G_{1n}\\ \vdots & \vdots & & \vdots & \vdots & & \vdots\\ p_n & G_{n1} & \ldots & G_{n\,i-1} & G_{n\,i+1} & \ldots & G_{nn}\end{vmatrix}$$

(the cofactor of p_i), and

$$A_{ij}=\begin{vmatrix} 0 & p_1 & \ldots & p_{i-1} & p_{i+1} & \ldots & p_n\\ \vdots & \vdots & & \vdots & \vdots & & \vdots\\ p_{j-1} & G_{j-1\,1} & \ldots & G_{j-1\,i-1} & G_{j-1\,i+1} & \ldots & G_{j-1\,n}\\ p_{j+1} & G_{j+1\,1} & \ldots & G_{j+1\,i-1} & G_{j+1\,i+1} & \ldots & G_{j+1\,n}\\ \vdots & \vdots & & \vdots & \vdots & & \vdots\\ p_n & G_{n1} & \ldots & G_{n\,i-1} & G_{n\,i+1} & \ldots & G_{nn}\end{vmatrix}.$$

(the cofactor of G_{ji}).

Consider now the symmetry of K_{ij}. If $u_{ij}=u_{ji}$ as presupposed, then $A_{ij}=A_{ji}$ since $G_{ij}=G_{ji}$. Thus, $K_{ij}=-\mu\frac{A_{ij}}{A}=-\mu\frac{A_{ji}}{A}=K_{ji}$, and K_{ij} is symmetric. Since K_{ij} and K_{ji} are observable, the equation $K_{ij}=K_{ji}$ is testable, and thus Lancaster's theory is testable.

As noted the (unconditional) negativity of K_{ii} is lost in Lancaster's theory. We now show this. $K_{ii}=-\mu\frac{A_{ii}}{A}$ and A is of order $n+1$. The second order conditions require that any border preserving principal minor of A of order n be of opposite sign of A. Also, by the definition of a cofactor, the sign of a cofactor of an element of the principal diagonal of a determinant is equal to the sign of the minor of that element. Thus $\frac{A_{ii}}{A}<0$[26]. From the $j+1$st equation in the first order conditions we have

$$\mu=\frac{-(u_1 b_{1j}+\ldots+u_m b_{mj})}{p_j}$$

This equality holds for all $j=1,\ldots,n$. If $u_1,\ldots,u_m>0$ (i. e., if all characteristics are "good"), then $\mu<0$. If $\mu<0$, then $K_{ii}<0$ as per the standard theory. However, it is possible that $\mu\geqq 0$. If $\mu\geqq 0$, then clearly $K_{ii}\geqq 0$. The important point here is that the sign of the Lagrangian multiplier μ is *not* unconditionally determined by the model, and thus the negativity of K_{ii} is not unconditionally testable. Furthermore, in order to get $\mu<0$, one must guarantee that

$$\sum_{i=1}^{m} u_i b_{ij}>0.$$

Clearly, $u_i>0$, $i=1,\ldots,m$ is sufficient. There are, of course, infinitely many more sufficient conditions. Note, however, that all of these conditions involve the consumer's (subjective) utility and are therefore not objectively determinable; that is, an experimenter can manipulate prices and income, but he cannot guarantee that $u_i>0$ for any particular $i=1,\ldots,m$. He can, of course, choose the characteristics so that $u_i>0$ ought to hold or will probably hold, but the experimenter cannot guarantee that $u_i>0$ will hold. Furthermore, the experimenter cannot guarantee that $u_i>0$ will hold for all i for all the subjects in an (experimental) economy, which is what is required. Thus Lancaster's theory, while testable, is not

[26] See, e. g., Lloyd [33], p. 72.

as testable as the traditional theory, and the only difference between the two theories is the form of the matrix B.

The preceding discussion of consumer theory is related to the earlier analysis of theory absorption as follows: the matrix B is regarded as a complex theoretical predicate[27]; the standard theory of consumer behavior is the complex generalization g_1 and Lancaster's theory is g_2; each individual possesses a (perhaps subjective) theory T which relates B to the predicates in λ; and the experimenter's instructions, i. e., the information catalyst, is the evidential statement e. The earlier results on absorption and maintenance of an economic theory can now be brought to bear in the specific context of testing the theory of consumer behavior.

If the agents in an experimental economy have each absorbed the traditional theory g_1, then, regardless of their inductive philosophies, if B is explicitly defined, then the traditional theory will be maintained. This is due, of course, to condition (iii) of Theorem 1. The same results hold if either condition (i) or (ii) of Theorem 1 is met. If all the members of an experimental economy adhere to an instrumentalist inductive philosophy, then any of conditions (i)—(v) of Theorem 1 will guarantee the maintenance of g_1, i. e., of the traditional theory.

However, if B is introduced by a piecewise definition T, i. e., is inexactly specified or is contextual for an individual, then depending upon his inductive philosophy, maintenance of g_1 can be guaranteed for that agent by invoking the appropriate condition, either (i) or (iv) from Theorem 2. If, as is more likely, the predicate B is introduced to separate individuals via piecewise definitions and these individuals adhere to different inductive philosophies, then Theorem 2 shows that there is no single condition which guarantees that g_1 will be maintained.

The experimenter faces a further difficulty. The theory T which governs the definitional status of B for a specific individual is subjectively possessed by that individual and is not under the experimenter's control. Various techniques might be employed to determine the nature of T for each individual. However, short of mind reading, there is no technique which will objectively determine an individual's theory T. The experimenter can present B via an explicit definition T, but there is no guarantee that that definition will be adopted by each individual as his theory T. The absence of the experimenter's capacity to guarantee the explicit definability of a

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[27] On a matrix as a complex theoretical predicate see Tuomela [43], p. 102 f.

predicate determines, at least in part, the inherent limitations to conducting objective experiments in economics, and economics, especially the theory of consumer behavior, provides the most overtly testable propositions of all the social sciences[28].

The foregoing impossibility of objective testing in the social sciences invites a parallel with Heisenberg's uncertainty principle from physics. The parallel is, however, spurious. An experimenter in the social sciences faces the difficulty of not being able to guarantee that observable behavior is generated within the context of the (proffered) theory under test. The experimenter has no difficulties beyond those inherent in his instruments in making observations. The Heisenberg uncertainty principle, on the other hand, announces that it is fundamentally impossible to make certain observations, even with the most ideal instruments that could conceivably be constructed[29]. Vis-a-vis observations, social science experimenters face measurement problems in fact, whereas physical science experimenters face measurement problems in principle. The impossibility of objective testing in the social sciences stems not from measurement problems but from the inherent inability of an experimenter to control a test[30].

Recall now Morgenstern's final comment as cited at the outset. "I believe that the study of the degree of 'theory absorption' by the members of the economy ... will make us all more modest in judging how far we have penetrated into the economic problems" [35, p. 707]. Our modesty is well founded. The human subjects of social science theorizing are different from their physical science counterparts exactly to the extent that human agents can and do learn. Furthermore, the catalyst in a social science experiment, information, is exactly the reactive agent of learning. If learning occurs among economic agents with mixed cognitivist logics, then the general impossibility of objective testing of theories must be admitted and our modesty converted to humility.

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[28] See Battalio, et al., [1], [2], on the control of error in an economic experiment.

[29] On Heisenberg's principle see, e. g., Halliday and Resnick [13], p. 1118. The connection with Heisenberg's uncertainty principle is weak at best. A much stronger connection can be made to the (general) notion of validity. See Tuomela [42] for the interrelationship of problem indefinability and the meta-psychological concept of validity.

[30] There are, of course, very real difficulties surrounding measurement in the social sciences. See, e. g., Morgenstern [34].

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