LEONE BURTON

MOVING TOWARDS A FEMINIST EPISTEMOLOGY OF MATHEMATICS

ABSTRACT. There is, now, an extensive critical literature on gender and the nature of science three aspects of which, philosophy, pedagogy and epistemology, seem to be pertinent to a discussion of gender and mathematics. Although untangling the inter-relationships between these three is no simple matter, they make effective starting points in order to ask similar questions of mathematics to those asked by our colleagues in science. In the process of asking such questions, a major difference between the empirical approach of the sciences, and the analytic nature of mathematics, is exposed and leads towards the definition of a new epistemological position in mathematics.

1. INTRODUCTION

Received science has been criticised on three grounds from a gender perspective. The first is its reductionism and its claim to be objective and value-free (e.g. Harding, 1986, 1991; Keller, 1985; Rose and Rose, 1980). Second, the conventional style of learning and teaching in science, its pedagogy, has been challenged. It is suggested that enquiry methods used by scientists are often intrusive and mechanistic, separating observer and observed, and reinforcing competition. Further, these methods are presented not only as 'correct' but also as the only way possible (e.g. Kelly, 1987; Whyte et al., 1985). Third, having rejected objectivity as an untenable criterion for judging science, a new scientific epistemology was required and has been derived (see Rosser, 1990) by examining the connections between the discipline and those who use it, and the society within which it develops. This line of reasoning is consistent with a broad range of thinking in the sociology of science.

The old certainties about science, the old belief in its cultural uniqueness and the old landmarks of sociological interpretation have all gone. (Mulkay, 1981, p. vii).

Mathematics and mathematics education have been subject to a similar challenge from within on philosophical, pedagogic and epistemological grounds. The philosophical arguments for a rejection of absolutism in mathematics have been explored elsewhere (see Ernest, 1991). Lakatos (1976, 1983), Bloor (1976, 1991) and Davis and Hersh (1983) have all made similar philosophical and epistemological criticisms to those out-

Educational Studies in Mathematics **28:** 275–291, 1995.

lined in the science literature with respect to the so-called objectivity of mathematics.

Likewise, a gender critique similar to that found in science has been made of mathematical pedagogy (see, for example, Burton, 1986, 1990a; Fennema and Leder, 1990; Leder and Sampson, 1989). Despite

> many reports calling for curriculum reform in mathematics and science ... the reforms suggested do not take feminist concerns into account; in fact, in the case of mathematics they tend to put added emphasis on curricular areas in which young women regularly perform less well than their male counterparts. (Damarin, 1991, p. 108).

Mathematics tends to be taught with a heavy reliance upon written texts which removes its conjectural nature, presenting it as inert information which should not be questioned. Predominant patterns of teaching focus on the individual learner and induce competition between learners. Language is pre-digested in the text, assuming that meaning is communicated and is non-negotiable. In Hull's terms this defines

> knowledge as an object and so equates knowing, and coming to know, with its possession; it effaces the crucial distinction between the learner's subjective experience of moving towards knowledge and the objectifying of a knowledge finally achieved. (1985, pp. 45–50).

Like science, therefore, mathematics is perceived by many students and some teachers as "*a body of established knowledge accessible only to a few extraordinary individuals*" (Rosser, op.cit. p. 89). Indeed, the supposed 'objectivity' of the discipline, a cause for questioning and concern by some of those within it, is often perceived by non-mathematician curriculum theorists as inevitable (see, for example, Hirst, 1965 and 1974 and, for a critique expanding the points being made here, Kelly, 1986). But

> the processes of knowing (and so also of science) in no way resemble an impersonal achievement of detached objectivity. They are rooted throughout ...in personal acts of tacit integration. They are not grounded on explicit operations of logic. Scientific inquiry is accordingly a dynamic exercise of the imagination and is rooted in commitments and beliefs about the nature of things. (Polanyi and Prosch, 1975, p. 63).

Adopting an objectivist stance within mathematical philosophy means accepting that mathematical 'truths' exist and the purpose of education is to convey them into the heads of the learners. This leads to conflicts both in the understanding of what constitutes knowing, and of how that knowing is to be achieved through didactic situations. For example, such conflicts can be found between the U.K. mathematics national curriculum, expressed in terms of a hierarchy of mathematical truth statements, and the support documentation given to teachers which includes such relativistic statements as:

Each person's "map" of the network and of the pathways connecting different mathematical ideas is different, thus people understand mathematics in different ways (Non-statutory Guidance to the Mathematics National Curriculum, para. 2.1, p. C1).

The teacher's job is to organise and provide the sorts of experiences which enable pupils to construct and develop their own understanding of mathematics, rather than simply communicate the ways in which they themselves understand the subject (Ibid, para. 2.2, p. C2).

Although "*the ideal of pure objectivity in knowing and in science has been shown to be a myth*" (Polanyi and Prosch, op.cit. p. 63) it is a philosophical myth which continues to exercise enormous power over mathematics both in curricular and in methodological terms.

Proposed as an alternative, social constructivism is a philosophical position which emphasises the interaction between individuals, society and knowledge out of which mathematical meaning is created. It has profound implications for pedagogy. Classroom behaviours, forms of organisation, and roles, rights and responsibilities have to be re-thought in a classroom which places the learner, rather than the knowledge, at the centre. Epistemology, too, requires reconsideration from a theoretical position of knowledge as given, as absolute, to a theory of knowledge, or perhaps better, of knowing, as subjectively contextualised and within which meaning is negotiated.

With respect to science, Rosser stated:

If science is socially constructed, then attracting a more heterogeneous group of scientists would result in different questions being asked, approaches and experimental subjects used, and theories and conclusions drawn from the data (Rosser, 1990, p. 33).

How might including many of those currently outside the mainstream of mathematical development influence its conjectures, its methods of enquiry and the interpretation of its results? In turn, how might any changes which resulted from a philosophical shift, affect the pedagogy and epistemology of the discipline? In particular, what are the epistemological questions which are sharpened by bringing a feminist critique to bear on the discipline of mathematics? These issues are the focus of this paper.

2. ADOPTING A CULTURAL VIEW OF MATHEMATICS

In writing about mathematics, Harding drew attention to its cultural dependency:

Physics and chemistry, mathematics and logic, bear the fingerprints of their distinctive cultural creators no less than do anthropology and history. A maximally objective science, natural or social, will be one that includes a self-conscious and critical examination of the relationship between the social experience of its creators and kinds of cognitive structures favoured in this inquiry ... whatever the moral and political values and interests responsible for selecting problems, theories, methods, and interpretations of research, they reappear at the other end of the inquiry as the moral and political universe that science projects as natural and thereby helps to legitimate. (Harding, 1986, pp. 250–1).

Despite the stance taken by many mathematicians on the objectivity and value-free nature of the discipline, Bloor convincingly argued from a historical perspective that it is possible to conceive of alternative mathematics differently derived at different periods:

Seeing how people decide what is inside or outside mathematics is part of the problem confronting the sociology of knowledge, and the alternative ways of doing this constitute alternative conceptions of mathematics. The boundary (between mathematics and meta-mathematics) cannot just be taken for granted in the way that the critics do. One of the reasons why there appears to be no alternative to our mathematics is because we routinely disallow it. We push the possibility aside, rendering it invisible or defining it as error or as nonmathematics. (Bloor, 1991, pp. 179–80).

More recently, Harding (1991) has pushed the argument further to locate mathematics firmly within its interpretative context despite its overtly comparable formalistic expression:

There can appear to be no social values in results of research that are expressed in formal symbols; however, formalisation does not guarantee the absence of social values. For one thing, historians have argued that the history of mathematics and logic is not merely an external history about who discovered what when. They claim that the general social interests and preoccupations of a culture can appear in the forms of quantification and logic that its mathematics uses. Distinguished mathematicians have concluded that the ultimate test of the adequacy of mathematics is a pragmatic one: does it work to do what it was intended to do? Moreover, formal statements require interpretation in order to be meaningful...Without decisions about their referents and meanings, they cannot be used to make predictions, for example, or to stimulate future research. (Harding, 1991, p. 84).

In his discussion of mathematical epistemology, Joseph (1993) drew attention to two major philosophical pre-suppositions which underlie Western (European) mathematics. These are, first, that mathematics is a body of absolute truths which are, second, argued (or 'proved') within a formal, deductive system. However, he pointed out that dependence upon an axiomatically deduced system of proof was a late nineteenth century development which was pre-dated by 'proofs' closer in style to that of non-European mathematicians:

The Indian (or, for that matter, the Chinese) epistemological position on the nature of mathematics is very different. The aim is not to build up an imposing edifice on a few self-evident axioms but to validate a result by any method, including visual demonstration. (Joseph, 1993, p. 9).

Joseph further stated that:

None of the major schools of Western thought ... gives a satisfactory account of what indeed is the nature of objects (such as numbers) and how they are related to (other) objects in everyday life. It is an arguable point ... that the Indian view of such objects ... may lead to some interesting insights on the nature of mathematical knowledge and its validation. Irrespective of whether this point can be substantiated or not, a more balanced discussion of different epistemological approaches to mathematics would be invaluable. However, a different insight into some of the foundational aspects of the subject is hindered by the prevalence of the Eurocentric view on the historical development of mathematics. (pp. 11–12).

Joseph is criticising the dominance of a Eurocentric (and male) mathematical hegemony which has created a judgmental situation within the discipline whereby, for example, deciding what constitutes powerful mathematics, or when a proof proves and what form a rigorous argument takes, is dictated and reinforced by those in influential positions. How often do we hear statements, often made about a geometric proof, dismissing it as 'merely a demonstration' or the suggestion that computer-assisted proofs are not quite as 'good' as those developed without a computer? How frequently are students encouraged to believe that the mathematico-scientific and technological development of the West has been made independently of a systematic knowledge and resource exploitation of the rest of the world? The colonisation of mathematics has been so successful that the history of their own mathematical culture and its contribution to knowledge is often unknown to students in Africa, Asia and Latin America. Such bias is increasingly under attack (see, for example, Needham, 1959; Zaslavsky, 1973; van Sertima, 1986; Joseph, 1991; Nelson *et al.*, 1993) as researchers uncover the richness and power of mathematical and scientific development in the non-European world which has been obscured by the re-writing of history from a European perspective. If the body of knowledge known as mathematics can be shown to have been derived in a manner which excluded non-Europeans and their mathematical knowledge, why not conjecture that the perceived male-ness of mathematics is equally an artefact of its production and its producers?

Since I am arguing that mathematics is socio-cultural in nature, the conditions under which it is produced are factors in determining the products. "Important" mathematical areas are identified, value is accorded to some results rather than others, decisions are taken on what should or should not be published in a society determined by power relationships, one of which is gender. Mathematical products can then be seen as the outcome of the influence of a particular 'reading' of events at a given time/place. Such readings are referred to by Sal Restivo as

stories about commercial revolutions and mathematical activity, as in Japan, or about the 'mathematics of survival' that is a universal feature of the ancient civilizations. And they can be stories about how conflict and social change shape and reflect mathematical developments. (1992, p. 20).

In a Plenary lecture given at the 1994 American Educational Research Association Conference, Jerome Bruner pointed out that explanation as causal is a post-nineteenth century phenomenon. A longer history can be found for interpretation whose objective is understanding and not explanation. He made out a case for understanding being viewed as both contextualising and systematising and he advocated a route to contextualising in a disciplined way through narrative. From this perspective, codified mathematics can be viewed as reified narrative and it no longer seems so absurd to ask how different narratives, or stories, might constitute alternative mathematics (in the plural). Mathematics as a particular form of story about the world *feels*, to me, very different from mathematics as a powerful explanation or tool. Re-telling mathematics, both in terms of context and person-ness, would consequently demystify and therefore seem to offer opportunities for greater inclusivity.

3. KNOWING SCIENCE AND MATHEMATICS

The feminist literature on the philosophy of science I find very valuable for the clarity with which it has sharpened the critical debate on the nature of knowledge in science and how that knowledge is derived. However, it is noticeable that the content criticisms of science are rooted in the empirical disciplines. For example, female primatologists such as Goodall (1971), Fossey (1983) and Hrdy (1986) challenged conceptions of interactive behaviour by refusing to accept the prevailing (male) views on dominance and hierarchy in sexual selection. Keller (1985) highlighted McClintock's approach to her study of maize as a symbiotic relationship between the plant and its environment which was distinctively different to the more usual 'objective' investigation undertaken by botanists. Carson (1962) is frequently cited for her early work on ecology and the broad view that she took about the environmental effects of pesticides. In all these cases, the results of the science were different from what had, formerly, been expected because different questions were asked about what was being observed and different methods were used to make the observations.

However, criticisms of, for example, nuclear physics are more likely to focus upon the social effects of the science, rather than the science itself. (See, for example, Easlea, 1983). This is not to diminish the importance of developing models of scientific use and abuse which criticise the purposes, products and implications of scientific developments. But, as with mathematics, it is difficult to confront the abstractions which are the substance and tools of the discipline and the methods used in their derivation especially where these are analytic and non-observational in order to ask what differences a female perspective would make to them.

In what ways might the questions, or the styles of enquiry or the mathematical products differ if mathematics were to be admitted to be a sociocultural construct? Part of the difficulty in responding to this question resides in the highly successful socialisation experiences through which we all go in order to achieve success at mathematics. It is exceedingly difficult to dismantle the beliefs which have been integral to our learning experiences of mathematics and almost impossible to construct in our imaginations alternatives to the processes which we have been taught and with which we have gained 'success'. Hence, scratch a pedagogical or philosophical constructivist and underneath you are likely to expose an absolutist. In other words, it might be acceptable to negotiate a curriculum or introduce a collaborative, language-rich environment within which to make the learning of mathematics more accessible, but the mathematics itself is considered non-negotiable. However, to be consistent in our critiques, we cannot avoid addressing the nature of knowing mathematics along with the philosophy and pedagogy of the discipline.

Knowing mathematics, and science, has traditionally required entry into a community of knowers who accord the status of 'objective', *"in some sense eternal and independent of the flux of history and culture"* (Restivo, 1992, p. 3), to the knowledge items as well as to the means by which these items are derived. However,

> objectivity is a variable; it is a function of the generality of social interests. Aesthetic and truth motives exist in the realm of ideas, but they are grounded in individual and social interests ranging from making one's way in the world (literally, surviving) to exercising control over natural and cultural environments. (Restivo, 1992, p. 135).

A consequence of this is that

> a mathematical object ... like a hammer or a screwdriver, is conceived, constructed, and put to use through a social process of collective representation and collective elaboration. (Restivo, 1992, p. 137).

If we are to argue for a different conception of mathematical knowing from that traditionally accepted, we must address the meaning which is to be understood by 'objectivity' since the truth status accorded to mathematical objects underpins the pervading epistemology. Criticising scientific 'objectivity' along similar lines, Harding (1991) called for an epistemology of the sciences which requires a more robust standard than that currently in use. This would include the critical examination, *"within scientific research"* (original italics, p. 146)

> of historical values and interests that may be so shared within the scientific community, so invested in by the very constitution of this or that field of study, that they will not show

up as a cultural bias between experimenters or between research communities. (Harding, 1991, p. 147).

and she further noted that:

the difficulty of providing (such) an analysis in physics or chemistry (and, I would add, mathematics) does not signify that the question is an absurd one for knowledge-seeking in general, or that there are no reasonable answers for those sciences too. (p. 157)

Rosser, in her book *Female-Friendly Science* (1990), used women's experience of knowing and doing science to draw out differences from what she called the conventional androcentric approaches. Amongst many of the inclusionary methods she listed are:

* expanding the kinds of observations beyond those traditionally carried out;
* increasing the numbers of observations and remaining longer in the observational stage of the scientific method;
* accepting the personal experience of women as a valid component of experimental observation;
* being more likely to undertake research which explores questions of social concern than those likely to have applications of direct benefit to the military;
* working within research areas formerly considered unworthy of investigation because of links to devalued areas;
* formulating hypotheses which focus on gender as an integral part;
* defining investigations holistically.

This list, useful as it is for science, does not generalise easily to mathematics although the links to the history, philosophy and pedagogy of mathematics are more obvious. But help appears to be at hand.

4. BEING A MATHEMATICIAN

In *The Emperor's (sic) New Mind*, Penrose (1990) arguing from the powerful position of a research mathematician at the top of his profession, claimed that the mathematician's "consciousness" is a necessary ingredient to the comprehension of the mathematics. He said:

We must 'see' the truth of a mathematical argument to be convinced of its validity. This 'seeing' is the very essence of consciousness. It must be present whenever (original emphasis) we directly perceive mathematical truth. When we convince ourselves of the validity of Gödel's theorem we not only 'see' it, but by so doing we reveal the very non-algorithmic nature of the 'seeing' process itself. (p. 541)

Elsewhere in his book, and in contradiction to the above, Penrose supported a Platonic approach to mathematics in that he propounded a discovery, rather than an invented, perspective on the discipline. That is, mathematics is out there waiting to be uncovered rather than within the head (and possibly the heart?) of the mathematician. And yet, Penrose himself admitted that 'seeing' the validity of a mathematical argument must be a personal experience and one which, it seems reasonable to me to assert, can be assumed to differ between individuals. By arguing that 'seeing' is non-algorithmic, Penrose permitted the personalisation of the process. He reinforced this with the statement:

There seem to be many different ways in which different people think – and even in which different mathematicians think about their mathematics. (Penrose, 1990, p. 552).

However, for me, far from accepting that the outcomes of mathematical thinking are discovered mathematical 'truths', the inevitable conclusion of his statement is that there are potentially many different mathematics.

The contradiction would appear to lie in a different perspective on the mathematician than on mathematics itself. Penrose viewed a mathematical statement, once articulated, as being absolute, that is either right or wrong, and its status verifiable by any interested party. But he said, in the

conveying of mathematics, one is *not* simply communicating *facts*. For a string of (contingent) facts to be communicated from one person to another, it is necessary that the facts be carefully enunciated by the first, and that the second should take them in individually ... the *factual* content is small. Mathematical statements are necessary truths ... and even if the first mathematician's statement represents merely a groping for such a necessary truth, it will be that truth itself which gets conveyed to the second mathematician ... The second's mental images may differ in detail from those of the first, and their verbal description may differ, but the relevant mathematical idea will have passed between them. (original emphases, p. 553).

Despite the assumed personal nature of the communication and the expectation of differences in human images and descriptions, there is an assumption that the 'mathematics' , the essential 'truth' of the statement, can and will be the same for all. This is repeatedly refuted by the message of many of the anecdotes which are recounted by and about mathematicians. For example, how is it possible to interpret the kind of intuitive insights which Penrose himself, and other mathematicians such as Poincaré, Hadamard, Thom, claim to have had and which have led to their finding particular, personal resolutions of a mathematical problem? Given that it is reasonable to expect that any one problem might be amenable to a number of different routes for solution, an individual is likely to fall on the one which matches her or his experience, approach, preferences, possibly making the mathematical outcome different from that which would be offered by another individual. Of course, once articulated, the internal consistency of the mathematical argument is claimed to be verifiable. However, the most recent attempt to prove of Fermat's Last Theorem provided an example of the unverifiability, by most mathematicians, of the claims being made and, consequently, both the potential non-uniqueness and fragility of their status. And, even if the internal consistency is substantiated, this does not additionally encompass any objective status nor any implication of uniqueness, it seems to me. The social context within which the mathematics is placed does, however, offer one explanation for apparent uniqueness, or at least convergence of 'solutions', given that it describes and constrains the 'possible' . Thus, a piece of mathematics is both contributory to, and defined by, the context within which it is derived.

A belief in the world of mathematical concepts existing independently of those who develop or work with them is attached to embracing the 'objective' truths of mathematics. An image of 'variable' truth, that is degrees of correctness, or solutions responsive to different conditions, is unacceptable to many within the discipline despite the support from the history of mathematics that understandings change over time as the foci and the current state of knowledge change. The social context of a mathematical statement, the impact upon it of the interests, drives and needs of the person deriving and then communicating it, are dismissed by many mathematicians as inappropriate to the product. Thus, the distinction is made between the person who is working at the mathematics, and the mathematics itself. But I believe that Penrose failed to sustain this distinction particularly in his discussion of intuition, insight and the aesthetic qualities of mathematical thinking. He underlined person-ness by reiterating an argument (see, for example, Thom, 1973, pp. 202–206) that:

> the importance of aesthetic criteria applies not only to the instantaneous judgements of inspiration, but also to the much more frequent judgements that we make all the time in mathematical (or scientific) work. Rigourous argument is usually the *last* step! (original emphasis, Penrose, p. 545).

In drawing a close analogy between mathematical thought and intuition and inspiration in the arts, Penrose added:

> The globality of inspirational thought is particularly remarkable in Mozart's quotation (from Hadamard, 1945) 'It does not come to me successively ... but in its entirety' and also in Poincaré's 'I did not verify the idea; I should not have had time'. (p. 347).

Any de-personalisation of the mathematical process and reification of the product pushes mathematics back into the absolutist position by objectivising the 'truths'. However, accepting a mathematics which is not absolute, is culturally defined and influenced by individual and social differences is not only of great interest to those who have argued for an inclusive mathematics but challenges the discipline epistemologically as well as philosophically and pedagogically.

It does not seem untimely to suggest a theory of knowing that draws attention to the knower's responsibility for what the knower constructs. (von Glasersfeld, 1990, p. 28)

Once we re-focus from knowing *that* a particular mathematical outcome exists to knowing *why* that outcome is likely under particular circumstances, we are distinguishing between the 'objective' knowledge of the outcome and the 'subjective' knowing which underlies how to achieve that outcome. This begins to be familiar as the old debate between product and process. However, by attempting to construct a theory of knowing, I am moving past the false dichotomy of product/process which polarised the how and the what, towards a re-conceptualisation and integration of the how with the what. The value to pedagogues of such an approach is obvious. As teachers, we can recognise when learners mimic a piece of mathematical behaviour rather than acquiring it as their own. The articulation of an epistemological position on knowing mathematics which is predicated on mathematical enquiry, rather than receptivity, challenges teacher behaviour. Rather than demanding evidence of the acquisition of mathematical objects by students, it assumes that mathematical behaviours and the changes in behaviour that might signify learning are products of, and responsive to, the community within which the learning is situated. Recounting different narratives, speculating about their similarities and differences, querying their derivations and applications, denies 'objectivity' and reinstates the person and the community in the mathematics. Such re-consideration of the characteristics of science and mathematics has under-pinned much of the feminist work in the philosophy of science already referenced and is exemplified in the work of Damarin. She presented a table of generalised descriptors

not as a definitive description of feminist science, but rather as defining a tentative framework for examining whether and how the teaching of science might be made more consistent with feminist conceptions of science. (Damarin, 1991, p. 112).

As stated above, the philosophical challenge, while not necessarily acceptable to a large number of mathematicians, has been well formulated. (Reference has already been made to the work of Bloor, Davis and Hersh, Harding, Lakatos and Restivo). Gadamer (1975) added his argument that:

all human understanding is contextual, perspectival, prejudiced, that is hermeneutic (and) fundamentally challenges the conception of science as it has been articulated since the Enlightenment (cited in Hekman, 1990, p. 107);

as did Fee (1982) to:

attack the objectivity that is part of the 'mythology' of science ... (and) ...re-admit the human subject into the production of scientific knowledge. (also cited in Hekman, 1990, p. 130).

Much of the pedagogic challenge is focussed on the dysfunctional nature of the continuum between an absolutist philosophy of mathematics and a transmissive pedagogy and the poverty of the product/process distinction:

On the one hand, authors and publishers produce textbooks that do not have to be read before doing the exercises; on the other hand, teachers acquiesce by agreeing that this is the way mathematics ought to be taught. ... the real importance lies not in the students' ability to conceptualize, but rather in their ability to compute. Teachers tend to underscore this by their rapt attention to correctness, completeness, and procedure. Students comply with the grand scheme by establishing as their local goal the correct completion of a given assignment and as their global goal receiving their desired grade in the course. For most, once it's over, it's over. (Gopen and Smith, 1990, p. 5).

Compare this with a student-centred problem-solving approach:

An instructor should promote and encourage the development for each individual within his/her class of a repertoire of powerful mathematical constructions for posing, constructing, exploring, solving and justifying mathematical problems and concepts and should seek to develop in students the capacity to reflect on and evaluate the quality of their constructions. (Confrey, 1990, p. 112).

Researchers have argued that creative mathematicians are more likely to develop by encountering and learning mathematics in a classroom climate which supports individuals within social groupings; that the negotiation of meaning both within the group and between the group and conventional social understandings needs to be encouraged. (See for example Davis, Maher and Noddings, 1990) These philosophical and pedagogical critiques, in my view, would be strengthened by the focus, structure and consistency which is gained from an epistemological stance, that is, a formulation of the nature of knowing mathematics.

5. THE EPISTEMOLOGICAL CHALLENGE

I believe that we can discern the outline of an epistemological challenge to mathematics which, potentially, incorporates approaches consistent with and familiar to broader constituencies than European, middle-class males. These approaches are inclusive, rather than exclusive, accessible rather than mystifying, encompassing of as wide a range of styles of understanding and doing mathematics as possible rather than reducible to those styles currently validated by the powerful. I am claiming that knowing, in mathematics, cannot be differentiated from the knower even though the knowns ultimately become public property and subject to public interrogation within the mathematical community. Knowing, however:

> involves encouraging rebellious spirits to blossom with free rein to the imagination, preserving a certain nimbleness of mind while affording it the means of being creative. The 'training' procedures, as we conceive them and ordinarily practice them, hardly lend themselves, one must admit, to that kind of enticement, since they more often emphasize the transmission of acquired knowledge and apprenticeship in proven methods. And considering that those procedures resemble an obstacle course where the competition is tighter and tighter, this hardly encourages departing from the beaten path. (Flato, 1992, p. 75).

I am speculating that five categories, drawn from the work already cited and consistent with the above critique, might distinguish the ways in which (creative) mathematicians come to know mathematics and that, in their choice of mathematical areas to pursue, more women (and men) might feel comfortable with an epistemology of mathematics described in this way. The assumption is that such an epistemology would displace dualisms such as the relativist/absolutist dichotomy and expectations of a value-free mathematics with an hermeneutic and pluralist approach . It would open the way towards an inclusive perspective on mathematics by challenging our understanding of what constitutes knowing in mathematics.

I propose defining knowing in mathematics in relation to the following five categories derived from the reading reviewed above in the philosophical, pedagogical and feminist literature:

* its person- and cultural/social-relatedness;
* the aesthetics of mathematical thinking it invokes;
* its nurturing of intuition and insight;
* its recognition and celebration of different approaches particularly in styles of thinking;
* the globality of its applications.

Knowing mathematics would, under this definition, be a function of who is claiming to know, related to which community, how that knowing is presented, what explanations are given for how that knowing was achieved, and the connections demonstrated between it and other knowings (applications). What evidence we have, usually sited in the learning and assessing of school mathematics, suggests that inviting students to define and describe their knowing in mathematics in these ways does have gender implications. (For example, see Burton, 1990b; Forgasz, 1994; Stobart *et al.*, 1992)

The similarities with Rosser's (1990) and Damarin's (1991) lists of the differences between male- and female-friendly science are encouraging. For example, both refer to the expansion of the kinds of observations carried out, the recognition of and concern for personal responsibility and the consequences of actions. I have listed a valuing of intuition and insight and the recognition and celebration of different approaches. Globality, or in both Rosser's and Damarin's terms holism, is a feature. A need to accept the personal experience of women as a valid component of experimental observations is acknowledged where I have pointed to person-relatedness which is important to knowing mathematics. Hekman's (1990) analysis of the relationship between gender and post-modernism was also supportive of this approach both in drawing out the similarities in argument between feminists and post-modernists as well as pointing out the pervading influence of absolutism in affecting these stances. In Rose's words:

> A feminist epistemology ... transcends dichotomies, insists on the scientific validity of the subjective, on the need to unite cognitive and affective domains; it emphasises holism, harmony, and complexity rather than reductionism, domination and linearity. (1986, p. 72).

The next step is to open a dialogue with practising mathematicians with a view to discussing the appropriateness of my description to their understanding of the nature of knowing in mathematics. This would be done in a style which would be rich in ethnographic data, encouraging the expression of feelings, aesthetics, intuitions, and insights. It would also attempt to challenge the effects of socialisation into the mathematical culture in order to untangle differences from cultural similarities. Outcomes which are supportive of the suggested epistemological framework, especially where these emphasise impact on gender inclusivity, would provide a strong argument in favour of a re-perception and re-presentation of mathematics. The resulting narrative would have an internal consistency which should please all mathematicians.

Such anecdotal approaches as have already been made confirm the validity of the five categories in describing how mathematicians come to know. Arguments in favour of humanising and demystifying the mathematics curriculum in schools have long been made with an implication that such attempts change perceptions of mathematics, and subsequent performance by formerly under-represented groups. However, these suggestions are rarely connected to epistemological frameworks of the discipline more frequently relating either to constructivist philosophy or empowering pedagogy. And Suzanne Damarin criticises curriculum reformers for their

> reliance on the models of expertise and information processing, which are popular in current research on the cognitive bases of teaching and learning of science and mathematics (*and*) appear to be diametrically opposed to first-order implications of feminist pedagogical research. (Damarin, 1991, p. 108).

If the nature of knowing mathematics were to be confirmed as matching the description given in this paper, the scientism and technocentrism which dominate much thinking in and about mathematics, and constrain many mathematics classrooms, would no longer be sustainable. Mathematics could then be re-perceived as humane, responsive, negotiable and creative. One expected product of such a change would be in the constituency of learners who were attracted to study mathematics but I would also expect changes in the perception of what *is* mathematics and of how mathematics is studied and learned. That such a possibility, in schools, is not outside the realms of possibility is suggested in Boaler (1993). We can also learn from experiences in other disciplines. English, for example, attracts predominantly female constituencies of learners at the undergraduate level, many of whom have been successful in developing academic careers.

> English was constructed as a liberal humanist discipline which demanded personal and thoughtful response...The most important characteristic of English, in the view of students and staff, is its individualism: the possibility of holding different views from other people. (Thomas, 1990, p. 173).

Providing a new epistemological context would enable the questioning of what mathematics is taught, how it is learned and assessed within a consistent treatment.

> By adopting an epistemological view of mathematical knowledge that stresses change, development, and its social foundations generally, and by consciously relating this to the curriculum process, the result would be to make the subject more open in its nature and more easily accessible. (Nickson, 1992, p. 131).

My aim in attempting this work is to question the nature of the discipline in such a way that the result of such questioning is to open mathematics to the experience and the influence of members of as many different communities as possible, thereby, I hope, not only enriching the individuals but also the discipline.

NOTES

This is a version of a paper first given at the ICME7 theme group of the International Organisation on Women and Mathematics Education, Quebec, 1992. Its present content owes much to discussion with and comments from members of that network. In addition, I would particularly like to thank Mary Barnes, Leonie Daws, Stephen Lerman and the anonymous reviewers for challenging and provoking re-working of the ideas.

REFERENCES

Bloor, D.: 1976, 1991, *Knowledge and Social Imagery*, 2nd ed., University of Chicago Press, London.

Boaler, J.: 1993, 'Encouraging the transfer of 'school' mathematics to the 'real world' through the integration of process and content, context and culture', *Educational Studies in Mathematics*, **25**(4), 341–373.

Bruner, J.: 1994, *The Humanly and Interpretively Possible,* Plenary address given to the AERA Annual Meeting, New Orleans, U.S.A.

Burton, L.(ed.): 1990a, *Gender and Mathematics: An International Perspective,* Cassell, London.

Burton, L.: 1990b, 'Passing through the mathematical critical filter – implications for students, courses and institutions', *Journal of Access Studies* **5**(1), 5–17.

Burton, L. (ed.): 1986, *Girls into maths can go,* Cassell, London.

Carson, R.: 1962, *Silent Spring,* Fawcett Press, New York.

Confrey, J.: 1990, 'What Constructivism Implies for Teaching', R.B. Davis, C.A. Maher and Nel Noddings (eds.,) Constructivist View on the Teaching and Learning of Mathematics, *Journal for Research in Mathematics Education,* Monograph No. 4, NCTM, Reston, VA.

Damarin, S.: 1991, 'Rethinking Science and Mathematics Curriculum and Instruction: Feminist Perspectives in the Computer Era', *Journal of Education* 173(1), 107–123.

Davis, P. and R. Hersh: 1983, *The Mathematical Experience.* Penguin, Harmondsworth.

Davis, R.B., C.A. Maher and N. Noddings (eds.): 1990, Constructivist Views on the Teaching and Learning of Mathematics, *Journal for Research in Mathematics Education,* Monograph No. 4, NCTM, Reston, VA.

Department of Education and Science, 1989, *Mathematics in the National Curriculum,* HMSO, London.

Easlea, B.: 1983, *Fathering the unthinkable: Masculinity, scientists and the nuclear arms race,* Pluto Press, London.

Ernest, P.: 1991, *The Philosophy of Mathematics Education,* Falmer Press, Basingstoke.

Fee, E.: 1981, 'A feminist critique of scientific objectivity', *Science for the People* **14**, 30–3.

Fennema, E. and G. Leder (eds.): 1990, *Mathematics and Gender,* Teachers College Press, New York.

Flato, M.: 1992, *The Power of Mathematics,* McGraw-Hill, London.

Forgasz, H.: 1994, *Society and Gender Equity in Mathematics Education,* Deakin University Press, Geelong, Vic.

Fossey, D.: 1983, *Gorillas in the Mist,* Houghton Mifflin, Boston.

Gadamer, H.-G.: 1975, *Truth and Method,* Continuum., New York.

Goodall, J.: 1971, *In the Shadow of Man,* Houghton Mifflin, Boston.

Gopen, G.D. and D.A. Smith: 1990, 'What's an Assignment Like You Doing in a Course Like This? Writing to Learn Mathematics', *The College Mathematics Journal* **21**(1), 2–19.

Hadamard, J.: 1945, *The psychology of invention in the mathematical field,* Princeton University Press, Princeton.

Harding, S.: 1986, *The science question in feminism,* Open University Press, Milton Keynes.

Harding, S.: 1991, *Whose Science? Whose Knowledge?* Open University Press, Milton Keynes.

Hekman, S.: 1990, *Gender and Knowledge – Elements of a Postmodern Feminism,* Northeastern University Press, Boston.

Hirst, P.H.: 1974, *Knowledge and the Curriculum,* Routledge & Kegan Paul, London.

Hirst, P.H.: 1965, 'Liberal Education and the Nature of Knowledge', R.D. Archambault (ed.), *Philosophical Analysis and Education,*Routledge and Kegan Paul, London.

Hrdy, S.B.: 1986, 'Empathy, polyandry, and the myth of the coy female', R. Bleier (ed.), *Feminist approaches to science,* Pergamon Press, Oxford.

Hull, R.: 1985, *The Language Gap,* Methuen, London.

Joseph, G.G.: 1993, 'A Rationale for a Multicultural approach to Mathematics', D. Nelson, G.G. Joseph and J. Williams (eds.) *Multicultural Mathematics*, Oxford University Press, Oxford.

Joseph, G.G.: 1991, *The Crest of the Peacock*, Tauris & Co., London.

Keller, E.F.: 1985, *Reflections on gender and science*, Yale University Press, New Haven, CT.

Keller, E.F.: 1983, *A feeling for the organism: The life and work of Barbara McClintock*, W.H. Freeman, New York.

Kelly, A. (ed.): 1987, *Science for Girls?*, Open University Press, Milton Keynes.

Kelly, A.V.: 1986, *Knowledge and Curriculum Planning*, Harper and Row, London.

Lakatos, I.: 1976, *Proofs and Refutations*, Cambridge University Press, Cambridge.

Lakatos, I.: 1983, *Mathematics, Science and Epistemology*, Cambridge University Press, Cambridge.

Leder, G.and Sampson, S.N. (eds.): 1989, *Educating Girls*, Allen & Unwin, Sydney.

Mulkay, M.: 1981, 'Preface', in A. Brannigan, *The Social Basis of Scientific Discoveries*, Cambridge University Press, Cambridge.

Needham, J.: 1959, *Science and Civilisation in China*, Cambridge University Press, Cambridge.

Nelson, D., G.G. Joseph and J. Williams: 1993, *Multicultural Mathematics*, Oxford University Press, Oxford.

Nickson, M.: 1992, 'Towards a Multi-cultural Mathematics Curriculum', M. Nickson and S. Lerman (eds.) *The Social Context of Mathematics Education: Theory and Practice*, South Bank Press, London.

Penrose, R.: 1990, *The Emperor's New Mind*, Vintage, London.

Polanyi, M. and H. Prosch: 1975, *Meaning*, University of Chicago Press, London.

Restivo, S.: 1992, *Mathematics in Society and History*, Kluwer, Dordrecht.

Rose, H.: 1986, 'Beyond Masculinist Realities: A feminist epistomology for the sciences', R. Bleier (ed.), *Feminist Approaches to Sciences*, Pergamon, Oxford.

Rose, H. and S. Rose: 1980, 'The myth of the neutrality of science', R. Arditti, P. Brennan and S.Cavrak (eds.), *Science and Liberation*, South End Press, Boston.

Rosser, S.V.: 1990, *Female-Friendly Science*, Pergamon, New York.

Stobart, G. Elwood, J. and Quinlan, M.: 1992, 'Gender Bias in Examinations: how equal are the opportunities?', *British Educational Research Journal* **18**(3), 261–276.

Thom, R.: 1973, 'Modern Mathematics: does it exist?', A.G. Howson (ed.), *Developments in Mathematics Education*, Cambridge University Press, Cambridge.

Thomas, K.: 1990, *Gender and Subject in Higher Education*, SRHE & Open University Press, Buckingham.

van Sertima, I. (ed.): 1986, *Blacks in Science: Ancient and Modern*, Transaction, New Brunswick, N.J.

von Glasersfeld, E.: 1990, 'An Exposition of Constructivism: Why Some Like It Radical', R.B. Davis, C.A. Maher and N. Noddings (eds.) 'Constructivist Views on the Teaching and Learning of Mathematics', *Journal for Research in Mathematics Education*, Monograph N. 4, NCTM, Reston, Va.

Whyte, J. *et al.* (eds.): 1985, *Girl Friendly Schooling*, Methuen, London.

Zaslavsky, C.: 1973, *Africa Counts*, Lawrence Hill Books, New York.

The University of Birmingham,
School of Education, Edgbaston,
Birmingham B15 2TT, UK