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Shape optimization of interior cutouts in composite panels

B.G. Falzon; G.P. Steven

Department of Aeronautical Engineering, University of Sydney, NSW 2006, Australia

Y.M. Xie

Department of Civil and Building Engineering, Victoria University of Technology, Melbourne, VIC 8001, Australia

Abstract This paper presents validated results of the optimization of cutouts in laminated carbon-fibre composite panels by adapting a recently developed optimization procedure known as Evolutionary Structural Optimization (ESO). An initial small cutout was introduced into each finite element model and elements were removed from around this cutout based on a predefined rejection criterion. In the examples presented, the limiting ply within each plate element around the cutout was determined based on the Tsai-Hill failure index. Plates with values below the product of the average Tsai-Hill number and a rejection ratio (RR) were subsequently removed. This process was iterated until a steady state was reached and the RR was then incremented by an evolutionary rate (ER). The above steps were repeated until a cutout of a desired area was achieved.

Nomenclature

σ_1	=	stress in fibre direction
σ_2	=	stress in transverse direction
τ_{12}	=	shear stress
X	=	tensile/compressive strength in fibre direction
Y	=	tensile/compressive strength in transverse direction
S	=	shear strength
e	=	plate element number lying on edge of cutout
p	=	plate element number
P	=	total number of plates
TH	=	Tsai–Hill number
$TH(l)$	=	limiting Tsai–Hill number of plate e
$TH(p)$	=	limiting Tsai–Hill number of plate p
RR	=	rejection ratio
$[\mathbf{K}]$	=	global stiffness matrix
$\{\delta\}$	=	displacement vector
$\{\mathbf{F}\}$	=	force vector
$\{\sigma\}$	=	stress vector
ℓ	=	ply number
j	=	number of plies per plate
FI	=	failure index
T	=	specified condition on which to terminate evolution (e.g. area of cutout)
σ_A	=	stress at major vertex of ellipse
σ_B	=	stress at minor vertex of ellipse
a	=	ellipse major axis
b	=	ellipse minor axis
i	=	increment number
ER	=	evolutionary rate

1 Introduction

The inevitable use of holes in aircraft structures introduces stress intensity factors, which may limit the operational life of the aircraft. As a result, the study of holes and their influence has received much attention since the theory of elasticity was fully enunciated in the first half of the 19th century. Research in this field has recently been given a renewed impetus with the emergence of advanced composite material technology.

A major objective in introducing cutouts in structures is to minimize the resulting stress concentrations. If the hole is to be neutral, giving rise to no variation in the prevailing stress field, reinforcement of varying sectional area is required as shown by Senocak and Waas (1995). If reinforcement is not a desirable option, then the inverse problem statement is to ask what cutout shape would give rise to the least perturbation in the stress field. This class of problems has been addressed through various shape optimization schemes.

Optimization of composite structures presents an added level of complexity due to the choice of lay-up for a structure. Wherever shape optimization is also involved, the search for an optimum becomes even more difficult and such a problem is usually approached by fixing one of these objectives. To demonstrate this added complexity, Vanderplaats and Weisshaar (1989) have shown that the seemingly simple case of determining the optimum lay-up for a panel under a given set of loading conditions is neither obvious nor easily found.

Most modern optimization techniques utilize finite element analysis combined with various numerical search techniques. A compendium of the development of shape optimization of structures is given by Vanderplaats (1982) and Ding (1986). Bäcklund and Isby (1988) used a point stress criterion at a characteristic distance of 1 mm from the edge of the hole along with a Tsai-Hill failure index to determine the stress field in the vicinity of a cutout in composite panels. The hole was defined using spline curves and these were allowed to change with the objective of minimizing the weight without increasing the maximum Tsai-Hill value. Vellaichamy *et al.* (1990) investigated the optimum orientation and aspect ratio of elliptical cutouts in composite structures for various lay-ups and load cases with the objective being that the maximum failure criterion around the hole was a minimum. The effect of ellipse orientation on buckling strength was also examined and found not to be significant. Hyer and Lee (1991) used a fibre placement technique to optimize the buckling strength of composite panels with a hole.

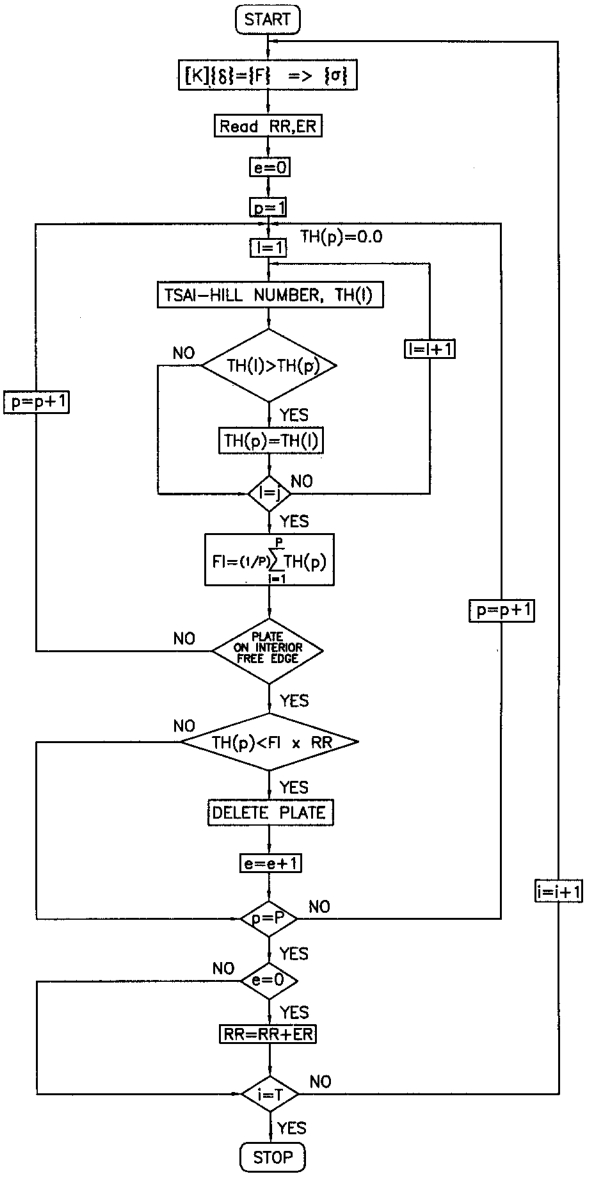
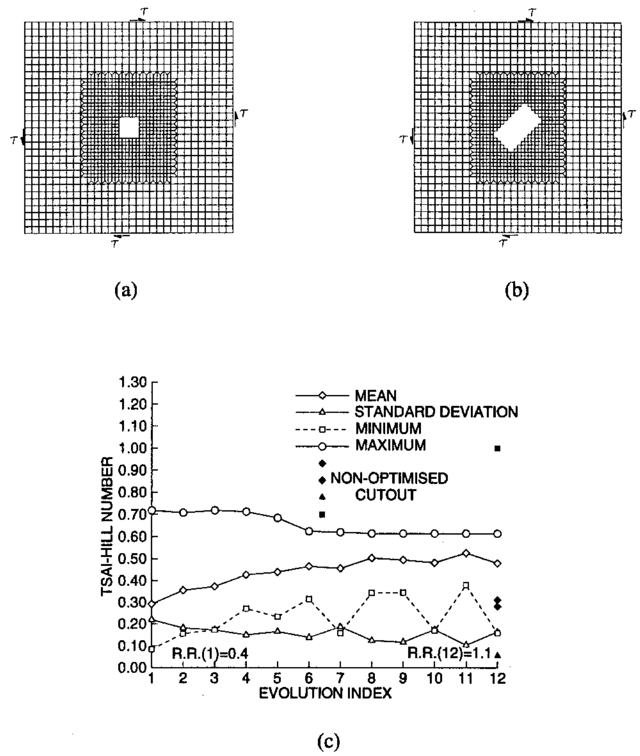


Fig. 1. ESO flowchart for the optimization of a cutout in a composite panel



	Opt.	Non-Opt.	% diff.
$TH(e)_{max}$	0.61	1	39
s	0.16	0.28	43

(d)

Fig. 2. Case 1A: panel loaded in pure shear $[\pm 45/0/90]_S$: (a) initial cutout, (b) final optimized cutout, (c) evolutionary history, (d) comparison table

Han and Wang (1993) investigated the location, size and orientation of a hole in a panel by minimizing the maximum tangential strain along the hole boundary. This was achieved by formulating the above as a linear programming problem coupled with finite element capability.

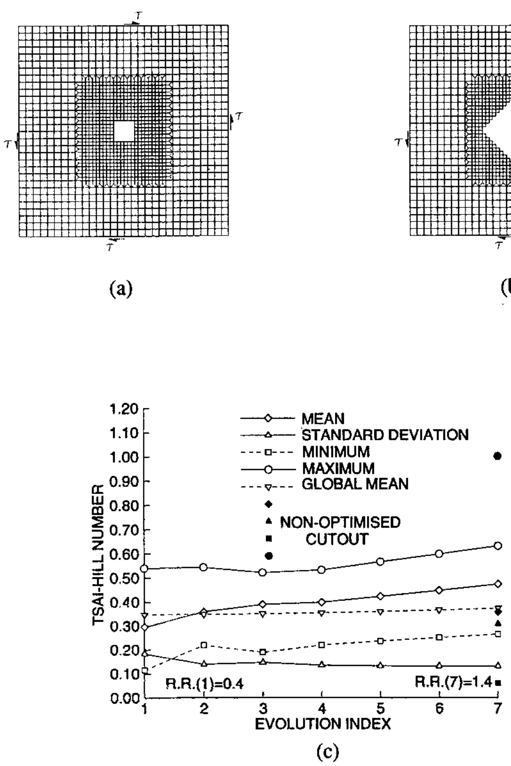
Recently, Xie and Steven (1993) developed a simple evolutionary procedure for structural and layout optimization (Evolutionary Structural Optimization, ESO). The strength of this method lies in its simplicity and is easily incorporated into existing finite element packages. The essence of this evolutionary procedure is to remove parts of a structure based on a rejection criterion, e.g. a stress-based criterion which removes elements in a state of low stress. This process is iterated until an optimum shape is evolved. Other evolutionary procedures which mimic nature have also been put forward by Baumgartner *et al.* (1992), who varied Young's modulus for a given structure, and Chen and Tsai (1993) using a simulated biological growth approach. Steven *et al.* (1993) and Xie and Steven (1994a,1994b,1995) have successfully demonstrated the applicability of their procedure to various optimization objectives, which include strength optimization of structures under single and combined load cases, frequency optimization of plate structures and shape optimization as applied to the repair of thin corroded metal surfaces.

The use of cutouts in aircraft structures for access panels and lightening holes introduces areas of stress concentrations and the major objective of their introduction is to minimize their influence on the overall integrity of the structure. Carbon-fibre composite material in aircraft is also finding increased use in primary structures and owing to the laminated nature of most composite structures, an effective optimized cutout is more readily achieved by adopting a layerwise approach. In this present study Evolutionary Structural Optimization (ESO) was applied to the optimization of a cutout in a square composite flat panel under various edge loading conditions as validation for this technique.

2 Evolutionary structural optimization

Evolutionary structural optimization is an attempt to simulate evolution, as observed in nature, through the use of numerical methods. This method has now been extended to include the optimization of cutouts in composite structures. Using finite element analysis, the panel was modelled with a small interior cutout to be optimized and the stress distribution was calculated from a linear static analysis resulting from a predefined loading. A failure criterion number was, in turn, assigned to each composite layer within each element. In this study a Tsai-Hill failure criterion with plane-stress assumptions was chosen although other polynomial-type failure criteria could have been used,

$$\left(\frac{\sigma_1}{X}\right)^2 + \left(\frac{\sigma_2}{Y}\right)^2 - \left(\frac{1}{X^2} + \frac{1}{Y^2}\right)\sigma_1\sigma_2 + \left(\frac{\tau_{12}}{S}\right)^2 \geq 1 . \quad (1)$$



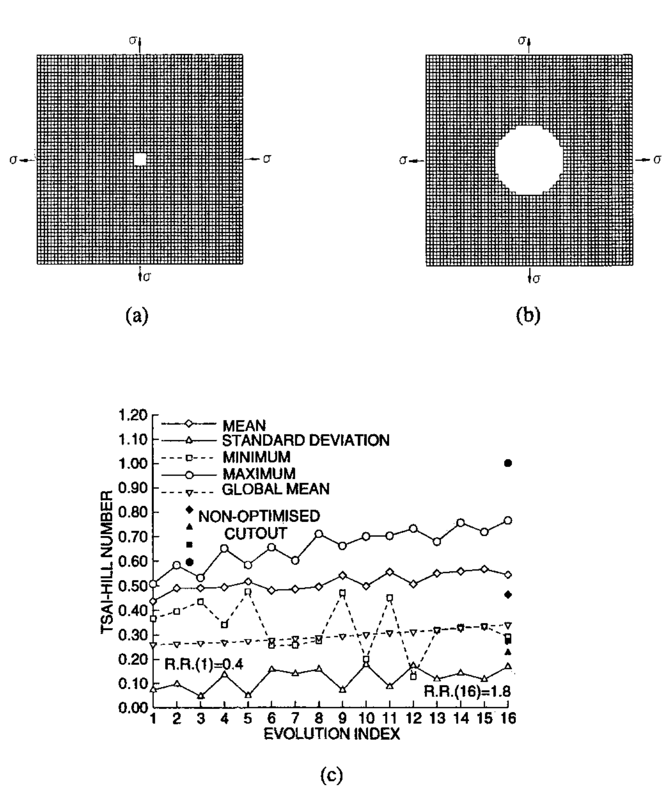
	Opt.	Non-Opt.	% diff.
$TH(e)_{max}$	0.63	1	37
s	0.13	0.31	58

(d)

Fig. 3. Case 1B: isotropic panel loaded in pure shear: (a) initial cutout, (b) final optimized cutout, (c) evolutionary history, (d) comparison table

By adopting a first-ply-failure philosophy, the ply with the maximum Tsai-Hill number within each plate on the interior free-edge was chosen as the limiting ply for the plate element. Plates were then rejected from this edge, subject to a user-specified rejection criterion. This rejection criterion was chosen such that a plate was rejected if and only if its representative Tsai-Hill number was less than the product of the user-specified rejection ratio (RR) and the average Tsai-Hill number of all the plates in the finite element model,

$$\text{reject plate } p \text{ if } TH(e) < \frac{1}{P}\sum_{p=1}^{P} TH(p) \times RR . \quad (2)$$



	Opt.	Non-Opt.	% diff.
$TH(e)_{max}$	0.76	1	24
s	0.17	0.28	39

(d)

Fig. 4. Case 2A: panel loaded in equibiaxial stress $[\pm 45/0/90]_S$: (a) initial cutout, (b) final optimized cutout, (c) evolutionary history, (d) comparison table

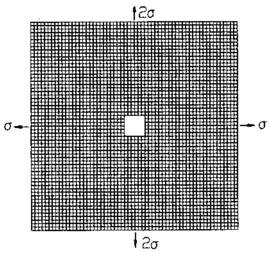
The linear static analysis was repeated for the updated structure using the same RR until a steady state was reached, i.e. no more plates were rejected. The RR was then updated by an evolutionary rate (ER), also specified by the user,

$$RR_{i+1} = RR_i + ER , \quad (3)$$

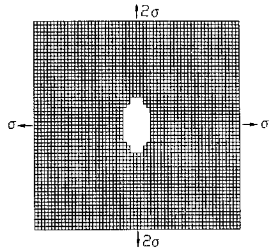
and the process repeated until a cutout of a desired size was achieved. A flowchart of this procedure is given in Fig. 1.

3 Results

In all cases presented, the initial RR was set at 4.0 with an ER of 0.05. Bounds for these variables were easily deduced from an initial solution to the problem and by observing the stress distribution in the panel. An initial cutout was introduced into this panel on which the evolutionary procedure could proceed. As mentioned earlier, the objective of introducing an improved or optimized cutout was to minimize its influence on the overall structure. One way to measure this improvement was to record the change in maximum Tsai-Hill number around the cutout through its evolution and to compare this value with that of a non-optimized hole (e.g. having the same shape as the initial cutout) of equivalent area. In further assessing the optimality of the resulting cutout, the standard deviation of the representative Tsai-Hill numbers of all the plates at the free-edge was also recorded and compared



(a)



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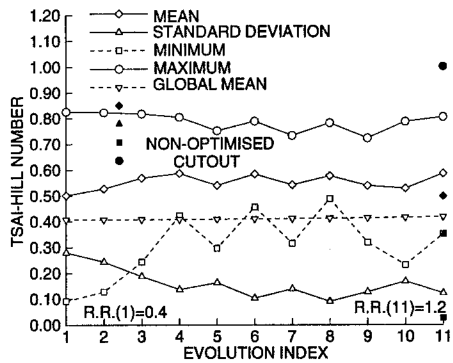
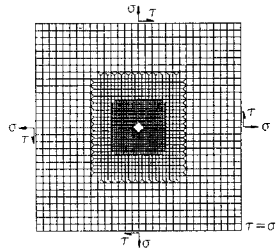


(c)

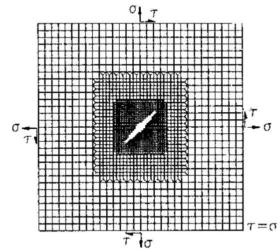
	Opt.	Non-Opt.	% diff.
$TH(e)_{max}$	0.8	1	20
s	0.12	0.35	66

(d)

Fig. 5. Case 2B: panel loaded in (2:1) biaxial stress $[\pm 45/0/90]_S$: (a) initial cutout, (b) final optimized cutout, (c) evolutionary history, (d) comparison table



(a)



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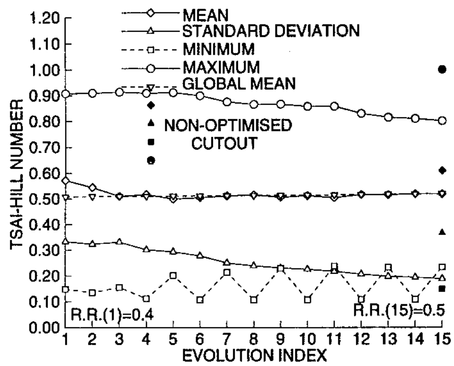


(c)

	Opt.	Non-Opt.	% diff.
$TH(e)_{max}$	0.8	1	20
s	0.19	0.36	47

(d)

Fig. 6. Case 3A: panel loaded in (1:1:1) biaxial and shear stress $[\pm 45/0/90]_S$: (a) initial cutout, (b) final optimized cutout, (c) evolutionary history, (d) comparison table

to that obtained from the non-optimized case. Whereas the first measurement concerns a localised phenomenon, the second pertains to the whole boundary of the cutout and a lower standard deviation implies less variability between the distribution of maximum and minimum Tsai-Hill values about the mean Tsai-Hill value in this region. This, in turn, suggests that the material around the interior free edge is being used more effectively as the shape of the cutout is evolved.

The initial and final shape of the cutout are presented as well as a graph which provides a history of the maximum, minimum, mean and standard deviation around the interior free-edge. These values are marked with hollow symbols connected by line-segments. A global mean Tsai-Hill number is also given although, as expected, this value remains fairly constant. Non-optimized values are shown in corresponding solid symbols at the last evolution index and the pertinent data has been tabulated showing percentage differences with respect to the non-optimized values. In each case the applied loading has been scaled such that the maximum non-optimized Tsai-Hill value corresponds to 1. The carbon fibre composite material data used in the present study is given in Table 1.

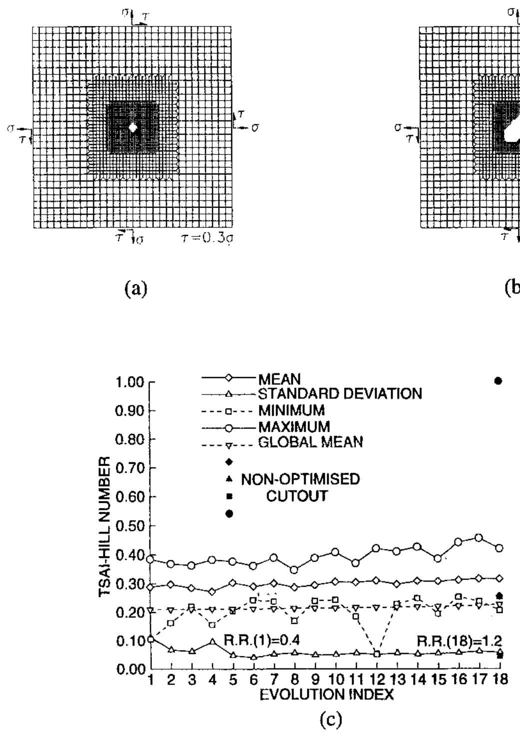
Table 1. Material data

Nominal material data for graphite-epoxy unidirectional preimpregnated tape		
E_1	128 GPa	
E_2	11.3 GPa	
G_{12}	6.0 GPa	
μ_{12}	0.3	
ply thickness	0.16 mm	
Strength	Tensile	Compressive
Longitudinal	1.45 GPa	1.25 GPa
Transverse	52 MPa	100 MPa
Shear	93 MPa	

4 Discussion

4.1 Case 1

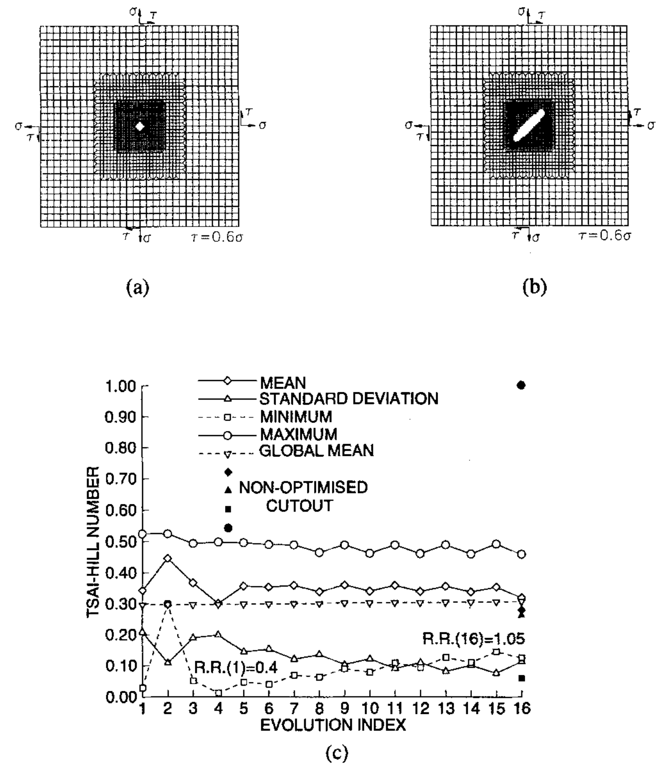
In this example, a square panel was subjected to a shear loading as shown in Fig. 2a. The resulting cutout in the quasi-isotropic laminated panel of lay-up $[\pm 45/0/90]_s$ was a rectangle of aspect ratio 1.86 and oriented at 45° to the horizontal axis as shown in Fig. 2b. The cutout for an identical isotropic panel was a diamond (a square oriented at 45°) as shown in Fig. 3b. The elongation of the cutout in Fig. 2b was a result of the directional strength of each ply within



	Opt.	Non-Opt.	% diff.
$TH(e)_{max}$	0.46	1	54
s	0.06	0.25	76

(d)

Fig. 7. Case 3B: panel loaded in (1:1:0.3) biaxial and shear stress $[\pm 45/0/90]_S$: (a) initial cutout, (b) final optimized cutout, (c) evolutionary history, (d) comparison table



	Opt.	Non-Opt.	% diff.
$TH(e)_{max}$	0.48	1	52
s	0.07	0.26	73

(d)

Fig. 8. Case 3C: panel loaded in (1:1:0.6) biaxial and shear stress $[\pm 45/0/90]_S$: (a) initial cutout, (b) final optimized cutout, (c) evolutionary history, (d) comparison table

the laminate. By referring to Fig. 3c it is interesting to note that despite the increasing size of the cutout, the maximum TH value initially remained fairly constant and reduced to a constant value halfway through the evolutionary process. The standard deviation also went through an initial decrease and underwent moderate oscillations right up to the end of the optimization procedure. More important are the considerable reductions of the Maximum TH ($TH(e)_{\max}$) and standard deviation (s) values as compared to a square cutout of equivalent area (limited by the descretisation of the finite element mesh). The percentage differences with respect to the non-optimized results were 39% and 43%, respectively. This compares to a reduction of 37% and 58%, respectively, for the isotropic case. The grading of the mesh was a matter of convenience in an attempt to reduce the amount of run-time for each solution. Subsequent results are presented with differing mesh gradings although these variations have an insignificant bearing on the results presented.

4.2 Case 2

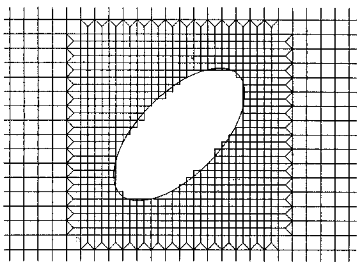
In Case 2a, the square panel was subjected to an equibiaxial stress. The resulting optimized cutout was a circle and this corresponded to the cutout expected from an isotropic panel under identical loading conditions. The percentage reductions with respect to the non-optimized cutout were 24% in $TH(e)_{\max}$ and 39% in s. Again it may be observed in Fig. 4c that there is a modest increase in the afore-mentioned values as the cutout size is increased. In Case 2b the stress in the longitudinal direction was set at double the stress in the transverse direction. The optimum cutout for this loading condition closely resembles an ellipse of aspect ratio (AR) 2. This may be compared to the expected aspect ratio of an elliptical cutout in an equivalent infinite isotropic sheet. The stresses at the vertices of this ellipse are given by

$$\sigma_A = \sigma_1 \left(1 + \frac{2a}{b}\right) - \sigma_2 \,, \quad \sigma_B = \sigma_2 \left(1 + \frac{2b}{a}\right) - \sigma_1 . \qquad (4)$$

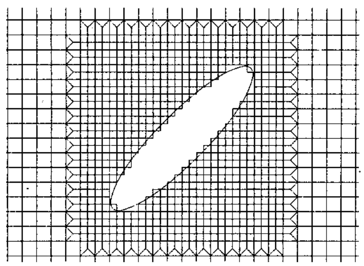
By making the assumption that an optimum configuration is one which yields $\sigma_A = \sigma_B$ and by setting $\sigma_2 = 2\sigma_1$, an ellipse of $AR = 2$ is obtained. This is identical to the AR obtained for the quasi-isotropic composite panel. The shape of the cutout shown in Fig. 5b is limited by the coarseness of the mesh. The percentage reductions were 20% for $TH(e)_{\max}$ and 66% for s.

4.3 Case 3

The optimal shape of a cutout in an equibiaxial and shear stress field was investigated for three values of shear stress; (a) $\tau = \sigma$, (b) $\tau = 0.3\sigma$, (c) $\tau = 0.6\sigma$. In all cases the optimum cutout was a super-ellipse oriented at 45°. To verify



(a)



(b)

Fig. 9. (a) Elliptical curve fit around cutout for Case 3b, (b) elliptical curve fit around cutout for Case 3c

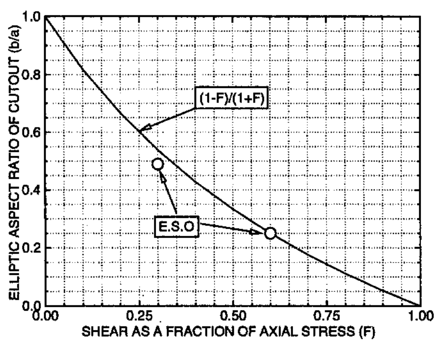


Fig. 10. Elliptic aspect ratio as a function for shear fraction

the AR under each loading condition, reference was again made to the isotropic case. In an infinite isotropic plate under equibiaxial stress σ and shear stress $F\tau$, where $0 \leq F \leq 1$ and $\sigma = \tau$, it can be easily shown that the optimum orientation of an ellipse is 45° with AR,

$$AR = \frac{a}{b} = \frac{1+F}{1-F}\,. \tag{5}$$

Thus for Case 3a we would expect a slit (i.e. $AR = \infty$), for Case 3b $AR = 1.85$ and for Case 3c $AR = 4$. With reference to Fig. 6b the resulting cutout is indeed a slit with the finite aspect ratio being due to the finite width of the initial cutout. The percentage differences were 20% for $TH(e)_{\max}$ and 47% for s. The cutout for Case 3b resembles a super-ellipse. The AR was estimated by fitting an ellipse through the cutout (Fig. 9a) and measuring the AR of this ellipse. This was measured at 2.04. A percentage difference of 54% in $TH(e)_{\max}$ and 76% in s were recorded. In Case 3c the AR of the cutout was estimated at 4.07 using the same procedure as in Case 3b and shown in Fig. 9b. The percentage difference in $TH(e)_{\max}$ was 52% and 73% for s. The two values of AR for Cases 3b and 3c were plotted against the curve obtained for the isotropic case and shown in Fig. 10.

5 Conclusion

The examples presented in this paper demonstrate the validity of evolutionary structural optimization as applied to the optimization of cutouts in laminated composite panels. This optimization was achieved by removing plates from around the cutout which were in a state of low stress. The strength of this method lies in its simplicity and is easily incorporated into commercially available finite element packages. This procedure is not restricted to two-dimensional problems and is easily applied to complex engineering-type composite structures.

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Preliminary Announcement

The 2-nd World Congress of Structural and Multidisciplinary Optimization

will be held in Zakopane, Poland, May 26–30, 1997.

For details contact:
Prof. Zenon Mróz, Prof. Witold Gutkowski
Institute of Fundamental Technological Research
Świętokrzyska 21, PL-00-049 Warsaw, Poland
FAX: (48) 22 26 98 15